

Time domain measurement of phase noise in a spin torque oscillator

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We measure oscillator phase from the zero crossings of the voltage versus time waveform of a spin torque nanocontact oscillating in a vortex mode. The power spectrum of the phase noise varies with Fourier frequency f as $1/f^2$, consistent with frequency fluctuations driven by a thermal source. The linewidth implied by phase noise alone is about 70% of that measured using a spectrum analyzer. A phase-locked loop reduces the phase noise for frequencies within its 3 MHz bandwidth. © 2009 American Institute of Physics. [DOI: 10.1063/1.3133356]

In a spin torque oscillator (STO), dc current transfers spin angular momentum from a thick “fixed” ferromagnetic layer to a thin “free” ferromagnetic layer. For sufficient current in the appropriate direction, the spin torque counteracts the intrinsic damping torque in the free layer, giving rise to coherent oscillations above a threshold current. As the magnetization precesses along a stable trajectory it produces an oscillating voltage, which in all-metallic devices is due to the giant magnetoresistance effect. The STO frequency can be tuned over a wide range by varying the dc current and a (static) applied magnetic field. Because of their small size (~ 100 nm), frequency agility and compatibility with silicon complementary metal-oxide semiconductor processing, STOs may be used for applications such as mixing and active phase control in integrated microwave circuits. Further details about spin transfer torque and STOs can be found in recent reviews.^{1,2}

STOs differ from classical electronic oscillators in several ways, but the most important is the essential dependence of frequency on oscillation amplitude.³ Because of this, an STO cannot be described by standard circuit models comprising a linear resonator and a feedback amplifier. In particular, Leeson’s model for phase noise⁴ does not apply to STOs. Although oscillator properties such as frequency modulation⁵ and phase locking⁶ have been demonstrated in STOs, phase noise has not been directly addressed by previous experiments. The time domain measurement of phase is particularly important because all other quantities that characterize the precision of an oscillator can be derived from it.⁷ When the output voltage of an STO is measured in the frequency domain using a spectrum analyzer, the linewidth is determined by both phase noise and amplitude noise. Although a recent time domain study⁸ provided direct evidence for effects previously inferred from frequency domain data, it did not address phase noise. In this paper we report measurements of phase versus time for an STO, compare the linewidth due to phase noise alone with that measured using a spectrum analyzer, and demonstrate the reduction of STO phase noise using a phase-locked loop.

We studied a nanocontact device from the same batch as those used for a previous study.⁹ The magnetic layer structure is Ta(3 nm)/Cu(15 nm)/Co₉₀Fe₁₀(20 nm)/Cu(4 nm)/Ni₈₀Fe₂₀(5 nm) and the region of electrical contact to the layers is a circle of 60 nm nominal diameter. As described

previously,^{9,10} spin torque can excite oscillations in these devices that are well below the frequency of uniform ferromagnetic resonance in the extended, continuous film below the contact. Micromagnetic simulations of these modes¹⁰ indicate the magnetization has a vortexlike pattern and the oscillations are due to gyrotropic motion of the vortex core about the contact.

To measure the oscillations, we used a bias tee to separate the dc bias current from the ac response of the device, which we measured using a real-time oscilloscope (1.5 GHz bandwidth and 8 GS/s sampling rate) or a spectrum analyzer. Two stages of preamplification were used to make the ac signal much larger than the input noise of the oscilloscope (the net power gain of 45 dB for the signal path was not subtracted from the data shown here). Harmonics were attenuated by a roll-off in amplifier gain above 150 MHz. As is typical of nanocontact STOs in the vortex mode, our device has several possible modes of oscillation and we adjusted the bias current I_b and applied field $\mu_0 H$ (μ_0 is the magnetic constant) to find modes with linewidth ≤ 1 MHz. Data for two modes are presented here. The 142.5 MHz mode used $I_b = 6.7$ mA and $\mu_0 H = 19$ mT at 85° from the film plane. The 128 MHz mode used $I_b = 6.6$ mA and $\mu_0 H = 66$ mT at 89° from the film plane. (Positive I_b corresponds to electrons flowing from the free layer to the fixed layer.) For both modes, the frequency varied monotonically with I_b at ≈ 11 MHz/mA and with $\mu_0 H$ at ≤ 1 MHz/mT. All measurements were done at room temperature.

The black traces in Fig. 1 show selected subsets of the waveform data. The apparent amplitude variations in these data are primarily due to noise in the amplifiers and do not represent intrinsic amplitude noise in the STO (similar variations were seen for the clean sine wave described below). We calculate the phase of the oscillation at each zero crossing as follows.¹¹ The waveform may be written as

$$V(t) = [V_0 + \epsilon(t)] \sin[2\pi\nu_0 t + \phi(t)], \quad (1)$$

where $\epsilon(t)$ is the deviation from the nominal amplitude V_0 , ν_0 is the nominal frequency, and $\phi(t)$ is the deviation from the nominal phase $2\pi\nu_0 t$. Note that $\phi(t)$ includes noise due to changes in precession amplitude combined with the coupling of amplitude and frequency in an STO.^{3,12} Zero crossings of $V(t)$ occur when $2\pi\nu_0 t + \phi(t) = n\pi$, where n is an even (odd) integer for crossings with a positive (negative) slope. The set of values $\{n_i, t_i\}$ for a waveform gives measurements of phase deviation at discrete times: $\phi(t_i) = 2\pi(n_i/2 - \nu_0 t_i)$. For

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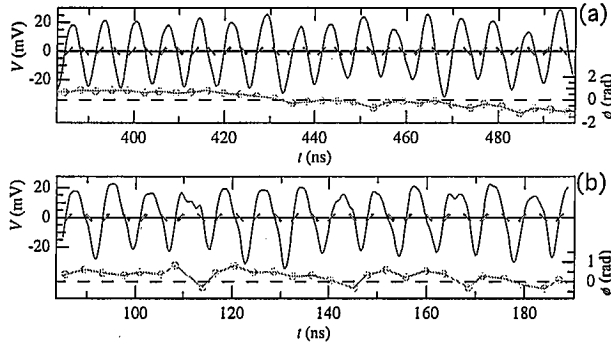


FIG. 1. (Color online) Waveforms and phase from zero crossings for (a) 142.5 MHz mode and (b) 128 MHz mode. The mean value over the entire 4 μ s waveform has been subtracted to remove a dc offset of a few millivolts. Slash marks show the expected zero crossings ("/" symbol for positive and "\\" symbol for negative) relative to the first crossing near $t=0$.

ν_0 we use the mean frequency determined from the total number of zero crossings during the entire waveform. Returning to Fig. 1, the slash marks along the line $V=0$ show the expected zero crossings (relative to the first crossing near $t=0$)¹³ and the lower curve shows $\phi(t_i)$. Figure 2 shows $\phi(t_i)$ for the entire span of 4 μ s acquired with the oscilloscope. The noise floor of our measurement is illustrated by $\phi(t_i)$ for a clean 130 MHz sine wave from a signal generator with negligible phase noise. This signal was attenuated to have the same amplitude as the raw STO signal and measured after passing through the same signal path.

The time domain data in Figs. 1 and 2 illustrate some typical characteristics of the phase noise for the vortex modes considered here. Figure 1(a) shows an example of several periods with ϕ nearly constant, then a relatively gradual change in about 1.5 rad between $t=420$ ns and 490 ns. Figure 1(b) shows more abrupt changes in ϕ , including multiple changes within a half period that cannot be detected by our zero crossing method (near $t=110$ and 143 ns). Figure 2 shows the random walk character of the phase variations over long times, with a rms fluctuation over 4 μ s of 1.2 rad for the 142.5 MHz mode and 1.1 rad for the 128 MHz mode. The rms fluctuation for the sine wave over 2 μ s is 0.10 rad.

Figure 3 shows the one-sided power spectral density (PSD) of $\phi(t_i)$,¹² which falls off as $1/f^2$ until it saturates at $(1.0 \pm 0.07) \times 10^{-9}$ rad²/Hz for both modes. (All uncertainties in this paper are one standard deviation.) The PSD for the sine wave saturates at $(9.4 \pm 0.6) \times 10^{-11}$ rad²/Hz, which is consistent with the noise floor for zero crossings set by the Johnson noise of a 50 Ω impedance propagated through the

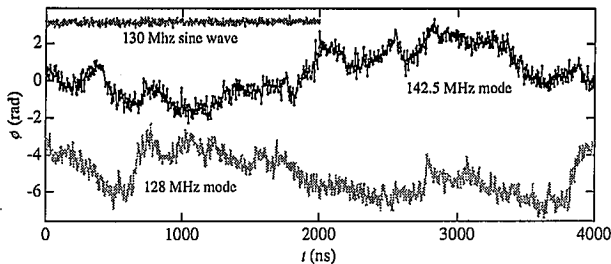


FIG. 2. (Color online) $\phi(t_i)$ from a 4 μ s waveform for each mode (lower curve offset by -3 rad for clarity), and from a 2 μ s waveform for a 130 MHz sine wave (offset by 3 rad for clarity).

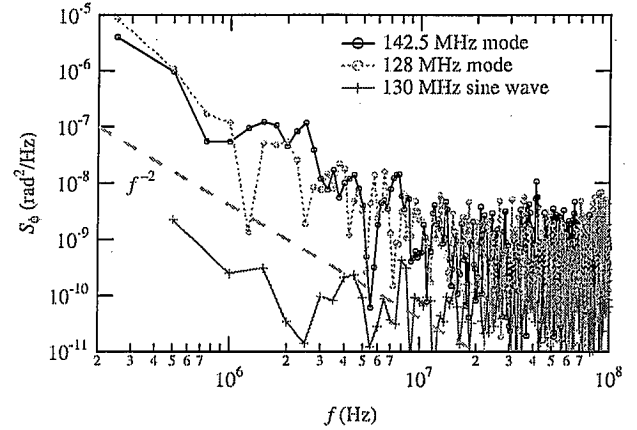


FIG. 3. (Color online) PSD of phase fluctuations for the data in Fig. 2. Dashed line is a guide to the eye showing $1/f^2$ dependence.

signal path.¹² The saturation of the STO phase noise at a higher value could be explained by additional white noise on $V(t)$ (before amplification) of about 2 nV/Hz^{1/2}, a level too small to measure directly in our setup. The phase added along the signal path varies slightly with frequency, but the noise floor due to this effect is $\leq 10^{-11}$ rad²/Hz.

In Eq. (1), the total phase $\theta(t) \equiv 2\pi\nu_0 t + \phi(t)$ defines an instantaneous frequency $\nu(t) \equiv (1/2\pi)d\theta/dt = \nu_0 + (1/2\pi)d\phi/dt$. Phase noise and frequency noise are equivalent ways of expressing the instability of an oscillator,⁷ and because frequency is the derivative of phase, their PSDs are related by $S_\nu(f) = f^2 S_\phi(f)$. The data in Fig. 3 thus imply that S_ν is independent of f (below 10^7 Hz), suggesting that frequency fluctuations in our vortex mode STO are driven by a white noise source such as thermal fluctuations. The mean value of S_ν for $f \leq 10^7$ Hz is $(2.3 \pm 0.3) \times 10^5$ Hz²/Hz for the 142.5 MHz mode and $(1.7 \pm 0.2) \times 10^5$ Hz²/Hz for the 128 MHz mode.

The value of S_ν allows us to calculate the contribution of phase noise to linewidth as follows.¹⁴ Neglecting amplitude noise, the ensemble-averaged $V(t)$ for an ensemble of oscillators whose phases undergo a diffusive random walk is a damped sinusoid. The power spectrum of $V(t)$ (the quantity measured by a spectrum analyzer) has a full-width-at-half-maximum $\Delta\nu$ that is proportional to the PSD of the white frequency noise: $\Delta\nu = \pi S_\nu$,¹⁴ where S_ν is the one-sided PSD.¹² Thus our values of S_ν above imply linewidths of (0.72 ± 0.10) MHz for the 142.5 MHz mode and (0.54 ± 0.08) MHz for the 128 MHz mode. Using a spectrum analyzer, we measured linewidths of (1.05 ± 0.14) MHz for the 142.5 MHz mode and (0.78 ± 0.11) MHz for the 128 MHz mode. We conclude that phase noise is responsible for about 70% of the linewidth in both cases, with the rest presumably due to amplitude noise. (As mentioned above, STO amplitude noise is masked by amplifier noise in the waveform data presented here.)

A common way to improve the phase noise of an oscillator is to use a phase-locked loop (PLL). Although abrupt phase jumps such as those seen in Fig. 1(b) cannot be removed, a PLL can stabilize the STO against slower phase variations such as those in Fig. 1(a). We built a PLL in which $V(t)$ from both the STO and a signal generator at the refer-

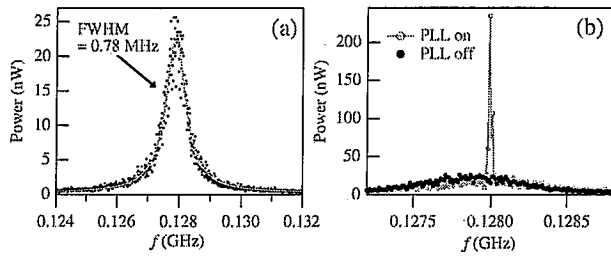


FIG. 4. (Color online) Spectrum analyzer measurements of amplified STO signal for the 128 MHz mode (resolution bandwidth=10 kHz). (a) PLL off, with Lorentzian fit. (b) PLL on, with same data as in (a) for comparison.

ence frequency are converted into square waves using digital comparators. Timing jitter between these square waves produces an error signal that is fed into an integrator, and the integrated error signal is added to the dc bias current of the STO using a dual-input bias tee. Distortion in the digital comparators becomes excessive above 200 MHz, and the overall feedback bandwidth is limited to 3 MHz by the input of the bias tee.

The effect of the PLL as measured by the spectrum analyzer is shown in Fig. 4. The left plot is the spectrum of the 128 MHz mode before locking the PLL, as well as a Lorentzian fit giving $\Delta\nu=0.78$ MHz. The right plot is the spectrum with the PLL on, showing a large increase in power at the 128 MHz reference frequency. Comparing the area of the narrow peak with the total area over a 6 MHz span, we find that 20% of the power occurs within ± 1 resolution bandwidth of the reference frequency for resolution bandwidths as low as 10 Hz. We obtained a similar result for the 142.5 MHz mode.

The effect of the PLL on S_ϕ is shown in Fig. 5, where one can see a modest decrease in noise for frequencies below the 3 MHz limit mentioned above. The variance obtained by integrating S_ϕ over f is 1.2 rad^2 with the PLL off and 0.53 rad^2 with the PLL on. The effect of the PLL is more dramatic in Fig. 4 than in Fig. 5 because the spectrum analyzer spends $\sim 100 \mu\text{s}$ at each frequency while acquiring the spectrum, whereas the waveforms used to obtain S_ϕ last only $4 \mu\text{s}$. Longer waveforms (using an oscilloscope with more memory) should reveal a more significant decrease in S_ϕ at lower frequencies.

The technique described here for directly measuring phase and its PSD offers a valuable characterization tool for STOs. It gives information that is not available from a linewidth measurement and it can discriminate among models

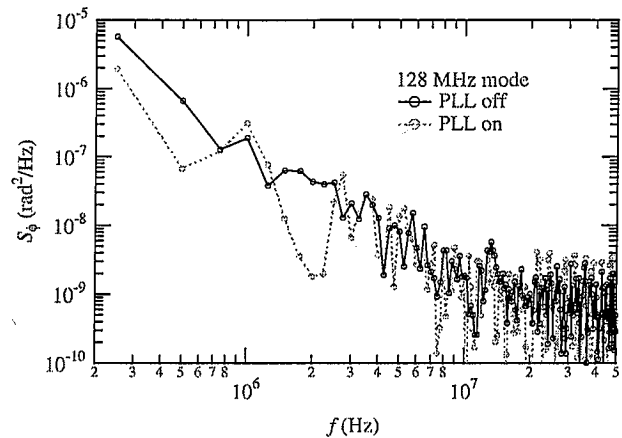


FIG. 5. (Color online) PSD of phase fluctuations with PLL off and on.

for noise that predict similar linewidths but different forms for $S_\phi(f)$. Such tests will be useful in understanding the behavior of STOs and in pushing their performance toward that required by applications.

- ¹D. C. Ralph and M. D. Stiles, *J. Magn. Magn. Mater.* **320**, 1190 (2008).
- ²W. H. Rippard and M. R. Pufall, in *Handbook of Magnetism and Advanced Magnetic Materials*, edited by H. Kronmüller and S. Parkin (Wiley, New York, 2008).
- ³J.-V. Kim, V. Tiberkevich, and A. N. Slavin, *Phys. Rev. Lett.* **100**, 017207 (2008).
- ⁴D. B. Leeson, *Proc. IEEE* **54**, 329 (1966).
- ⁵M. R. Pufall, W. H. Rippard, S. Kaka, T. J. Silva, and S. E. Russek, *Appl. Phys. Lett.* **86**, 082506 (2005).
- ⁶W. H. Rippard, M. R. Pufall, S. Kaka, T. J. Silva, S. E. Russek, and J. A. Katine, *Phys. Rev. Lett.* **95**, 067203 (2005).
- ⁷S. R. Stein, in *Precision Frequency Control*, edited by E. A. Gerber and A. Ballato (Academic, New York, 1985), Vol. 2, pp. 191–416.
- ⁸I. N. Krivorotov, N. C. Emley, R. A. Buhrman, and D. C. Ralph, *Phys. Rev. B* **77**, 054440 (2008).
- ⁹M. R. Pufall, W. H. Rippard, M. L. Schneider, and S. E. Russek, *Phys. Rev. B* **75**, 140404 (2007).
- ¹⁰Q. Mistral, M. van Kampen, G. Hrkac, J.-V. Kim, T. Devolder, P. Crozat, C. Chappert, L. Lagae, and T. Schrefl, *Phys. Rev. Lett.* **100**, 257201 (2008).
- ¹¹L. Cosart, L. Peregrino, and A. Tamba, *IEEE Trans. Instrum. Meas.* **46**, 1016 (1997).
- ¹²See EPAPS Document No. E-APPLAB-94-059919 for supplemental material. For more information on EPAPS, see <http://www.aip.org/pubserv/epaps.html>.
- ¹³The strong harmonic content of vortex mode oscillations makes our waveforms asymmetric, with the peaks 10%–20% wider than the valleys. The expected zero crossings include this asymmetry to avoid a systematic offset in phase between positive and negative crossings.
- ¹⁴A. Blaquiere, *Nonlinear System Analysis* (Academic, New York, 1966).

Supplementary Material for “Time domain measurement of phase noise in a spin torque oscillator”

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Coupling between amplitude and phase

In a linear oscillator, perturbations that change the phase but do not cause the oscillator to deviate from its stable trajectory, *i.e.*, perturbations *along* the stable trajectory, cause phase noise only. Similarly, perturbations *transverse* to the stable trajectory cause amplitude noise only, since the frequency of oscillation is independent of amplitude. Unlike in a linear oscillator, the frequency of an STO depends on the amplitude of oscillation [1, 2], with important consequences for phase noise [3]. During the time the STO is perturbed away from its stable trajectory, the magnetization moves at a different rate, so when it returns to the stable trajectory it will be either advanced or retarded relative to its unperturbed motion. As a consequence, deviations from the stable trajectory result in both amplitude and phase noise. In the equation for $V(t)$, both $\epsilon(t)$ and $\phi(t)$ are affected by transverse perturbations, but it is sufficient to measure either one to detect these perturbations.

Power spectral density

Computation of the Fourier transform of $\phi(t)$ is slightly complicated by the fact the the times t_i at which ϕ is measured are not uniformly spaced. For oscillators with low phase noise it is appropriate to proceed by assuming that the phase change within one period is small ($\ll 1$ rad) and then interpolating from the actual times t_i to uniformly spaced times. This assumption is clearly not valid for our waveforms (see Fig. 1 of the paper). We therefore used the Lomb periodogram described in [4], which is designed to handle nonuniform data. As is common for real signals [4], we define the PSD over positive frequencies only and normalize so that its integral over these frequencies gives the variance of the waveform. With this definition, the PSD at each value of f is twice as large as for the two-sided spectrum often used in theoretical work, and we took this into account when applying the results of Blaquiére for the relation between f and S_ν . We multiplied $\phi(t)$ by a Hann window before computing the PSD in order to remove spurious effects due to the non-periodic nature of the data, and the effect of this window was included in the normalization of the PSD [4].

Noise floor

The output of an oscillator having no phase noise but with an additive noise voltage δv is

$$V(t) = V_0 \sin(2\pi\nu_0 t) + \delta v. \quad (1)$$

At its zero crossings, $V(t)$ has a slope of

$$\frac{dV}{dt} = \pm 2\pi\nu_0 V_0, \quad (2)$$

A weak voltage noise $\delta v \ll V_0$ will cause a timing noise given approximately by

$$\delta t \approx \frac{\delta v}{2\pi\nu_0 V_0}, \quad (3)$$

which implies a phase noise

$$\delta\phi = 2\pi\nu_0 \delta t \approx \frac{\delta v}{V_0}. \quad (4)$$

The same result is derived in a slightly different way in [5] (see equations 30 through 34). This result implies that the PSD of phase noise due to additive voltage noise is

$$S_\phi = \frac{S_v}{V_0^2}. \quad (5)$$

We note that Eq. 5 differs by a factor of two from the often-cited result by Leeson in [6].

The noise in our signal path is nearly independent of the impedance seen by the input of the first amplifier for loads less than $50\ \Omega$. Therefore, we estimate the minimum S_v presented to the first amplifier as that of a $50\ \Omega$ load at 290 K. Using the gains and noise temperatures of our amplifiers, we propagate this noise through our $50\ \Omega$ signal path in the standard way [7] and find an output noise of $S_v = 8.1 \times 10^{-14}\ \text{V}^2/\text{Hz}$. Combining this with $V_0 = 22\ \text{mV}$ gives $S_\phi = 1.7 \times 10^{-10}\ \text{rad}^2/\text{Hz}$ (with an uncertainty of about a factor of 2) as the noise floor due to Johnson noise in our measurement.

References

- [1] S. M. Rezende, F. M. de Aguiar, and A. Azevedo, "Spin-wave theory for the dynamics induced by direct currents in magnetic multilayers," *Phys. Rev. Lett.*, vol. 94, p. 037202, 2005.
- [2] A. Slavin and P. Kabos, "Approximate theory of microwave generation in a current-driven magnetic nanocontact magnetized in an arbitrary direction," *IEEE Trans. Magn.*, vol. 41, pp. 1264–1273, 2005.
- [3] J.-V. Kim, V. Tiberkevich, and A. N. Slavin, "Generation linewidth of an auto-oscillator with a nonlinear frequency shift: Spin-torque nano-oscillator," *Phys. Rev. Lett.*, vol. 100, p. 017207, 2008.
- [4] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*. Cambridge Univ. Press, 3 ed., 2007.
- [5] E. Baghdady, R. Lincoln, and B. Nelin, "Short-term frequency stability: Characterization, theory, and measurement," *Proc. IEEE*, vol. 53, pp. 704–722, 1965.
- [6] D. Leeson, "A simple model of feedback oscillator noise spectrum," *Proc. IEEE*, vol. 54, pp. 329–330, 1966.
- [7] D. M. Pozar, *Microwave Engineering*. Wiley, 3rd ed., 2005.