

Long distance frequency transfer through an optical carrier

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ABSTRACT

Fiber optic networks are an attractive means for the remote distribution of highly stable frequencies from optical clocks. The highest performance is achieved by use of the frequency of the optical carrier itself as the transfer frequency. We will review our measurements on the transfer of optical frequency (a stabilized 1550 nm laser) over fiber optic links with lengths ranging from 38 km to 251 km. We discuss experimental details important for optimum performance and relate our measured performance to the theoretical limit on the phase and frequency noise of the transmitted signal as a function of the transmission distance.

Keywords: Doppler cancellation, optical frequency transfer, phase noise

1. INTRODUCTION

As atomic clocks continue to improve their stability, their complexity increases as well, requiring comparisons between optical clocks in different laboratories to be conducted remotely. A natural means of comparison is to send a stable frequency between laboratories over an optical fiber link. The frequency transferred can be an RF modulation on an optical carrier, or the frequency of the optical carrier itself. The second approach yields the best fractional frequency instabilities. The optical transport frequencies are converted back to RF frequencies by the use of optical frequency combs.

There are two dimensions involved in optical frequency transfer, and each must be optimized. The frequency is transported both in distance (between one laboratory and another) and across the optical spectrum from the RF frequency of one clock to the optical transport frequency and then down to the frequency of the second clock. This process might involve multiple steps and conversions between optical frequencies. Also, for frequency transfer stabilities at a level useful for optical clock comparison, these processes must be completed over a phase-stabilized fiber link. In this talk, we will discuss our work focusing on the latter problem where we successfully transported optical frequencies over distances ranging from 38 km to 251 km. We will discuss the fundamental limits to phase stabilization as well as practical limits that depend on the geometry of the optical fiber network.^{1,2}

2. FREQUENCY TRANSPORT ACROSS THE OPTICAL SPECTRUM

The basic components for stable optical frequency comparison are Doppler-cancelled transport over fiber optic links,^{3,4} high stability, low-phase-noise optical frequency combs,⁵⁻⁷ and well stabilized lasers.^{3,8-12} As a demonstration of the capabilities and vulnerabilities involved in the transport of optical frequencies across the optical spectrum, researchers at NIST previously coherently transmitted an optical carrier over 750 m of optical fiber with conversions to wavelengths of 657, 767, 1126, and 1535 nm within one building at NIST.¹³ Using Doppler fiber phase stabilization techniques, they demonstrated an overall timing jitter of 590 attoseconds, and a frequency instability of 12 mHz for the 195 THz carrier in 1 s and 250 μ Hz in 1000 s. Also, importantly, they found that limitations to the stability were purely “technical”, set by uncompensated “out-of-loop” fiber paths. And, with the Doppler phase cancellation, the fiber link effects themselves provided negligible degradation. This prompted our investigation into the behavior and magnitude of compensated phase noise as a function of length of the fiber optic link.

3. FREQUENCY TRANSPORT ACROSS LONG DISTANCES

Transporting optical frequencies over long distances relies on stabilization of the fiber optic link against phase noise caused by acoustic and thermal vibrations. From experimental and theoretical studies,^{1,2} we find that the residual phase noise after Doppler cancellation is in two regimes (Figure 1a). At low frequencies, interferometer noise (excess phase noise from the measurement system) dominates. For higher frequencies the noise suppression is limited by what we call “delay-unsuppressed” residual phase noise due to the transit time of the light in the fiber link (the delay). This is not the well-known feedback loop delay, which limits the bandwidth and therefore the corresponding noise suppression. Rather, this delay effect causes the “signal” light to exit the link at the far end before any correction can be sensed and applied at the transmit end. As a result, it is impossible to fully suppress the fiber link noise, and even if the round-trip noise is fully suppressed, the one-way, signal light will still suffer from some unsuppressed fiber noise. In general, the phase noise on the transmitted (remote) signal for the phase-locked system can be expressed as

$$S_{remote}(f) = S_{Int}(f) + S_D(f) + \text{other terms}, \quad (1)$$

where S_{Int} and S_D are the interferometric and delay-unsuppressed noise, respectively, and dominate other noise contributions for frequencies f below $1/(4\tau)$ where τ is the one-way propagation delay along the link. Assuming low enough Fourier frequencies and spatially uncorrelated noise along the fiber, we derive the delay-unsuppressed phase noise as

$$S_D(f) \approx a(2\pi f\tau)^2 S_{fiber}(f), \quad (2)$$

where, $a=1/3$, f is the Fourier frequency, and S_{fiber} is the phase noise spectrum of the uncompensated fiber link. We observed the scaling of phase noise in these two regimes as $S_{Int} \propto f^{-2}$ and $S_D \propto f^0$. This provides a scaling for the modified Allan variance,

$$\sigma_{v,remote}^2 = \kappa_{Int}^2 / t_g + \kappa_D^2 L^3 / t_g^3 + \text{other terms}, \quad (3)$$

where t_g is the gate time (averaging time of the frequency counter) in seconds and L is the link length in kilometers. The empirical constant κ_D characterizes the instability due to “delay-unsuppressed” fiber noise and is equal to $\sim 8 \times 10^{-20} (\text{s/km})^{3/2}$ for our fiber link. The constant κ_{Int} characterizes the instability noise floor from the transmitter and receiver interferometers and is equal to $\sim 2 \times 10^{-17} \text{s}^{1/2}$ for our setup. This is illustrated in Figure 1b.

Our test setup is shown in Figure 2. In order to test stability, our fiber link looped back so that the frequency transmitter and receiver portions of the link were located together on the same optical bench (Figure 3). We measured phase noise and Allan variance for various fiber lengths including sections of installed fiber (up to 76 km) and spooled fiber (up to 175 km). Our fractional frequency uncertainty results are shown as a function of distance in Figure 4 and compared to our theoretical estimate of modified Allan variance

$$\sigma_D^2(t_g, L) = \chi \frac{ah_L}{v^2 c_n^2} \left(\frac{L^3}{t_g^3} \right) = \kappa_D^2 L^3 / t_g^3. \quad (4)$$

The factor χ is 3/2 for the modified Allan variance and 8 for the triangle Allan variance^{14,15}, h_L is the empirical phase noise coefficient of our fiber link (normalized to length, L), v is the optical frequency, κ_D is as defined in Equation (1.2), c_n is the speed of light in the optical fiber, and t_g is the gate time.

In studying other sources of phase noise (RF electronics, fiber nonlinear effects, and fiber polarization-mode dispersion), we found them all negligible (under our test conditions) compared to the delay-unsuppressed noise. At sufficiently low frequencies, noise in the interferometer itself dominated. We expect this is the fundamental factor limiting the stability of long-distance frequency transfer. Fortunately, it can be reduced with averaging time.

4. FIBER LINK DESIGN LIMITATIONS

Fundamental to our Doppler phase cancellation approach has been the propagation of light to the remote end of the link and its partial reflection back to the local end by travelling along the same fiber. This allows a measure of twice the phase noise of the fiber, which is then cancelled by a Doppler shift from an acousto-optic modulator. However, most installed long fiber links have erbium-doped fiber amplifiers that operate unidirectionally with built in optical isolation to prevent light from travelling backwards in the fiber. In typical optical telecommunications light transport is accomplished through duplexed fiber pairs where light travels in the "forward" direction on one fiber and in the "reverse" direction on the other. Clearly, the phase noise in the two fibers will not be identical, so we performed frequency transfer tests on duplexed fibers of 400 m and 38 km lengths. The duplexed fiber geometry is illustrated in Figure 5a. We found a significant degradation in performance from the bidirectional cases. Figures 5b and 5c show the transmitted frequency error for the 38 km fiber link in the duplexed and bidirectional cases. While for bidirectional transport, the frequency error is on the order of one or two millihertz, the duplexed fiber case increases the frequency error up to 0.7 Hz. Similarly, the Allan deviations degraded by 1 to 2 orders of magnitude when going from bidirectional propagation to unidirectional (duplexed fiber).

5. CONCLUSIONS

The purpose of our work was to explore fundamental limits to transmission of optical frequencies over fiber optic links. We find good agreement between the theoretical noise predicted for compensated fiber links (based on uncompensated fiber noise measurements) and the experimental results. Recent efforts show rapid progress in long-haul optical frequency transfer¹⁶⁻¹⁸, and we hope that these fundamental descriptions of noise performance will be useful for predicting optimum link performance.

6. REFERENCES

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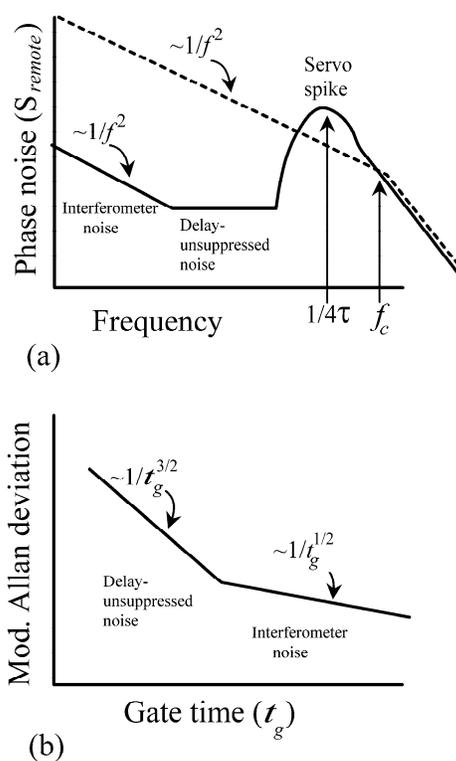


Figure 1. Schematic illustration of (a) phase noise and (b) frequency instability behavior for optimized optical frequency transfer over optical fiber. Dashed line indicates unlocked fiber phase noise, solid lines indicate phase noise and modified Allan deviation when the system is phase-locked.

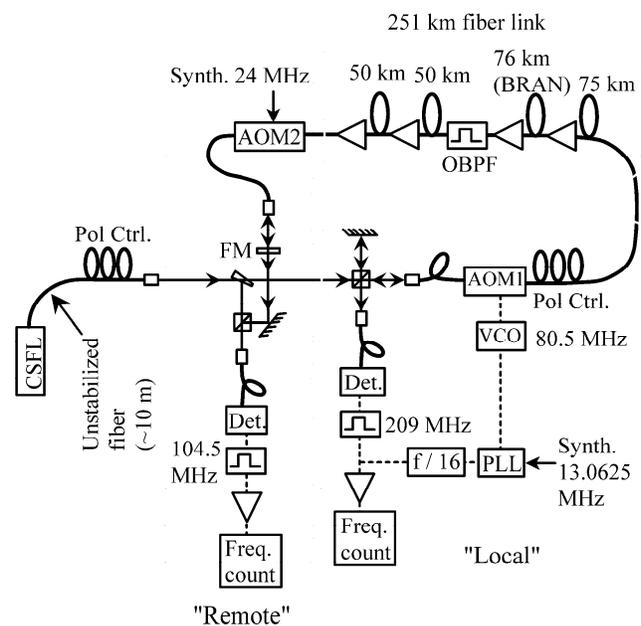


Figure 2. Experimental setup for frequency transfer and fiber stabilization. AOM, acousto-optic modulator; CSFL, cavity stabilized fiber laser; Det, detector, FM, Faraday mirror; OBPF, optical bandpass filter; PLL, phase-locked loop; Pol Ctrl, fiber optic polarization controller; VCO, voltage controlled oscillator. Thick lines are optical fiber, thin lines are free space propagation and dashed lines are electrical paths. Laser light is split by a wedge and cube beamsplitters to act as the LO on the remote and local detectors. The remaining light is frequency shifted by AOM1 and traverses the amplified link and receives a second frequency shift from AOM2. The FM transmits 50 % of the light to mix with the LO at the remote detector. The remaining light is reflected by the FM back through the link and mixes with the LO at the local detector.

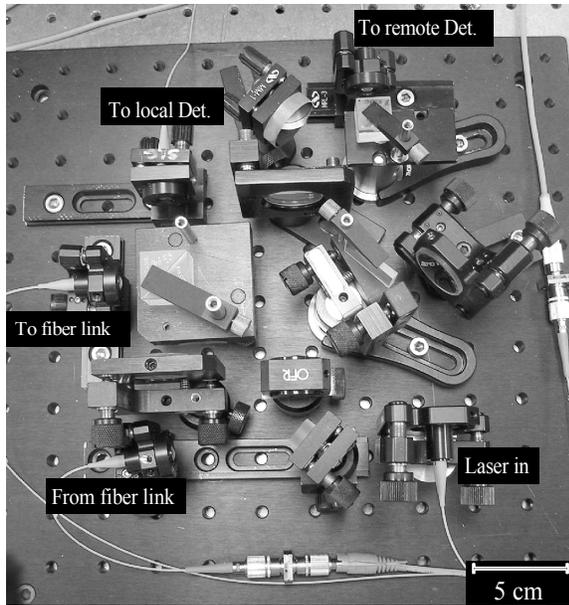


Figure 3 Photo of co-located free-space transmit and receive interferometers. Dimensions $\sim 25 \text{ cm} \times 25 \text{ cm}$.

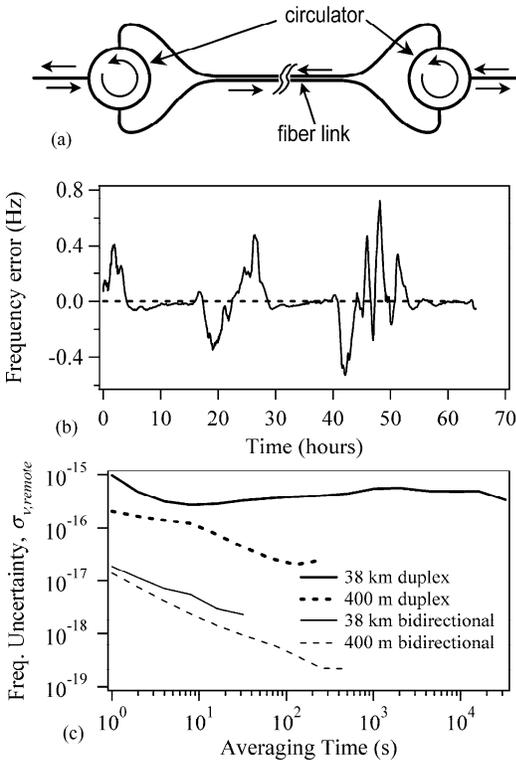


Figure 5. (a) Experimental setup to replace bidirectional fiber link (single fiber) with a duplexed link and a pair of circulators. (b) Counted frequency error (difference from transmitted frequency) for a 1 hour running average on the one-way locked signal after the 38 km link for duplexed (solid) and bidirectional (dashed) configurations. (c) Relative frequency instability (modified Allan deviation) for the duplexed configuration of the 38 km BRAN link and a 400 m spooled fiber compared to the instability for a bidirectional configuration for the 38 km BRAN link and a 400 m spool.

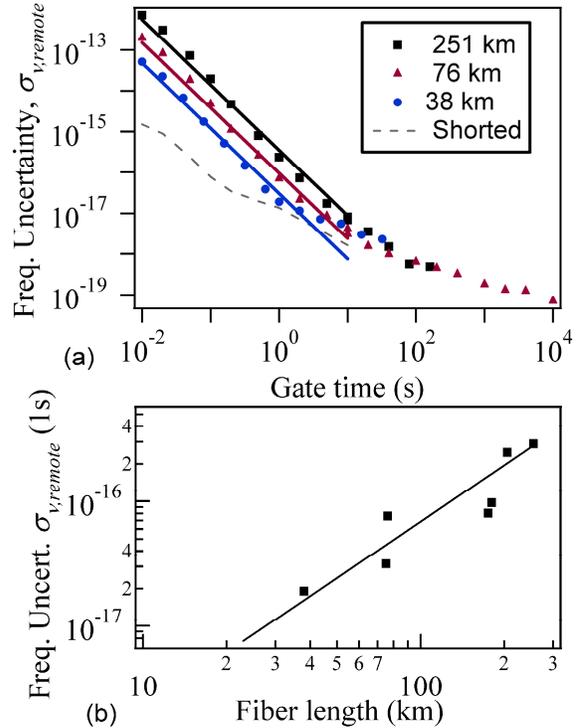


Figure 4. (a) Residual fractional frequency uncertainty (modified Allan deviation) vs. gate time for various fiber link lengths. “Shorted” indicates a link length of less than 2 m. Note: as gate time t_g approaches $\sim 100 \text{ s}$, the interferometric noise S_{int} begins to dominate. The thick solid lines illustrate power-law scaling with a $t_g^{-1.6}$ dependence. (b) Residual fractional frequency uncertainty at a fixed 1 s gate time vs. fiber link length (square points) and the prediction.¹ Noise differences between spooled and installed fiber contribute to the deviations from theory. All points above the curve are data from links that include installed fiber; points below the curve include data only from spooled fiber.