RESOURCE LETTER

Resource Letters are guides for college and university physicists, astronomers, and other scientists to literature, websites, and other teaching aids. Each Resource Letter focuses on a particular topic and is intended to help teachers improve course content in a specific field of physics or to introduce nonspecialists to this field. The Resource Letters Editorial Board meets at the AAPT Winter Meeting to choose topics for which Resource Letters will be commissioned during the ensuing year. Items in the Resource Letter below are labeled with the letter E to indicate elementary level or material of general interest to persons seeking to become informed in the field, the letter I to indicate intermediate level or somewhat specialized material, or the letter A to indicate advanced or specialized material. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one Resource Letter on a given subject. A complete list by field of all Resource Letters published to date is at the website www.kzoo.edu/ajp/letters.html. Suggestions for future Resource Letters, including those of high pedagogical value, are welcome and should be sent to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, MN 55455; e-mail: rstuewer@physics.umn.edu

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Resource Letter FC-1: The physics of fundamental constants

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This Resource Letter provides a guide to the literature on the physics of fundamental constants and their values as determined within the International System of Units (SI). Journal articles, books, and websites that provide relevant information are surveyed. Literature on redefining the SI in terms of exact values of fundamental constants is also included.

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I. INTRODUCTION

This Resource Letter is a guide to the literature describing both experimental and theoretical works on the determination of values of certain fundamental physical constants.

In this Resource Letter, fundamental physical constants are taken to be the experimentally determined parameters in the equations that describe the basic laws of physics as they are currently understood. The numerical values of these constants are needed to make quantitative predictions by theory for comparison to the results of measurements. In fact, numerical values of the constants are periodically determined as being those that give the best agreement between theoretical predictions and experimental measurements.

Some of these constants have dimensions and others do not. For example, the speed of light, a fundamental constant associated with special relativity and electromagnetism, has the dimension of velocity, or distance divided by time. In order to give a unique meaning to the value of such a constant, it is necessary to specify the system of units in which it is expressed since the numerical value will depend on the unit definitions. For example, the speed of light is about 3×10^8 meters/second, which is the same as about 6.7×10^8 miles/hour. For such constants with dimensions, the International System of Units (SI) is widely accepted in science and technology and is presently the standard agreed to through a treaty among 54 nations. On the other hand, dimensionless constants, such as the fine-structure constant, the constant that characterizes the strength of electromagnetic interactions, are independent of the unit system.

Many of the constants considered here are of use in practical metrological applications, while others are included because they have traditionally been evaluated in the periodic least-squares adjustments of the constants. In general, these constants have values that have been accurately determined, and they have a well-defined place in a fundamental theory such as quantum mechanics or relativity. This includes universal constants that apply to broad physical laws as well as properties of particular particles such as their mass. In view of the critical role of the system of units for expressing the values of constants, literature on changes that may be made to the SI definitions in the near future is also surveyed. Not included here is the broad topic of possible time variation of the constants.

Besides the references given in this Resource Letter, relevant papers may be located on the searchable database of the NIST Fundamental Constants Data Center at http://physics.nist.gov/constantsbib.

II. BASIC RESOURCES

A. Journals

Most of the physics leading to values of the fundamental constants is reported in the following journals. The listing is in decreasing order of the number of times articles in the journal are cited in the latest adjustment of the values of the constants.

Physical Review Letters
Physical Review A
Metrologia
IEEE Transactions on Instrumentation and Measurement
Physical Review D
Physics Letters B
Canadian Journal of Physics
Zhurnal Eksperimental' noi i Teoreticheskoi Fiziki [Journal of Experimental and Theoretical Physics (JETP)]

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Philosophical Transactions of the Royal Society of London
Physics Letters A
International Journal of Mass Spectrometry
Reviews of Modern Physics
Measurement Science and Technology
Reports on Progress in Physics
Physical Review
Nuclear Physics B (Proceedings Supplements)
Modern Physics Letters A
Journal of Physics G
Journal of Physics B
Izmeritel'naya Tekhnika [Measurement Techniques]
Annals of Physics (N.Y.)

B. Conference proceedings

A number of conference series are focused on work that is often relevant to the physics and values of fundamental constants. Published proceedings of these conferences are listed here. Other relevant conference proceedings that are not part of a series are also given.

1. Conference on Precision Electromagnetic Measurements (CPEM)

This series is focused on measurements and theory that are of high relevance to knowledge of the fundamental constants.

1. **2008 Conference on Precision Electromagnetic Measurements (CPEM)**, edited by T. E. Lipe and Y.-H. Tang, IEEE Trans. Instrum. Meas. **58**(4) 748–1267 (2009). (A)
2. **Special issue on CPEM 2006**, edited by G. M. Reedtz, IEEE Trans. Instrum. Meas. **56**(2) 209–676 (2007). (A)
3. **Special issue on CPEM 2004**, edited by G. Jones, IEEE Trans. Instrum. Meas. **54**(2) 473–936 (2005). (A)
4. **Special issue on CPEM 2002**, edited by U. Feller, IEEE Trans. Instrum. Meas. **52**(2) 221–651 (2003). (A)
5. **Special issue on Selected Papers CPEM'2000**, edited by B. W. Ricketts, IEEE Trans. Instrum. Meas. **50**(2) 173–655 (2001). (A)
6. **Special issue on Selected Papers CPEM'98**, edited by B. A. Bell, IEEE Trans. Instrum. Meas. **48**(2) 145–671 (1999). (A)
7. **Special issue on Selected Papers CPEM'96**, edited by R. J. Cook, IEEE Trans. Instrum. Meas. **46**(2) 89–646 (1997). (A)
8. **Special issue on Selected Papers CPEM/94**, edited by M. Young, IEEE Trans. Instrum. Meas. **44**(2) 77–622 (1995). (A)
9. Editor J. McA Steele, IEEE Trans. Instrum. Meas. **42**(2) i–679 (1993). (A)
10. **Special issue on Selected Papers CPEM '90**, edited by N. B. Belecki, IEEE Trans. Instrum. Meas. **40**(2) 65–535 (1991). (A)

2. Enrico Fermi Schools on Metrology and Fundamental Constants

This series is of general interest for fundamental constants.

11. **Metrology and Fundamental Constants**, edited by T. Hänsch, S. Leschiutta, and A. J. Wallard (IOS, Amsterdam, 2007). (A)
12. **Recent Advances in Metrology and Fundamental Constants**, edited by T. J. Quinn, S. Leschiutta, and P. Tavella (IOS, Amsterdam, 2001). (A)
13. **Metrology at the Frontiers of Physics and Technology**, edited by L. Crovini and T. J. Quinn (North-Holland, Amsterdam, 1992). (A)
14. **Metrology and Fundamental Constants**, edited by A. F. Milone, P. Giacomo, and S. Leschiutta (North-Holland, Amsterdam, 1980). (A)

3. International Conference on Atomic Physics (ICAP)

These conferences cover a broader range of topics but include work relevant to the constants.

15. **Proceedings of the XXI International Conference on Atomic Physics, Pushing the Frontiers of Atomic Physics**, edited by R. Côté, P. L. Gould, M. Rozman, and W. W. Smith (World Scientific, Singapore, 2009). (A)
16. **Atomic Physics 20: XX International Conference on Atomic Physics**, edited by C. Roos, H. Häffner, and R. Blatt, AIP Conference Proceedings 869 (AIP, Melville, NY, 2006). (A)
17. **Atomic Physics 19: XIX International Conference on Atomic Physics, ICAP 2004**, edited by L. G. Marcassa, V. S. Bagnato, and K. Helmerson, AIP Conference Proceedings 770 (AIP, Melville, NY, 2005). (A)
18. **Proceedings of the XVIII International Conference on Atomic Physics: The Expanding Frontier of Atomic Physics**, edited by H. R. Sadeghpour, E. J. Heller, and D. E. Pritchard (World Scientific, Singapore, 2003). (A)
19. **Atomic Physics 17: XVII International Conference, ICAP 2000**, edited by P. De Natale, M. Inguscio, and E. Arimondo, AIP Conference Proceedings 551 (AIP, Melville, NY, 2000). (A)

4. Precision Physics of Simple Atomic Systems

As the titles indicate, these conferences are focused on precision work on atomic structure.

20. **Precision Physics of Simple Atoms and Molecules**, edited by S. G. Karshenboim, Lecuture Notes in Physics 745 (Springer, Berlin, 2008). (A)
21. **Precision Physics of Simple Atomic Systems**, edited by S. G. Karshenboim and V. B. Smirnov, Lecture Notes in Physics 627 (Springer, Berlin, 2003. (A)
22. **The Hydrogen Atom: Precision Physics of Simple Atomic Systems**, edited by S. G. Karshenboim, F. S. Pavone, F. Bassani, M. Inguscio, and T. W. Hänsch, Lecture Notes in Physics 570 (Springer, Berlin, 2001). (A)

5. Other conferences

23. **The Fundamental Constants of Physics, Precision Measurements and the Base Units of the SI**, edited by T. Quinn and K. Burnett, Philos. Trans. R. Soc. London, Ser. A **363**(1834), 2097–2327 (2005). (A)

24. **Gravitational Measurements, Fundamental Metrology and Constants**, edited by V. De Sabbata and V. N. Melnikov, NATO ASI Series C, Vol. 230 (Kluwer Academic, Dordrecht, 1988). (A)

25. **Precision Measurement and Fundamental Constants II**, edited by B. N. Taylor and W. D. Phillips, NBS Special Publication 617 (U.S. Government Printing Office, Washington, DC, 1984). (A)

26. **Quantum Metrology and Fundamental Physical Constants**, edited by P. H. Cutler and A. A. Lucas, NATO ASI Series B Vol. 98 (Plenum, New York, 1983). (A)

C. Books and monographs

27. H. Fritzsch, **The Fundamental Constants: A Mystery of Physics** (World Scientific, Singapore, 2009). (I)

28. **Quantum Metrology and Fundamental Constants**, edited by F. Piquemal and B. Jeckelmann, Eur. Phys. J. Special Topics **172**, 1–408 (2009). (A)

29. **Atomic Clocks and Fundamental Constants**, edited by S. G. Karshenboim and E. Peik, Eur. Phys. J. Special Topics **163**, 1–332 (2008). (A)

30. J.-P. Uzan and B. Leclercq, **The Natural Laws of the Universe: Understanding Fundamental Constants** (Springer, Berlin, 2008). (I)

31. **Astrophysics, Clocks and Fundamental Constants**, edited by S. G. Karshenboim and E. Peik, Lecture Notes in Physics 648 (Springer, Berlin, 2004). (A)

32. J. D. Barrow, **The Constants of Nature** (Random House, New York, 2003). (I)

33. H. Bachmair *et al.*, **Fundamental Constants in Physics and Chemistry (Numerical Data and Functional Relationships in Science and Technology)** (Springer, Berlin, 1992). (A)

34. B. W. Petley, **The Fundamental Physical Constants and the Frontier of Measurement** (Adam Hilger, Bristol, 1985). (A)

35. B. N. Taylor, W. H. Parker, and D. N. Langenberg, **The Fundamental Constants and Quantum Electrodynamics** (Academic, New York, 1969). (A)

D. Electronic archives and websites

Papers on fundamental physical constants may be found in various electronic archives. The more frequently used ones are the following:

36. <http://arxiv.org/archive/physics.gen-ph/GeneralPhysics> (A)

37. <http://arxiv.org/archive/physics.atom-ph/AtomicPhysics> (A)

38. <http://arxiv.org/archive/astro-ph/Astrophysics> (A)

39. <http://arxiv.org/archive/hep-th/HighEnergyPhysics-Theory>. (A)

40. <http://arxiv.org/archive/hep-ph/HighEnergyPhysics-Phenomenology>. (A)

41. <http://arxiv.org/archive/gr-qc/GeneralRelativityandQuantumCosmology> (A)

Relevant websites are the following:

42. ⟨http://www.aps.org/units/gpmfc/⟩, The website for the American Physical Society's Topical Group on Precision Measurement and Fundamental Constants. (E)

43. ⟨http://www.codata.org/taskgroups/TGfundconst/⟩, A description of the CODATA Task Group on Fundamental Physical constants within the home page of the Committee on Data for Science and Technology (CODATA). (E)

44. ⟨http://www.bipm.org/extra/codata/⟩, The home page of the CODATA Task Group on Fundamental Physical constants within the home page of the International Bureau of Weights and Measures (BIPM). (E)

45. ⟨http://physics.nist.gov/cuu/⟩, The National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty. (E)

46. ⟨http://amdc.in2p3.fr/web/amdcw_en.html⟩, The home page for the Atomic Mass Data Center which provides the 2003 Atomic Mass Evaluation. (I)

47. ⟨http://www.iupap.org/commissions/c2/⟩, The website of the International Union of Pure and Applied Physics (IUPAP) Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUNAMCO). (E)

III. SURVEYS OF CONSTANTS AND UNITS

A. Overviews of the constants

48. "Adjusted recommended values of the fundamental physical constants," S. G. Karshenboim, Eur. Phys. J. Spec. Top. **172**, 385–397 (2009). (I)

49. "Universal constants, standard models and fundamental metrology," G. Cohen-Tannoudji, Eur. Phys. J. Spec. Top. **172**, 5–24 (2009). (I)

50. "New recommended values of the fundamental physical constants (CODATA 2006)," S. G. Karshenboim, Phys. Usp. **51**(10), 1019–1026 (2008). (I)

51. "The fundamental physical constants," P. J. Mohr, B. N. Taylor, and D. B. Newell, Phys. Today **60**(7), 52–55 (2007). (I)

52. "Fundamental physical constants: Looking from different angles," S. G. Karshenboim, Can. J. Phys. **83**(8), 767–811 (2005). (I)

53. "Adjusting the Values of the Fundamental Constants," P. J. Mohr and B. N. Taylor, Phys. Today **54**(3), 29–34 (2001). (I)

B. Evaluations of fundamental constants

Starting with the pioneering work of Raymond Birge at the University of California, Berkeley, in 1929, various evaluations of the fundamental constants, based on the data available at the time, have been made. The Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants was founded in 1969, and since then, they have recommended values for the constants that are internationally recognized and used in research and publications.

54. "CODATA recommended values of the fundamental physical constants: 2006," P. J. Mohr, B. N. Taylor, and

D. B. Newell, J. Phys. Chem. Ref. Data **37**(3), 1187–1284 (2008). (A)

55. "CODATA recommended values of the fundamental physical constants: 2006," P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**(2), 633–730 (2008). (A)

56. "CODATA recommended values of the fundamental physical constants: 2002," P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **77**(1), 1–107 (2005). (A)

57. "CODATA recommended values of the fundamental physical constants: 1998," P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**(2), 351–495 (2000). (A)

58. "CODATA recommended values of the fundamental physical constants: 1998," P. J. Mohr and B. N. Taylor, J. Phys. Chem. Ref. Data **28**(6), 1713–1852 (1999). (A)

59. "Recommended values of the fundamental physical constants: A status report," B. N. Taylor and E. R. Cohen, J. Res. Natl. Inst. Stand. Technol. **95**(5), 497–523 (1990). (A)

60. "The 1986 adjustment of the fundamental physical constants," E. R. Cohen and B. N. Taylor, Rev. Mod. Phys. **59**(4), 1121–1148 (1987). (A)

61. "The 1973 least-squares adjustment of the fundamental constants," E. R. Cohen and B. N. Taylor, J. Phys. Chem. Ref. Data **2**(4), 663–734 (1973). (A)

62. "Determination of e/h using macroscopic quantum phase coherence in superconductors: Implications for quantum electrodynamics and the fundamental physical constants," B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. **41**(3), 375–496 (1969). (A)

63. "Our knowledge of the fundamental constants of physics and chemistry in 1965," E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. **37**(4), 537–594 (1965). (A)

64. "Status of knowledge of the fundamental constants of physics and chemistry as of January 1959," J. W. M. DuMond, Ann. Phys. (N.Y.) **7**(4), 365–403 (1959). (A)

65. "Résumé of atomic constants," J. A. Bearden and J. S. Thomsen, Am. J. Phys. **27**(8), 569–576 (1959). (A)

66. "A survey of the systematic evaluation of the universal physical constants," R. T. Birge, Nuovo Cimento, Suppl. **6**(1), 39–67 (1957). (A)

67. "A survey of atomic constants," J. A. Bearden and J. S. Thomsen, Nuovo Cimento, Suppl. **5**(2), 267–360 (1957). (A)

68. "Present status of the atomic constants," J. A. Bearden, M. D. Earle, J. M. Minkowski, and J. S. Thomsen, Phys. Rev. **93**(3), 629–630 (1954). (A)

69. "Least-squares adjustment of the atomic constants, 1952," J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. **25**(3), 691–708 (1953). (A)

70. "A Re-evaluation of the fundamental atomic constants," J. A. Bearden and H. M. Watts, Phys. Rev. **81**(1), 73–81 (1951). (A)

71. "Our knowledge of the atomic constants, F, N, m, and h in 1947, and of other constants derivable therefrom," J. W. M. DuMond and E. R. Cohen, Rev. Mod. Phys. **20**(1), 82–108 (1948); Erratum **21**(4), 651–652 (1949).

72. "Probable values of the general physical constants," R. T. Birge, Rev. Mod. Phys. **1**(1), 1–73 (1929). (A)

C. The International System of Units (SI)

The following are various resources describing the International System of Units and related concepts:

73. **International System of Units (SI)**, 8th ed. (Bureau International des Poids et Mesures, Sèvres, France, 2006), ⟨http://www.bipm.org/en/si/si_brochure/⟩. (I)

74. B. N. Taylor and A. Thompson, **The International System of Units (SI)**, 2008 ed., NIST Spec. Pub. 330 (U.S. Government Printing Office, Washington, DC, 2008) . (I)

75. A. Thompson and B. N. Taylor, **Guide for the Use of the International System of Units (SI)**, 2008 ed., NIST Spec. Publ. 811 (U.S. Government Printing Office, Washington, DC, 2008). (I)

76. E. R. Cohen, T. Cvitaš, J. G. Frey, B. Holmström, K. Kuchitsu, R. Marquardt, I. Mills, F. Pavese, M. Quack, J. Stohner, H. L. Strauss, M. Takami, and A. J. Thor, **Quantities, Units and Symbols in Physical Chemistry, IUPAC (International Union of Pure and Applied Chemistry)**, 3rd ed. (Royal Society of Chemistry, Cambridge, 2007). (I)

77. E. R. Cohen and P. Giacomo, **Symbols, Units, Nomenclature and Fundamental Constants in Physics, IUPAP-25 (IUPAP-SUNAMCO 87-1)** (International Union of Pure and Applied Physics, 1987); also published in Physica (Utrecht) **146A**(1–2), 1–68 (1987). (I)

See also Ref. 469.

IV. EXACT CONSTANTS

A number of fundamental constants have exact values either by the definition of the constant itself or as a consequence of the definitions of the SI units. Because of the possible dependence on SI unit definitions, which are modified from time to time, the set of constants that are exact may also change.

A. Speed of light c

The primary example of an exact constant is the speed of light $c=299\,792\,458\ \text{m/s}$. Before 1983, the speed of light was a measured quantity based on the meter and the second, which were defined independently. In 1983, the SI meter was redefined to be the distance light travels in a certain interval of time. From then on, the speed of light has been exactly one meter divided by that specified time interval. This definition takes into account the result of special relativity, that the speed of light in vacuum is the same in all inertial frames. This redefinition represented a new concept in units in which a general property of nature, namely, the invariability of the speed of light, could be used as the basis for measuring and calibrating any other velocity or, in combination with the definition of the second, as the basis for defining the meter.

B. Hyperfine frequency in cesium ν_{Cs}

Another constant that is exact as the result of a definition of a unit is the ground-state hyperfine frequency in cesium $\nu_{\text{Cs}}=9\,192\,631\,770\ \text{Hz}$. This constant is qualitatively different from the speed of light in that it refers to a particular property of a particular atom. As such, it is not a universal constant like c. In the SI, the second is defined to be the period of time for a certain number of cycles of this transition frequency to elapse. In view of this definition, the hyperfine frequency is an exact number. Other frequencies are determined by comparing them to ν_{Cs}.

C. Magnetic constant μ_0 and electric constant ϵ_0

The magnetic constant, or the permeability of free space, is fixed by the SI definition of the ampere to be $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}. From electromagnetic theory in SI units, the electric constant, or permittivity of free space, is $\epsilon_0 = 1/\mu_0 c^2$ so that ϵ_0 is also exact.

V. DIRECTLY MEASURED CONSTANTS

As mentioned in the introduction, the values of the fundamental constants are obtained by comparing experimental results to the predictions of theory in which the constants appear as parameters. The best values of the constants are taken to be those for which the theoretical predictions best match the experimental results as determined by the method of least squares. That work and its methodology are described in the papers cited in Sec. III B. In this section, references for the more recent theoretical and experimental work relevant to the determination of the values of particular constants are given and organized according to the constant to which the research is relevant. The focus is on recent papers and some older papers that are of particular significance.

A. Fine-structure constant α

The most accurate determination of the fine-structure constant α is from the combination of measurements of the anomalous magnetic moment of the electron, a_e, and quantum electrodynamics (QED) theory. Also given are determinations of α from atomic recoil, neutron diffraction, and electromagnetic measurements.

1. Electron anomalous magnetic moment

The QED part of the theoretical expression for the anomalous magnetic moment of the electron a_e is

$$a_e(\mathrm{QED}) = C_e^{(2)}\left(\frac{\alpha}{\pi}\right) + C_e^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_e^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_e^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \ldots .$$

There are additional contributions from weak and strong interaction effects, but these are relatively small. A value for α may be obtained by equating the theoretical expression to the measured a_e.

The first coefficient $C_e^{(2)}$ was calculated about 60 years ago by Schwinger, and subsequent work has led to values for the other coefficients. Work is still underway refining the value of $C_e^{(8)}$, and calculations have only begun on the next term in the series.

78. "Renormalization of QED and its experimental test over 60 years," T. Kinoshita, Prog. Theor. Phys. Suppl. **167**, 62–75 (2007). (I)

79. "Automated calculation scheme for α^n contributions of QED to lepton $g-2$: New treatment of infrared divergence for diagrams without lepton loops," T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Nucl. Phys. B **796**(1–2), 184–210 (2008). (A)

80. "Revised value of the eighth-order QED contribution to the anomalous magnetic moment of the electron," T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. D **77**, 053012 (2008), 24 pp. (A)

81. "Tenth-order lepton anomalous magnetic moment: Second-order vertex containing two vacuum polarization subdiagrams, one within the other," T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. D **78**, 113006 (2008), 7 pp. (A)

82. "Eighth-order vacuum-polarization function formed by two light-by-light-scattering diagrams and its contribution to the tenth-order electron $g-2$," T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, and N. Watanabe, Phys. Rev. D **78**, 053005 (2008), 14 pp. (A)

83. "Revised value of the eighth-order contribution to the electron $g-2$," T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, Phys. Rev. Lett. **99**, 110406 (2007), 4 pp. (A)

84. "Precise mass-dependent QED contributions to leptonic $g-2$ at order α^2 and α^3," M. Passera, Phys. Rev. D **75**, 013002 (2007), 6 pp. (A)

85. "Improved α^4 term of the electron anomalous magnetic moment," T. Kinoshita and M. Nio, Phys. Rev. D **73**, 013003 (2006), 28 pp. (A)

86. "New results on the hadronic contributions to $\alpha(M_Z^2)$ and to $(g-2)_\mu$," M. Davier and A. Höcker, Phys. Lett. B **435**(3–4), 427–440 (1998). (A)

87. "Higher-order hadronic contributions to the anomalous magnetic moment of leptons," B. Krause, Phys. Lett. B **390**(1–4), 392–400 (1997). (A)

88. "The analytical value of the electron $(g-2)$ at order α^3 in QED," S. Laporta and E. Remiddi, Phys. Lett. B **379**(1–4), 283–291 (1996). (A)

89. "The analytical value of the corner-ladder graphs contribution to the electron $(g-2)$ in QED," S. Laporta, Phys. Lett. B **343**(1–4), 421–426 (1995). (A)

90. "Progress in the analytical evaluation of the electron $(g-2)$ in QED: The scalar part of the triple-cross graphs," S. Laporta and E. Remiddi, Phys. Lett. B **356**(2–3), 390–397 (1995). (A)

91. "Analytical value of some sixth-order graphs to the electron $g-2$ in QED," S. Laporta, Phys. Rev. D **47**(10), 4793–4795 (1993). (A)

92. "The Analytical Contribution of the sixth-order graphs with vacuum polarization insertions to the muon $(g-2)$ in QED," S. Laporta, Nuovo Cimento **106**(5), 675–683 (1993). (A)

93. "Precise mass-ratio dependence of fourth-order lepton anomalous magnetic moments: Effect of a new measurement of m_τ," G. Li, R. Mendel, and M. A. Samuel, Phys. Rev. D **47**(4), 1723–1725 (1993). (A)

94. "The analytic value of the light-light vertex graph contributions to the electron $g-2$ in QED," S. Laporta and E. Remiddi, Phys. Lett. B **265**, 182–184 (1991). (A)

95. "Improved analytic theory of the muon anomalous magnetic moment," M. A. Samuel and G. Li, Phys. Rev. D **44**(12), 3935–3942 (1991); Erratum **48**(4), 1879–1881 (1993). (A)

96. R. Z. Roskies, E. Remiddi, and M. J. Levine, "Analytic evaluation of sixth-order contributions to the electron's g factor," in **Quantum Electrodynamics**, edited by T. Kinoshita (World Scientific, Singapore, 1990), Chap. 6, pp. 162–217. (A)

97. T. Kinoshita, "Theory of the anomalous magnetic moment of the electron—numerical approach," in **Quantum Electrodynamics**, edited by T. Kinoshita (World

Scientific, Singapore, 1990), Chap. 7, pp. 218–321. (A)

98. "Fourth-order magnetic moment of the electron," A. Petermann, Nucl. Phys. **5**(4), 677–683 (1958). (A)

99. "The magnetic moment of the electron," C. M. Sommerfield, Ann. Phys. (N.Y.) **5**(1), 26–57 (1958). (A)

100. "Fourth-order magnetic moment of the electron," A. Petermann, Helv. Phys. Acta **30**(5), 407–408 (1957). (A)

101. "Magnetic dipole moment of the electron," C. M. Sommerfield, Phys. Rev. **107**(1), 328–329 (1957). (A)

102. "Quantum electrodynamics III. The electromagnetic properties of the electron—radiative corrections to scattering," J. Schwinger, Phys. Rev. **76**(6), 790–817 (1949). (A)

103. "On quantum-electrodynamics and the magnetic moment of the electron," J. Schwinger, Phys. Rev. **73**(4), 416–417 (1948). (A)

Accurate measurements of the anomalous magnetic moment of the electron are made by confining electrons in a Penning trap.

104. "The anomalous magnetic moment of the electron," M. Vogel, Contemp. Phys. **50**(3), 437–452 (2009). (I)

105. "Why is sideband mass spectrometry possible with ions in a penning trap?," G. Gabrielse, Phys. Rev. Lett. **102**, 172501 (2009), 4 pp. (A)

106. "The true cyclotron frequency for particles and ions in a Penning trap," G. Gabrielse, Int. J. Mass Spectrom. **279**(2–3), 107–112 (2009). (A)

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The quotient of the Planck constant h to the mass of a cesium or rubidium atom has been obtained by measuring the frequency shift due to recoil of the atoms when photons are absorbed or emitted. The quotient of the Planck constant to the mass of the neutron has been obtained by Bragg reflection of neutrons from a crystal, which gives the neutron de Broglie wavelength. All of these measurements lead to a value of the fine-structure constant from the definition of the Rydberg constant, given in Sec. V C, by writing

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_x}{m_e}\frac{h}{m_x},$$

where m_x is the mass of the particle in the measurement and m_e is the mass of the electron. The Rydberg constant and the mass ratios are very accurately known, so the quotient gives a value for α limited mainly by the accuracy of the measurement of h/m_x.

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3. Quantum Hall effect experiments and the Thompson–Lampard theorem

When calibrated with an independent value for the ohm, the quantum Hall effect provides a condensed-matter determination of α. In terms of the von Klitzing constant R_K, α is given by

$$\alpha = \frac{\mu_0 c}{2R_K},$$

(see also Sec. VI). An accurate SI value of the ohm can be determined by means of a cross capacitor and a theorem in electrostatics developed by Thompson and Lampard that allows the capacitance per unit length to be calculated.

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The gyromagnetic ratio γ' of a bound particle with a magnetic moment μ can be determined using a low magnetic field generated by a precision single-layer solenoid carrying an electric current, where the dimensions of the solenoid and the current are accurately measured. The results can be related to the fine-structure constant and other constants by

$$\alpha^3 = \frac{4\mu_0 R_\infty}{g_{\mathrm{e}} K_{\mathrm{J}} R_{\mathrm{K}}} \frac{\mu_{\mathrm{e}}}{\mu} \gamma',$$

where R_∞ is the Rydberg constant, μ_{e} is the magnetic moment of the electron, g_{e} is the g-factor of the electron, and K_{J} is the Josephson constant.

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B. Newtonian constant of gravitation *G*

The Newtonian constant of gravitation G is the proportionality constant in the universal law of gravitational attraction force F between two masses m_1 and m_2 separated by a distance r

$$F = G\frac{m_1 m_2}{r^2}.$$

Mainly due to the intrinsically weak nature of gravity, the Newtonian constant continues to be a relatively poorly determined physical constant.

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C. Rydberg constant R_∞

The Rydberg constant is obtained by comparing theory and experiment for transition frequencies in hydrogen and deuterium atoms. Many of the recent experiments are based on use of a laser frequency comb to make precise frequency measurements. The quantity actually measured is cR_∞, which has the dimension of frequency. The Rydberg constant is related to other constants by

$$R_\infty = \frac{\alpha^2 m_e c}{2h}.$$

1. Theory

The frequency is expressed as cR_∞ times a factor that depends on the detailed theory, including effects from the finite size of the nucleus and quantum electrodynamics (QED). The papers cited in this section describe the various calculations that contribute to that factor.

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See also Refs. 161 and 160.

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The Rydberg constant is determined from precision frequency measurements on hydrogen and deuterium atoms. Frequencies are the physical quantities that can be measured with the highest precision. The quest to improve the precision of frequency measurements was motivated largely by efforts to improve the accuracy of the Rydberg constant. This effort eventually led to the development of the frequency comb, which resulted in a recent Nobel Prize in physics.

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D. Planck constant *h*

The Planck constant h appears in various roles in quantum mechanics. It is the conversion factor between frequency ν and energy E of electromagnetic radiation

$$E = h\nu,$$

and it appears in the von Klitzing and Josephson relations associated with electrical phenomena in condensed-matter physics. The reduced Planck constant $\hbar = h/2\pi$ is two times the quantum unit of angular momentum, is a proportionality factor for action, and appears in the Heisenberg uncertainty relation.

The most accurate measurements of the Planck constant are made with the watt-balance experiment. Measurements on crystalline silicon, discussed in a subsequent section on the Avogadro constant, are also capable of providing information on the Planck constant through a theoretical relation.

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E. Avogadro constant N_A

The Avogadro constant gives the number of entities in a mole. According to the definition of the mole in the SI, this number is the same as the number of carbon-12 atoms in 12 g of carbon. The main effort to directly determine this constant has been focused on determining the lattice spacing, mass, volume, and isotopic composition of spherical crystals of silicon.

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F. Molar gas constant R

The molar gas constant is the macroscopic link between energy and thermodynamic temperature expressed in units of $\mathrm{J\,K^{-1}\,mol^{-1}}$. To date, the most accurate value of R has been determined by measuring the speed of sound in argon in a spherical acoustic resonator.

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G. Boltzmann constant *k*

The Boltzmann constant is the microscopic link between energy and thermodynamic temperature expressed in units of J K^{-1}. In practical determinations of temperature, it is the product kT that is proportional to the energy associated with the temperature that is measured. Although there are direct measurements of k, the most accurate determination is through the measurement of the molar gas constant R and use of the relation

$$k = \frac{R}{N_A}.$$

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H. Josephson constant K_J

Josephson junctions are used as highly precise voltage standards. The Josephson constant K_J is the constant of proportionality that relates the frequency of microwave radiation applied to a superconducting Josephson junction to the induced voltage across the junction. It is related to the elementary charge and the Planck constant by

$$K_J = \frac{2e}{h} = \frac{1}{\Phi_0},$$

where Φ_0 is the magnetic flux quantum.

The universality tests mentioned below provide supporting evidence for the validity of the equation for K_J, which is relevant to the possible redefinition of the SI units.

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I. von Klitzing constant R_{K}

The quantum Hall effect provides a highly precise resistance standard. The quotient of the voltage (at a plateau) across a quantum Hall device and the current flowing through it is proportional to the von Klitzing constant R_{K}, which is given by

$$R_{\mathrm{K}} = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}.$$

See also Sec. V A 3.

The universality tests mentioned below provide supporting evidence for the validity of the equation for R_{K}, which is relevant to the possible redefinition of the SI units.

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See Sec. V A 3.

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J. Electron mass m_e

The electron mass is most accurately known in atomic mass units (in which the relative atomic mass of the carbon-12 atom is exactly 12) denoted by $A_r(e)$. Three methods of determining the mass in these units are described in the following three sections. The subsequent section gives an expression for the electron mass in terms of other constants, which provides the best value in kilograms.

1. Bound electron g-factor

The best value of the electron mass in relative atomic mass units is provided by measurements on hydrogenlike ions of the ratio of the electron spin-flip resonance frequency f_s to the cyclotron frequency f_c of the ion in a uniform magnetic field. The electron mass follows from the relation

$$A_r(e) = -\frac{g_e(I)}{2(Z-1)}\frac{f_c}{f_s}A_r(I),$$

where $g_e(I)$ is the g-factor of the electron in the ion I, Z is the nuclear charge number of the ion, and $A_r(I)$ is its relative atomic mass. The atomic masses, from which $A_r(I)$ is readily obtained, are generally known with sufficient accuracy for this purpose (see Sec. V L). The g-factor is calculated from the atomic theory of the bound electron, with the following references. References for the experiments are given next.

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2. Transitions in antiprotonic helium

An antiprotonic helium atom is the atomic state of a helium nucleus, an antiproton, and an electron. Transition frequencies between highly excited states of the antiproton in the atom have been measured and compared to the theoretical values, which depend on the mass ratios of the three particles. This comparison provides a determination of the electron mass since the masses of the helium nucleus and the antiproton are more accurately known from other measurements. This assumes that *CPT* (combined charge conjugation, parity reflection, and time reversal) invariance holds, in which case the masses of the antiproton and the proton are the same.

The theory is described in the following references.

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The experiments use precision laser spectroscopy.

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3. Mass measurement in a Penning trap

403. "Determination of the electron's atomic mass and the proton/electron mass ratio via Penning trap mass spectroscopy," D. L. Farnham, R. S. Van Dyck, Jr., and P. B. Schwinberg, Phys. Rev. Lett. **75**(20), 3598–3601 (1995). (A)

4. Electron mass in kilograms

The electron mass in kilograms is evaluated with the relation

$$m_{\mathrm{e}} = \frac{2hR_\infty}{\alpha^2 c}$$

from the definition of the Rydberg constant in Sec. V C using values of the constants obtained as discussed elsewhere in this Resource Letter.

K. Electron-muon mass ratio m_{e}/m_{μ}

Muonium is the atomic state of a positive muon and an electron. The ground state has two hyperfine levels, depending on the relative alignment of the spins of the muon and electron. The electron-muon mass ratio is determined by a comparison of the measured and calculated values of the hyperfine splitting, where the mass ratio is one of the parameters in the theory. The expression is

$$\Delta\nu_{\mathrm{Mu}} = \frac{16}{3} c R_\infty \alpha^2 \frac{m_{\mathrm{e}}}{m_{\mu}} \left(1 + \frac{m_{\mathrm{e}}}{m_{\mu}}\right)^{-3} \mathcal{F}(\alpha, m_{\mathrm{e}}/m_{\mu}),$$

where the function $\mathcal{F}$, which is 1 to lowest order in the variables, contains the details of the theory but depends only weakly on the mass ratio.

The value of the fine-structure constant α can also be determined from this relation if an independent value of the mass ratio is used. However, the value for α obtained this way is less accurate than values obtained from other sources.

1. Theory

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See also Ref. 206.

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L. Atom, ion, and nucleon masses

Masses of atoms, ions, and nuclei are most accurately known in atomic mass units (in which the mass of the carbon-12 atom is exactly 12). Mass measurements are generally made with ions rather than neutral atoms. For an ion I with n electrons and nucleus N, the mass is

$$m_I = m_N + n m_{\mathrm{e}} - E_{\mathrm{b}}/c^2,$$

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VI. DERIVED CONSTANTS

The best values of the constants in this section are currently not obtained from direct measurements but instead are calculated from theoretical identities, relating them to directly measured constants considered in the previous section.

A. Elementary charge *e*

In the classic Millikan oil-drop experiment, the elementary charge e was measured directly, but the current best value is based on the definition of the fine-structure constant as it appears in the SI system, which yields the relation

$$e = (2\epsilon_0 hc\alpha)^{1/2} = \left(\frac{2\alpha h}{\mu_0 c}\right)^{1/2}.$$

The elementary charge is a universal constant because the charge of any isolated system is an integer multiple of e, as far as it has been tested experimentally. Limits can be placed on possible differences between the magnitude of positive and negative elementary charges from measurements of the neutrality of matter.

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B. Bohr radius a_0

The Bohr radius, which is also the atomic unit of length, is given by

$$a_0 = \frac{\alpha}{4\pi R_\infty}.$$

C. Reduced Compton wavelength of the electron $\lambda\!\!\!^{-}_{\mathrm{C}}$

The reduced Compton wavelength, *i.e.* the Compton wavelength divided by 2π, is given by

$$\lambda\!\!\!^{-}_{\mathrm{C}} = \frac{\hbar}{m_{\mathrm{e}}c} = \alpha a_0.$$

D. Bohr magneton μ_{B}

The Bohr magneton follows from

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m_{\mathrm{e}}} = \left(\frac{c\alpha^5 h}{32\pi^2\mu_0 R_\infty^2}\right)^{1/2}.$$

E. Nuclear magneton μ_{N}

The nuclear magneton is given by

$$\mu_{\mathrm{N}} = \frac{A_{\mathrm{r}}(\mathrm{e})}{A_{\mathrm{r}}(\mathrm{p})}\mu_{\mathrm{B}}, \tag{1}$$

where $A_{\mathrm{r}}(\mathrm{e}) = m_{\mathrm{e}}/m_{\mathrm{u}}$ and $A_{\mathrm{r}}(\mathrm{p}) = m_{\mathrm{p}}/m_{\mathrm{u}}$ are the relative atomic masses of the electron and proton and m_{u} is the relative atomic mass unit given in Sec. VI G.

F. Electron volt, eV

The electron volt is a unit of energy given by

$$1\ \text{eV} = \frac{e}{1\ \text{C}} 1\ \text{J},$$

where C is a coulomb and J is a joule.

G. Relative atomic mass unit, u

The relative atomic mass unit is

$$1\ \text{u} = m_{\text{u}} = \tfrac{1}{12} m(^{12}\text{C}) = 10^{-3}\ \text{kg mol}^{-1}/N_{\text{A}},$$

where $m(^{12}\text{C})$ is the mass of the carbon-12 atom.

VII. POSSIBLE REDEFINITION OF SI UNITS

As methods of measurement have advanced over the years, the International System of Units has also evolved. The latest change in the definition of an SI unit was made in 1983 when the meter was redefined by the statement: "The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second." This definition implies that the speed of light c is exactly 299 792 458 m/s. (See Ref. 73.)

Currently there is interest in providing similar definitions of other SI units in terms of fundamental constants. The Consultative Committee for Units has recommended to the International Committee for Weights and Measures of the International Bureau of Weights and Measures that the kilogram, ampere, kelvin, and mole be redefined by specifying exact values for the Planck constant h, the elementary unit of charge e, the Boltzmann constant k, and the Avogadro constant N_{A}. The change is likely to take place when it is felt that the experiments needed to determine mass based on the proposed new definition are sufficiently accurate.

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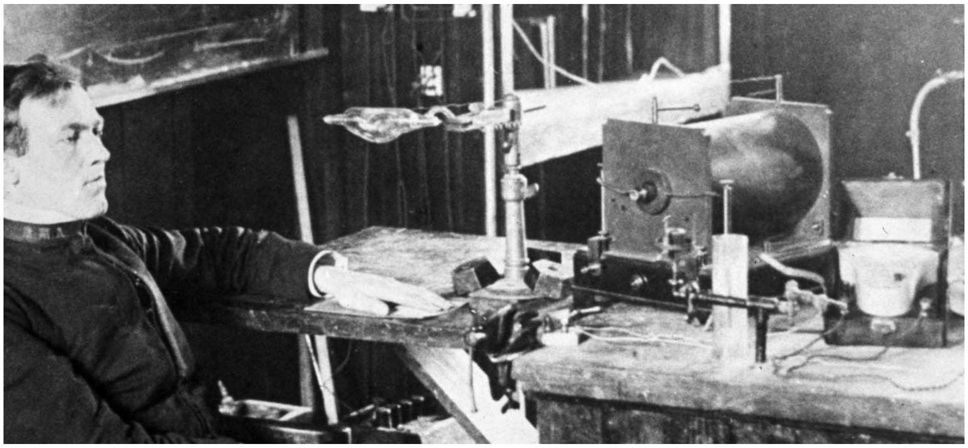

X Ray Photograph. The photograph shows an instructor at the Kenyon Military Academy using Kenyon College apparatus to make an X ray photograph of his hand. The date is about 1905, and the picture was taken in Ascension Hall on the Kenyon campus. The tube is still in existence and the glass in front of the cathode, through which the X rays passed, has been colored a light purple due to the formation of color centers. The photographic plate is wrapped up in opaque paper and is underneath the instructor's hand. Note the complete absence of shielding from stray radiation. (Photograph and Notes by Thomas B. Greenslade, Jr., Kenyon College)