

ECONOMICS OF MEASUREMENT UNCERTAINTY AND TOLERANCES

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INTRODUCTION

Economic growth, whether that of a national economy or of a particular industry, is often modeled as a production function relating output (Y) to input factors of production such as hours of labor (L) and capital investment (C). The Cobb-Douglas production function [1] given in equation (1) is a simple and common example, and is sufficient to illustrate our discussion. The exponents α and β describe the influence of the input factors; with $\alpha, \beta > 0$ and typically $\alpha + \beta \approx 1$. The magnitude of the coefficient A , known as total factor productivity (TFP), scales with human capital (worker training and education) and the state of employed technology.

$$Y = AL^\alpha C^\beta \quad (1)$$

Developing economies grow rapidly because they can increase all three of these quantities: the productive labor force increases as rural population become factory workers, capital spending creates new infrastructure such as improved roads, bridges, and machinery, and TFP increases as workers gain better training, education, and adopt new technology.

In advanced economies, such as the United States, the labor force is largely developed and is changing only incrementally. Similarly, domestic infrastructure is well developed and while additional capital investments can improve output – and increase the national competitive advantage – it is not a strategic advantage because any nation with access to capital can create infrastructure, i.e. there is no barrier to replication of this advantage.

In developed economies, a strategic competitive advantage is gained primarily by adopting new technology (including new management methods) that increase output and are not easily duplicated. That is, a strategic advantage arises from working smarter, as opposed to working

harder (increasing L) or by constantly purchasing new tools (increasing C). Although competitive advantage creates high value-added products, even the best defended advantage eventually “leaks out” and becomes nullified. Hence, today’s high value-added manufacturing becomes tomorrow’s commoditized product. High value products imply manufacturing strategies and practices that exclude competitors. This involves intellectual capital that is protected either as knowledge (patents, trade secrets) or processes (unique facilities, deep institutional knowledge within the organization). In this paper, we are primarily interested in developing and exploiting deep knowledge about the manufacturing process in order to optimize it and increase profits.

MEASUREMENT ADDED VALUE

In manufacturing, a persistent factor in the exclusion of competitors is the ability to produce increasingly complex components, with both higher accuracy and lower cost, which also drives product quality metrics such as improved function and higher reliability. One aspect of higher accuracy and lower cost in precision manufacturing involves optimizing the metrology process on an economic basis. This involves knowledge of the product production distribution, the measurement process distribution, and the consequences of bad decisions – e.g. rejecting in-specification components or accepting out-of-specification components.

Recently a number of national and international standards including the ASME B89.7 series [2] and the ISO 14253 [3] series have addressed some of the probabilistic aspects of this issue. We use this particular concept as an example of working smarter by optimizing this aspect of the measurement process.

While it is sometimes claimed that measurements “do not add value” this is only true in the case that *a priori* there is 100 %

certainty that all components are produced within specification. In the more typical case, some fraction of components are out-of-specification and identifying and eliminating them reduces a host of potential costs such as increased warranty expense, loss of customers due to dissatisfaction, decreased brand value, and potential lawsuits. Ideally, all of these effects can be captured in cost functions that express the cost of making various incorrect decisions. Hence, in reality, metrology's role is to identify and minimize "negative value" components produced in the manufacturing process, whether that is expensive scrap or – generally more expensive – downstream problems created from out-of-specification components.

In addition to the basic function of culling out-of-specification components from the manufacturing output, measurements are increasingly being used to control the manufacturing process. Process control is highly desirable as it not only reduces the number of measurements (and hence costs), but it proactively adjusts the process and thus can reduce the number of out-of-specification components, i.e. improves the production distribution. The cost functions are now "leveraged up" as a few process control measurements may affect several hundred manufactured components. Additionally, an erroneous measurement system now has the opportunity to misadjust the manufacturing parameters creating out-of-specification components and then pass them onto the customer. Hence the stakes (cost functions) are much higher for process control measurements than for inspection measurements.

MANUFACTURING OPTIMIZATION

In an ideal world, the entire manufacturing endeavor, from design, production, inspection, to post-purchase support, would be economically optimized. For example, there are obvious cost tradeoffs between design tolerances and production costs, or between production quality and post-purchase expenses (warranty costs etc.). As manufacturing processes become increasingly understood, and as data acquisition, storage, and computation costs have dramatically declined, new opportunities for optimizing the manufacturing process are emerging. By recognizing this new technological environment, strategic competitive advantage can be captured through increasing

the TFP to yield more and higher quality output for the same labor and capital costs.

Recently, some aspects of the above scenario are being realized. In this paper we focus on progress on optimizing the "gauging limits" that set the measurement threshold for accepting or rejecting components. Additionally, we discuss the current progress on optimizing the entire measurement process to further reduce costs.

GAUGING LIMITS

The industrial revolution accelerated with the invention of interchangeable parts. For the majority of the nineteenth century this involved hard gauges (at that time often called limit gauges) that were the physical embodiment of workpiece tolerances. Toward the end of the nineteenth century analog measuring instruments such as calipers and micrometers became increasingly available and thus required comparing the instrument's output value against numerical limits (i.e. tolerances) on the workpiece drawing. Acceptable workpieces had measured values at or within the tolerance zone – a decision rule now known as "simple acceptance"; see Figure 1 [4].

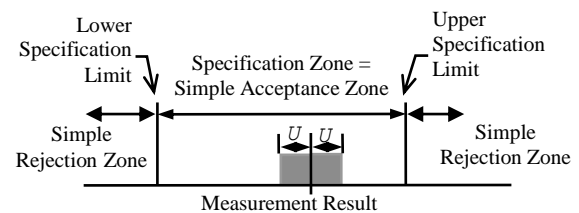


FIGURE 1. An example of simple acceptance and rejection. The measurement uncertainty interval is of width $2U$, where U is the expanded ($k = 2$) uncertainty.

In the latter half of the twentieth century the uncertainty of measurements became more important as requirements on tolerances steadily decreased and global interchangeability requirements increased. More recently, the distinction between the acceptance limits (also known as the gauging limits) and the specification limits (tolerance limits) have become more significant. The difference between the two sets of limits is known as the guard band. If the gauging limits are inside the specification limits the resulting decision rule is known as "stringent acceptance"; see Figure 2 [4]. If the gauging limits are outside the specification zone then this increases the size of

the acceptance zone and is known as “relaxed acceptance”; see Figure 3 [4].

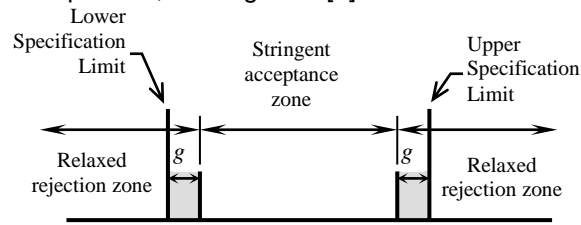


FIGURE 2. A stringent acceptance zone for measured components, the offsets between the specification limits and the acceptance limits is the guard band g .

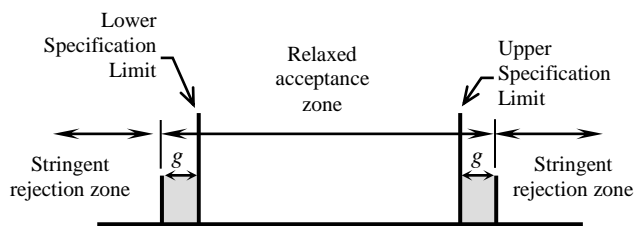


FIGURE 3. A relaxed acceptance zone for measured components; components with measured values beyond the specification limit may be accepted.

Allowing this new degree of freedom, where specification and gauging limits are no longer identical, creates the opportunity to increase profits by exploiting our knowledge about the metrology and manufacturing process, as we illustrate next.

OPTIMIZING DECISION RULES

To illustrate the advantage of working smarter by making better decisions we provide the following example. Consider a manufacturing process that has a production capability index of $2/3$, i.e. $C_p = 0.66$, where $C_p = T/(6u_p)$, T is the component tolerance and u_p is one standard deviation of the production distribution. Suppose also that the measurement capability index is 2 , i.e. $C_m = 2$, where $C_m = T/(4u_m)$, and u_m is one standard uncertainty of the measurement system. (This is sometimes referred to as a 2:1 gauging ratio.) For example, the inspection might be done with a high speed automated digital camera that rapidly images and computes the measurement result of the component.

The cost model for this example is given in Table 1. In this example, the net profit of accepting an in-specification component is \$1.

The net loss of rejecting a component is \$1 (it does not matter if the component is in-specification or out-of-specification, in either case the loss is the same). We examine three cases of accepting an out-of-specification component. In case A, the consequences are minor and the loss is \$2, e.g. minor customer dissatisfaction costs. In case B, the loss is \$10, perhaps due to increased warranty costs as a result of defective components. In case C, the loss is \$20 which might represent both warranty costs and customer dissatisfaction.

TABLE 1. The profit matrix showing the profit or loss for each decision

	In-specification	Out-of-specification
Accepted	+1	Case A: -2 Case B: -10 Case C: -20
Rejected	-1	-1

In Table II we present six different decision rules ranging from no inspection – which is equivalent to 100 % acceptance (i.e. relaxed rejection with an infinite guard band) – to stringent acceptance with a guard band equal to 100 % of the expanded measurement uncertainty. This latter rule is the ISO 14253-1 default rule. For each rule, the probabilities of the four decision outcomes are presented and the economic consequences of the cost functions (given in Table I) are shown.

For this particular example, when the costs associated with accepting an out-of-specification component are minor (case A) then relaxed (75 % U) acceptance is the economically optimal decision. Conversely, when the cost of accepting an out-of-specification component is high (case C), then the economics shifts to stringent (25 % U) acceptance.

OPTIMIZING METROLOGY SYSTEMS

We have illustrated through the previous example how increased profits can be derived by optimizing the decision rule, i.e. setting the gauging limits. But this example naturally gives rise to more questions such as “how is the information needed to compute the process and measurement capability indices acquired?” and “what additional parameters can be profitably optimized?”

TABLE 2. The outcome matrix showing the probability of accepting or rejecting either an in-specification or out-of-specification component for various decision rules. The net profit for each of the rules and cases are also shown, the most profitable outcome is shown in bold.

	100 % U Stringent Acceptance	25 % U Stringent Acceptance	0 % U Simple Acceptance	25 % U Relaxed Acceptance	75 % U Relaxed Acceptance	No Inspection
Accept / In-specification	0.625	0.873	0.911	0.934	0.9499	0.9545
Accept / Out-of-specification	0.00035	0.007	0.013	0.019	0.0354	0.0455
Reject / In-specification	0.328	0.080	0.041	0.021	0.0024	NA
Reject / Out-of-specification	0.047	0.041	0.035	0.027	0.0123	NA
Net Profit per 1000 pieces Case A: cost = 2	249.1	738.3	809.4	846.7	864.4	863.5
Case B: cost = 10	246.3	683.1	706.4	682.8	581.2	499.5
Case C: cost = 20	242.8	614.1	577.7	477.9	227.2	44.5

Fortunately, a confluence of computer hardware, software and sensor technology have recently converged to address these issues. Hence the ability to understand and reconfigure complex manufacturing systems – previously requiring human thought and intuition – is now becoming accessible to more rigorous and accurate computation.

The measurement capability index, C_m , can now be computed in detail for complex 3D coordinate measuring machine (CMM) measurements using computer simulation. This technology can take into detailed account the specific parametric errors of the measuring instrument performance, the geometric imperfections of the workpiece (i.e. form errors), the thermal environment's influence on both the instrument and the workpiece, the metrologist's selected measurement plan (the number and location of measurement points), and the complex nature of the feature under measurement which is often related to other features via datum reference frames. A rigorous evaluation of all of these effects is beyond a single human's cognitive capabilities, but can be computed using Monte Carlo simulation [5].

Given that the details of the measurement process can be simulated, it follows that the cost functions of this process can also be determined. For example, different CMMs have different accuracies and also operational costs, e.g., due to costs of capital equipment and the level of operator training. Similarly, different measurement plans have different costs since increasing the number of measurement points increases the inspection time; conversely, increasing the CMM speed reduces the run time but also (potentially) the accuracy. These and other costs can be estimated and therefore

included in the optimization process. Hence, we anticipate in the near future that the capability to compute the optimal CMM, measurement plan, thermal environment, and gauging limits (i.e. decision rule) that yields the maximal economic profits.

This type of deep insight into the fundamental nature of the manufacturing endeavor becomes a strategic competitive advantage for the firm that can capture and develop this knowledge. By successively increasing and exploiting these insights, the TFP can be steadily increased thus yielding higher productivity in a manner that is not easily replicated. We view this approach as the vanguard of the future high value-added manufacturing firm.

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