

## Collisional cooling of ultracold-atom ensembles using Feshbach resonances

L. Mathey, Eite Tiesinga, Paul S. Julienne, and Charles W. Clark

*Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland,  
Gaithersburg, Maryland 20899, USA*

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We propose a cooling mechanism for ultracold fermionic atom ensembles, which capitalizes on the energy dependence of inelastic collisions near a Feshbach resonance. We first discuss a single magnetic resonance and find that the final temperature and cooling rate is limited by the resonance width. A concrete example for a  $p$ -wave resonance of  $^{40}\text{K}$  is given. We then improve upon this setup by using both a sharp optical or radio-frequency-induced resonance and a broad magnetic resonance and show that one can improve upon temperatures reached with current technologies.

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### I. INTRODUCTION

The technology of cooling atomic ensembles has been one of the most important developments in physics over the last decades [1]. Two methods have had the most widespread impact, laser cooling, and evaporative cooling. In the former, the heat of the atoms is removed by careful absorption and emission of photons. The mechanism is typically limited by the photon recoil energy from the last photon. Lower temperatures are reached with evaporative cooling. This mechanism has two ingredients. The first is a “knife” that selectively removes atoms with the largest kinetic energy. Second, elastic collisions between the atoms thermalize the remaining atoms. It has been a critical ingredient in creating Bose-Einstein condensates [2].

The push toward lower temperatures is fueled by the study of ever complex collective phenomena. For instance, in Fermi gases the BEC-BCS crossover and other pairing mechanisms can be further elucidated. The lowest temperatures that have been achieved in bosonic gases are below a nano-Kelvin [3], whereas for fermionic systems, temperatures of tens of nano-Kelvin have been reported [4], corresponding to temperatures as low as 0.05 of the Fermi temperature  $T_F$ . Theoretical studies on evaporative cooling are given in Refs. [5,6]. Sympathetic cooling of fermions using a much cooler Bose gas was studied in Ref. [7]. Laser cooling of two-component Fermi gases was studied in Ref. [8]. Cooling Feshbach molecules was reported in [9]. In principle, these could be adiabatically dissociated to produce a cold Fermi gas.

We propose an alternative cooling mechanism that uses inelastic scattering processes due to a narrow magnetic Feshbach resonance [10,11]. Here, a molecular state that is resonantly coupled to two scattering atoms acts as a knife that changes the internal state of colliding atoms in a strongly energy-selective manner. These states are then either untrapped or have gained sufficient kinetic energy to quickly leave the trap. Elastic scattering near the resonance leads to thermalization. For a resonance to generate loss, the internal state of the atoms cannot be the lowest-energy state, as there need to exist states into which they can scatter. For samples of atoms in their lowest state, we further propose combining a narrow optical [12] or radio-frequency (rf) induced [13]

resonance with a broad magnetic resonance. Here, the narrow resonance generates the loss processes, whereas the broad magnetic resonance drives thermalization. We show that we can reach lower temperatures than with a single resonance.

Figure 1 shows a schematic representation of the cooling process. We assume a gas of Fermi atoms in a single state although the ideas can be generalized to bosonic or multispecies and multistate fermionic gases. In the degenerate regime the atoms with mass  $m$  form a Fermi sea, as shown in the figure, with Fermi energy  $E_F = k_F^2 / (2m)$ , Fermi momentum  $k_F$ , and a temperature  $T$  less than  $E_F / k_B$ . We consider a narrow Feshbach resonance that for two colliding atoms with momenta  $\vec{k}$  and  $\vec{p}$  induces atom loss with a rate coefficient [14–17]

$$K_{in}(\vec{k}, \vec{p}) = v_r \frac{\pi \hbar^2}{k_r^2} \frac{\Gamma(E) \Gamma_0}{(E - E_{res})^2 + \Gamma_{tot}^2 / 4}. \quad (1)$$

This coefficient is only a function of the relative collision energy  $E$  and is strongly localized around the resonance energy  $E_{res}$ , which can be controlled by external magnetic field.

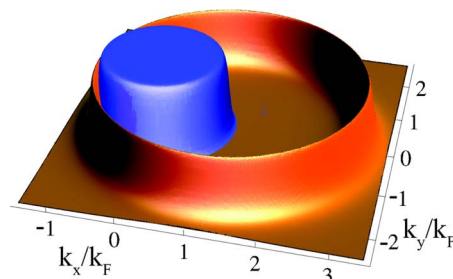


FIG. 1. (Color online) Schematic representation of the loss processes that lead to cooling. We show the momentum distribution  $(k_x, k_y)$  of a degenerate Fermi gas (blue inner feature) and the inelastic scattering rate  $K_{in}(\vec{k}, \vec{p})$  (orange ring-shaped feature), both in arbitrary units, with a scattering partner with a momentum slightly above the Fermi surface at  $\vec{p} = (p_x, p_y) = (1.05k_F, 0)$  and with a resonance energy that corresponds to a relative momentum of  $1.05k_F$ . The losses are largest where the two surfaces approach each other. This corresponds to atoms with momenta located on opposite sides of the Fermi sea. (The momenta along  $z$  are not shown for clarity.)

The energy  $E$  is given by  $E = \vec{k}_r^2 / (2m_r) = m_r v_r^2 / 2$ , where  $\vec{k}_r = (\vec{k} - \vec{p}) / 2$  is the relative momentum and  $m_r = m / 2$  is the reduced mass. Finally, the total energy width is  $\Gamma_{tot} = \Gamma_0 + \Gamma(E)$ , where  $\Gamma_0 / \hbar$  is the linewidth of the resonant state and  $\Gamma(E) / \hbar$  is the collision-energy-dependent stimulated width.

To generate cooling, we need to choose the resonance energy so that only atoms with momenta larger than  $k_F$  are lost. The largest relative momentum is about  $k_F$  for a pair of atoms on opposite sides of the Fermi sea. Consequently,  $E_{res}$  must be set slightly above  $k_F^2 / (2m_r) = 2E_F$ . The exact amount that the resonance energy lies above the Fermi energy will be determined by the temperature and  $\Gamma_{tot}$ . Moreover, the resonance energy needs to be gradually lowered in time by changing the magnetic field as the atom number decreases due to the losses and thus the Fermi energy decreases.

At the beginning of the cooling process the temperature is much larger than  $\Gamma_{tot} / k_B$ . In our simulations, we will find that the smallest achievable temperature is approximately  $4\Gamma_{tot} / k_B$ . This suggests the use of arbitrarily narrow resonances. However, the thermalization rate is also influenced by the resonance. In fact, the total elastic scattering rate coefficient  $K_{el}(\vec{k}, \vec{p})$  is

$$K_{el}(\vec{k}, \vec{p}) = v_r \frac{\pi \hbar^2}{k_r^2} \frac{\Gamma^2(E)}{(E - E_{res})^2 + \Gamma_{tot}^2 / 4}. \quad (2)$$

Typically, during the cooling process it is preferable that the Fermi gas is close to thermal equilibrium, and therefore we require the ratio  $K_{el} / K_{in} = \Gamma(E) / \Gamma_0 \gg 1$  for  $E \approx 2E_F$ , for a nondegenerate gas. Consequently we have  $\Gamma_{tot} \approx \Gamma(E)$ . Since we want a temperature that is as low as possible, this leads to competing requirements on  $\Gamma_0$  and  $\Gamma(E)$ .

For fermionic atoms in the same internal state, only odd partial wave scattering exists. In fact, for ultracold atoms we only need to include the  $p$  or  $\ell = 1$  partial wave. Moreover, the energy dependence of  $\Gamma(E)$  is enforced by the Wigner threshold laws. Here, this leads to  $\Gamma(E) = AE^{3/2}$ , where  $A$  is an intrinsic property of the resonance.

In Ref. [18], a  $p$ -wave resonance was characterized in the collisions of fermionic  $^{40}\text{K}$  atoms in the hyperfine state  $|9/2, -7/2\rangle$ . For the  $m_l = 0$  component of the  $p$ -wave resonance, they find  $\Gamma_0 / k_B = 0.9$  nK and  $\Gamma(E) / k_B = 1.4 \times 10^{-3}$  nK at  $E / k_B = 1$  nK.

## II. CLASSICAL LIMIT

Before we consider cooling a Fermi gas in the degenerate regime, we treat the classical limit of a three-dimensional homogeneous gas, in which we assume that the momentum distribution  $f(\vec{p}, t)$  remains a Maxwell-Boltzmann distribution throughout the cooling process [19]. In fact

$$f(\vec{p}, t) = \varpi(n, T) \exp[-p^2 / (2mk_B T)], \quad (3)$$

where  $\varpi(n, T) = (2\pi\hbar)^3 n / (2\pi mk_B T)^{3/2}$  is the phase-space density, and only the particle density  $n$  and the temperature  $T$  are time dependent. Its time evolution is

$$\partial_t f(\vec{p}, t) = - \int \frac{d^3k}{(2\pi\hbar)^3} K_{in}(\vec{k}, \vec{p}) f(\vec{k}, t) f(\vec{p}, t). \quad (4)$$

In the limit that  $\Gamma_0 + \Gamma(E) \ll k_B T$ , we find that the approximate time evolution for  $n$  and  $T$  is given by

$$\partial_t n = -\gamma_{in}(t)n, \quad \partial_t T = -\gamma_{in}(t) \left( \frac{E_{res}}{3k_B} - \frac{T}{2} \right), \quad (5)$$

with the rate

$$\gamma_{in}(t) = \frac{1}{\hbar} 2^{3/2} \varpi(n, T) \frac{\Gamma_0 \Gamma(E_{res})}{\Gamma_0 + \Gamma(E_{res})} e^{-E_{res} / (k_B T)} \quad (6)$$

being linear in  $n$ , and time dependent through  $n$ ,  $T$ , and  $E_{res}$ . For this approach to be consistent, the thermalization rate during the evolution needs to be larger than the rate  $\gamma_{in}(t)$ . Within the classical theory we find that the thermalization rate is equal to Eq. (6) with  $\Gamma_0$  replaced by  $\Gamma(E)$  in the numerator. Therefore we have to require  $\Gamma(E) / \Gamma_0 \gg 1$ , and  $\gamma_{in}(t)$  becomes independent of  $\Gamma(E)$ .

As an example for the dynamics described by Eqs. (5), we consider a process in which the resonance energy tracks the temperature at a fixed ratio  $\lambda$ , i.e.,  $E_{res}(t) = \lambda k_B T(t)$ . We find the solution

$$n(t) = n_0 (1 - t/t_{cl})^{4/(2\lambda-7)}, \quad T(t) = T_0 (1 - t/t_{cl})^{2(2\lambda-3)/(2\lambda-7)}, \quad (7)$$

where  $n_0$  and  $T_0$  are the initial density and temperature, and the classical cooling time  $t_{cl}$  is given by  $1/t_{cl} = [(2\lambda - 7)/4] \gamma_{in}(t=0)$ . The phase-space density increases as  $\varpi(n, T) = \varpi_0 (1 - t/t_{cl})^{-1}$ , where  $\varpi_0$  is the initial phase-space density. When this approaches one, the system reaches degeneracy and the classical limit breaks down. This occurs shortly before  $t_{cl}$  if  $\varpi_0 \ll 1$ .

For the  $^{40}\text{K}$  example,  $\lambda = 11/2$ ,  $T_0 = 1$   $\mu\text{K}$ , and  $n_0 = 10^{13}$   $\text{cm}^{-3}$ , we find  $t_{cl} \approx 2$  s. This is a realistic time scale for current experimental setups, motivating the subsequent more in-depth analysis.

## III. SINGLE RESONANCE COOLING

We now consider cooling of a gas of spin-polarized fermions in the degenerate regime using a single Feshbach resonance. We use the quantum kinetic theory of Refs. [20,21]. The quantum dynamics is then fully given by the evolution of the momentum state occupation, which satisfies a homogeneous quantum Boltzmann equation. We further assume a spherically symmetric momentum distribution, and the momentum distribution  $f$  becomes a function of kinetic energy  $e = p^2 / (2m)$  only. This function  $f(e)$  satisfies

$$\rho(e_1) \partial_t f(e_1) = -\kappa(e_1) \rho(e_1) f(e_1) - I(e_1), \quad (8)$$

where the density of states per unit volume is  $\rho(e) = 4\pi m^{3/2} \sqrt{2e} / (2\pi\hbar)^3$ , the loss rate  $\kappa$  is given by

$$\kappa(e_1) = \frac{1}{2} \int \rho(e_2) de_2 f(e_2) \int d \cos \theta K_{in}(E). \quad (9)$$

We denote  $K_{in}(E) \equiv K_{in}(\vec{k}_1, \vec{k}_2)$ , as the inelastic loss rate coefficient only depends on the relative energy  $E = e_1 / 2 + e_2 / 2$

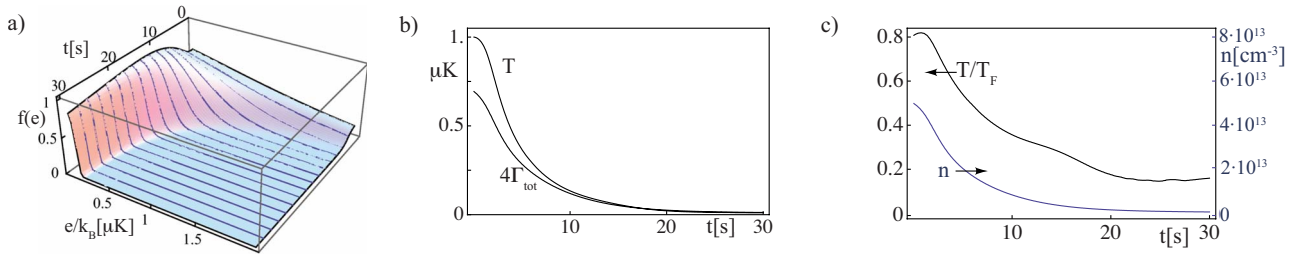


FIG. 2. (Color online) Collisional cooling of a Fermi gas with the resonance described in [18]. Panel (a) shows the distribution  $f(e)$  as a function of time. Panel (b) shows the temperature  $T$  and  $4\Gamma_{tot}(2E_F)$ . Finally, panel (c) shows  $T/T_F$  and the density  $n$ .

$-\sqrt{e_1 e_2} \cos \theta$ , and  $\theta$  is the angle between  $\vec{k}_1$  and  $\vec{k}_2$ . The collision integral is given by

$$I(e_1) = \int de_2 de_3 W(e_1, e_2, e_3, e_4) \{f(e_1)f(e_2)[1-f(e_3)][1-f(e_4)] - [1-f(e_1)][1-f(e_2)]f(e_3)f(e_4)\}, \quad (10)$$

where  $e_4 = e_1 + e_2 - e_3$ , and the collision kernel  $W$  is

$$W(e_1, e_2, e_3, e_4) = \frac{32\pi^2 m^2}{(2\pi\hbar)^6} \int_{P_{\min}}^{P_{\max}} dP \sigma(E). \quad (11)$$

The elastic cross section  $\sigma(E) = K_{el}(E)/v_r(E)$ , and we use  $K_{el}(E) \equiv K_{el}(\vec{k}_1, \vec{k}_2)$  as it only depends on the relative energy  $E = e_1 + e_2 - P^2/(4m)$ , where  $\vec{P}$  is the total momentum in the collision. The integration bounds are given by  $P_{\min} = \max(|p_1 - p_2|, |p_3 - p_4|)$  and  $P_{\max} = \min(p_1 + p_2, p_3 + p_4)$ , where  $p_i = \sqrt{2me_i}$ .

We estimate the thermalization rate in the quantum degenerate regime based on Eq. (8). As outlined in [22], we linearize the Boltzmann equation around a Fermi distribution  $f_0(e)$  with Fermi energy  $E_F$  and temperature  $T$ , that is,  $f(e) \rightarrow f_0(e) + f_0(e)[1 - f_0(e)]\psi(e)$ , where  $\psi(e)$  is a small deviation and the functional form ensures that the fluctuations are localized around the Fermi energy. Then we find that the thermalization rate for states close to the Fermi energy is given by

$$\frac{1}{\tau_{th}} \sim \frac{(k_B T)^2}{\rho(E_F)} W(E_F, E_F, E_F, E_F), \quad (12)$$

reflecting that the only contributions to the collision integral of Eq. (10) are processes close to the Fermi energy.

$W(E_F, E_F, E_F, E_F)$  can be estimated by realizing that the integral in Eq. (11) runs from zero to  $2k_F$ , and therefore the relative energy from zero to  $2E_F$ . As the resonance energy will be larger than  $2E_F$  by an amount of the order of  $\Gamma(2E_F)$  or equivalently  $k_B T$  toward the end of the cooling cycle, we find that  $W \propto \sqrt{\Gamma(2E_F)/E_F}$  and the thermalization rate is

$$\frac{1}{\tau_{th}} \sim (k_B T)^2 A^{1/2} E_F^{-3/4} / \hbar. \quad (13)$$

The quadratic temperature dependence is typical for a Fermi gas in the degenerate limit [22]. For the losses we similarly find a time scale  $1/\tau_l \sim \Gamma_0 \Gamma(2E_F) / (\hbar E_F)$ .

We now solve the quantum Boltzmann equation numerically to study our cooling process, starting from a Fermi distribution for the atoms. The initial resonance energy  $E_{res}$  is set well above twice the Fermi energy. We then gradually lower  $E_{res}$  to eliminate atoms with large kinetic energy. The final  $E_{res}$ , and thus  $E_F$ , and the time scales of the cooling process can be estimated from our expectation that the smallest  $T/T_F$  is of the order  $\Gamma_{tot}/E_F$ . Minimizing this with respect to  $E_F$  gives  $E_F = (2\Gamma_0/A)^{2/3}$ . The time scales  $\tau_{th}$  and  $\tau_l$  will be largest at that final value. They are approximately  $\tau_{th} \sim \hbar / (A\Gamma_0^{3/2})$  and  $\tau_l \sim \hbar / (A^{2/3}\Gamma_0^{4/3})$ .

Figure 2 shows an example of cooling for the  $^{40}\text{K}$  resonance described before. The atomic ensemble has an initial temperature of  $1.0 \mu\text{K}$  and a chemical potential of  $\mu(0)/k_B = 0.5 \mu\text{K}$ , corresponding to an initial density of  $n \approx 5 \times 10^{13} \text{ cm}^{-3}$ . We choose  $E_{res}(0)/k_B = E_{res,i}/k_B = 8 \mu\text{K}$ , well above  $2E_F/k_B \approx 2.5 \mu\text{K}$  and  $T$ . We choose  $E_{res,f}/k_B = 0.2 \mu\text{K}$  as the final value for the resonance energy based on the above estimate for the optimal  $T/T_F$ . We use an exponential time dependence  $E_{res}(t) = (E_{res,i} - E_{res,f}) \exp(-t/t_0) + E_{res,f}$ , with a time scale  $t_0 = 5 \text{ s}$ , that is larger than the estimates for  $\tau_{th}$  and  $\tau_l$ . In Fig. 2(a) we show the distribution  $f(e)$  as a function of energy and time. It becomes visibly colder in the process. Throughout the simulation  $f(e)$  is fairly close to thermal. Thus one can fit  $f(e)$  to a Fermi distribution and assign a temperature and a chemical potential at any point in time. Figure 2(b) shows the fitted temperature as a function of time. It gradually approaches the temperature of  $4\Gamma_{tot}(2E_F)/k_B$ . The fastest cooling rate or slope, around  $t \approx 3 \text{ s}$ , is consistent with the rate expected in the classical limit. The cooling is slower for lower temperatures consistent with Eq. (13). The temperature has decreased by two orders of magnitude in the process, from  $1 \mu\text{K}$  to  $10 \text{ nK}$ . In Fig. 2(c) we show  $T/T_F$  and  $n/T_F$  is reduced by a factor of 6, from  $0.8$  to  $0.15$ , while simultaneously the density has decreased from  $5 \times 10^{13} - 0.7 \times 10^{12} \text{ cm}^{-3}$ . We have performed other cooling simulations with other parameters and found that the final temperature always approaches  $\approx 4\Gamma_{tot}/k_B$ .

We have performed multiple simulations with various initial densities, temperatures, resonant energies, and  $t_0$ . We observe the same qualitative behavior, except for very small values of  $t_0$ , where we lose atoms too quickly and the system does not equilibrate. Most importantly, we find that  $k_B T$  approaches  $4\Gamma_{tot}$ . By optimizing the functional form of  $E_{res}(t)$ , this temperature could be achieved in a shorter time. However, we do not expect to reach significantly lower temperatures.



#### IV. COOLING WITH TWO RESONANCES

The elastic and inelastic scattering rate coefficients are governed by  $\Gamma(E)$ , which leads to contradictory requirements. To overcome this limitation, we propose the use of two  $p$ -wave resonances: a narrow optical or rf induced one, which acts as the knife, and a broad lossless magnetic one which thermalizes. We locate the magnetic resonance such that the elastic rate coefficient is unitarity limited at  $K_{el} = v_r \pi \hbar^2 / k_r^2$ . To ensure a lossless magnetic resonance the atoms must be in the lowest hyperfine state and the field driving the narrow transition must not couple to the Feshbach molecular state [23]. On the other hand, the loss from and the width of the optical (rf) resonance can be controlled by the laser intensity (rf field). This creates a very narrow knife with a negligible contribution to the elastic rate coefficient. Its resonance location  $E_{res}^{opt}$  is lowered in time.

We have repeated the analysis of the previous section. The thermalization rate is driven by the magnetic resonance and given by  $1/\tau_{th} \sim (k_B T)^2 / (\hbar E_F)$ . Hence, unlike for the single resonance case, thermalization does not constrain  $\Gamma^{opt}(E)$  and  $\Gamma_0^{opt}$ . In fact we find that  $\Gamma^{opt}(2E_F) \approx \Gamma_0^{opt}$  leads to the fastest cooling for a given  $\Gamma_{tot}^{opt}$  and that the lowest temperature is again given by  $4\Gamma_{tot}^{opt}/k_B$ . Figure 3 shows an example of the cooling process. The initial density is  $4 \times 10^{13} \text{ cm}^{-3}$ , and the initial temperature is  $0.3 \text{ } \mu\text{K}$ . The system is cooled down to a final density  $1.9 \times 10^{13} \text{ cm}^{-3}$  and a final temperature  $0.017 \text{ } \mu\text{K}$ . This demonstrates that  $T/T_F \approx 2.5 \times 10^{-2}$  can be achieved with this cooling process.

In conclusion, we have proposed a cooling mechanism for fermions which uses the energy selectivity of a Feshbach resonance. We first discussed the limit of a classical thermal gas before we turned to a quantum kinetic simulation of a degenerate Fermi gas. We find that the temperatures achieved by the cooling process are limited by the linewidth of the resonance, and we give estimates of the thermalization time

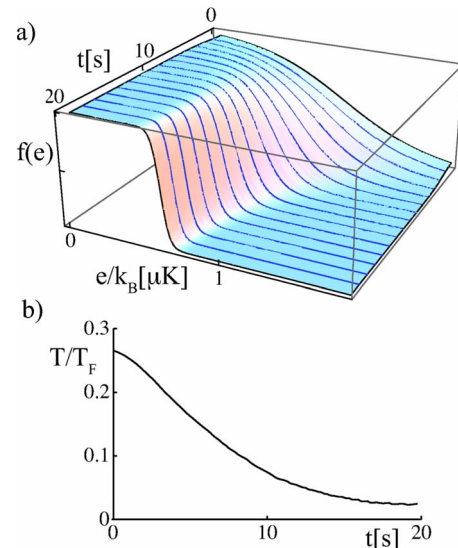


FIG. 3. (Color online) Collisional cooling with an optical resonance as a knife and a magnetic resonance for thermalization. We use the mass of  $^{40}\text{K}$ ,  $\Gamma^{opt}(E)/k_B = 40 \cdot 10^{-6} \text{ nK}[E/(1 \text{ nK})]^{3/2}$ , and  $\Gamma_0^{opt}/k_B = 1.8 \text{ nK}$ . For the time dependence of the resonance energy we use  $E_{res,i}^{opt}/k_B = 4.1 \text{ } \mu\text{K}$ ,  $E_{res,f}^{opt}/k_B = 1.2 \text{ } \mu\text{K}$ , and  $t_0 = 6 \text{ s}$ . (a)  $f(e, t)$ . (b)  $T/T_F$ .

scales. We show that for  $^{40}\text{K}$  we can reach  $T/T_F \approx 0.1$  for an easily accessible resonance. We then improve on this setup by using one narrow resonance for the loss process and a broad magnetic resonance for thermalization.

#### ACKNOWLEDGMENTS

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