

# Single-photon propagation through dielectric bandgaps

Natalia Borjemscaia,<sup>1,2\*</sup> Sergey V. Polyakov,<sup>2</sup> Paul D. Lett,<sup>2</sup> and Alan Migdall<sup>2</sup>

<sup>1</sup>Department of Physics, Georgetown University, 37th and O Streets, NW, Washington, D.C. 20057, USA

<sup>2</sup>Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, MD 20899, USA

\*nrutter@nist.gov

**Abstract:** Theoretical models of photon traversal through quarter-wave dielectric stack barriers that arise due to Bragg reflection predict the saturation of the propagation time with the barrier length, known as the Hartman effect. This saturation is sensitive to the addition of single dielectric layers, varying significantly from sub-luminal to apparently super-luminal and vice versa. Our research tests the suitability of photonic bandgaps as an optical model for the tunneling process. Of particular importance is our observation of subtle structural changes in dielectric stacks drastically affecting photon traversal times, allowing for apparent sub- and super-luminal effects. We also introduce a simple model to link HOM visibility to wavepacket distortion that allows us to exclude this as a possible cause of the loss of contrast in the barrier penetration process.

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## 1. Introduction

The original prediction of Hartman [1] indicates that the asymptotic saturation of the transit time for an opaque barrier is smooth, however it has been shown [2–4] that for non-evanescent barriers such as dielectric stacks this is no longer true. Small changes in a dielectric stack, which can alter the details of the corresponding bandgap properties, have been predicted to cause rather dramatic variations in the transit time. While there have been measurements of optical traversal times with classical optical or RF pulses, the only measurement for single photons was presented by Steinberg et al [5]. They observed an apparent superluminal propagation for photons that traversed a dielectric bandgap structure. The single-particle nature of the photons used to probe the structure suggests an analogy with particle transit times. The measurement is based on a post-selection of a set of successful tunneling events and can thus be viewed as a "weak measurement" [6]. The fact that even a single photon can be described as an electromagnetic pulse, however, means that a wave picture remains valid. All of the conflicting explanations and pictures for such superluminal behavior have added to the interest that such experiments hold for the community studying quantum physics, classical analogies, and fundamental physical phenomena.

Transit time measurements through a series of dielectric stacks have been performed by Spielmann, et al. [7], using femtosecond laser pulses. These authors reported that the transit time became monotonically independent of the barrier thickness as the barrier became more opaque. The measurements were made, however, on a set of similar dielectric stacks - ones that varied only by adding more high- and low-index layers in pairs.

It has been pointed out [8], that a model based on bandgap properties of a dielectric quarter-wave-stack is equivalent to quantum mechanical tunneling, but only if a slowly varying envelope approximation (SVEA) is applied. In general, the SVEA simplifies theoretical treatment of wave phenomena by removing the rapidly oscillating optical phase, and can adequately describe the propagation only if the complex amplitude (that serves as both the envelope and a correction to the phase) changes slowly and continuously. In our case a SVEA substitutes a single equation that is mathematically equivalent to one describing an evanescent field for the interference between two counter-propagating waves reflected off of the boundaries of dielectric layers. The key difference between the underlying process and its SVEA treatment is that the interfering waves are oscillatory, while a true evanescent wave is not. We emphasize that upon reflection from boundaries and defects of quarter-wave structures, the phases of the counter-propagating waves experience jumps that have to be treated in an ad hoc fashion to make the SVEA work, thus compromising the mathematical analogy with particle tunneling [8]. Indeed, theoretical calculations with no approximations applied show that if the structure of dielectric stacks is slightly altered, the propagation times may be significantly changed [3]. For alternating high and low index quarter-wave dielectric stacks, the addition of single layers causes an alternation; the transit time saturating to different values for even and odd numbers of layers [2–4]. Therefore, while the experiments of Ref [5], indeed demonstrate an apparent superluminality of traversal, it does not establish a quantitative estimate for delay times in true particle-barrier experiments, hence other optical models of tunneling should be used. In our experimental work we confirm that minute changes to a dielectric stack yield dramatic changes in traversal time and put an upper bound on photon wavefunction distortion via barrier traversal.

## 2. Experimental setup

To measure propagation delays through dielectric stacks where the overall transit times, estimated simply using the thickness of the stack, are expected to be on the scale of several

femtoseconds, we chose a Hong-Ou-Mandel (HOM) interferometer [9] and a parametric down conversion source of photon pairs (Fig. 1), first used for this purpose by Steinberg et. al [5].

To generate identical photons for the HOM, we use parametric down conversion (PDC) in a  $\text{LiIO}_3$  crystal with a continuous wave pump at a wavelength of 351 nm. We generate photon pairs with a bi-photon wavefunction that is  $\approx 140$  fs in duration (full width at half maximum). Detectors in the two arms are connected to a start-stop board, producing a second-order correlation function. The output of the detector that produces stop events is delayed by  $\approx 50$  ns. The detector response, limited by jitter, ( $\approx 0.5$  ns) is much slower than a bi-photon wavefunction duration, but because a parametric downconversion process creates the two photons simultaneously (i.e. they are correlated), the peak due to such coincidences clearly stands out over the background created by uncorrelated events. The coincidences are found by subtracting the background from the second-order correlation function. Such a definition makes the measurement largely insensitive to the timing resolution of the detectors [10]. One of the photons is sent through a dielectric structure, while the other travels through a path of known length. The two photons are then recombined at the beamsplitter. A movable prism in one of the arms of the interferometer provides coarse adjustment of the photon's path length, while a piezoelectric actuator on a mirror in the second arm of the interferometer provides fine path length adjustments. Because a HOM interferometer is insensitive to second-order dispersion [11], the introduction of a prism does not reduce the accuracy. By overlapping the photons' wavefunctions at a beam splitter, we can observe a drop in the coincidence counts between detectors in the two output ports of the beamsplitter. This is because quantum interference causes both photons of an exactly matched pair entering the beamsplitter to exit from the same port of the beamsplitter. If the spatial or spectral overlap of the wavefunctions does not exactly match, this interference will be incomplete. The reduction in coincidence counts as the relative path length difference of the two arms is scanned, is referred to as the HOM dip. Degenerate downconverted photons at 702 nm are selected in the present experiment by interference filters. Because the transmittance of our sample is relatively low, we wanted to avoid any additional losses in our measurement system that would further reduce our data acquisition rate. To minimize this loss, we chose the highest transmittance filters available to us that had similar center wavelengths and spectral widths. Specifically, we used a 9 nm bandwidth interference filter, with  $>90\%$  transmittance in the sample arm, and a 12 nm bandwidth filter (with 63% transmittance) in the reference arm, see Fig. 2. (Note that filter bandwidths are measured as full width at half maximum.) This bandwidth asymmetry did fundamentally limit the visibility of the HOM dip in our experiments [12], but this limitation does not significantly affect our measurement.

With no stack in place, our setup produces  $\approx 200,000$  photon counts per second for each channel and  $\approx 3500$  coincidences per second (background rate  $\approx 400$  counts per second). The HOM dip visibility is  $\approx 80\%$ . To test the interferometric stability of the setup, we positioned the one of the sample's reference regions in the beam. We then scanned over the HOM dip 84 times during a 3 hour period. We fit the dip minimum for each of the scans, obtaining a standard deviation of 0.33 fs. To measure tunneling times through each of the 4 stacks, we found the HOM dip position with the stack in place and compared it with one for a reference region. This measurement was performed  $\approx 300$  times over a 5 hour period, obtaining  $\approx 300$  data points for each stack structure (with the exception of  $(\text{HL})^{15}\text{H}$ , for which  $\approx 800$  scans were taken to compensate for the lower transmission). To reduce the effect of various drifts, we averaged reference scans taken before and after each data scan. The background-subtracted HOM visibility was measured for all stacks and reference points.

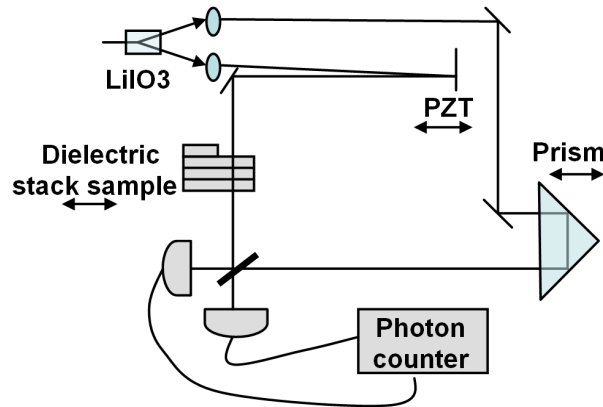


Fig. 1. Experimental apparatus. Two photons created via parametric down conversion are sent to a Hong-Ou-Mandel interferometer. The sample holder positions the reference and sample regions into and out of the beam. A mirror on a piezo actuator (PZT) provides fine motion control of the path length for scanning over the Hong-Ou-Mandel dip.

### 3. Sample preparation

We study single photon traversal in stop bands of 1D Bragg gratings made of alternating quarter wave layers of higher (H) and lower (L) refractive index dielectrics. The sample is made of a fused silica substrate with four stack configurations deposited by coating:  $(HL)^{15}H$ ,  $(LH)^{15}L$ ,  $(LH)^{15}$ , and  $(HL)^{16}$ . The difference between the saturation values of the traversal times (i.e. when  $N \rightarrow \infty$ ) is proportional to the geometric average index of refraction of the two layers and is inversely proportional to its difference [3,13]. Hence, to maximize the effect, one would want dielectric materials with large and nearly equal indices of refraction. However, there are other constraints on the choice of materials. For instance, for low index contrast one needs more layers to get close to the Hartman saturation limit. For example, a refractive index difference of 0.1, requires about 30 bilayers ( $N = 30$ ) to make the difference in saturation times apparent, whereas for an index difference of 0.2, the number of bilayers needed is reduced to  $<20$ . Another constraint is related to the spectral width of the gap. Because we need high resolution for time delay measurements, the bandwidth of the single photons should be large, but all of this bandwidth must be contained inside the bandgap. Ultimately, an index difference between 0.2 and 0.3 is fairly optimal for our conditions.

To satisfy the constraints above, we chose samples with  $\approx 15$  bilayers ( $N = 15$ ) of  $TiO_2$  and  $HfO_2$  with nominal indices of refraction of  $\approx 2.19$  and  $\approx 1.97$ , respectively, when deposited as amorphous films. Both materials have a negligible absorption at the wavelength of interest (702 nm). Along with the four stacks  $(HL)^{15}H$ ,  $(LH)^{15}L$ ,  $(LH)^{15}$ , and  $(HL)^{16}$ , the fused silica substrate includes 2 uncoated regions that are used as reference points for the measurements. The other side of the substrate is antireflection coated to reduce the effects of fringing produced by the substrate-to-air interface on the photon delay times.

Nominally, all four stack regions have a total thickness of  $2.5 \mu m$ , with individual layer thicknesses  $d_{Ti}$  and  $d_{Hf}$  being  $0.080 \mu m$  and  $0.089 \mu m$ , respectively. It should take  $0.585$  fs for a photon to traverse those distances in either material, as they both have the same optical path-length, i.e.  $\lambda/4$ . Note that a photon traveling the same distances ( $0.080 \mu m$  and  $0.089 \mu m$ ) in vacuum would traverse the regions in  $0.27$  fs or  $0.30$  fs, respectively. By measuring the difference between the transit time of a photon traveling through a single stack and a photon transiting a reference path, a  $0.315$  fs delay would be observed for a  $TiO_2$  layer and a  $0.285$  fs delay through a  $HfO_2$  layer. Interestingly, the measured delays through various stack structures do not correspond to a simple addition of the propagation delays through the individual layers. This is true even though  $(HL)^N H$  and  $(LH)^N L$  have the same optical (but not physical) thickness. All four stack structures appear nearly structurally identical, while their

overall optical properties differ significantly. Indeed, all structures exhibit similar stop-bands centered at around 708nm, as shown in Fig. 2.

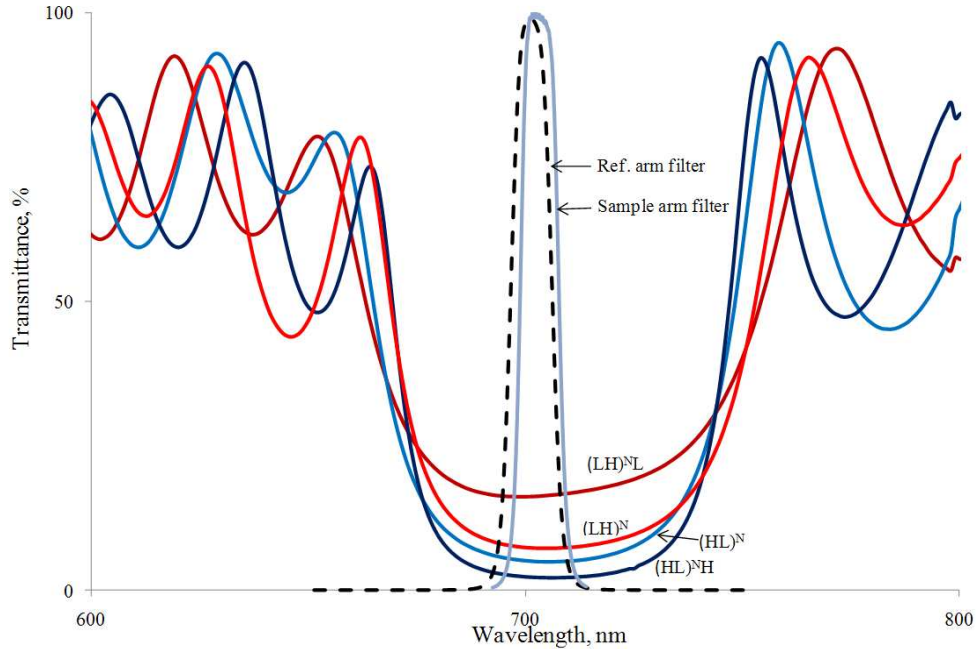


Fig. 2. Stop-bands of the four structures studied, measured with a spectrophotometer. The photons are filtered, before entering HOM interferometer by the two filters, whose measured normalized transmittances are also shown.

#### 4. Results and discussion

An HOM interferometer measures the time difference between the arrival of the two parts of a biphoton wavepacket, which corresponds to a relative group delay [10]. It can be shown [14] that a time delay of the same nature is measured in forbidden gap traversal experiments. The theoretical treatments of structures via characteristic matrix approach [15] (which is similar to the approach of [2,3]) shows that different structures yield very different traversal times. It turns out that all four structures individually exhibit Hartman-like saturation, as seen in Fig. 3(a). Because we consider a stack grown on a glass substrate, propagation delays of photons through the two even-number layer structures  $(HL)^N$  and  $(LH)^N$  are not equal, as the substrate breaks the time-reversal symmetry. Hartman-like saturation can only be observed by adding layers to any given structure in pairs. If single layers are added to the structure, jumps in the propagation delay times occur. By the same token, for the  $(LH)^N L$  structure, the saturation time is significantly higher than for the  $(HL)^N H$  structure even though both structures are of the same optical length as each layer is designed to be a quarter wave. If any of these odd layer structures is changed by adding one more layer, yielding  $(LH)^{N+1}$  and  $(HL)^{N+1}$  respectively, the traversal time abruptly changes, as is evident in Fig. 3(a). Specifically, adding an extra layer to the  $(LH)^N L$  structure produces a  $(LH)^{N+1}$  structure which would surprisingly *decrease* the propagation delay for the thicker structure, while adding an extra layer to the  $(HL)^N H$  structure produces a  $(HL)^{N+1}$  structure, which would increase the propagation delay. This phenomenon can also be viewed as light trapping by localized modes due to structure termination (i.e. surface modes) [16,17]. These modes reside in stop bands of the structure, increasing the transmittance through structures at the expense of increased traversal times, but this is not the focus of this current work.

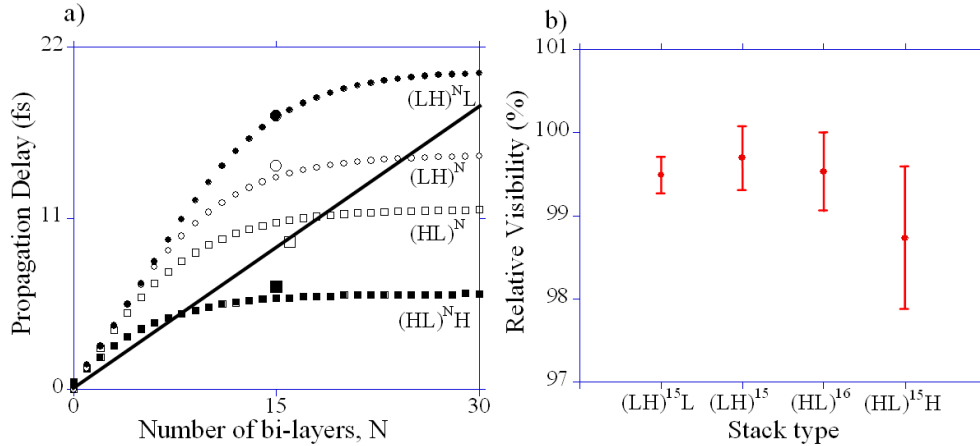


Fig. 3. a) Theoretical prediction of propagation delay times for different dielectric stack configurations. Each of the structures is responsible for a different propagation delay branch of the graph. The line represents propagation delay of a photon in an equivalent thickness of vacuum. Values below this luminal line represent superluminal propagation. Values above the luminal line represent subluminal propagation. Large symbols show experimental measurements with uncertainties being about half the size of the symbols; b) Relative HOM dip visibilities for the four stack types.

For each of the four stack types and two reference regions we took over 300 scans across the HOM dip. The averaged and background-subtracted coincidence signal for the two odd-layer stack structures and reference regions can be seen in Fig. 4. By finding the average location of the HOM dip minima for the reference and sample regions, we calculated apparent pathlength differences and obtained corresponding propagation delays,  $\Delta\tau$ . The measured  $\Delta\tau$  is the delay relative to the propagation of light in vacuum through a distance,  $l$  equal to the stack thickness,  $\Delta\tau = \tau - l/c$ . The measured  $\Delta\tau$  for all the structures are presented in Table 1 and Fig. 3. One sees that the experimental results are consistent with the theoretical model. Note that the theoretical calculation used no fitting parameters. Thus, we confirm experimentally that the traversal times can be very sensitive to subtle changes in the dielectric stack structure. This highlights the failure of the mathematical analogy between dielectric stack traversal and tunneling. In particular, the two unpaired-layer structures, with the same number of layers and exactly equal optical path lengths, produce propagation delays that differ by  $10.99 \pm 0.58$  fs, solely because of the different first and last layers. Note that this delay difference is larger than the propagation delay through the entire  $(HL)^N H$  structure. Interestingly, the shorter time corresponds to an apparent superluminal propagation delay, while the longer time is  $\approx 2x$  longer than the luminal delay.

An important question is the extent to which wavepacket distortion affects the measured transit times and results in apparent superluminality. Such distortion can present itself as a change in the measured visibility of the HOM dip [14]. Visibility is defined as  $V = (C_{\text{baseline}} - C_{\text{dip}}) / (C_{\text{baseline}} + C_{\text{dip}})$ , where  $C_{\text{baseline}}$  is the level of the baseline coincidence rate, that occurs when the two photons are separated in time such that the interference does not occur and  $C_{\text{dip}}$  is the coincidence count at a minimum of best theoretical fits to the data. We use a simple model that ties such a visibility change to the wavepacket distortion (in frequency domain,  $\omega$ ) by introducing a sine modulation  $\varphi = \varphi_0(1 - A \sin(f\omega))$  onto the initial wavepacket generated in a PDC process. Here,  $\varphi_0$  is the unmodulated two-photon spectral function [18],  $A$  and  $f$  are the amplitude and rate of the modulation, respectively. We have found that the HOM visibility for  $f$  much less than the inverse packet spectral width is of the form

$V \propto (2 - A^2) / (2 + A^2)$ . We note that  $V$  decreases monotonically with the modulation amplitude  $A$ , and does not depend on  $f$ . Thus, by observing changes in dip visibility, we can assess the

overall effects of distortion of the wavepacket caused by the barrier traversal under this model [19]. This means that relevant wavepacket changes can be traced and characterized in a general fashion without prior knowledge of the nature or shape of such change. This technique is useful for studying distortion in HOM experiments that traverse any unknown medium, and is particularly helpful in studying distortions in long potential barriers, the situation in our experiment.

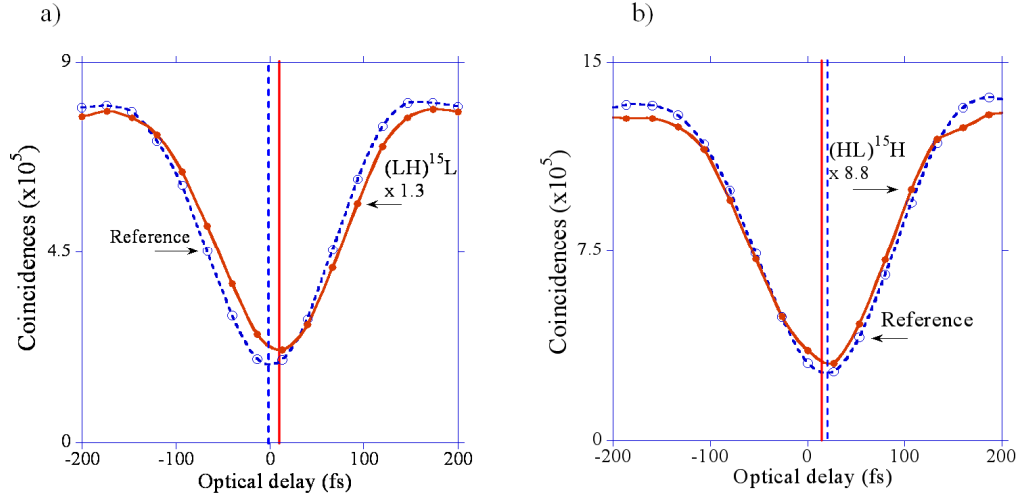


Fig. 4. Measured HOM dip profiles. Propagation (group) delays are the mean difference between minima of the fits (indicated by vertical lines). The reference is an uncoated substrate (open circles and dotted lines). Sample regions are a)  $(LH)^{15}L$  and b)  $(HL)^{15}H$  (closed circles and solid lines). The sample profiles are scaled as indicated on the figures. The uncertainties associated with the datapoints are smaller than the size of the symbols

**Table 1. Predicted and measured delays and dip visibilities**

Sample	Predicted delay, fs	Experimental delay, fs	Reference visibility, %	Sample visibility, %
$(LH)^{15}L$	7.70	$7.60 \pm 0.35$	$78.60 \pm 0.13$	$78.20 \pm 0.19$
$(LH)^{15}$	3.59	$4.38 \pm 0.40$	$79.06 \pm 0.14$	$78.82 \pm 0.36$
$(HL)^{16}$	0.68	$-0.49 \pm 0.43$	$72.56 \pm 0.19$	$72.22 \pm 0.42$
$(HL)^{15}H$	-3.89	$-3.39 \pm 0.46$	$80.45 \pm 0.23$	$79.43 \pm 0.82$

To detect any distortions of our wavepacket due to tunneling, we measured a background-corrected visibility of the dip, presented in Table 1 and Fig. 3(b). We see that the difference in the visibilities observed is on the order of the uncertainties. This result allows us to bound the wavepacket distortion to  $A < 0.185$  (or 18.5%), with a 95% probability under the model discussed above. This limit on wavepacket distortion indicates that a quasi-stationary approximation, as introduced in [20,21], may still give valid results under these conditions.

## 5. Results and discussion

In conclusion, we have presented a measurement of propagation delays of photons that traverse a stop band of quarter-wave dielectric stacks that have both even and odd numbers of layers. We find experimentally that, as predicted in [2–4], dielectric stack samples of the same approximate thickness and with similar structure can yield very different traversal times for photons (Hartman saturation level). Indeed, the addition of a single layer can alter these times up or down by many times what would be expected from the transit time across that single layer in isolation, a picture consistent with light trapping in surface modes that appear due to differences in structure termination [16,17]. This shows the failure of the mathematical

analogy [8] between traversing stop bands and particle tunneling. Hence, quarter-wave stacks are not suitable as an optical equivalent of particle tunneling through a single barrier. We have also examined the change in HOM visibility due to distortion caused by traversal through our dielectric structures and find the effects to be insignificant.

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