The Benefits of Network Coding over a Wireless Backbone

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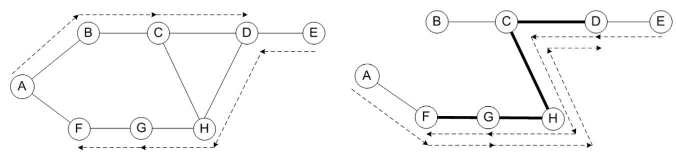
Abstract— **Network coding is a promising technology that can effectively improve the efficiency and capacity of multi-hop wireless networks by exploiting the broadcast nature of the wireless medium. However, current packet routing schemes do not take advantage of the network coding, and the benefits of network coding have not been fully utilized. To improve the performance gain of network coding, in this paper, we apply network coding over a wireless backbone and investigate the performance of this approach from a theoretical perspective. Our analysis shows that, compared to network coding over ad hoc networks with traditional routing schemes, network coding over the backbone structure exhibits significant advantages. This is because all packets are transmitted over the constructed backbone with pre-specified routes, and consequently the opportunity for coding packets at intermediate nodes can be substantially improved. To further enhance the performance, we also present an optimized link scheduling protocol for network coding over a wireless backbone. The performance results show that with proposed approach, the coding gain can achieve the theoretical bound in some scenarios.**

I. Introduction

Network coding has recently emerged as a new paradigm that has demonstrated a practical way for improving the capacity of a network. In the literature, a number of network coding schemes have been proposed, including linear coding [1] and randomized coding [2]. Compared to these schemes, XOR-based coding (COPE) [3] is a simpler but effective way, which can enhance the capacity of multiple unicast traffic flows in wireless networks.

In a chain topology, assume nodes A and C need to exchange two packets P_1 and P_2. Due to communication range limitations, the intermediate node B serves as a relay node to forward the packets to their destinations. Because of radio interference, with traditional relay schemes, this information exchange takes four transmission slots to complete. The following is a possible sequence: 1) $A \mapsto B : P_1$; 2) $C \mapsto B : P_2$; 3) $B \mapsto C : P_1$; 4) $B \mapsto A : P_2$. In comparison, with a simple XOR-based network coding (as employed in COPE), this information exchange can be implemented by using three slots as in the following: 1) $A \mapsto B : P_1$; 2) $C \mapsto B : P_2$; 3) $B \mapsto A, C : P_1 \oplus P_2$. Because node A already has packet P_1, A can decode P_2 through the operation $P_2 = P_1 \oplus (P_1 \oplus P_2)$. Similarly, node C can decode P_1 through $P_1 = P_2 \oplus (P_1 \oplus P_2)$. With this network coding scheme, the number of transmission slots is reduced from 4 to 3, thus producing a coding gain of $\frac{4}{3} \cong 1.33$.



(a) Shortest path routing (b) Coding with backbone routing

Fig. 1. Example of network coding over a multi-hop wireless network

Despite the promising potential of network coding, a number of challenges have yet to be addressed. One of the most important issues is that current packet routing schemes in wireless ad hoc networks do not take advantage of network coding. Note that in COPE-type network coding schemes, when a transmission opportunity occurs at an intermediate node, the relay node would check the coding opportunity to combine multiple packets in the output queue together to broadcast with a single transmission. Because the route of the packets has multiple choices and the traffic flows are dynamic, the coding opportunity could be less with traditional ad hoc routing schemes. In Fig. 1, we illustrate the drawback of route selection with traditional routing schemes by a simple example. In this example, there are two flows in a multi-hop wireless network, one from node A to node D and the other from node E to node F. If the shortest path routing strategy is employed, the paths for the two flows are shown in Fig. 1(a). We can see that there is no coding opportunity if the routes shown in Fig. 1(a) are used.

To improve the efficiency of network coding, in this paper, we propose to apply network coding on a wireless backbone. In the literature, backbone construction has been investigated extensively to enhance the efficiency of the multi-hop wireless networks [4] [5]. To build a wireless backbone, some nodes with superior communication range, computational capability, or greater energy resources are selected as the *backbone nodes* (BNs). The main purpose of the BNs is to provide a wireless infrastructure facilitating network-wide efficient and resilient communications. Usually, a backbone construction algorithm utilizes a "connected dominating set" (CDS) of nodes to find a tree over the network graph, and a node not in the backbone has at least one backbone node neighbor (i.e., the BNs form a dominating set). Once a backbone has been constructed, any non-backbone node can access the nearest BN and achieve end-to-end communications through the backbone structure. Because the constructed backbone has a tree topology over the

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network, the route of each packet can be simply determined at the source node.

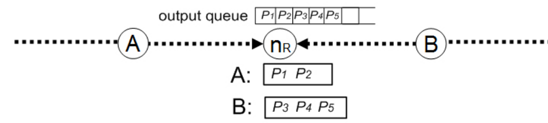
Our study is motivated by the observation that additional coding gain can be obtained if the packet forwarding is done in a way to maximize the coding opportunities at each hop of the backbone structure. In general, a backbone construction scheme will produce a connected backbone with a tree topology over the network, which provides a unique path for each pair of nodes. Hence, there will usually be a number of two-way flows over the backbone. Thus the coding opportunity increases dramatically because of the bi-directional nature of traffic. As shown in Fig. 1(b), if we construct a wireless backbone where the nodes F, G, H, C, D are selected as backbone nodes, then the two flows can "overlap" at the common nodes G, H and C, and then network coding can occur at these nodes. If the source nodes A and E continuously send packets to their destinations D and F respectively, network coding can be done continuously at the common nodes G, H and C. In addition, once a backbone is constructed, it is a semi-permanent wireless highway that can be established and maintained for some period of time, which reduces the computational cost of coding decisions.

In this paper, we explore the benefits of network coding over wireless backbones. The major contributions of this paper are: 1) To the best of our knowledge, this is the first work to investigate the gain of network coding over a wireless backbone; 2) We prove that the wireless backbone routing structure exhibits unique advantages for network coding, including increased coding opportunity and reduced overhead; 3) We conduct theoretical analysis of the proposed scheme and present numerical results; 4) To fully utilize the coding gain and avoid radio interference, we propose a simple optimized link scheduling algorithm and investigate its impact on the performance of wireless ad hoc networks.

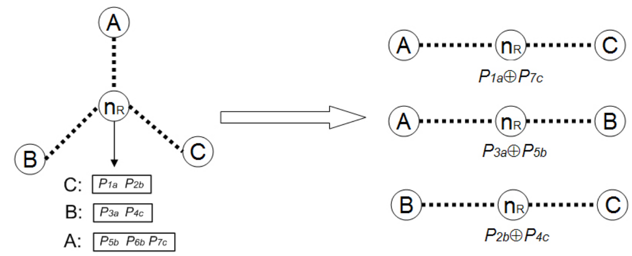
The rest of this paper is organized as follows. In Section II, we give an overview of the related work. In Section III, we present and analyze our network coding scheme over a wireless backbone. In Section IV, we propose an optimized link scheduling algorithm for a wireless backbone and analyze its performance. Finally, we conclude the paper in Section V.

II. Related Work

The use of network coding to enhance the capacity of a network was first proposed in [6] in the context of multicast communications. Subsequently, linear network coding [1] was shown to be sufficient for achieving optimal capacity based on the max-flow min-cut theorem. Since then the work [7] gave an algebraic characterization of linear encoding schemes, and distributed random network coding schemes were investigated in [2]. In the context of wireless networks, [8] studied the problem of minimum cost multicast involving a single session with a single source node. These works are all focused on multicast traffic. COPE is a pioneering work that presented a practical network coding scheme for multiple unicast traffic flows over multi-hop wireless networks [3].



(a) XOR coding for chain topology: n_R broadcasts $P_1 \oplus P_3$, $P_2 \oplus P_4$. The packet P_5 is left for the next round to be sent



(b) XOR coding for 3-star topology

Fig. 2. Examples of coding strategy for backbone nodes

However, the capacity improvement of COPE depends on the existence of coding opportunities, which in turn depends on the traffic patterns. To improve the coding opportunities in COPE-type coding approaches, several coding-aware routing schemes are proposed. Coding-aware routing is a routing paradigm that takes coding opportunities into account when performing route selection in network nodes. [9] and [10] proposed linear-programming-based coding-aware routing protocols. Although these routing protocols can improve the resource utilization and throughput, their centralized characteristic causes scalability problems. Therefore they are not practical to be employed in multi-hop wireless networks. In [11] and [12], the authors proposed distributed hop-by-hop coding-aware routing protocols, in which routing decisions are made based only on limited local view information. However, because of the dynamic routing strategy, when each node sends a packet, the node will locally calculate the coding opportunities to decide the next hop of that packet. Because packet transmission is the most frequent event, the computation at each node at high frequency is costly when conducting nexthop selection, which is especially true in dense wireless networks and heavy traffic scenarios. In addition, some of these schemes even require two-hop local view for information exchange, which results in even higher communication overhead.

Our proposed scheme for network coding over a wireless backbone can effectively overcome this issue. That is, it increases coding opportunity while not incurring any additional computational cost and communication overhead. To the best of our knowledge, this work is the first to propose using opportunistic network coding over a wireless backbone structure and to investigate the benefits of the scheme.

III. Network Coding over A Wireless Backbone

A. Coding Strategy

In earier work [5], we proposed a backbone construction scheme for heterogeneous wireless ad hoc networks. In our scheme, each BN has no more than three BN neighbors within its communication range, i.e., for each BN serving as a relay, it is part of either a chain topology or a 3-star topology, as shown in Fig. 2. In our scheme, we have assumed that the multi-channel technique [13] is employed at the BNs, and thus there is no interference between BNs and regular nodes in network. As a result, for the channel used for backbone communications, there are at most three neighbors for each BN. Therefore, the capacity of the backbone can be improved compared with traditional network routing schemes with a single-channel system.

Fig. 2 shows two examples of *coding decision* (i.e., deciding which packets are encoded together to broadcast) in a chain topology (two BN neighbors) and a 3-star topology (three BN neighbors) respectively. In Fig. 2(a), there are 5 packets P_1 through P_5 buffered in the output queue of a relay node n_R, where the next hop of P_1, P_2 is node A and the next hop of P_3, P_4, P_5 is node B. Because the relay packets P_1 through P_5 either come from the node to *left side* of n_R or the node to *right side* of n_R, the previous hop of P_1, P_2 must be the node B and the previous hop of P_3, P_4, P_5 must be the node A. Also, it is certain that a packet P_i must exist in a neighbor of n_i if the neighbor is the previous hop of that packet. Therefore, n_R can broadcast the packets $P_1 \oplus P_3$ and $P_2 \oplus P_4$ in two transmission slots, and each packet is decodable at its intended nexthop. Because there is high likelihood of coding opportunities in bi-directional traffic, the remaining packet P_5 is left for the next round to be encoded with another packet.

A 3-star topology (which is possible at the backbone structure) can be decomposed into three chain topologies, as shown in Fig. 2(b). This example has seven packets queued at node n_R, where the next hop of P_1, P_2 is node C, the next hop of P_3, P_4 is node B and the next hop of P_5, P_6, P_7 is node A. In addition, the symbol P_{1a} means the previous hop of P_1 is the node A, and so forth. For each decomposed chain, n_R just combines two packets moving in opposite directions, thus the encoded packet can be decoded at their intended nexthops. Note that the remaining packet P_{6b} is left for the next round to be encoded with another packet.

We can see that this network coding scheme do not need the *reception reports* (which is required by the COPE) to let a node know what packets the neighbors have, because this information can be extracted by checking the previous-hop field in the packet header. Therefore, it dramatically reduces the overhead for information synchronization. In addition, the route between any source-destination pair is unique and predetermined with a virtual-backbone routing approach. Effectively, this results in a chain topology over which bidirectional traffic passes through. This increases coding opportunities. In summary, our proposed network coding over a wireless backbone scheme increases the coding opportunity while reducing the computational cost and overhead for coding decisions.

B. Coding Opportunity

In this subsection, we present a theoretical analysis of coding opportunity for both COPE and our scheme. Assume a given network node n_R has $N \geq 2$ neighbors within its communication range, and q packets buffered in its output queue. Using COPE-type opportunistic coding, the node n_R may find one or more packets in $\{P_2, P_3, ..., P_q\}$ to encode with the head of queue, P_1, and then broadcast them collectively, with the guarantee that the combined packets are decodable at their intended nexthops. We define the coding opportunity as the following:

Definition 3.1: If there exists at least one packet in the output queue that can be encoded with P_1, the head of output queue, and the encoded packet can be decoded to the native packets at corresponding intended nexthops, then there is a coding opportunity at node n_R.

We denote E_r as the event that there is a coding opportunity at node n_R. Because our scheme of network coding over a backbone does not need any reception reports to know what packets its neighbors have, for fair comparison (with the same computational cost and communication overhead), we assume n_R does not have access to reception reports and thus only two flows with reverse directions can be encoded together for transmission. In our analysis, we consider the scenario of uniform unicast traffic distribution, and we assume all nodes are independently and randomly deployed. We denote the previous hop of P_1 as N_i and the next hop of P_1 as N_j, and denote C_m the event that packet P_m has the reverse direction of P_1, i.e., $prev\_hop(P_m) = N_j$, $next\_hop(P_m) = N_i$. In addition, we define $P(C_m|n = N)$ as the probability of event C_m given that there is N number of neighbors around a node n_R, and $P(E_r|n = N, q)$ as the probability of event E_r given that there are N number of neighbors around a node n_R and the length of its output queue is q. Because of the uniform traffic distribution among the nodes, we have $P(C_m|n = N) = \frac{1}{N(N-1)}$, and the probability of its complement is given by $P(\overline{C_m}|n = N) = 1 - \frac{1}{N(N-1)}$. Hence, for an output queue of length q, we have $P(\overline{E_r}|n = N, q) = [1 - \frac{1}{N(N-1)}]^{q-1}$, and then

$$P(E_r|n = N, q) = 1 - [1 - \frac{1}{N(N-1)}]^{q-1} \qquad (1)$$

Note that with the backbone structure, the number of BN neighbors is no more than three, i.e., $N \leq 3$.

Since the number of neighbors, N, is related to node density in a network without a backbone structure, next we investigate the coding opportunity with different node densities. Let ρ be the node density of the network, and R be the communication range of node n_R. We assume the nodes are deployed randomly according to Poisson point distribution, thus the probability of having n neighbors around n_R within a finite area (πR^2) is give by

$$P(n = N) = \frac{e^{-\rho\pi R^2}(\rho\pi R^2)^{(N+1)}}{(N+1)!} \qquad (2)$$

then we have the probability of coding opportunity as a function of ρ and R

$$\begin{aligned} P(E_r|n \geq 2, q) &= \sum_{N=2}^{\infty} P(E_r, n = N|n \geq 2, q) \\ &= \frac{2\sum_{N=2}^{\infty}(1 - (1 - \frac{1}{N(N-1)})^{q-1})\frac{e^{-\rho\pi R^2}(\rho\pi R^2)^{(N+1)}}{(N+1)!}}{2 - 2e^{-\rho\pi R^2}\rho\pi R^2(1 + \rho\pi R^2)} \end{aligned} \qquad (3)$$

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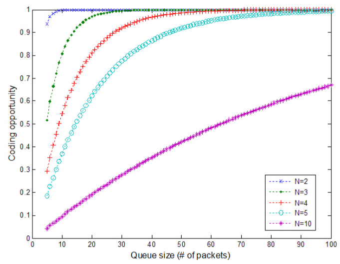

Fig. 3. Probability of coding opportunity as a function of queue size

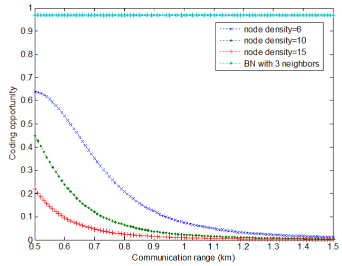

Fig. 4. Probability of coding opportunity as a function of communication range when $q = 20$

C. Numerical Results

Some numerical results are presented in Fig. 3, which is a plot of Eq. (1). We can see that the probability of coding opportunity increases with the output queue size. This indicates that there is a higher chance of coding opportunity in a network with heavy traffic. Thus network coding has more advantageous in heavy traffic scenarios, which is consistent with the intuition that network coding is not necessary in light traffic situations. Also, we can see that the probability of coding opportunity decreases with the number of neighbors. This demonstrates that chain topology (two neighbors) and 3-star topology (three neighbors) are more likely to lead to network coding opportunities, which is the case with our backbone topology presented in [5].

Fig. 4 shows the numerical results of Eq. (3) with $q = 20$. The results demonstrate that the probability of coding opportunity decreases with communication range. This is due to the fact that the number of neighbors increases with communication range. For fixed traffic intensity, the increase in number of nodes within communication range would decrease the coding opportunity. Whereas, for a backbone structure, the number of BN neighbors is not affected by the communication range. That is, the selected BNs within the communication range are always no more than four, because the number of BN neighbors is no more than three. Thus the probability of coding opportunity remains the same as communication range increases. In addition, we can see that for a network with no backbone structure, the probability of coding opportunity decreases with the node density.

IV. Link Scheduling and Observed Throughput

In this section, we investigate the scheduling issues of the backbone links and then derive an expression for *observed throughput* for relay nodes with an optimized link scheduling protocol. Here the term *observed throughput* means the amount of traffic passing through a specific relay node in a unit of time. In real-world, the theoretical coding gain 1.33 can only be achieved with optimal link scheduling. If the link scheduling is such that the transmitters always rotate as the following cycle: $A, C, B, ...$ (or $C, A, B, ...$), then node B can always encode two packets in each transmission and maximize the total throughput. However, if the link scheduling is $A, B, C, B, A, B, ...$, then node B cannot encode any packets. In practical situations, most of the wireless link scheduling

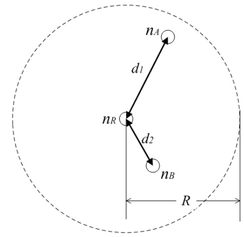

Fig. 5. An example for a BN (n_R) with two BN neighbors

if *the relay buffer of* n_R *is empty* **then**
  n_R pulls n_A to send a new packet $P_{(A->B)}$ to itself until the transmission is received successfully;
end
if *the relay buffer of* n_R *has a packet* $P_{(A->B)}$ *to be relayed to* n_B*, but has no packet* $P_{(B->A)}$ **then**
  n_R pulls n_B to send a new packet $P_{(B->A)}$ to itself until the transmission is received successfully;
end
if *the relay buffer of* n_R *has a packet* $P_{(B->A)}$ *to be relayed to* n_A*, but has no packet* $P_{(A->B)}$ **then**
  n_R pulls n_A to send a new packet $P_{(A->B)}$ to itself until the transmission is received successfully;
end
if *the relay buffer of* n_R *has both the packets* $P_{(A->B)}$ *and* $P_{(B->A)}$ **then**
  n_R broadcasts $P_{(A->B)} \oplus P_{(B->A)}$, which leads with the following probabilities to the situations listed:
  1) $(1 - p_{eA})(1 - p_{eB})$: both n_A and n_B receive the broadcast packet successfully, after which the relay buffer of n_R is empty;
  2) $p_{eB}(1 - p_{eA})$: n_A receives the broadcast packet successfully, but n_B does not, after which the relay buffer of n_R keeps $P_{(A->B)}$ for the next transmission;
  3) $p_{eA}(1 - p_{eB})$: n_B receives the broadcast packet successfully, but n_A does not, after which the relay buffer of n_R keeps $P_{(B->A)}$ for the next transmission;
  4) $p_{eA}p_{eB}$: neither n_A nor n_B receives the broadcast packet successfully, after which n_R rebroadcasts the combined packets in the next time slot.
end

Algorithm 1: A network coding aware link scheduling

protocols (such as a contention-based or random access link scheduling mechanism) are probabilistic in nature and not coding-oriented. Here we present a simple equal access protocol for backbone nodes in a chain topology that outperforms other sophisticated scheduling schemes by fully utilizing the potential coding opportunities. We then investigate the impact of lossy characteristics of a wireless channel on observed throughput of a relay node based on the proposed scheduling scheme.

Consider the general case where a BN (n_R) has two BN neighbors (n_A and n_B) and each one has packets that need to be relayed through n_R (i.e., n_R is an intermediate node to relay packets from n_A or n_B). The distance between n_A and n_R is d_1, and the distance between n_B and n_R is d_2, as shown in Fig. 5. This example also applies to the 3-star topology since a 3-star can be decomposed into three chain topologies, and the packets for exchanging between the two neighbors in a chain are a subset of the packets in the output queue based

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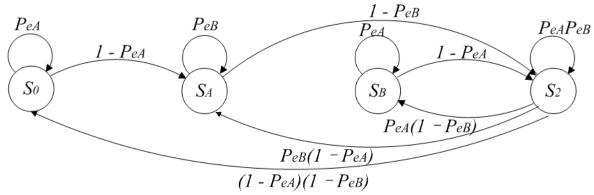

Fig. 6. Markov chain for state transitions of n_R

on their intended nexthops. Because the neighbors n_A, n_B usually have different distances from n_R ($d_1 \neq d_2$), when n_R broadcasts an encoded packet, n_A and n_B will receive that packet with different packet error rates (PER). Here we define the following terminology. p_{eA}: *PER of the link between* n_A *and* n_R; p_{eB}: *PER of the link between* n_B *and* n_R; pb_{eA}: *bit error rate (BER) of the link between* n_A *and* n_R; pb_{eB}: *bit error rate (BER) of the link between* n_B *and* n_R.

It is desirable that the relay node n_R always send XOR-ed packets in each transmission slot to fully utilize the broadcast nature of wireless medium. Here we present a simple pull-based link scheduling scheme in which n_R acts as a coordinator and pulls data from selected neighbors. This protocol is assumed to be used in heavy traffic scenarios, where the neighbor nodes n_A, n_B always have generated packets that need to be relayed by n_R. The protocol is described in Algorithm 1. The term *relay buffer* means a buffer space especially reserved for relay packets (i.e., n_R is neither the source nor the destination of the packets), the symbol $P_{(A->B)}$ denotes a packet with the direction from n_A to n_B. In this scheduling scheme, n_R coordinates its neighbors to satisfy the requirements of XOR-coding at each time slot. In this protocol, node n_R has four possible states, S_0: the relay buffer of n_R is empty; S_A: a packet $P_{A->B}$ is in n_R; S_B: a packet $P_{B->A}$ is in n_R; S_2: both packets $P_{A->B}$ and $P_{B->A}$ are in n_R. We can analyze the state transitions of n_R through the Markov chain shown in Fig. 6. Let $P(S_0)$, $P(S_A)$, $P(S_B)$ and $P(S_2)$ be the steady-state probabilities that n_R is in the states S_0, S_A, S_B and S_2 at a given time slot, respectively. Then we have

$$\begin{cases} P(S_0) = p_{eA}P(S_0) + (1-p_{eA})(1-p_{eB})P(S_2) \\ P(S_A) = p_{eB}P(S_A) + (1-p_{eA})P(S_0) \\ \qquad\quad +p_{eB}(1-p_{eA})P(S_2) \\ P(S_B) = p_{eA}P(S_B) + p_{eA}(1-p_{eB})P(S_2) \\ P(S_0) + P(S_A) + P(S_B) + P(S_2) = 1 \end{cases} \tag{4}$$

In each time slot, only when n_R is in the state S_2, it contributes to the observed throughput. Therefore the observed throughput is proportional to $P(S_2)$. From Eq. (4) we have

$$P(S_2) = \frac{1}{3 + (P_{eA} - P_{eB})^2/(1-P_{eA})(1-P_{eB})} \tag{5}$$

Here we assume that the duration of a time slot is T_0 seconds, and n_R can broadcast a packet of size L bits in a single transmission; then we define R_0 as $R_0 = \frac{L}{T_0}$. In a given time slot, if n_R is in state S_2 and the encoded packet $P_{A->B} \oplus P_{B->A}$ has been successfully received by both n_A and n_B, the achieved throughput in this time slot is $\frac{2L}{T_0} = 2R_0$. If either n_A or n_B receives the encoded packet successfully, the throughput in this slot is R_0. Therefore, we obtain the overall throughput with network coding as:

$$\begin{aligned} R_{nc} &= P(S_2)(p_{eA}(1-p_{eB}) + p_{eB}(1-p_{eA}) \\ &\quad + 2(1-p_{eA})(1-p_{eB}))R_0 \\ &= \frac{(2-p_{eA}-p_{eB})R_0}{3 + (P_{eA}-P_{eB})^2/(1-P_{eA})(1-P_{eB})} \end{aligned} \tag{6}$$

We then compute the throughput with a traditional transmission scheme for the example of Fig. 5, i.e., the intermediate node n_R just performs relay-and-forward for passing through packets. With a traditional transmission scheme, the node n_R will pull one of its neighbors (such as n_A) to send (or resend) the packet $P_{A->B}$ until n_R receives it correctly. Then n_R sends (or resends) the packet $P_{A->B}$ to n_B until n_B receives it successfully. The transmission from n_A to n_B takes $\frac{T_0}{1-p_{eA}} + \frac{T_0}{1-p_{eB}}$ seconds on average. This result is also applicable to the $P_{B->A}$ packet in the reverse direction of the same link. Thus, the observed throughput with a traditional transmission scheme is given by

$$R_{tr} = L/[\frac{T_0}{1-p_{eA}} + \frac{T_0}{1-p_{eB}}] = \frac{R_0(1-p_{eA})(1-p_{eB})}{(2-p_{eA}-p_{eB})} \tag{7}$$

and then the coding gain can be calculated by

$$C_{gain} = \frac{R_{nc}}{R_{tr}} = \frac{(2-p_{eA}-p_{eB})^2}{3(1-p_{eA})(1-p_{eB}) + (P_{eA}-P_{eB})^2} \tag{8}$$

We can see that when $p_{eA} = p_{eB} = p_e$, $R_{nc} = \frac{2}{3}(1-p_e)R_0$ and $R_{tr} = \frac{(1-p_e)R_0}{2}$, which confirms the previous analysis that the coding gain (R_{nc}/R_{tr}) is 1.33. In addition, we conclude that C_{gain} is always larger than 1 because $(2-p_{eA}-p_{eB})^2 - 3(1-p_{eA})(1-p_{eB}) - (p_{eA}-p_{eB})^2 = (1-p_{eA})(1-p_{eB})$, which is always larger than 0.

In this paper we model the lossy characteristics of wireless channel by the results in [14], where the authors derived the successful reception probability (SRP) as a function of distance (x) between a transmitter and a receiver under the log normal shadow fading model. The successful reception probability $SRP(x)$ can be approximated by

$$SRP(x) = \begin{cases} 1 - ((\frac{x}{K})^{2\beta})/2, & \text{if } \; x \leq K \\ ((\frac{2K-x}{K})^{2\beta})/2, & \text{if } \; K < x \leq 2K \\ 0, & \text{if } \; x > 2K \end{cases} \tag{9}$$

where K is the distance such that $SRP(K) = 0.5$, and β is the power attenuation factor ranging from 2 to 6. We define the communication range R as the following: within distance R from n_R, the $SRP(d)$ is larger than a threshold p_0. Because $SRP(R)$ (i.e., p_0) should be larger than 0.5 (i.e., $R < K$), for a fixed R, we have

$$K = \frac{R}{\sqrt[2\beta]{2(1-p_0)}} \tag{10}$$

In our analysis we assume that there is no error correction coding scheme employed. That is, if one of the bits in a packet is transmitted in error, the whole transmission is regarded as a failure. For a packet with L bits, we have $p_{eA} = 1 - (1 -$

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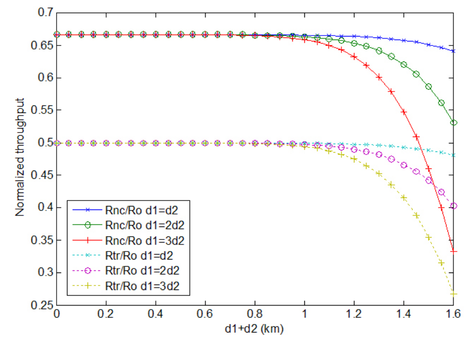

Fig. 7. Normalized observed throughput as a function of $d_1 + d_2$

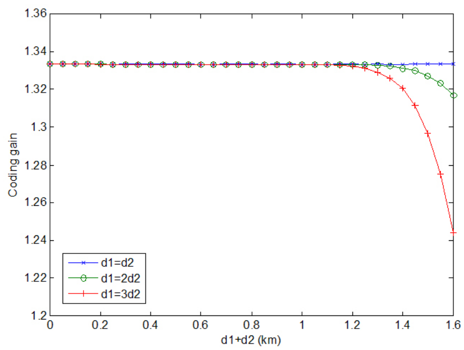

Fig. 8. Coding gain as a function of $d_1 + d_2$

$pb_{eA})^L$ and $p_{eB} = 1-(1-pb_{eB})^L$ where $pb_{eA} = ((\frac{d_1}{K})^{2\beta})/2$, and $pb_{eB} = ((\frac{d_2}{K})^{2\beta})/2$ based on Eq. (9).

Fig. 7 shows the results of the normalized observed throughput R_{nc}/R_0 and R_{tr}/R_0 as a function of $d_1 + d_2$. Here we assume the communication range (R) of n_R is 1.2 *Km*, $p_0 = 0.99$, L = 100 (*bits*) and $\beta = 4$. We also examine the impact of the asymmetry of the links $n_A \leftrightarrow n_R$ and $n_B \leftrightarrow n_R$ on the observed throughput by changing the ratio d_1/d_2. We can see that the observed throughput with network coding is always larger than that of a traditional transmission scheme (i.e., C_{gain} is always larger than 1), and when the two links are symmetric (i.e., $d_1 = d_2$), the system achieves the best observed throughput. In addition, the observed throughput is decreasing with the increasing of d_1+d_2, and with larger ratio d_1/d_2. In Fig. 8, we plot the results of Eq. (8) as a function of d_1+d_2. We can see that with asymmetric links (i.e., $d_1 \neq d_2$), the coding gain decreases with the increasing of d_1+d_2, which demonstrates that the asymmetry of lossy wireless links has significant impact on coding gain when d_1+d_2 is large enough. We can also see that in case of symmetric links, the coding gain always achieves the theoretical bound of 1.33.

V. Conclusions

In this paper, we have proposed a new paradigm for network coding over a wireless backbone and studied the benefits of the scheme. In our scheme, we construct a wireless backbone that facilitates end-to-end communications, and then employ an XOR-based network coding scheme over the backbone to further enhance the network efficiency and capacity. Because the wireless backbone provides bi-directional chain topology, which is particularly suitable for XOR-based network coding, the coding opportunity can be significantly improved compared with a COPE-type scheme over multi-hop wireless networks. In addition, our scheme does not introduce any additional computational cost and communication overhead compared with other coding-aware routing schemes. The theoretical analysis of coding opportunity in COPE is also presented and compared with our scheme. Furthermore, to fully utilize the coding gain, a simple optimized link scheduling protocol is presented and the observed throughput for a relay node is derived. The results demonstrate that with backbone routing and an optimized link scheduling scheme, network coding can effectively enhance the observed throughput for relaying nodes, and the coding gain achieves the theoretic bound 1.33 in symmetric ($d_1 = d_2$) or error-free links ($p_{eA} = p_{eB} = 0$). In addition, we find that the asymmetry of lossy wireless links has significant impact on network performance when the sum of d_1 and d_2 is large enough.

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