Generation of GKP states with optical states

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Abstract. Classical information can be carried by either a discrete signal or by a continuous signal. Quantum information can also be carried by a discrete, finite-dimensional system, such as a two-level atom or an electron spin, or by continuous, infinite-dimensional system, such as a harmonic oscillator. In Gottesman *et al.* [1] a qubit is encoded in the continuous position and momentum degrees of freedom of an oscillator (Gottesman - Kitaev - Preskill or GKP qubit states). One likely realization of GKP states is in the quadrature variables of traveling light waves. Quantum computation can be performed on GKP states using relatively simple linear optical devices, squeezing, and homodyne detection. However, the initial GKP states are extremely difficult to prepare. Here we propose the generation of an approximate GKP state by using superpositions of optical coherent states (sometimes called "Schrödinger cat states"), linear optical devices, squeezing, and homodyne detection. We initially consider two optical modes containing Schrödinger cat states. A displacement followed by a squeezing is applied to both modes and then the two modes are sent into a beam splitter. The action of the beam splitter entangles the two modes. An approximate GKP state is obtained when we perform a measurement of the p-quadrature in one of the beam splitter output modes.

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Many quantum computation schemes propose encoding qubits using natural two level systems such as spin 1/2 particles. Others exploit only two states of a larger discrete Hilbert space such as the energy excitation levels of an ion. An alternative method for encoding a qubit in the continuous position and momentum degrees of freedom of an oscillator is proposed in Gottesman *et al.* [1]. One of the advantages of this proposed scheme is the possibility to perform error correction to repair shifts on the optical variables [2].

The ideal GKP qubit state with logical value of zero has an x-quadrature wave function that is an infinite series of delta function peaks occurring whenever $x = 2\sqrt{\pi}s$ for all integers s. Because these states are unphysical, GKP described approximate states whose x-quadrature is a series of Gaussian peaks with width Δ contained in a larger Gaussian envelope of width $1/k$:

$$G(x) = N \sum_{s=-\infty}^{\infty} e^{-\frac{1}{2}(2sk\sqrt{\pi})^2} e^{-\frac{1}{2}\left(\frac{x-2s\sqrt{\pi}}{\Delta}\right)^2} \tag{1}$$

where N is a normalization factor. We plot the x-quadrature probability distribution in figure 1 for the case $\Delta = k = 1/4$. To closely approximate the ideal GKP states and avoid errors in quantum computations we desire Δ and k to be small, so there are many sharp Gaussian peaks contained in a wide envelope.

The GKP states are extremely difficult to prepare. In [1] GKP proposed preparing these states by coupling an optical mode to an oscillating mirror. Travaglione and Milburn [3] describe a method that prepares the qubit states in the oscillatory motion of a trapped ion rather than the photons in an optical mode. Pirandola *et al.* [4] discusses the preparation of optical GKP states using a two mode Kerr interaction followed by a homodyne measurement of one of the modes. The same authors also describe a fourth method for GKP state production in [5, 6]. No experiments have yet demonstrated preparation of GKP states.

Here we propose the generation of an approximate GKP state by using superpositions of optical coherent states ("cat state"), linear optical devices, squeezing, and homodyne detection. The basic recipe is first prepare two cat states (each of which contains two Gaussian peaks in its x-quadrature wave function), squeeze both cats (to reduce the width of the Gaussian peaks), interfere them at a beam splitter, then perform homodyne detection on one of the beam splitter's output ports. Depending on the measurement result, we will find an approximate GKP state (with three Gaussian peaks) in the beam splitter's other output port.

A superposition of coherent states can be written as:

$$|\psi(\alpha)\rangle = \frac{|-\alpha\rangle + |\alpha\rangle}{\sqrt{2(1+e^{-2\alpha^2})}} \tag{2}$$

where α is the amplitude of the coherent state. A

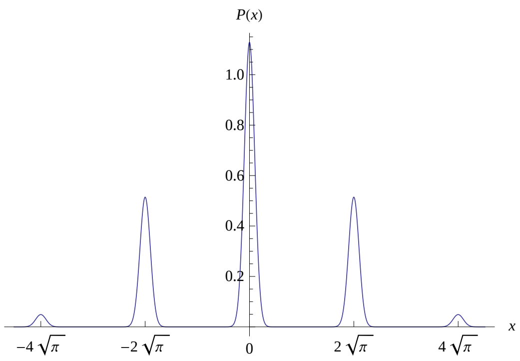

FIGURE 1. Approximate GKP state x-quadrature probability distribution for $\Delta = k = 1/4$.

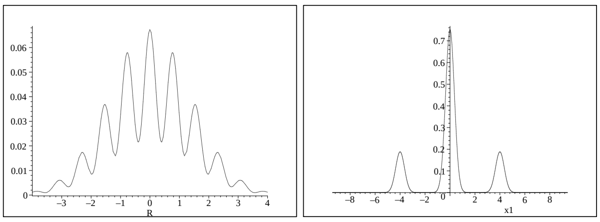

FIGURE 2. Left: Probability for measuring $p_2 = R$ as function of R. Right: Square of the wave function as a function of x_1 for the state of mode 1 at $R = 0$. In both cases $\alpha_1 = \alpha_2 = 4$, $\zeta_1 = \zeta_2 = \ln 4$ and transmissivity $\eta = 0.5$.

Schrödinger cat state such as the one in equation (2) can be created using several techniques [7], including a scheme that sends traveling optical modes through a Kerr medium [8]. In the x-quadrature basis, we can write the cat state in equation (2) as:

$$\psi_\alpha(x) = \frac{e^{-\frac{1}{2}(x+\sqrt{2}\alpha)^2} + e^{-\frac{1}{2}(x-\sqrt{2}\alpha)^2}}{\sqrt{2\sqrt{\pi}(1+e^{-2\alpha^2})}} \qquad (3)$$

The initial state of the system has two modes, both of which have cats. We may write the initial state as

$$A(x_1, x_2) = \psi_{\alpha_1}(x_1) \times \psi_{\alpha_2}(x_2) \qquad (4)$$

This initial state must be displaced by α_1 and $-\alpha_2$, in modes 1 and 2, respectively. Mode 1 is then squeezed by $S(\zeta_1)$ and mode 2 by $S(\zeta_2)$. The two modes are now sent into a beam splitter with transmissivity η. Finally, we want to measure mode 2 in the p basis, and for that we do a Fourier transform on mode 2, project the state of the system onto $p_2 = R$, and then discard the second mode.

We show our results in figure 2. At left, we plot the probability for measuring a certain value R as a function of R. The maximum value of this function is obtained for $R = 0$. If we measure $p_2 = 0$, we obtain a state with three Gaussian peaks, figure 2 at right, whose widths are determined by the degree of squeezing applied to the initial cat states and whose heights are proportional to the third row of Pascal's triangle (1,2,1). This is similar to the GKP state in figure 1, but we would like to create states with more peaks in the x-quadrature. This can be achieved by using the above procedure iteratively. If we produce two copies of the state, then they can be combined to create a state with 4 peaks, whose heights are proportional to the fourth row of Pascal's triangle (1,3,3,1). As the number of iterations increases, we approach a wave function of Gaussian peaks contained in a Gaussian envelope, as originally proposed by GKP.

As described above, the success of the procedure requires measuring $p_2 = 0$ in the homodyne detection step. In a future work we will investigate how the fidelity of our GKP states depends on this measurement result. Also we will investigate the possibility of using linear optical operations to salvage cases in which $p_2 \neq 0$.

Although our scheme is built of apparently simple, well understood linear optical operations, it will certainly be very difficult to achieve in an experiment. We expect the greatest difficulty will be matching the transverse and longitudinal shapes of all of the optical modes especially during the squeezing stage [9, 10].

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