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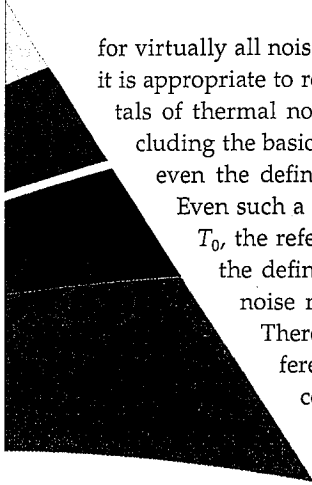
*Anthony R. Kerr
and James Randa*

Thermal Noise and Noise Measurements— A 2010 Update

As high-frequency equipment becomes more sensitive, and as demands on measurement accuracy increase, it is no longer safe to assume that $hf \ll kT$ (h is Planck's constant, f the frequency, k Boltzmann's constant, and T the temperature). This inequality underlies the familiar Rayleigh-Jeans noise equation, $P = kTB$, which is the basis

*Anthony R. Kerr is with National Radio Astronomy Observatory, Charlottesville, Virginia 22903 USA.
James Randa is with the Electromagnetics Division, National Institute of Standards and Technology, Boulder, Colorado 80305 USA
and the Physics Department, University of Colorado, Boulder, Colorado 80309 USA.*

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for virtually all noise measurements in electronics, so it is appropriate to reexamine some of the fundamentals of thermal noise and noise measurements, including the basic conventions and definitions, and even the definition of noise temperature itself.

Even such a seemingly innocuous quantity as T_0 , the reference temperature that occurs in the definitions of noise figure and excess noise ratio (ENR), is subject to change.

There are potentially significant differences between common, old procedures, many written into IEEE or other standards, and newer procedures and conventions being adopted by increasing numbers of engineers and scientists. Practices may

also vary by frequency range—for example, between the microwave and the optical. An understanding of thermal noise in the microwave, millimeter-wave, and submillimeter-wave bands allows the engineer to choose a consistent set of definitions and procedures.

A judiciously chosen set of definitions can give results not much different from those obtained with the older standard procedures, while a less judicious choice of definitions can lead to substantially different results. A good example is the noise temperature of an amplifier or receiver obtained by the standard Y-factor method. Depending on whether one uses the Rayleigh-Jeans or the Planck form for the noise temperature, and whether or not one accounts for the effect of vacuum (zero-point) fluctuations (see “Thermal Noise at Absolute Zero”), one can get significantly different results. There are differences of opinion (even between the present authors) over just how to include the vacuum fluctuations, but as long as a given scheme is used consistently, the results are the same. There is, however, a danger of incorrect results if a given scheme is not used consistently throughout a measurement or analysis.

In one sense, our discussion presumes that hf/kT is not small enough to ignore in a particular application.

However, even if it is negligible, the fact that essentially all computations are now done by machine means that there is little reason not to use the correct, exact expressions just to be safe, in case one wanders into a regime where it matters.

Noise Temperature as a Measure of Noise Power

It is appropriate to start this discussion by clarifying what we mean by *noise temperature*, since this term will be used throughout this article. The key to the concept of noise temperature is the Rayleigh-Jeans approximation to the available noise power spectral density of a resistor

$$P^{R-J} \approx kT, \quad (1)$$

where k is Boltzmann's constant (1.38065×10^{-23} J/K) and T is the absolute physical temperature of the resistor. The Rayleigh-Jeans law is useful in the range of temperatures and frequencies for which hf is substantially less than kT , h being Planck's constant (6.626068×10^{-34} m² · kg/s).

The widespread use of the Rayleigh-Jeans approximation, with its simple association of a temperature with a given noise power density, led to the concept of noise temperature, even for active devices or systems such as receivers whose noise powers are not simply related to their physical temperatures. There are basically two different ways to define noise temperature, which can be called the *physical-temperature* definition and the *power* definition. In the physical-temperature definition, the noise temperature is equal to the physical temperature of a resistor whose thermal noise would result in the given spectral density of (available) noise power. With such a definition, the noise temperature of a passive device is equal to its physical temperature. In the power definition, the noise temperature is defined as the given spectral density of (available) noise power divided by Boltzmann's constant. This latter definition is now the most commonly used (except in the microwave remote-sensing community), and it is the one we use in this article. Thus, by definition, the noise temperature is given by

$$T_n \equiv \frac{P_n}{k}, \quad (2)$$

where the subscript n indicates the noise temperature or spectral density of available noise power. (It is interesting to note that the 1996 *IEEE Standard Dictionary of Electrical and Electronic Terms* [1] (the most recent edition) defines noise temperature both ways, as if they were equivalent.) In practice, the power definition of (2) is much more convenient

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when dealing with amplifiers, although with this definition, the noise temperature of a passive device such as a resistor is not exactly equal to its physical temperature.

Available Noise Power from a Resistor

The Rayleigh-Jeans expression for the spectral density of the available noise power from a resistor at physical temperature T is given by (1). The Planck equation for this quantity is

$$P_{\text{Planck}} = kT \left[\frac{\frac{hf}{kT}}{\exp\left(\frac{hf}{kT}\right) - 1} \right] \quad (3)$$

This is widely considered to be the correct expression, to which the Rayleigh-Jeans form is an approximation valid for small values of hf/kT . Equations (2) and (3) are equivalent to the well known Nyquist equations [2] for the noise voltage of a resistor at temperature T . However, there is a further complication. In very-low-noise systems or at very high frequency, the zero-point vacuum fluctuations contribute significantly to the overall noise of an amplifier or mixer. This vacuum fluctuation term is given by

$$P^{\text{vac}} = \frac{hf}{2} \quad (4)$$

Although it is generally recognized that one must account for the vacuum fluctuation term, there is no general agreement on how this should be done. One way is to include it in the resistor noise power [3] so that the power available from a resistor takes the form

$$P^{C-W} = P^{\text{Planck}} + P^{\text{vac}} = kT \left[\frac{\frac{hf}{kT}}{\exp\left(\frac{hf}{kT}\right) - 1} \right] + \frac{hf}{2} \quad (5)$$

This is referred to as the Callen-Welton form [4]. Another method of dealing with P^{vac} is to treat it as a separate input to an amplifier or other device [5]. As shown in the following, the two methods yield the same results in amplifier (or mixer) measurements.

By applying (2) to the three expressions for the available noise power from a resistor at temperature T , we get the corresponding expressions for the noise temperature of a resistor.

$$T_n^{\text{R-J}} = T, \quad (6)$$

$$T_n^{\text{Planck}} = T \left[\frac{\frac{hf}{kT}}{\exp\left(\frac{hf}{kT}\right) - 1} \right], \quad (7)$$

and

$$T_n^{\text{C-W}} = T \left[\frac{\frac{hf}{kT}}{\exp\left(\frac{hf}{kT}\right) - 1} \right] + \frac{hf}{2k} \quad (8)$$

Figure 1 shows the noise temperature of a resistor at 100 GHz and 1 THz according to (6)–(8), as a function of its physical temperature. The corresponding noise

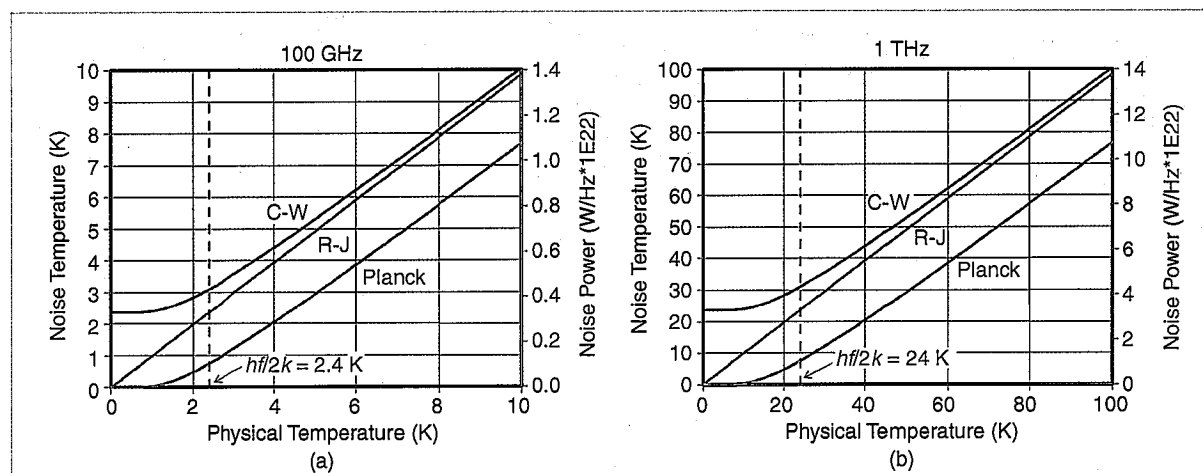


Figure 1. Noise temperature T_n (left axis) and noise power spectral density (right axis) versus physical temperature for a resistor, according to the Rayleigh-Jeans, Planck, and Callen-Welton equations, (a) at 100 GHz and (b) at 1 THz. (These graphs can be scaled for other frequencies simply by scaling their axes by the ratio of frequencies.)

Thermal Noise at Absolute Zero—The Vacuum Fluctuations

The classical vacuum is as boring as possible; there's just nothing there. The quantum mechanical vacuum is far more interesting; the vacuum is a bubbling cauldron of virtual particle-antiparticle pairs, appearing and disappearing. In its ground state, or lowest energy state, even a simple harmonic oscillator has a nonzero energy, the *zero-point* energy. Heuristically, the uncertainty principle allows a virtual pair to violate energy conservation very briefly under the condition $\Delta E \Delta t \leq h/(2\pi)$.

While it is impossible to extract any energy from these fluctuations (it is the lowest possible energy state), the vacuum fluctuations can affect a system connected to an external energy source, such as an amplifier. In that case, the vacuum fluctuations at the input are a source of noise that gets amplified and appears at the output. At microwave frequencies, the effect is small, though not always negligible. In optical amplifiers, the vacuum fluctuations can be the dominant source of noise.

The lowest possible equivalent input noise of an amplifier is $(1-1/G)hf/2$ W/Hz. With a resistive load connected to the input of the amplifier, the input noise contribution of the vacuum or zero-point fluctuations is $hf/2$ W/Hz. If the source were at absolute zero temperature, and the amplifier had the lowest possible equivalent input noise,

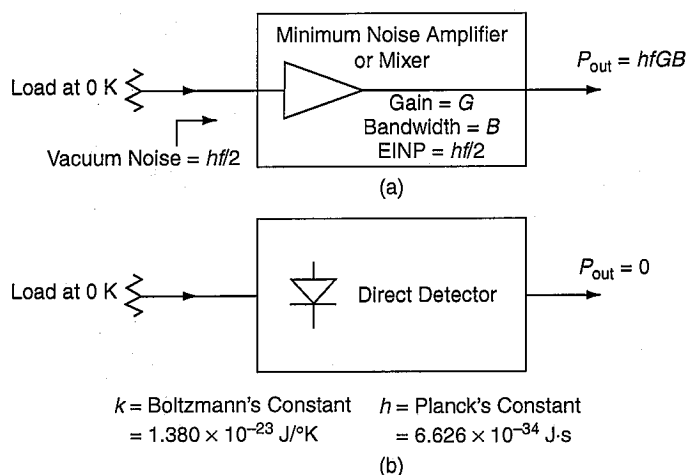


Figure S1. Minimum possible noise of (a) a high-gain amplifier and (b) a direct detector, with input terminations at absolute zero temperature.

the total output noise power would then be $(G-1)hf/2 + Ghf/2 = hf(G-1/2)$ as required by the Heisenberg uncertainty principle. In the case of a high gain amplifier, the output noise power would be hfG W/Hz, as shown in Figure S1(a).

The vacuum fluctuation noise is not detected by a direct detector (e.g., a detector diode, as in a power meter). It is not possible to collect net power from the vacuum fluctuations, which would violate the conservation of energy. This is indicated in Figure S1(b).

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power spectral densities are shown on the right axes. There are distinct regions above and below physical temperature $hf/2k$. At temperatures substantially above $hf/2k$, the Rayleigh-Jeans curve converges on the Callen-Welton curve, while at temperatures substantially below $hf/2k$, both Planck and Callen-Welton curves become independent of temperature. At all physical temperatures, the Callen-Welton curve is above the Planck curve by exactly $hf/2k$ (or $hf/2$ on the power spectral density scale). Note that, contrary to the widely held belief, the Rayleigh-Jeans curve does not approach the Planck curve at high temperature, but is always above it by almost $hf/2k$ on the noise temperature scale. In the normally encountered range of frequency and temperature, $hf/kT \ll 1$ and the Rayleigh-Jeans value of the noise temperature is close to the Callen-Welton value and can be used with little error. This is apparent in Figure 2, which shows

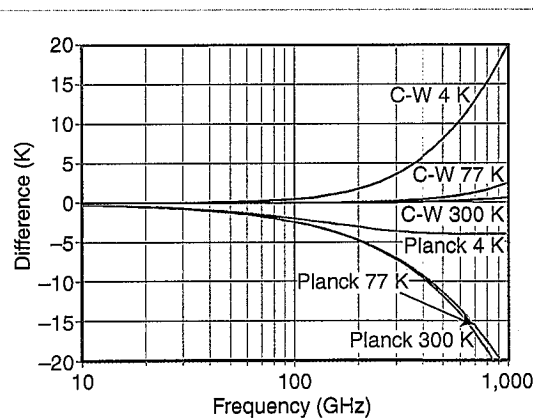


Figure 2. Difference between the noise temperature and the physical temperature, for physical temperatures 4 K, 77 K, and 300 K—Callen-Welton (red curves), and Planck (green curves).

There are basically two different ways to define noise temperature—the physical-temperature definition and the power definition.

the difference between physical temperature and the Callen-Welton and Planck noise temperatures as functions of frequency for physical temperatures 4 K, 77 K, and 300 K.

The Zero-Point Noise Term and the Minimum Noise of an Amplifier

Before moving on, it is interesting to consider the nature of the zero-point vacuum fluctuations. As indicated in "Thermal Noise at Absolute Zero," the vacuum fluctuations are sensed by an amplifier connected to an input resistor and contribute to the output noise of the amplifier. However, a direct detector (e.g., square-law or photon counter) will not respond to the vacuum fluctuations. This may seem to cast doubt on the physical reality of the vacuum fluctuations, but their tangible nature is clear from the radiation pressure they exert on conductors—the Casimir force—which was measured directly in a very elegant experiment by Lamoreaux in 1997 [6]. Essentially, when two conductors are brought close together, the small spacing reduces the number of electromagnetic modes in the space between them. Vacuum fluctuations associated with each mode exert a small radiation pressure on each conductor, so as the number of modes between the conductors is reduced there is less radiation pressure and a resulting net attractive force which can be measured with a mechanical balance.

It is generally understood that the Rayleigh-Jeans equation is a convenient approximation applicable over the large range of frequencies and physical temperatures familiar to most RF and microwave engineers. There is also agreement that the Planck-plus-vacuum or Callen-Welton form is correct for the input noise to an amplifier with a passive termination at its input. To shed more light on the role of the vacuum term, it is informative to consider the fundamental physical limit on the equivalent input noise of an amplifier. It is well established [7], [8] that the minimum possible equivalent input noise power spectral density of an amplifier is $(1 - 1/G)hf/2$. This is a result of the Heisenberg uncertainty principle, which also requires that the minimum possible noise at the output of an amplifier with high gain, connected to a source at absolute zero temperature, is hfW/Hz referred to the input [8], [9]. This implies an additional input contribution of $hf/2$ from the source, which is provided by the vacuum term.

Opinions differ on how to treat this zero-point vacuum contribution. One possibility (Scheme 1) is to say that the vacuum fluctuations are necessarily present at the terminals of a resistor, and therefore they should be included in the resistor's noise temperature [3], leading to the Callen-Welton form for the resistor noise temperature (8). Another possibility (Scheme 2) is to say that, since the vacuum fluctuations are present everywhere, including at the terminals of a resistor, but their noise power is detectable only through interaction with an active device, it is more appropriate to include them with the active device rather than with the resistor. In this scheme they are treated as additional input noise to the amplifier [5] (or other active device), and the resistor's noise temperature is given by the Planck form (7). In either case, a vacuum contribution of $hf/2$, referred to the input, is present at the amplifier's output. Of course, it is also possible to ignore the vacuum contribution entirely and use only the Planck form. Although this is not strictly correct at the input of an amplifier, it can be an acceptable approximation at low frequencies. We shall see the magnitude of the effect in the following.

Noise-Temperature Standards and Measurement of One-Port Noise Sources

The primary noise-temperature standards used at National Measurement Institutes (NMIs) are passive loads held at a constant, known temperature. The noise temperature of the standard is calculated from the Planck form, with corrections for losses between the reference plane and the plane at which the physical temperature is known. The common practice at NMIs is to define the noise temperature by the power definition of (2) and to use the Planck equation (7) for the noise temperature of a passive device at physical temperature T . The vacuum term is not included in the noise temperature of a noise source. Therefore, if one wishes to include the vacuum fluctuations with the noise source (as in Scheme 1), $hf/2k$ must be added to the calibrated noise temperature. Since $hf/2k = 0.24 \text{ K}$ at 10 GHz, this is usually considerably less than the uncertainty in the noise temperature of a one-port noise source at typical calibration frequencies (50 GHz and below).

Beyond the question of whether the calibrated noise temperature includes the vacuum-fluctuation term, there is the issue of whether the neglect of the vacuum fluctuations compromises the accuracy of the calibration process. We have not examined in detail the procedures at all NMIs performing noise-temperature calibrations, but general considerations indicate that the inclusion of vacuum fluctuations has no effect (beyond the addition of $hf/2k$ to the calibrated noise temperature, if one

so chooses). Basically, by using a radiometer, the calibration process removes their effect. As a specific example, consider a total-power radiometer with an isolator at its input, as described in [10]. The calibration procedure requires the radiometer output power P to be measured with the load under test ($P = P_x$), a noise standard ($P = P_s$), and a second standard at ambient temperature ($P = P_a$). This gives two Y-factors: $Y_x = P_x/P_a$ and $Y_s = P_s/P_a$, from which the noise temperature T_x of the load under test is deduced as

$$T_x = T_a + \left(\frac{Y_x - 1}{Y_s - 1} \right) (T_s - T_a), \quad (9)$$

where we have ignored inessential complications due to mismatch and differing measurement paths. The derivation of (9) neglects the vacuum fluctuations, but it is straightforward to introduce them, either by revisiting the derivation or by simply letting $T \rightarrow T + hf/2k$, as in Scheme 1. If we add the vacuum-fluctuation term $hf/2k$ to each noise temperature on the right hand side of (9), the result is simply to add $hf/2k$ to T_x , as discussed in the first paragraph of this section. Thus, the radiometer equation gives the correct value for the noise temperature in Scheme 2, and it can be converted to the value in Scheme 1 simply by adding $hf/2k$.

Measurement of Amplifier and Receiver Noise Temperature

We shall follow the approach used in [3] to compare amplifier noise-temperature measurements made assuming the Rayleigh-Jeans, Planck, and Callen-Welton noise temperature formulas. The noise temperature of an amplifier or (coherent) receiver is usually measured using the Y-factor method, in which hot and cold noise sources with known noise temperatures are connected to the input of the receiver and the ratio $Y = P_{\text{hot}}/P_{\text{cold}}$ of the receiver output powers is measured—see “Simple Y-Factor Method of Measuring Noise Temperature.” From the Y-factor, the intrinsic noise of the receiver can be deduced and expressed as an equivalent input noise power density, an equivalent input noise temperature, or a noise figure. Using the power definition of noise temperature (2), the noise power spectral density of the standard sources are kT_{hot} and kT_{cold} . Let the unknown equivalent input noise temperature of the device under test be T_e corresponding to power spectral density kT_e . Then

$$Y = \frac{T_e + T_{\text{hot}}}{T_e + T_{\text{cold}}}, \quad (10)$$

from which

Simple Y-Factor Method of Measuring Noise Temperature

In the commonly used Y-factor method for determining the noise temperature of an amplifier or receiver, the output noise power is measured with hot and cold loads connected to the input of the device under test. The ratio of these powers is the Y-factor

$$Y \equiv \frac{P_{\text{Hot}}}{P_{\text{Cold}}} = \frac{T_A + T_{\text{Hot}}}{T_A + T_{\text{Cold}}}.$$

From the measured Y-factor the desired noise temperature is obtained as

$$T_A = \frac{T_{\text{Hot}} - YT_{\text{Cold}}}{Y - 1}.$$

Note that this requires noise temperature to be a measure of noise power spectral density as opposed to a physical temperature, a subtle but sometimes significant distinction discussed in the article.

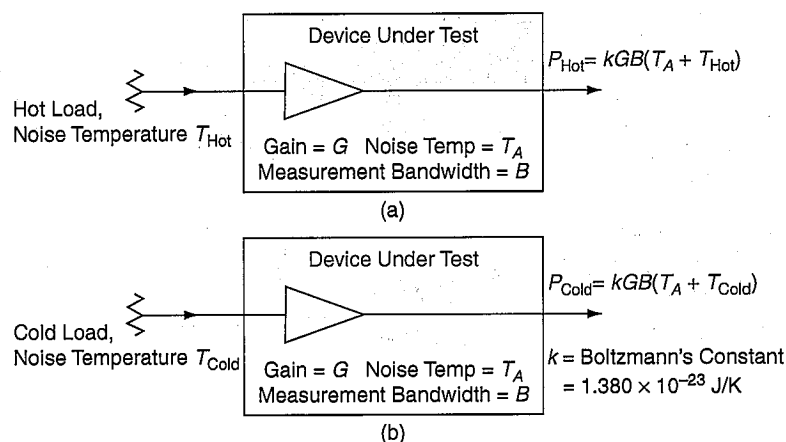


Figure S2. Device under test with (a) a hot load and (b) a cold load at its input.

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- [S4] G. Engen, *Microwave Circuit Theory*, London, U.K.: Peregrinus, 1992, ch. 19–21.
- [S5] A. Riddle, “Noise Measurements,” in *The RF and Microwave Handbook*, M. Golio, Ed. New York, NY: CRC Press, 2001, ch. 4–3.

In the normally encountered range of frequency and temperature, $hf/kT \ll 1$ and the Rayleigh-Jeans value of the amplifier noise temperature is very close to the Callen-Welton value.

$$T_e = \frac{T_{\text{hot}} - YT_{\text{cold}}}{Y - 1}. \quad (11)$$

Suppose, initially, that the noise temperatures of the hot and cold noise sources were obtained from their physical temperatures using the Planck formula (7), as would be the case for noise sources calibrated at a standards laboratory, and that vacuum fluctuations were not included in the input power or noise temperature. For this case (11) can be written

$$T_e^{\text{Planck}} = \frac{T_{\text{hot}}^{\text{Planck}} - YT_{\text{cold}}^{\text{Planck}}}{Y - 1}. \quad (12)$$

If we now use the Callen-Welton noise temperatures (8) for the hot and cold noise sources, (11) can be written

$$T_e^{C-W} = \frac{T_{\text{hot}}^{C-W} - YT_{\text{cold}}^{C-W}}{Y - 1}. \quad (13)$$

Since the Callen-Welton noise temperature differs from the Planck noise temperature by exactly $hf/2k$, we can substitute in (13):

$$T_{\text{hot}}^{C-W} = T_{\text{hot}}^{\text{Planck}} + \frac{hf}{2k}$$

and

$$T_{\text{cold}}^{C-W} = T_{\text{cold}}^{\text{Planck}} + \frac{hf}{2k},$$

which gives

$$T_e^{C-W} = \frac{T_{\text{hot}}^{\text{Planck}} - YT_{\text{cold}}^{\text{Planck}}}{Y - 1} - \frac{hf}{2k}, \quad (14)$$

or

$$T_e^{C-W} = T_e^{\text{Planck}} - \frac{hf}{2k}. \quad (15)$$

That is, the noise temperature of a receiver, measured by the Y-factor method, is lower by exactly $hf/2k$ when the noise temperatures of the hot and cold noise standards are obtained from their physical temperatures using the Callen-Welton equation (8) than when the Planck equation (7) alone is used. This is as expected from a consideration of the zero-point vacuum fluctuation noise $hf/2k$ —when the Planck formula is used for the hot and cold load noise temperatures, the vacuum

term is not included as part of the input source and must therefore be considered as part of T_e , the noise temperature of the device under test. On the other hand, when the Callen-Welton formula is used, the vacuum term is included as part of the hot and cold load noise and does not appear in T_e .

If the noise temperatures of the hot and cold loads are assumed equal to their physical temperatures (the Rayleigh-Jeans approximation) as is common practice, the receiver noise temperature deduced from the Y-factor measurement falls between the Planck and Callen-Welton values. In the normally encountered range of frequency and temperature $hf/kT \ll 1$ and the Rayleigh-Jeans value of the amplifier noise temperature is very close to the Callen-Welton value; this is as expected from Figure 2.

In a lighter vein, those interested in noisemanship will prefer to standardize on the Callen-Welton convention and include the zero-point vacuum noise as part of the noise of the source, as this will always result in lower receiver noise temperatures. For those bound to the Planck convention by a standards laboratory affiliation, it is always possible to lower the quoted noise temperature of an amplifier or receiver by regarding the vacuum term as an additional input to the device under test and subtracting it from the measured noise temperature; but then they must remember to add it back in any noise analysis.

It is straightforward to extend the preceding discussion to the noise parameters that characterize the dependence of the noise temperature of an amplifier or receiver on the impedance or reflection coefficient of the load connected to the input. If the vacuum fluctuations are included in the noise input to the amplifier, either through Scheme 1 or 2, then the measured value of T_{min} will be smaller by $hf/2k$ than the value obtained if the vacuum fluctuations are not taken into account.

Noise in Mixers

It is not widely realized that an ideal double-sideband (DSB) mixer need contribute no noise to a receiver. This is true of mixers with ideal (e.g., exponential) diodes and superconducting (SIS) tunnel junctions, and is predicted by quantum mixer theory [11], [12]. Although shot noise is generated by current flowing in the mixer diode, modulation of this noise by the local oscillator waveform causes the upper and lower sideband components of the shot noise to be correlated with the IF component—see "Noise in Mixers." When converted by the mixer to the intermediate frequency, the correlated components can cancel one another, resulting in zero output noise from the ideal mixer with optimum embedding impedances. The mechanism of this noise cancellation is explained in more detail in [13]. The following discussion is based on that in [14].

From a quantum mechanical point of view, a mixer is considered a linear amplifier but one that may

Noise in Mixers

It may seem counterintuitive that a diode, operating in a mixer with a substantial dc current, could contribute no noise to a system. But for the ideal case—a diode without series resistance in a mixer circuit with appropriate embedding impedances—this is true. It is a result of noise cancellation at the intermediate frequency (IF) f_0 caused by the action of the local oscillator (LO) waveform on the shot noise generated at all sideband frequencies $|f_0 \pm nf_{LO}|$ in the pumped diode.

To understand shot noise in a mixer, it is helpful to consider two steps: first, the generation of pulses of shot noise in a diode driven by a local oscillator; and second, the conversion of the frequency components of that shot noise, at all sideband frequencies, to the IF.

Figure S3(a) shows the noise equivalent circuit of a diode. The current i_d in the diode produces shot noise according to the usual shot noise formula:

$$\langle i_s^2 \rangle = 2ei_d\Delta f.$$

Figure S3(b) represents the shot noise current $i_s(t)$ in the diode with a steady dc bias current i_d . In the frequency domain, the noise can be represented as a multitude of pseudo-sinusoidal frequency components, $i_s(f)$, each with its own amplitude and phase. The amplitudes have a Gaussian distribution and the phases are random. A typical component, at frequency f_1 , is marked in red.

When the diode is periodically pumped at frequency f_{LO} by the local oscillator, the shot-noise current is indistinguishable (in the time domain) from the noise current in a dc-biased diode multiplied by a periodic modulating function to produce the pulses of shot noise shown on the left in Figure S3(c). This multiplication in the time domain corresponds to a convolution in the frequency domain; each of the frequency components of the noise of the dc-biased (unpumped) diode is convolved with the spectrum of the periodic modulating function. The frequency component at f_1 in Figure S3(b) thereby produces components at all the sideband frequencies $|f_0 \pm nf_{LO}|$, which are correlated with one another and with the original component at f_1 . This is depicted on the right in Figure S3(c).

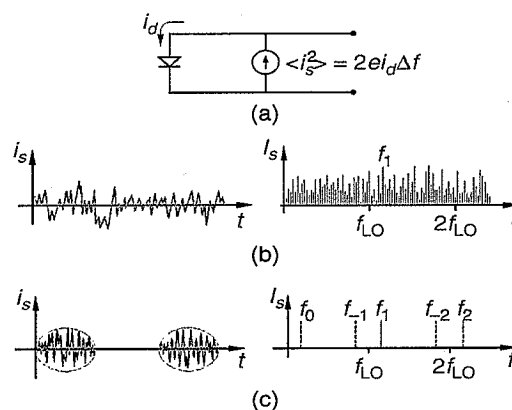


Figure S3. (a) The noise equivalent circuit of an ideal diode. (b) The shot noise current $i_s(t)$ in the diode with a steady dc bias current i_d . In the frequency domain, this noise can be represented as a multitude of pseudo-sinusoidal frequency components, $i_s(f)$. (c) The current in the diode is modulated by the local oscillator, producing correlated components at the sideband frequencies.

So far we have considered only the properties of the shot noise produced by the action of the local oscillator. When the correlated frequency components of the pulsed shot noise are converted by the mixer to the IF, they can either increase the overall IF noise or decrease it, depending on their amplitudes and phases. If the embedding impedances at the sideband frequencies have appropriate values, the resulting IF shot noise can approach zero.

For Further Reading on Noise Characterization

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For Further Reading on Noise in Mixers

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have multiple inputs (e.g., at $f_{LO} \pm f_{IF}$, $2f_{LO} \pm f_{IF}$, ...), and is subject to the same minimum noise limit as an amplifier with a single input [7]–[9]. To satisfy the uncertainty principle, the minimum possible output noise power spectral density of the complete system (which includes the source resistors at absolute zero temperature), referred to one input, is hf W/Hz. To understand where this noise originates, it is helpful to consider first a single-sideband (SSB) receiver con-

sisting of a simple DSB mixer with a resistive image termination as shown in Figure 3. Half a photon, $hf/2k$, of vacuum (zero-point) noise is present at the upper and lower sideband inputs of the DSB mixer, corresponding to one photon, hf/k , when referred to the input of the SSB receiver. If the Callen-Welton formula is used to evaluate the noise of the resistors, the vacuum noise is included as part of the noise of the resistor. If the Planck law is used, one must remember

Because the thermal noise in different resistors is not correlated, noise powers and noise temperatures are additive in the way familiar to those accustomed to using the Rayleigh-Jeans law.

to add the $hf/2$ W/Hz or $hf/2k$ K vacuum contribution from each resistor. The minimum noise of the SSB receiver itself (excluding the contribution associated with the source) is therefore just $hf/2$ W/Hz, or $T_{\min} = hf/2k$ K.

An interesting situation arises when the image termination in Figure 3 is replaced by a short circuit (or any reactive termination) as shown in Figure 4. Then there is no resistance in the image circuit to

provide the half photon of vacuum noise. In this case, the reactive image termination affects the conversion of shot noise from the image frequency to IF, and there can no longer be complete cancellation of the shot noise down-converted from the signal and image frequencies. Remarkably, nature arranges that shot noise now accounts for the necessary half photon of noise referred to the input of the receiver, a result predicted by Tucker's quantum mixer theory [11], [12].

A DSB receiver is often used for broadband continuum measurements in which both upper and lower sideband input signals contribute to the IF output. If the sideband gains are equal, this DSB mode of operation has the effect of doubling the receiver's apparent radiometric gain and reducing its equivalent input noise by a factor of two relative to a SSB measurement. An ideal DSB receiver is shown in Figure 5. It is apparent that the source provides a half photon of zero point noise in each sideband, so the mixer need contribute no noise to satisfy Heisenberg: the minimum possible noise temperature of the DSB mixer is zero.

Thermal Noise in Networks and Attenuators

The complex form of the Planck and Callen-Welton equations (3), (5) might be thought to complicate calculation of the

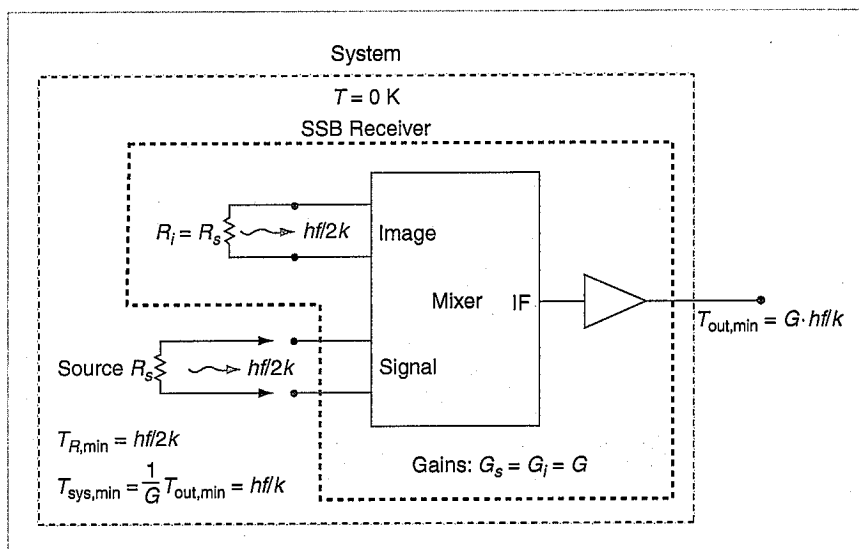


Figure 3. Minimum noise single-sideband receiver consisting of a double-sideband mixer with a resistive image termination. All resistors are at 0 K. The vacuum (zero-point) noise associated with the source resistance R_s contributes half a photon of noise, and half a photon comes from the image termination R_i . With equal signal and image gains, the minimum equivalent input noise of the single-sideband receiver is half a photon: $T_{\min} = hf/2k$.

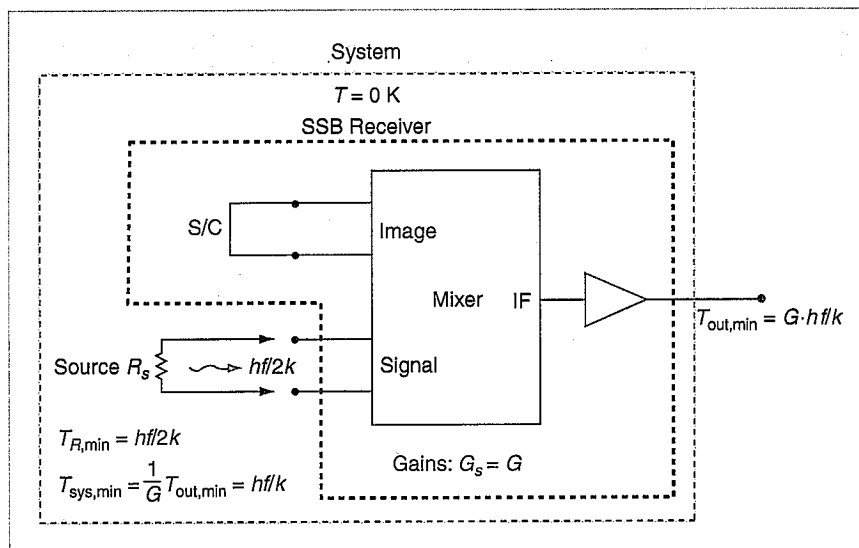


Figure 4. Minimum noise single-sideband receiver consisting of a double-sideband mixer with a short-circuit image termination. The source resistor is at 0 K. The zero-point noise associated with the source resistance R_s contributes half a photon of noise, and half a photon comes from the shot noise of the mixer. The minimum equivalent input noise of the receiver is half a photon: $T_{\min} = hf/2k$.

noise properties of cascaded lossy networks and transmission lines. In fact, this is not the case. Because the thermal noise in different resistors is not correlated, noise powers and noise temperatures are additive in the way familiar to those accustomed to using the Rayleigh-Jeans law (1). In calculating the noise at various stages through a network it is simply required that the noise temperatures of the elements be used and not their physical temperatures. The calculations then proceed just as when the familiar Rayleigh-Jeans law is used. Note that this is a consequence of using the power definition of noise temperature (2). If one were to use the equivalent-physical-temperature definition, the noise temperatures would not be additive; it would then be necessary to convert them to powers, then combine the powers, and finally convert the resulting power back to an equivalent noise temperature.

The thermal noise of a resistor can be represented in Thévenin or Norton form, as shown in Figure 6(a), the familiar Nyquist form, and (b). The noise temperature T_N is given as a function of the physical temperature by (7) or (8). The Twiss theorem [15] generalizes the Nyquist equations and states that the thermal noise of a lossy reciprocal linear network at a uniform physical temperature can be represented as shown in Figure 6(c), in which the noise currents and their correlations are given by $\langle i_{N1} i_{Nj}^* \rangle = 4kT_N \text{Re}[Y_{ij}] \text{ A}^2/\text{Hz}$. Again, the noise temperature T_N is given by (7) or (8). When the Planck law (7) is used, one must remember to add the $hf/2k$ zero-point noise before an amplifier. Using the Callen-Welton law does this automatically.

The example in Figure 7 shows how, when the Callen-Welton law (8) is used, the zero-point fluctuation noise from a load is attenuated by an attenuator, while the attenuator adds an amount equal to that lost, thereby maintaining a noise temperature $hf/2k$ at the output regardless of the attenuation, as required by thermodynamics. When the Planck law (7) is used, the vacuum noise is omitted, but must be added in at the input of any active device.

Noise Figure, Excess Noise Ratio, and the Standard Reference Temperature, T_0

The noise figure of an amplifier was defined by Friis in 1944 as "...the ratio of the available signal to noise ratio at the signal generator terminals to the avail-

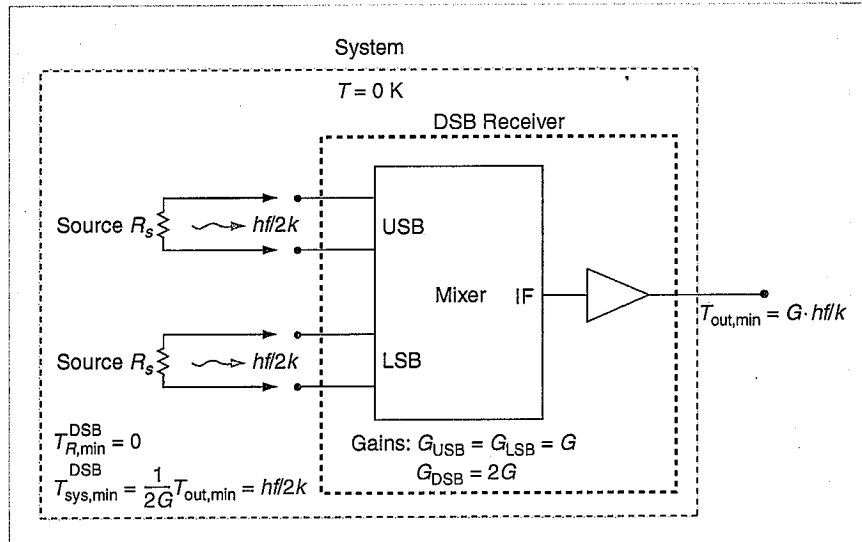


Figure 5. A double-sideband receiver with equal upper sideband (USB) and lower sideband (LSB) gains. The source at the USB and LSB inputs is physically the same resistor. Zero point noise associated with the source provides half a photon of noise in each input. The mixer itself need not contribute any noise.

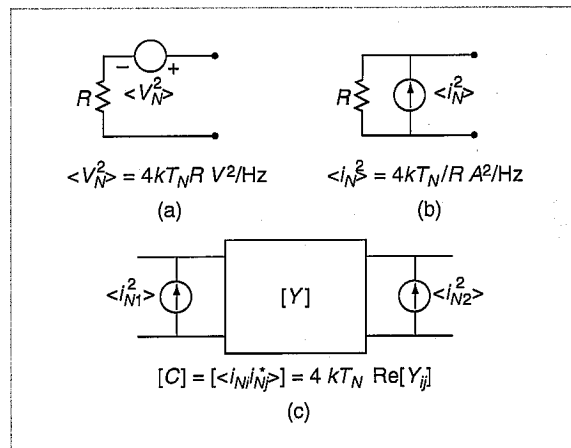


Figure 6. Thermal noise equivalent circuits: (a) Thévenin and (b) Norton equivalent circuits of resistors. (c) Twiss equivalent circuit of a reciprocal two-port network. The noise temperature T_N is related to the physical temperature by the Planck or Callen-Welton equations (7), (8).

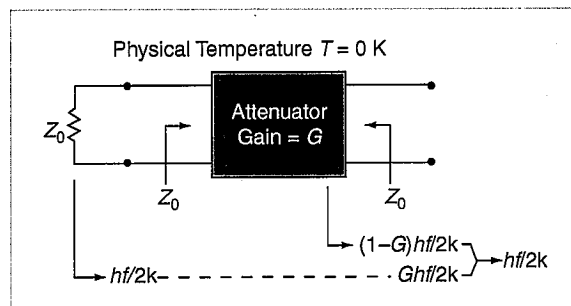


Figure 7. Effect of an attenuator on the vacuum fluctuation noise of a load when the Callen-Welton law (8) is assumed. The attenuator adds an amount of vacuum noise equal to the amount it attenuates from the load.

There is more than one correct way to deal with noise effects, but it is important to be consistent in one's approach and not to mix methods.

able signal-to-noise ratio at [the amplifier's] output terminals." [16] The source temperature is not explicitly given, but Friis goes on to say: "...it is suggested that... the noise figure be defined for a temperature of 290 K..." He does not say whether this is the physical temperature of the source or its noise temperature, but, as noise figure is a practical quantity to be measured on equipment at room temperature in a laboratory, the intent was almost certainly that the physical temperature of the source be 290 K. As most radio systems at that time operated at frequencies for which $hf \ll kT$ (that is, below a few hundred gigahertz), the distinction was of little consequence. From the Friis definition,

$$F = (S_i/N_i)/(S_o/N_o) = (1/G)(N_o/N_i), \quad (16)$$

where (S_i/N_i) and (S_o/N_o) are the signal-to-noise ratios at the input and output, and G is the available gain of the component being measured. For a system at 290 K physical temperature (16) can be expressed in terms of the amplifier's noise temperature T_A as

$$F = 1 + T_A/T_N(290), \quad (17)$$

where $T_N(290)$ is the noise temperature of the input termination at a physical temperature of 290 K. At very high frequencies, for which $hf \gg kT$, $T_N(290) \approx hf/2k$, which is also the minimum possible value of T_A . Then, from (17), $F_{\min} \approx 2$ (3 dB).

The definition of noise figure given in the 1957 IRE *Standards on Electron Tubes: Definitions of Terms* [17] has a subtle difference from the Friis definition in that the input termination is required to have a noise (not physical) temperature of 290 K. The definition given by the current IEEE *Standard Dictionary of Electrical and Electronics Terms* [1] is essentially the same:

... the ratio of (a) the total noise power per unit bandwidth (at a corresponding output frequency) delivered by the system into an output termination, to (b) that portion thereof engendered at the input frequency by the input termination, whose noise temperature is standard (290 K) at all frequencies.

By this definition:

$$F = [T_N(290) + T_A]/290. \quad (18)$$

In the RF and microwave bands $hf \ll kT$, so $T_N(290)$ can be taken as 290 K, and (18) gives the familiar noise figure expression

$$F = 1 + T_A/290. \quad (19)$$

At very high frequencies, for which $hf \gg kT$, $T_N(290) \approx hf/2k$, which is also the minimum possible value of T_A . Then, from (18), $F_{\min} \approx hf/290k$.

The difference between the Friis noise figure (17) and the IEEE definition (18) is shown in Figure 8 for an amplifier with the minimum noise temperature allowed by the uncertainty principle, $T_A = hf/2k$. In the case of an amplifier at 200 THz, e.g., an erbium-doped fiber amplifier, $T_N(290) \approx hf/2k = T_{A,\min} = 4,800$ K. Then $F_{\min|\text{Friis}} \approx 1 + 4,800/4,800 = 2$ (3 dB) and $F_{\min|\text{IEEE}} \approx (4,800 + 4,800)/290 = 33$ (15 dB).

While there is still debate in the photonics community [18] over the best definition of the noise figure of a photonic amplifier, it is generally agreed [19] that the minimum theoretical noise figure for an optical amplifier is 3 dB. It is clear from Figure 8 that this is not consistent with the IEEE definition of noise figure, while it is consistent with the Friis definition when the source has a physical temperature of 290 K and the Callen-Welton noise temperature definition (8) is used. In 1999 it was suggested [3] that the IEEE noise figure definition be modified to the Friis definition described above. This universal noise figure definition would differ to a negligible degree from the separate definitions currently in use at low and high frequencies, and would therefore not invalidate measurements made under the old definitions in the RF, microwave, millimeter wave, and submillimeter bands, or at optical wavelengths.

The standard reference temperature T_0 also occurs in the definition of the excess noise ratio (ENR) of a one-port noise source

$$\text{ENR} = \frac{T - T_0}{T_0}, \quad (20)$$

where T_0 is commonly taken to be 290 K (ENR is usually expressed in dB: $\text{ENR}(\text{dB}) = 10\log_{10}((T - T_0)/T_0)$). Again the question arises whether one should use the physical temperature, the Planck form corresponding to 290 K, or the Callen-Welton form corresponding to 290 K. The common practice is to use the physical temperature, that is, to use $T_0 = 290$ K, independent of frequency. This has the peculiar, but seldom noted, consequence that a resistor at 290 K has an ENR (relative to 290 K) that is greater than 0 if the Callen-Welton form of the noise temperature T is used, and is negative if the Planck form is used. The ENR is usually used for noise temperatures and frequencies at which the vacuum fluctuations are negligible, but consistency suggests that the same form of the noise temperature

(Rayleigh-Jeans, Planck, or Callen-Welton) should be used for T_0 as is used for T in the ENR.

Microwave Brightness Temperature

The brightness temperature in microwave remote sensing and radio astronomy is closely related to the noise temperature in microwave circuits and systems. The brightness temperature is a measure of spectral radiance [20]–[22], and its definition raises the same issues raised by the definition of noise temperature. The principal issue is whether to define the brightness temperature as equal to the physical temperature of a black body that would give rise to the observed spectral radiance (similar to the physical-temperature definition of noise temperature in Section 2), or to define the brightness temperature directly as the spectral radiance density divided by $2k/\lambda^2$, where λ is the wavelength and equal radiance is assumed in each polarization [similar to the power definition of noise temperature (2)]. The common practice in the microwave remote-sensing and radio astronomy communities [20]–[22] is to use the physical-temperature definition, which could also be called the *equivalent blackbody temperature* definition. This convention is convenient because the quantity of interest is often the physical temperature of the source of the radiation, but there is also a significant drawback, which we discuss below. As for a vacuum-fluctuation contribution to the brightness temperature, the vacuum fluctuations radiate no net power, and therefore this contribution is not included.

The problem with using the physical-temperature definition for microwave brightness temperature is that it must be converted to power or radiance before performing most (exact) calculations. When combining radiation from different sources, for example, it is powers that add incoherently, not equivalent blackbody temperatures (except in the Rayleigh-Jeans approximation). Therefore, in order to add radiation from different sources (exactly), it is necessary to convert equivalent blackbody temperatures into powers (radiances) and add those. Since integration is fundamentally an addition operation, the same applies to integrals.

A related situation occurs at the interface between antenna and radiometer. Since noise temperatures in the radiometer are defined with the power definition (2), it would be necessary to convert the antenna temperature to the power definition. Obviously, one could instead use the equivalent-physical-temperature definition for the noise temperature in the radiometer, but there is a good, even compelling, reason for using the power definition of noise temperature in dealing with amplifiers or receivers. The equation generally used to describe an amplifier's noise behavior is

The primary noise-temperature standards used at National Measurement Institutes are passive loads held at a constant, known temperature.

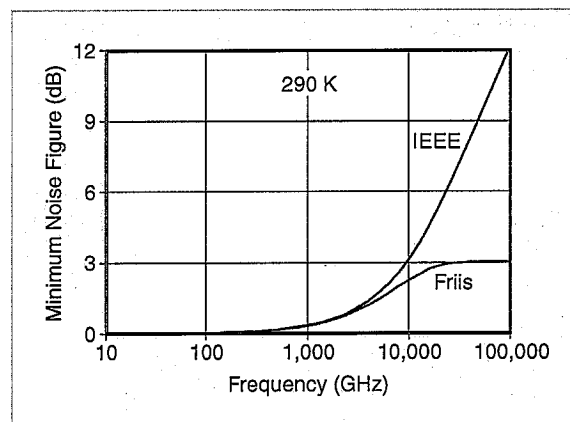


Figure 8. Minimum possible noise figure (in dB) of an amplifier as a function of frequency. The red curve uses the IEEE definition of noise figure in which the noise temperature of the source is 290 K. The blue curve uses the Friis definition with the source at a physical temperature of 290 K.

$$T_{\text{out}} = G(T_{\text{in}} + T_e), \quad (21)$$

where T_e is the equivalent input noise temperature of the amplifier, and G is the available-power gain of the amplifier. The derivation and validity of this equation rest on the equation for available power,

$$P_{\text{out}} = GP_{\text{in}} + P_{\text{amp}}, \quad (22)$$

where P_{amp} is the output spectral noise power density due to the amplifier. If noise temperatures are multiples of available spectral power density (21) follows from (22) without any approximation. If instead one insists on defining noise temperature as an equivalent physical temperature, the exact equation for T_{out} takes the form

$$T_{\text{out}} = \left(\frac{hf}{k} \right) \frac{1}{\ln \left(1 + \frac{1}{GA} \right)}$$

$$A = \frac{1}{2} + \frac{1}{(e^{hf/kT_{\text{in}}} - 1)} + \frac{1}{(e^{hf/kT_e} - 1)}, \quad (23)$$

which some might consider rather unwieldy. (The $1/2$ in A is the vacuum-fluctuation term, which could be subsumed into the T_e term, as discussed

in "Measurement of Amplifier and Receiver Noise Temperature.")

Thus, the situation in microwave remote sensing is that the common definition of brightness temperature, which is based on an equivalent blackbody (physical) temperature, requires that the brightness temperature be converted to power in order to do any exact calculations, unless one wishes to work with forms such as (23). If, on the other hand, a power-based definition is used, the exact calculations can be performed in terms of the brightness temperatures, but the brightness temperature does not correspond exactly to a physical temperature. (Of course, since most surfaces do not have unit emissivity and zero reflectivity, they are not perfect blackbodies, and the equivalent blackbody temperature also does not correspond exactly to the physical temperature.)

Conclusions

We have presented a close look at effects of order hf/kT in noise measurements and in the definition of noise quantities—noise temperature, noise figure, etc. Given the perennial push to higher frequency, lower noise, and smaller uncertainty, such effects are becoming significant in an increasing number of applications, and especially in radio astronomy where the noise temperature of a modern receiver can be within a factor of a few of the quantum limit. In particular, we have discussed issues arising from the definition of noise temperature and the treatment of contributions from vacuum fluctuations. There is more than one correct way to deal with these effects, but it is important to be consistent in one's approach and not to mix methods. A simple rule of thumb for whether such effects may be significant in a given application can be obtained by computing $hf_{\max}/k \approx (0.048 \text{ K/GHz}) \times f_{\max}$, where f_{\max} is the maximum frequency in the application. If hf_{\max}/k is laughably small compared to any noise temperature or noise-temperature uncertainty in the application, then one is probably safe in ignoring these effects and using the old, approximate forms for the relevant equations. On the other hand, it is always safe to account for these effects and use the exact equations.

Acknowledgment

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