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Statistical analysis of short term fading and shadowing in Ultra-Wideband systems

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***Abstract*—Statistical analysis of Ultra-Wideband (UWB) signals is undertaken to examine the existence of short term fading and shadowing. Based on the data collected from multiple locations and 4 different sites using a single transmitter and a 96-element receiving antenna in the 2-8 GHz range, hypothesis testing was performed to explore the statistical fit to composite probability density functions such the double Rayleigh, double Nakagami and the K distribution as well as the Nakagami and lognormal. The tests clearly showed that excellent match existed with the K distribution, with the Nakagami distribution being the next best. These results point to the existence of shadowing besides the short fading reported by other researchers. They can also assist in the development of fading and shadowing mitigation techniques.**

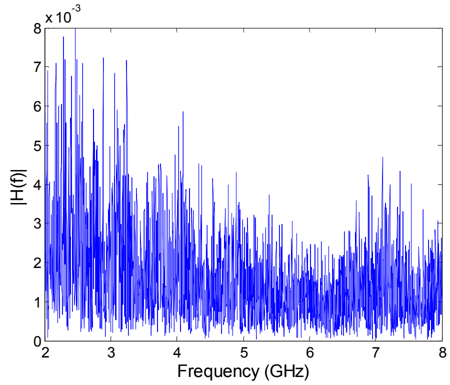
***Index Terms*—UWB. Wireless systems. Short term fading. Shadowing. K distribution. Chi-square tests**

I. Introduction

Ultra-Wideband (UWB) offers a great potential for high speed data communications in many indoor facilities such as the military, industrial, and medical establishments [1] - [8]. The very high bandwidth of the UWB signals (~500 MHz or more) makes them less susceptible to interference from other existing systems along with providing protection from frequency selective fading, often seen in traditional outdoor wireless systems employing low bandwidth systems. Another benefit of high bandwidth transmission is the increased difficulties in intercepting them.

While UWB systems offer an improved means for data transmission, their statistical behavior is not very well understood [1] - [5]. This is primarily due to the fact that the signal propagation takes place in a variety of indoor environments such as buildings and structures made up of very dissimilar elements ranging from natural materials such as wood, brick, stucco, glass, metal, etc. to synthetic materials such as those found in doors, ceilings, wall partitions, etc., making the characteristics of the signals more complex than that of typical narrowband wireless signals propagating outdoors. The modeling of transmission losses is also complicated by the environment consisting of media of differing properties [4], [8]. Another problem that inhibits understanding the statistics of the UWB signals is the limited ensemble data available for the analyses [2], [5], [6]. The ensemble data set essential to the understanding of statistics is provided by the multiple locations of the receiving antennas or spatial diversity provided by a multi-element antenna. Results reported include those based on sets of sizes 16-49, limiting the universal applicability of such analyses. Data size might also be inadequate to incorporate shadowing along with short term fading leading to statistics that seem to point to Rayleigh, Nakagami, Weibull, lognormal [2], [5], [6]. Another problem with some of the existing results and the interpretations and their applicability to a general UWB propagation is also a direct consequence of the limited separation (8-10 meters) between the transmitter and receiver.

Recent results reported by NIST scientists offer an excellent means to understand the statistics [7], [8]. The data sets were collected using a 96 element receiving antenna with transmission distances of up to 45 m and different directions, varying number of walls, a very diverse set of building materials, doors, walls, etc., making the data sufficiently useful for the statistical analysis. The frequency range of the data was 2-8 GHz. In this work, these data sets were analyzed to test several statistical models for the received signal power. These models included those for pure short term fading such as the Nakagami and those ideal for pure shadowing (lognormal) along with models which could describe shadowed fading channels (double Rayleigh, double Nakagami and the Rayleigh-Nakagami, or more commonly known as the K distribution) [9] - [14]. Note all these three density functions are different versions of the generalized K



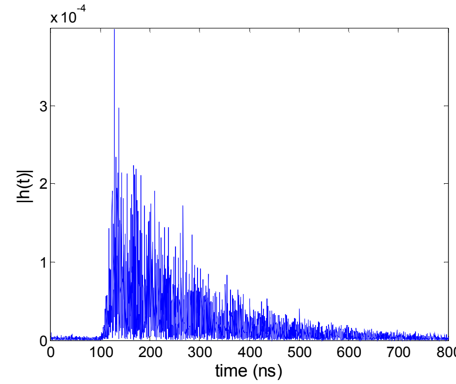


Figure 1 The frequency response of one of the antenna elements is shown on the left while the corresponding time delay profile (impulse response) is on the right.

(GK) distribution used for the modeling of shadowed fading channels [9], [10].

The rest of the manuscript is organized into three sections. The next section provides an overview of the procedures used for the data collection. This is followed by a description of the statistical models, model parameter estimation, and testing. The final section deals with the results and discussion, and potential applications of the results.

II. Measurement Setup and Data Collection

The spatial-temporal response measurements were undertaken using a uniform circular array antenna of radius 24 cm with 96 elements, providing spatial diversity. The frequency was swept from 2 to 8 GHz in steps of 1.25 MHz which resulted in a time delay spread of 800 ns. The complex frequency response was stored for each of the 96 elements [7], [8].

The data sets were collected in 4 buildings (Plant, Sound, NIST_North, and Child Care). The material properties of these four buildings are given in Table 1. They clearly illustrate the diverse nature of the environments in which propagation takes place. In each of these buildings, 50 sets of data were collected for different transmitter (TX) - receiver (RX) separation, number of walls in between and the angle between RX and TX. Additional details on measurements along with data collection procedures are available at http://www-x.antd.nist.gov/uwb. All the measurements were done at high signal-to-noise ratio (SNR) with the worst of them being 26 dB (Sound). The dynamic range was 140 dB, allowing for very reliable results. The measurements, which were taken during after hours to ensure channel invariance between spatial points on the array, provided a total set of 4x50x96 data, with each of these sets consisting of 4801 samples (complex frequency response).

Table 1 The material properties of the building/walls of the four sites. The diversity of the materials used for construction of the building can be seen.

Site	**wall material**
Plant	steel
Sound	cinder block
NIST_North	sheet rock / aluminum studs
Child Care	plaster / wooden studs

Figure 1 shows the frequency response and the corresponding temporal response from one of the receiving antenna elements. The data set was from the building (Plant) with steel walls. The RX-TX separation was 20.58 m and there were 3 walls in the path.

The impulse response for all the sites and antenna elements were calculated and stored for the statistical analyses. The different statistical distributions and hypothesis testing methodology are described in the next section.

III. Statistical Models and Hypothesis Testing

Statistics of short term fading in wireless systems is reasonably well understood and Nakagami distribution is one of the best ways to model the envelope of the received signal [14]. However, wireless channels are subject to short term fading and shadowing simultaneously. Such shadowed fading channels are modeled using the Nakagami-lognormal distribution. Since the Nakagami-lognormal probability density function (pdf) does not have a closed form solution [14], use of that distribution poses difficulties in the analyses of wireless system. It was shown that the generalized K (GK) distribution provides a simple analytical means to model the shadowed fading channels [9], [10], in particular

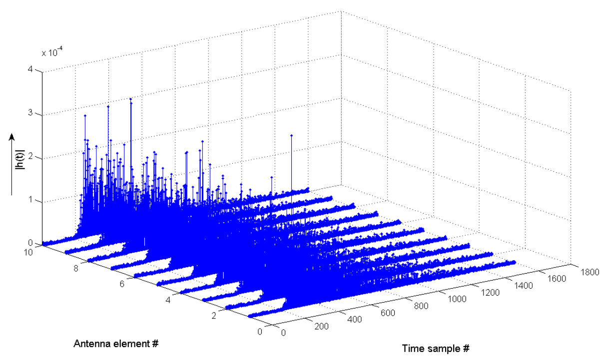


Figure 2 The impulse responses of ten antenna elements (of the total of 96) from one set of measurements are shown. The X axis (delay) is indicated in sample # from 1 to 1601 showing that every 3rd sample will be used for the statistical analyses.

cluttered scattering created by multiple walls/structures between the TX and RX in environments typical to UWB operation. The pdf of the signal power Z in such a channel is given by

$$f_{GK}(z)=\frac{2}{\Gamma(m)\Gamma(c)}\left(\frac{b}{2}\right)^{c+m} z^{\left(\frac{c+m}{2}\right)-1} K_{c-m}\left(b\sqrt{z}\right) \quad (1)$$

where $K_{c-m}(.)$ is the modified Bessel function of order (c-m) and $\Gamma(.)$ is the gamma function [15]. The subscript GK identifies the generalized K pdf. The parameter b is related to the average signal power Z_0 through

$$Z_0=\langle Z\rangle_{GK}=\frac{4mc}{b^2}. \quad (2)$$

The other two parameters m and c characterize the short term fading and shadowing respectively. While m is the Nakagami parameter, c is related to the standard deviation σ dB of shadowing, generally modeled using the lognormal pdf through the following equation [9],

$$\sigma=4.34\sqrt{\psi'(c)}\,dB. \quad (3).$$

In (3), ψ′ is the trigamma function [15]. The cumulative distribution function (CDF) associated with the pdf in (1) is given as

$$F_{GK}(z)=\frac{\Gamma(m\text{-}c)}{\Gamma(m)\Gamma(c+1)}\,{}_1F_2\left(c,[1\text{-}m+c,1+c],\frac{zb^2}{4}\right)\left(\frac{zb^2}{4}\right)^c+\frac{\Gamma(c\text{-}m)}{\Gamma(m+1)\Gamma(c)}\,{}_1F_2\left(m,[1\text{-}c+m,1+m],\frac{zb^2}{4}\right)\left(\frac{zb^2}{4}\right)^m \quad (4)$$

where ${}_1F_2$ is the hypergeometric function [15]. For shadowing levels of σ ranging from 2-9 dB, it was shown that (1) provides an excellent means to study wireless channels when both short term fading and shadowing are simultaneously present [9]. When c goes to ∞ (no shadowing), (1) becomes the pdf of the received power in pure short term fading channels, described using the Nakagami model as [14]

$$f_N(z)=\frac{z^{m-1}}{W^m\,\Gamma(m)}e^{-\frac{z}{W}}. \quad (5)$$

The subscript N identifies the pdf as the density function of the power in a Nakagami fading channel. In (5), W is related to the average power [14]. It is now possible to consider several special cases of the GK distribution which can also be seen as arising from the product of two gamma random variables of orders m and c [9],[10]. The first is the case of double Rayleigh which has been suggested as a model to describe the statistics of the signal power in wireless channels [11]. By putting m = c =1 in (1), the pdf of the signal power Z becomes

$$f_{dR}(z)=\frac{2}{\beta}K_0\left(2\sqrt{\frac{z}{\beta}}\right) \quad (6)$$

and the associated CDF is given as

$$F_{dR}(z)=\left[1-2\sqrt{\frac{z}{\beta}}K_1\left(2\sqrt{\frac{z}{\beta}}\right)\right]. \quad (7).$$

The subscript dR identifies (6) as the double Rayleigh pdf and β is the average signal power. Note that $K_0(.)$ and $K_1(.)$ are the modified Bessel functions of order 0 and 1 respectively. Another possibility of modeling the received power is through the use of the double Nakagami pdf [12]. In this case m = c and (1) now becomes (choosing M instead of m to distinguish the value of m from the Nakagami pdf in eqn. (5))

$$f_{dN}(z) = \frac{2}{\left[\Gamma(M)\right]^2}\left(\frac{a}{2}\right)^{2M} z^{M-1} K_0\left(a\sqrt{z}\right) \tag{8}$$

and the associated CDF is given by

$$F_{dN}(z) = \frac{MeijerG\left(\left[[1],[\ ]\right],\left[[M,M],[0]\right],\frac{za^2}{4}\right)}{\left[\Gamma(M)\right]^2} \tag{9}$$

Note that CDF in (9) is expressed in terms of the *MeijerG* function [15] and the subscript dN identifies (8) as the double Nakagami pdf. The average power is given by

$$\langle Z\rangle_{dN} = \frac{4M^2}{a^2}. \tag{10}$$

A slightly different pdf is obtained by choosing m = 1 (corresponds to Rayleigh pdf) in eqn. (1) resulting in the so called K distribution for shadowed fading channels as [13]

$$f_{RN}(z) = \frac{2}{\Gamma(c)}\left(\frac{\alpha}{2}\right)^{c+1} z^{\left(\frac{c-1}{2}\right)} K_{c\text{-}1}\left(\alpha\sqrt{z}\right) \tag{11}$$

and the associated CDF is given as

$$F_{RN}(z) = 1 - \frac{2}{\Gamma(c)}\left(\frac{\alpha\sqrt{z}}{2}\right)^{c} K_c\left(\alpha\sqrt{z}\right) \tag{12}$$

where the subscript RN identifies the Rayleigh-Nakagami pdf (also known as the K distribution) [13, 16, 17,18]. The average power is given by

$$\langle Z\rangle_{RN} = \frac{4c}{\alpha^2}. \tag{13.}$$

The lognormal pdf has been used to model shadowing in wireless channels and the associated pdf is given by [14]

$$f_{LN}(z) = \frac{4.34}{\sqrt{2\pi\sigma^2 z^2}} e^{-\frac{\left[10\log_{10}(z)-\mu\right]^2}{2\sigma^2}} \tag{14}$$

where μ is the average power in decibels (dB) and σ is the standard deviation of shadowing (dB) as expressed in (3) above. The subscript LN identifies (14) as the lognormal pdf.

The impulse response data sets were used to perform the statistical tests. For each location, every time delay instant provided 96 samples for the statistical testing, obtained from the 96 elements of the uniform circular antenna. Instead of performing the statistical analyses at each of the 4801 delay instants, every 3rd delay instant was used resulting in 1601 time delays for each impulse response. Choosing the 3rd sample in the delay was considered as a means to reduce the overall computational time and it could have been possible to take all the 4801 delays if necessary. Note that no time averaging was done. A typical set of data from 10 antenna elements (of the 96 elements) is plotted in Figure 2 and one can see that at each of the 1601 time delay instants will provide 96 ensemble values for statistical testing. These impulse responses plotted here belong to the same location as in Figure 1.

Two different statistical tests can be used for hypothesis testing, namely the K-S test and the chi-square (χ^2) test [19]. Since the latter can also account for an unknown number of parameters, the chi-square test is better suited to validate the hypothesis that the pdf of the data matched a chosen distribution from the ones indicated earlier: of all the distributions, the GK distribution requires the estimation of three parameters leading to complicated means to solve for parameters while most of the remaining ones require the estimation of two parameters. The double Rayleigh distribution in eqn. (6) only requires the estimation of a single parameter. Since all the distributions except the lognormal pdf are special cases of the GK distribution and they all required either two parameters or a single parameter to be estimated; these distributions, namely, the Nakagami, double Rayleigh, double Nakagami, K were chosen along with the lognormal distribution for hypothesis testing. All the processing and computations were undertaken using MATLAB (version R 2007b, MathWorks, Natick, MA, USA).

The parameters of the Nakagami distribution, namely m and W of (5) can be readily estimated using *gamfit* from MATLAB since (5) is also the gamma density. The parameter β of the double Rayleigh pdf in (6) is also easily estimated since it is the average power. The two parameters of the double Nakagami pdf were evaluated by first performing a maximum likelihood estimation technique [17] – [19] and solving for the parameters using (10) and

Table 2 The results of the hypothesis tests are given. The numbers shown are the success of the respective hypothesis (pdf) averaged over the 50 locations of the four sites. Note that the successes were counted out of a total of 1601. The results show that the K distribution is the best fit (Nakagami being the second best) regardless of the environment suggesting that the channel exhibits weak shadowing along with short term fading.

	Hypothesis pass				
Site	K	Nakagami	D. Rayleigh	D. Nakagami	Lognormal
Plant	1335	1271	1259	1066	34
Sound	1297	1264	1210	1030	30
NIST_North	1316	1276	1233	1057	29
Childcare	1303	1259	1211	1030	30

$$2\log_e\left(\frac{a}{2}\right)+\langle\log_e Z\rangle=2\psi(M) \tag{15}$$

where <.> is the average and $\psi(.)$ is the digamma function [15]. Replacing a in (15) using (10), the resulting transcendental equation can easily be solved using the *optimization* toolbox in MATLAB. The parameters of the K distribution can obtained from the solution of the transcendental equation

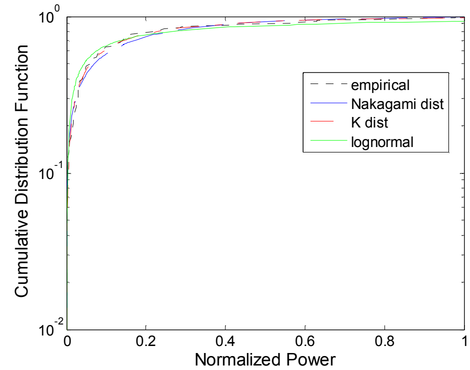
$$\log_e(c)-\psi(c)=\log\left[\langle Z\rangle\right]-\langle\log_e(Z)\rangle+\psi(1) \tag{16}$$

and (13) [16]-[18]. The parameters of the lognormal pdf were obtained using *lognfit* from MATLAB.

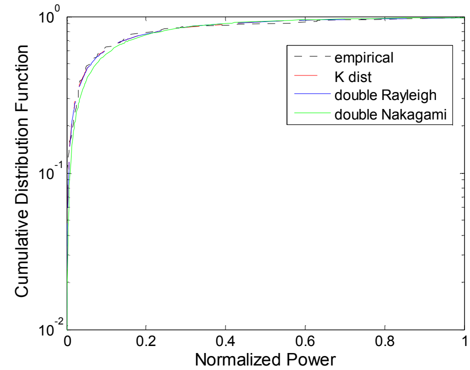
With the availability of the parameters of the five density functions (Nakagami, double Rayleigh, double Nakagami, K and lognormal) and the associated CDFs, χ^2 tests [19] were performed for all 1601 delays for all the sites (10 bins, 95% percentile). The CDFs of the Nakagami and lognormal random variables were directly available through MATLAB while the CDFs of other random variables were estimated using (7), (9) and (12). Note that there were 4 separate sites and each site had 50 different locations, each location with distinct values of the RX-TX separation, number of walls between RX and TX and the angle between RX and TX [7], [8]. For each location, the success of the test was measured in terms of the number of time delays at which the χ^2 test statistic values were less than the threshold values [19], which were the same for all density functions except for the double Rayleigh which only required the estimation of one parameter. A very high number (close to or equal to 1601) indicates acceptance of that hypothesis (pdf). For every site, the success of the hypothesis was estimated by averaging the values over the 50 locations. The results are shown in Table 2. It is seen that the K distribution (Rayleigh-Nakagami) of (11) provides the maximum number of matches with the Nakagami distribution coming close to it. The double Rayleigh and double Nakagami also appear to provide a reasonable degree of success while the lognormal pdf does not seem to be fit at all. Figure 3 shows the plots of the CDFs for one location. The plots seem to reflect the results of the χ^2 tests listed in Table 2.

IV. RESULTS AND DISCUSSION

A comprehensive study of the statistical characteristics of short term fading and shadowing in UWB systems was undertaken. The results covered measurements taken at several locations with different types of environments. The shadowed fading characteristics were modeled using the special cases of the generalized K (GK) distribution, namely, the double Rayleigh, double Nakagami and the K distribution. The hypothesis testing was also performed by including the Nakagami pdf (pure short term fading) and lognormal pdf (pure shadowing). The availability of 96 ensemble values provided a firmer basis for the modeling of the UWB channels compared to some of the previously published results [2]-[6]. The results obtained here support some of the conclusions of the previously published research where it was argued that the Nakagami pdf (or gamma pdf for the power) was a reasonable fit. The present analysis shows that the composite pdf obtained through the K distribution of (11), which incorporates short term fading and shadowing, is the best fit regardless of the geometry of the RX-TX locations and the environmental conditions (type of materials used in the building for walls, number of walls, etc.). The closeness of the fit of the K and Nakagami (Nakagami being the second best fit) suggests that the shadowing is present, but, the level of shadowing is weaker than what is typically observed in outdoor cellular wireless systems with coverage over larger distances (a few kilometers) and diverse topology. This is also seen from the poor match to the lognormal pdf which is the ideal fit for shadowing channels. It is worth mentioning that the gamma density function (pdf of the power in



Empirical data, Nakagami, K and lognormal.



Empirical data, K, double Rayleigh and double Nakagami.

Figure 3 The cumulative distribution of the data is fitted against the estimated CDFs for one of the antenna elements. The excellent fits of the K and Nakagami pdf's are seen.

Nakagami fading) has been shown to approximate the K distribution, and, some of the techniques used for the estimation of the parameters of the K distribution take advantage of such an association [16]-[18]. Thus, one can also interpret the results in this work, namely the K distribution being the best fit and the Nakagami being the close second in terms of the fit, due to the close relationship between these two distributions. The availability of a more exact model for the statistics of the UWB channel will make it easy to analyze the performance of the system and help formulate ways to mitigate the short term fading and shadowing present in UWB channels.

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