NONLINEAR CALIBRATION OF POLARIMETRIC RADAR CROSS SECTION SYSTEMS †

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Abstract

Polarimetric radar cross section systems are characterized by polarimetric system parameters ϵ_h and ϵ_v. These parameters can be measured with the use of rotating dihedrals. The full polarimetric dataset as a function of the angle of rotation can be analyzed with a nonlinear set of calibration equations to yield the system-parameter complex constants and the four polarimetric calibration amplitudes. These amplitudes appropriately reproduce the system drift and satisfy a drift-free system configuration criterion very accurately. The results indicate that the nonlinear approach is better than the previously studied linear approach, which yielded system parameters that are seriously distorted by drift.

Keywords: drift, nonlinear polarimetric calibration, polarimetric system parameters, radar cross section, reproducibility

1. Introduction

A complete set of monostatic radar cross section (RCS) scattering matrix calibration data, taken with a dihedral rotating about the line-of-sight to the radar, is used to obtain the cross-polarimetric system parameters and the channel calibration constants of a polarimetric RCS measurement system [1-7]. Our model assumes that the two cross-polarimetric hv and vh parameters are constants of the polarimetric RCS system and that the four channel amplifications are specified by complex constants. In the next section we see that each of the four polarimetric measurements M_{pq}, with p and q denoting either the horizontal h or vertical v polarization, can be written as

$$M_{pq} = c_{2,pq} \cos 2\theta + s_{2,pq} \sin 2\theta. \tag{1}$$

The Fourier coefficients $c_{2,pq}$ and $s_{2,pq}$ are proportional to the channel calibrations constants A_{pq} and are functions of the cross-polarization system parameters. A linear method of solving for the system parameters by means of only the copolar measurements seems to be theoretically sound [1,2]. However, two realistic observations challenge the appropriateness of the linear approach to calibration: (1) the system cross-polarimetric parameters are very small, and (2) a RCS system is subject to unpredictable and arbitrary drift in the channel amplifications. Because any method of analysis will be based on a least-squares technique, the solutions for the system parameters can be significantly distorted by the presence of drift in the data.

In more detail, we will see in the next section that

$$c_{2,hh} = A_{hh}(\epsilon_h^2 - 1), \quad s_{2,hh} = 2A_{hh}\epsilon_h. \tag{2}$$

Hence, the ratio $s_{2,hh}/c_{2,hh}$ is a simple function of ϵ_h that is independent of the channel amplitude A_{hh}. The linear approach to obtain the system parameters relies on this simple algebraic step. However, this assumes that the coefficients $c_{2,hh}$ and $s_{2,hh}$ are known exactly. Since we can obtain these coefficients only in the least-squares sense, the ratio of the coefficients will not, in general, be independent of the channel amplitude because of drift. Under the conditions (1) and (2) above, the distortion due to drift could be significant. In fact, we have found this to be the case.

Consequently, a set of constant system parameters cannot be obtained *repeatedly* without adequately controlling the drift. We cannot ignore the basic requirement of *repeatability* in this process. Obviously, when we obtain the system parameters from just a single dataset, we will not reveal any problems. However, when we repeatedly obtain the system parameters from $N \approx 10$ datasets, the linear approach will most certainly produce very large, hence unacceptable, variations in the system parameters. This is the problem that we address in this paper.

In a nonlinear polarimetric calibration model we use all four components M_{pq} in eq (1) and incorporate the nonlinear signal-path condition

$$\chi_A \equiv \frac{A_{hv}A_{vh}}{A_{hh}A_{vv}} = 1, \tag{3}$$

which is *valid even in the presence of drift*. The validity of this condition can be easily ascertained if we consider the indices to represent the transmit and receive signal paths (see Appendix). The nonlinear system of eqs (1) and (3) are then solved for the channel calibration amplitudes and the system parameters. The drift will be adequately reflected in the channel calibration amplitudes, and we can expect that the system parameters will be approximately constant with acceptable *repeatability*.

We show with real data that *the nonlinear technique is superior to the linear approach* in that we can exhibit drift-dependent amplitudes A_{pq} and constant cross-polarimetric ratios that are repeatable within small uncertainties.

2. Theoretical Model

We use a rotating dihedral to calibrate a polarimetric radar. The receive matrix is given by [1-7]

$$r = \begin{pmatrix} r_{hh} & r_{hv} \\ r_{vh} & r_{vv} \end{pmatrix}. \tag{4}$$

The dihedral scattering matrix (in the high frequency limit) is given by [1-7]

$$D(\theta) = k_D \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \tag{5}$$

where θ, with respect to the vertical, is the angle of rotation about the line-of-sight from the radar to the dihedral, and k_D depends on the dimensions of the dihedral and is assumed to be known. We define

$$\epsilon = \begin{pmatrix} 1 & \epsilon_h \\ \epsilon_v & 1 \end{pmatrix}, \tag{6}$$

where

$$\epsilon_h = \frac{r_{hv}}{r_{hh}} \quad \text{and} \quad \epsilon_v = \frac{r_{vh}}{r_{vv}} \tag{7}$$

are the cross-polarimetric system parameters that we wish to determine. After normalization (see Appendix), the polarimetric signal scattered from a dihedral $M \propto rDt$ [1-7] is given by the matrix product

$$M(\theta) = kA * \epsilon D(\theta)\tilde{\epsilon}, \tag{8}$$

where k is a complex constant, A is the channel amplification matrix, and we assumed reciprocity so that the transmit matrix is $t = \tilde{r}$ (transpose of r). In eq (8) the * denotes element-by-element multiplication. The matrix elements of M are

$$(kk_D A_{hh})^{-1} M_{hh} = (-1 + \epsilon_h^2)\cos 2\theta + 2\epsilon_h \sin 2\theta, \tag{9}$$

$$(kk_D A_{vv})^{-1} M_{vv} = (1 - \epsilon_v^2)\cos 2\theta + 2\epsilon_v \sin 2\theta, \tag{10}$$

$$(kk_D A_{hv})^{-1} M_{hv} = (\epsilon_h - \epsilon_v)\cos 2\theta + (1 + \epsilon_h \epsilon_v)\sin 2\theta, \tag{11}$$

and

$$(kk_D A_{vh})^{-1} M_{vh} = (\epsilon_h - \epsilon_v)\cos 2\theta + (1 + \epsilon_h \epsilon_v)\sin 2\theta. \tag{12}$$

For all polarizations, we can write

$$M_{pq} = c_{2,pq}\cos 2\theta + s_{2,pq}\sin 2\theta, \tag{13}$$

where p and q are either h or v. We can use Fourier analysis to obtain all coefficients $c_{2,pq}$ and $s_{2,pq}$ from measured data.

In the *linear* approach we obtain the system parameters ϵ_q with only the copolar expressions in eqs (9-10). We form the ratio of the $n = 2$ Fourier coefficients obtained from the data and express this ratio in terms of the system parameters by use of the theoretical expressions for the coefficients in eqs (9-10). Thus

$$r_{2,q} = \frac{s_{2,qq}}{c_{2,qq}} = \frac{2\epsilon_q I_q}{1 - \epsilon_q^2}, \tag{14}$$

where $I_h = -1$ and $I_v = 1$. We then easily get two solutions for each polarization,

$$\epsilon_q = \frac{-I_q \pm \sqrt{1 + r_{2,q}^2}}{r_{2,q}}. \tag{15}$$

The solutions are negative reciprocals of each other; we choose $|\epsilon_q| < 1$, which is true for RCS systems. In eq (14) we assumed that the ratio of coefficients are *independent of all amplitudes* in eqs (9-10), which implies that A_{qq} are constants throughout the measurements. Thus, we have assumed that *drift is insignificant*! This is generally untrue.

In a *nonlinear* approach we use all the component eqs (9-12) to obtain least-squares solutions to all A_{pq} and ϵ_q. We also demand that the signal-path configuration condition eq (3) be satisfied at every point of measurement. We note that eqs (11-12) differ only in the cross-polar channel-amplification constants, which, together with the increased complexity of the cross-polar dependence on the polarimetric system parameters, significantly reduces the least-squares solution space and decouples the independent parameteres A_{pq} and ϵ_q. In addition, we have the option to subdivide any full rotation into seqments and solve for different amplitudes A_{pq} in each segment as we keep ϵ_q constant throughout a rotation. We expect that drift-dependent amplitudes and a *repeatable* set of system parameters can be obtained with this approach.

3. Polarimetric calibration data

Full polarimetric measurements using a 12-inch-square rotating dihedral were obtained at 9.6 GHz for $N = 9$ rotations. The dihedral was located at approximately $12,800$ ns from the radar.

Polarimetric drift was monitored with a stationary trihedral approximately 550 ns behind the dihedral. Consequently the drift data could not be used to remove drift analytically from the dihedral data; here we merely demonstrate system drift and provide an estimate of the drift modifying the dihedral data.

Polarimetric clutter data were recorded in front of the dihedral, and polarimetric noise data were recorded behind the trihedral. All clutter and noise data exhibited a random character, and the recorded magnitudes were substantially below the dihedral and the trihedral responses. Hence, we assumed that clutter and noise do not introduce significant errors into the determination of the polarimetric system parameters and were ignored in the analysis. However, contributions from clutter and noise must still be included in the uncertainty in the system parameters, but we expect such contributions to be small.

In Figure 1, we show the polarimetric drift responses of the trihedral located behind the dihedral. We include only the copolar hh response; other polarimetric components of the data were qualitatively similar.

In Figure 2, we show the amplitude and phase of the hh polarimetric dihedral response as a function of the rotation angle θ as the dihedral underwent nine full rotations. The rotations are delineated by solid vertical lines; the broken lines indicate angles at multiples of 90°. Ideally, the rotations would be identical, but drift is seen to have introduced changes in both amplitude and phase from rotation to rotation. The other polarimetric components show qualitatively the same influence of drift.

In Figure 3, we show evidence that the $n = 4$ component of the drift has modified the data [3]. We compare the unprocessed hh data with its $n = 2$ Fourier component. We can explicitly verify that the features observed in the discrepancy at angles of multiples of 45° (dotted lines) are due to the presence of a small but significant $n = 6$ Fourier component in the data. This Fourier component is the harmonic produced by the dihedral response ($n = 2$) and the drift ($n = 4$). The other components of the data exhibit the same features.

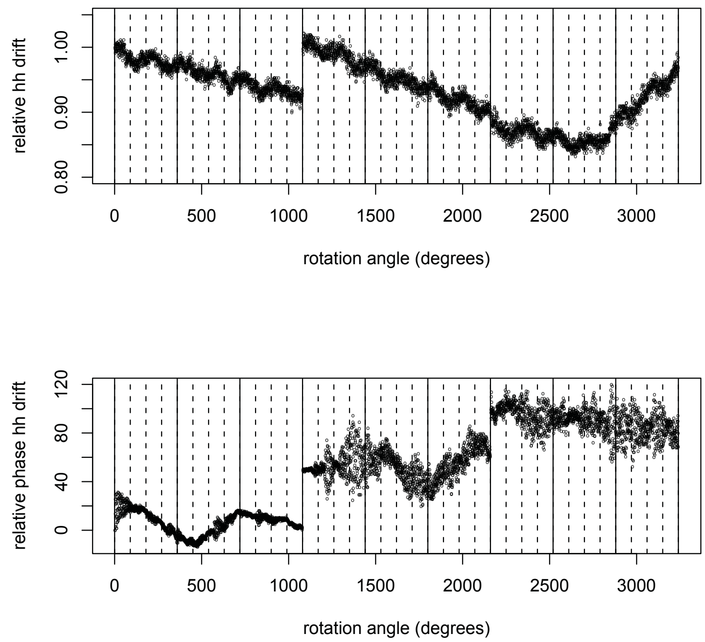

Figure 1. The amplitude and phase of the hh response of a trihedral located behind the rotating dihedral. Ideally the response would be constant. The drift is due to changing environmental conditions. The solid vertical lines delineate the nine rotations.

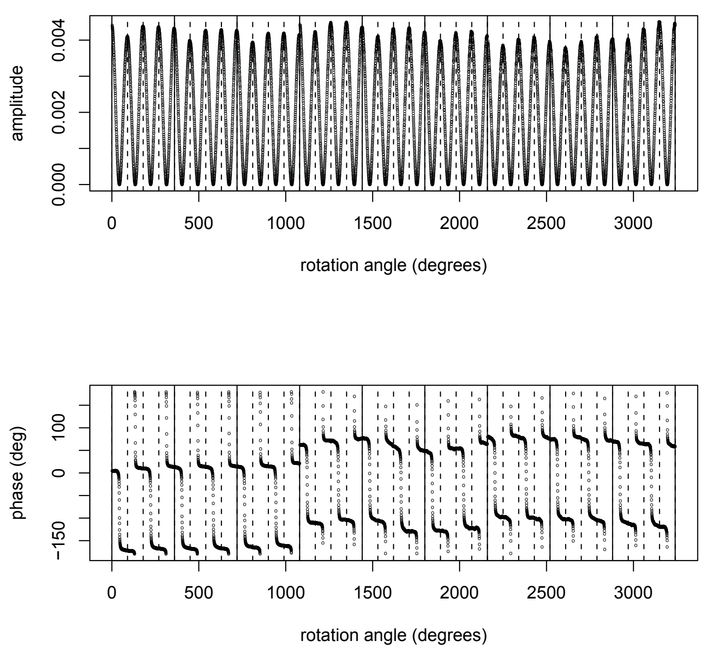

Figure 2. The amplitude and phase of the hh dihedral response as a function of rotation angle for nine rotations. The solid vertical lines delineate the rotations; broken lines are at multiples of 90°.

4. Polarimetric data analysis and results

We rely on the model eqs (9-12) to determine the channel-amplification amplitudes $kk_D A_{pq}$ and the cross-polarimetric system parameters ϵ_q defined in eqs (6-7). First, we have applied the linear model to obtain ϵ_q for each rotation [1]; these results are shown in Figure 4 with small circles. We observe a strong rotation-to-rotation variation in the imaginary ϵ_q and a weaker variation in the real part. As discussed above this is due to the inability of the linear model to decouple the drifting amplitudes and the system parameters. Next, we applied the nonlinear model to solve for the unknowns. Figure 4 shows these results with circles with dots in them. We observe that the system parameters obtained with the nonlinear model cluster within a small area of uncertainty. In Figure 5, we show amplitudes A_{pq} for each half rotation. The first 180° of each rotation is shown with a smaller circle. The drift in amplitudes is clearly evident. Figure 6 shows χ_A as defined in eq (3) for two cases: first, the amplitudes apply to each full rotation, and, second, the amplitudes apply to each half-rotation. The nine open circles indicate $\chi_A \approx 1$ in the first case, and the 18 X-s represent $\chi_A = 1$ for the second case. Because of the least-squares procedure, we see some deviation away from $\chi_A = 1$ in the first case, but the agreement is still excellent. For each half rotation, we demonstrate that the driftless condition $\chi_A = 1$ has been very accurately satisfied for 18 sets of A_{pq} as they drift during the rotations (see Figure 5).

5. Summary and suggestions

We have demonstrated that the nonlinear set of equations can decouple the polarimetric signal-path amplitudes A_{pq} and the cross-polarization system parameters ϵ_q that describe the full scattering-matrix RCS response of a rotating dihedral. In comparison, the previously recommended linear analysis procedure is seen to provide poorly determined system parameters. This is an important observation, since up to now all procedures reported in the polarimetric RCS literature are variants of the linear model. However, we still observe residual variations in the system parameters obtained with the nonlinear procedure that cannot be explained as due to clutter or noise. We need to revisit the model on drift published previously [3] to see whether the nonlinear model proposed in this study could be modified to include drift explicitly. Furthermore, additional datasets need to be obtained in various environments to further test and validate the procedure developed here and in future studies.

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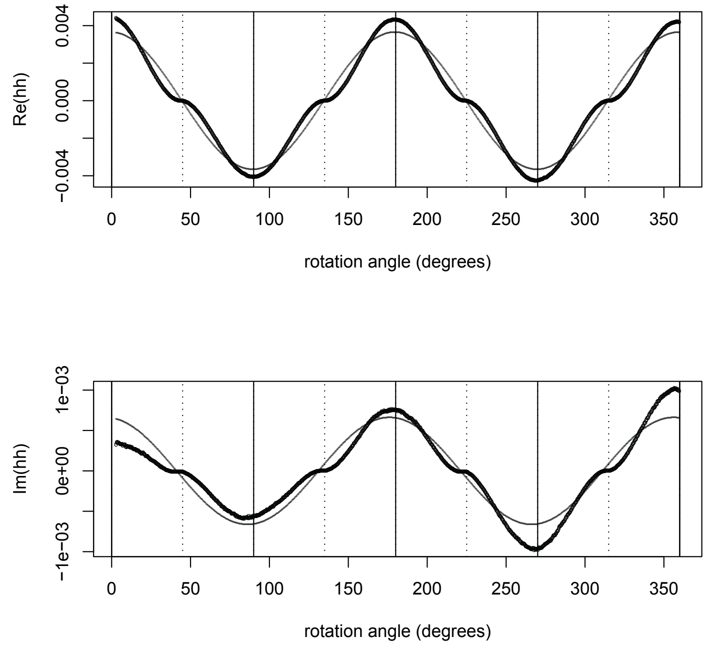

Figure 3. The real and imaginary components of the dihedral response (thick line) during the first rotation and its $n = 2$ Fourier component. The discrepancies in the neighborhood of multiples of 45° (dotted lines) and 90° (solid lines) are due to the presence in the data of a $n = 6$ Fourier component due to the interaction of drift and the dihedral response.

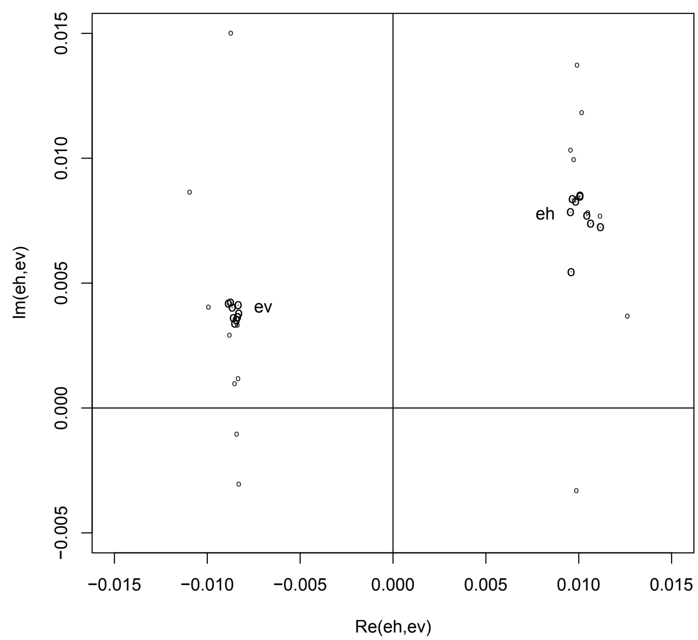

Figure 4. The cross-polarimetric system parameters ϵ_h and ϵ_v as determined by the linear model (small circles) and by the nonlinear model (large circles with dots). We expect the parameters to be constants of the measurement system. The linear solution is strongly affected by drift and produces unacceptable variations in the parameters.

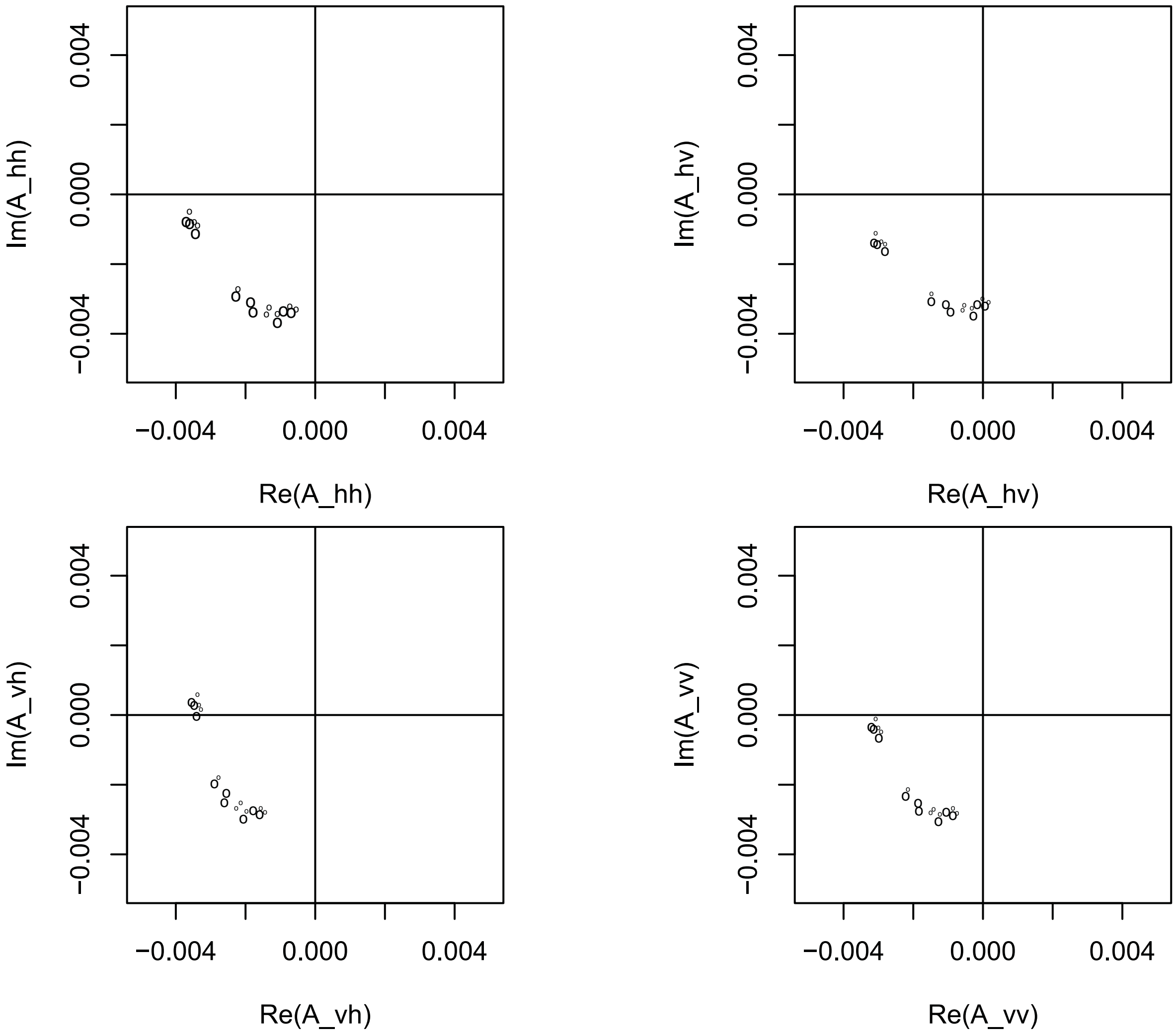

Figure 5. The 18 channel-amplification complex amplitudes A_{pq} for each 180° segment of rotation. The drift is due to changing environmental conditions.

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Appendix

Here we present in detail the derivation of eq (8). In general, the monostatic polarimetric signal scattered by the dihedral is given by [1-7]

$$M'(\theta) = kA' * rD(\theta)t, \qquad (A1)$$

where the transmit matrix $t = \tilde{r}$, the transpose of the receive matrix, and the elements of A' give the applied amplification (or attenuation) to each polarimetric channel, the * indicates element-by-element multiplication, and k is a complex constant that depends on the separation between the target and the radar. We can write

$$r = r_d \epsilon, \qquad (A2)$$

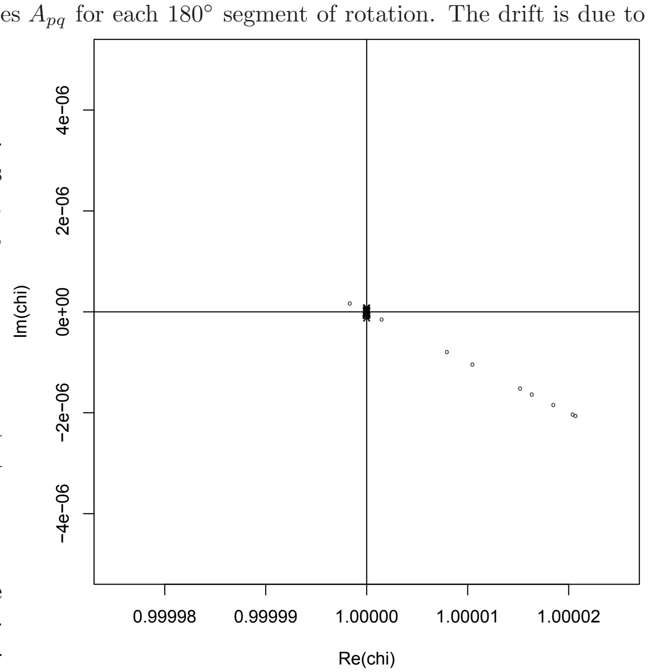

Figure 6. χ_A plotted when A_{pq} are formed using the amplitudes of each rotation (circles). The small deviations from 1 are due to averaging of drift by the least-squares analysis. When the amplitudes are determined for each 180° segment (X-s), the deviations from 1 almost vanish.

where

$$r_d = \begin{pmatrix} r_{hh} & 0 \\ 0 & r_{vv} \end{pmatrix}, \qquad (A3)$$

and ϵ is defned in eq (6). Equation (A1) now becomes

$$M(\theta) = kA' * r_d \epsilon D(\theta) \tilde{\epsilon} \tilde{r}_d. \qquad (A4)$$

We note that $\tilde{r}_d = r_d$, and that the inverse is given by

$$r_d^{-1} = \begin{pmatrix} r_{hh}^{-1} & 0 \\ 0 & r_{vv}^{-1} \end{pmatrix}. \qquad (A5)$$

We now rewrite eq (A4) by performing element-by-element division by A'; then we pre- and post-multiply by the inverse of r_d and perform element-by-element multiplication with

$$A = \begin{pmatrix} r_{hh}^2 A'_{hh} & r_{hh} r_{vv} A'_{hv} \\ r_{vv} r_{hh} A'_{vh} & r_{vv}^2 A'_{vv} \end{pmatrix}. \qquad (A6)$$

The result is

$$M(\theta) = kA * \epsilon D(\theta) \tilde{\epsilon}, \qquad (A7)$$

which is eq (8).

In general, the channel amplifications $A'_{pq} = \tau_q \rho_p$ are the products of the transmit τ_q and receive ρ_p channel amplifications, which can be adjusted arbitrarily and satisfy

$$\frac{A_{hv} A_{vh}}{A_{hh} A_{vv}} = 1. \qquad (A8)$$

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