Reducing the Effects of Record Truncation Discontinuities in Waveform Reconstructions

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Abstract—Record truncation discontinuities (RTD's) are artifacts in recorded data caused by the difference between the values of the data at the two ends of the record. The RTD causes errors in waveform reconstructions, in particular, in digital reconstructions that use a deconvolution process. Consequently, we examine the effects of these RTD's on reconstructions of discrete-time waveforms. Four previously proposed methods for reducing the effects of the RTD on the spectra of step-like waveforms are examined for application in deconvolution, and a comparison of their effects in deconvolution is given. An analysis of the errors is given for each case.

I. INTRODUCTION

Data acquired from the measurement of a given signal are affected by the measurement instrument (such as measuring an electrical pulse with an oscilloscope). These data represent the signal as recorded within the limitations of the measurement instrument and, therefore, can be described by the convolution of the instrument's impulse response with the signal. Consequently, it is necessary to remove the effects of the instrument on the data to obtain a more accurate representation of the original signal; this is done by deconvolution. In addition, the data may have different values at the ends of the record because of the finite record length. This difference is the record truncation discontinuity (RTD), and it will cause errors in a reconstruction that uses waveform deconvolution.

The deconvolution technique we use here uses the discrete Fourier transformation (DFT's) of the data and the instrument response and, therefore, requires periodicity of the data. Consequently, if the data is step-like (i.e., the waveform has zero or near-zero slope at either end of the record and that the values at the ends of the record are not equal), the RTD's will cause oscillations in the record of the deconvolved data. These oscillations are an undesirable artifact caused by the abrupt transition and the periodicity assumed by the DFT (see Fig. 1). Therefore, to perform deconvolutions using step-like waveforms, the effects of RTD's must be removed. In this paper, we provide derivations of the techniques [1]–[5] proposed for reducing the effects of RTD’s in the spectra of step-like waveforms and derive operator equations to facilitate the analysis of the effects of these techniques in deconvolution. This information will be useful for those who have a need for deconvolving step-like waveforms and who may, in addition, desire a mathematical description of these techniques as used in waveform deconvolution.

II. BACKGROUND

The techniques examined include time-domain windowing [4], the Nicolson ramp-subtraction method [1], the Nahman-Gans (NG) record-extension method [2], and using first differences [5]. The waveforms considered here are real-valued and either step-like or impulse-like (that is, the waveforms have zero or near-zero slope and the same value at both ends of the record). The analysis of the effects of the RTD on a reconstructed waveform is more easily calculated using the frequency-domain representation of the waveforms than the time-domain representation; therefore, we begin the analysis with the Fourier transform of the time-domain convolution.

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The discrete-time convolution process is described by
\[ f_\tau = \sum_{m=0}^{N-1} g_m h_{\tau-m}, \quad 0 \leq \tau \leq 2N-1 \] (1a)
and its frequency-domain equivalent is
\[ F_k = G_k H_k, \quad 0 \leq k \leq N-1 \] (1b)
where, for the purpose of illustration, \( g_m \) is the measured event; \( h_m \) is the impulse response of the measurement system; \( f_\tau \) is the acquired signal; \( m \) is the time index; \( N \) is the number of points in the time record; \( \tau \) is the delay; \( k \) is the frequency index; and \( G_k, H_k, \) and \( F_k \) are the DFT's of \( g_m, h_m, \) and \( f_\tau \). We assume \( h_m \) is known so that \( g_m \) can be obtained by deconvolving \( h_m \) from \( f_\tau \). In the frequency domain, the deconvolution becomes a division of the frequency components,
\[ G_k = \frac{F_k}{H_k} \] (2)
and \( g_m \) is recovered by an inverse DFT of \( G_k \). We will assume for the purpose of this paper that, for the functions used and for sufficiently long epochs, (2) would give the correct reconstruction of \( g_m \). However, here we will be concerned with the case when the epoch is too short, that is, for a step-like \( f_\tau \).

It should be pointed out that (1a) represents a linear convolution process whereas (1b) represents a cyclic convolution process. These two processes provide the same result as long as time-aliasing in the cyclic process is prevented; this will be assumed here. Furthermore, if \( g_m \) and \( h_m \) are both \( N \)-point records, the result of (1a) is a \((2N-1)\)-point record. For the purpose of this paper the convolution result will be of length \( 2N \) because the record length of the acquired data, \( f_m \), can be any length desired.

This study will be performed with a single-step deconvolution (division of spectra) and will assume complete knowledge of \( h_m \). A noise-free \( f_\tau \) is used to determine and compare the errors introduced into the reconstructions by the different techniques. In Section III we examine the effects of RTD-reducing techniques on waveform reconstructions and develop operator expressions to describe these techniques. These expressions are used to analyze the effect of these techniques on waveform deconvolutions. We will consider noise in \( f_m \) in Section IV.

III. THE EFFECTS OF RECORD TRUNCATION DISCONTINUITIES

A. Time Windowing

Windowing techniques have been used to reduce the effects of RTD's on either the time or frequency representations of data [4]. The problem with windowing is that two mutually nonassociative mathematical processes are being used. Consider windowing \( f_m \) by \( w_m \); the subsequent deconvolution is given by
\[ G'_k = \frac{(G_k H_k) \ast W_k}{W_k} \] (3)
where * denotes a convolution process, and \( G'_k \) is an approximation to \( G_k \). The desired signal \( G_k \) will not be recovered because \( G_k H_k \) remains convolved with \( W_k \). Therefore, any technique that uses windowing to remove the RTD of a waveform should be used cautiously in a reconstruction. Table I shows the mean and standard deviation of the differences between the values of the original waveform and the waveforms reconstructed using windowing or one of the other techniques examined here.

B. Nicolson Ramp-Subtraction Method

1) Derivation: The Nicolson method subtracts a ramp from the acquired data [Fig. 2(a)]; the result is a waveform where the two ends of the record have the same value [Fig. 2(b)]. The Nicolson-modified record is
\[ f'_m = f_m - r_m \] (4a)
and its frequency-domain equivalent is
\[ F'_k = F_k - R_k \] (4b)
where \( r_m \) is the ramp function and \( R_k \) its spectrum,
\[ R_k = \begin{cases} \frac{f_0}{2} & k = 0 \\ \frac{-f_0}{1 - \exp(-i2\pi k/N)} & k = 1, 2, \ldots, N-1 \end{cases} \] (5)
with
\[ f_0 = \frac{N}{N-1} (f_{N-1} - f_0) \] (6)
where \( f_{N-1} \) and \( f_0 \) are the values of the last and first points, respectively, of the original record. Using (5) in (4b), \( F'_k \) becomes
\[ F'_k = \sum_{m=0}^{N-1} f_m \exp(-i2\pi km/N) + \frac{f_0}{1 - \exp(-i2\pi k/N)}; \quad k = 0 \]
\[ k = 1, 2, \ldots, N-1 \] (7)
where the summation gives the DFT of \( f'_m \).

To develop an operator equivalent of (7), first consider \( r_m \). Let \( r_m \) be the result of a convolution of two step functions with a mutual delay of one sample, the first, \( s_m \), having unit height, and the second, \( f_{\beta,m} \), having height \( f_\beta/N \). Let \( f_{\beta,m} \) be the \( N \)-point truncated record of the convolution of \( g_{\beta,m} \) with \( h_m \) such that \( f_\beta = \sum f_{\beta,m} \). Let \( g_{N-1} = g_0 \) and \( g_{N-1} \) and \( g_0 \) be the averages of the last and first points of \( g_m \). Convolving \( f_{\beta,m} \) with \( s_m \) gives \( r_m \); its frequency representation is
\[ R_k = G_{\beta,k} H_k S_k \] (8)
where \( S_k \) is the Fourier transform of \( s_m \). Using (8) in (7) yields
\[ F'_k = G_k H_k + G_{\beta,k} H_k S_k \] (9)
Equation (9) will be used in the deconvolution analysis.
2) Deconvolution Effects: The deconvolution of \( h_m \) from the Nicolson-modified record of \( f_m \) is, in the frequency domain, given by

\[
G'_{k} = \frac{F'_{k}}{H_{k}} = \frac{G_{k}H_{k} + G_{h,k}H_{k}S_{k}}{H_{k}} = G_{k} + G_{h,k}S_{k},
\]

(10)

The \( g_m \) is recovered by an inverse transform of \( G'_k \) and then a subtraction of a ramp. This ramp is obtained by dividing \( r_m \) by \( \delta h_m \) for all \( m \). An offset correction is also necessary because \( f_0 \) and \( g_0 \) do not contain information on the absolute position. The offset correction is done by subtracting \( g_0 \) (\( g_{m} \) is the inverse DFT of \( G'_{k} \) from \( g_{m} \)) and then adding \( f_0/\delta h_m \) to \( g_{m} \), for all \( m \). The next equation shows the results for the deconvolution of one step-like waveform by another:

\[
G'_k = \frac{G_{k}H_{k} + G_{h,k}H_{k}S_{k}}{H_{k}} = G_{k}.
\]

(11)

Only the offset correction is required in this case.

C. Nahman–Gans Method

1) Derivation: The NG operation consists of negating \( f_m \) (\( f_m = 0 \) for \( m < 0 \) and \( m > N - 1 \)), delaying this negated version of \( f_m \) by \( N \) points, adding it to the original record, and then adding a square pulse to force the end values to be the same [Fig. 2(c)]. Call this new function \( f'_m \). The square pulse is zero-valued for \( m < N \) and for \( m \)

\[
\text{TABLE I}
\]

<table>
<thead>
<tr>
<th>Time-Window</th>
<th>Nicolson</th>
<th>Nahman–Gans</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{1,m} ) ( * q_{2,m} )/( q_{1,m} )</td>
<td>-1.26e-01</td>
<td>4.16e-10</td>
<td>4.46e-09</td>
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<tr>
<td>( q_{1,m} ) ( * q_{3,m} )/( q_{1,m} )</td>
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<td>3.75e-05</td>
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<tr>
<td>( q_{1,m} ) ( * q_{4,m} )/( q_{1,m} )</td>
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<td>9.52e-06</td>
</tr>
<tr>
<td>( q_{1,m} ) ( * q_{5,m} )/( q_{1,m} )</td>
<td>-1.83e-01</td>
<td>8.78e-06</td>
<td>9.64e-06</td>
</tr>
<tr>
<td>( q_{1,m} ) ( * q_{6,m} )/( q_{1,m} )</td>
<td>-2.24e-02</td>
<td>-9.76e-05</td>
<td>2.96e-05</td>
</tr>
<tr>
<td>( q_{1,m} ) ( * q_{7,m} )/( q_{1,m} )</td>
<td>6.93e-01</td>
<td>4.28e-05</td>
<td>3.76e-05</td>
</tr>
<tr>
<td>( q_{1,m} ) ( * q_{8,m} )/( q_{1,m} )</td>
<td>1.27e-01</td>
<td>4.50e-06</td>
<td>3.75e-05</td>
</tr>
<tr>
<td>( p_{1,m} ) ( * q_{2,m} )/( q_{1,m} )</td>
<td>4.89e-01</td>
<td>1.37e-04</td>
<td>1.44e-06</td>
</tr>
<tr>
<td>( p_{1,m} ) ( * q_{3,m} )/( q_{1,m} )</td>
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<td>-9.36e-04</td>
<td>1.87e-05</td>
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<tr>
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<td>4.81e-05</td>
<td>1.87e-05</td>
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<tr>
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<tr>
<td>( p_{1,m} ) ( * q_{6,m} )/( q_{1,m} )</td>
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<td>5.75e-04</td>
<td>3.31e-06</td>
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<tr>
<td>( p_{1,m} ) ( * q_{7,m} )/( q_{1,m} )</td>
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</tr>
<tr>
<td>Deviation</td>
<td>1.43e-00</td>
<td>1.99e-04</td>
<td>1.41e-05</td>
</tr>
</tbody>
</table>

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\( N \) has a value equal to the sum of the values of the first and last points of the original record. The DFT of \( f'_n \) is

\[
F'_k = \sum_{m=0}^{N-1} f'_m \exp\left(-\frac{i2\pi km}{2N}\right) + \sum_{m=N-1}^{2N-1} f'_m \exp\left(-\frac{i2\pi km}{2N}\right)
\]

\( = \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) + \sum_{m=N}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \exp(-i\pi k)
\]

\( = \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) - \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \exp(-i\pi k). \)  

(12a)

Letting \( f_a = f_{N-1} + f_0 \) and rewriting (12a):

\[
F'_k = \begin{cases} 
Nf_a; & k = 0 \\
0; & k = 2, 4, \ldots, 2N - 2 \\
2 \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) & k = 1, 3, \ldots, 2N - 1. 
\end{cases}
\]

(12b)

We now develop an operator expression to describe the NG technique that can be used to analyze the reconstruction results. The NG extension process can be described by a convolution of \( f_m \) and the step-like function with another function, \( d_m \), and then adding these two new waveforms. Let the function for the convolution process, \( d_{m,0} \), be equal to 1 for \( m = 0 \), -1 for \( m \geq N \), and zero-valued for all other \( m \). We want to examine

\[
f'_m = f_m * d_m + f_{a,m} * d_m + \frac{f_a}{2} = f_{1,m} + f_{2,m} + \frac{f_a}{2}.
\]

(13)

where \( f_{a,m} \) is the step-like function. The frequency representation of \( f_{1,m} \) is

\[
F_{1,k} = F_k D_k = \left( \sum_{m=0}^{2N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( \sum_{m=0}^{2N-1} d_m \exp\left(-\frac{i\pi km}{N}\right) \right) - \left( \sum_{m=N}^{2N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( \sum_{m=0}^{2N-1} d_m \exp\left(-\frac{i\pi km}{N}\right) \right) \]

\( = \left( \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( 1 - \exp(-i\pi k) \right) \]

(14)

\[
0; \quad k = 0, 2, 4, \ldots, 2N - 2,
\]

\[
N-1 \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right); \quad k = 1, 3, \ldots, 2N - 1.
\]

The change of the upper limit from \( 2N - 1 \) to \( N - 1 \) is possible because \( f_m \) is zero-valued for \( m \geq N \). Now consider \( F_{2,k} \). The step-like function \( f_{a,m} = f_a/2 \) for \( 0 \leq m \leq N - 1, 0 \) for all other \( m \), and arises from the convolution of \( g_{a,m} \) (a function with value \( g_a \) over some \( m \) so that \( f_a = g_a \sum h_m \), where \( g_a = g_{N-1} + g_0 \)) and \( h_m \). Convolving \( f_{a,m} \) with \( d_m \), we get

\[
F_{2,k} = F_{a,k} D_k.
\]

(15)

We now develop an operator expression to describe the NG technique that can be used to analyze the reconstruction results. The NG extension process can be described by a convolution of \( f_m \) and the step-like function with another function, \( d_m \), and then adding these two new waveforms. Let the function for the convolution process, \( d_{m,0} \), be equal to 1 for \( m = 0 \), -1 for \( m \geq N \), and zero-valued for all other \( m \). We want to examine

\[
f'_m = f_m * d_m + f_{a,m} * d_m + \frac{f_a}{2} = f_{1,m} + f_{2,m} + \frac{f_a}{2}.
\]

(13)

where \( f_{a,m} \) is the step-like function. The frequency representation of \( f_{1,m} \) is

\[
F_{1,k} = F_k D_k = \left( \sum_{m=0}^{2N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( \sum_{m=0}^{2N-1} d_m \exp\left(-\frac{i\pi km}{N}\right) \right) - \left( \sum_{m=N}^{2N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( \sum_{m=0}^{2N-1} d_m \exp\left(-\frac{i\pi km}{N}\right) \right) \]

\( = \left( \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right) \right) \left( 1 - \exp(-i\pi k) \right) \]

(14)

\[
0; \quad k = 0, 2, 4, \ldots, 2N - 2,
\]

\[
N-1 \sum_{m=0}^{N-1} f_m \exp\left(-\frac{i\pi km}{N}\right); \quad k = 1, 3, \ldots, 2N - 1.
\]

2) Deconvolution Effects: First consider the deconvolution where \( h_m \) is impulse-like,

\[
G'_k = \frac{G_k H_k D_k - G_{a,k} H_k D_k}{2} + N g_a \delta(k) \sum h_m.
\]

(17b)

Equation (17a) is identical to (12b); therefore, (17b) will be used to examine the errors associated with the NG-extended records on waveform reconstructions.

2) Deconvolution Effects: First consider the deconvolution where \( h_m \) is impulse-like,

\[
G'_k = \left[ G_k H_k D_k - \frac{G_{a,k} H_k D_k}{2} + N g_a \delta(k) \sum h_m \right] H_k
\]

(18)
This is the spectrum for an NG-extended record, and \( g_m \) is recovered by simply ignoring the reconstructed record for \( m \geq N \). The reconstruction error statistics are listed in Table I.

Now consider the case for \( h_m \) being step-like. Let \( H_i = 2H_k - H_{-k} \) be the Fourier transform of the NG-extended \( h_m \). Because \( F_k \) is described by (12a), \( F_k \) should have been produced by a convolution of one NG-extended waveform and one nonextended waveform. Therefore, either \( g_m \) or \( h_m \) should be NG-extended. Choosing \( h_m \) to be NG-extended gives the resultant deconvolution,

\[
G_k' = \frac{G_k H_k D_k - H_{0,k} G_k D_k}{2} + N g_m \delta(k) \sum_m h_m
\]

\[
= \frac{2G_k H_k - H_{0,k} G_k}{H_k^*}; \quad k = 1, 3, \ldots, 2N - 1
\]

\[
= \text{NG}_w \delta(k); \quad k = 0
\]

\[
= G_k' \neq G_k.
\]

The \( G_k' \neq G_k \) because \( G_k \) is undefined where \( H_k = 0 \), that is, for \( k = 2, 4, \ldots, 2N - 2 \). We can set \( G_k' = 0 \) for \( k = 2, 4, \ldots, 2N - 2 \), but in doing so \( g_m \) becomes NG-extended (to get \( g_m' \)). Consequently, \( f_m \) is the result of the convolution of two NG-extended records, \( g_m \) and \( h_m \). Therefore, we need to examine how this apparent convolution of two NG-extended records came about.

The linear-time convolution of \( g_m \) and \( h_m \), both of length \( 2N \), produces a \( 4N \)-point record, and its frequency representation is

\[
F_j^* = \sum_{m = 0}^{4N - 1} f_m \exp(-\text{j}2\pi jm/4N), \quad 0 \leq j \leq 4N - 1.
\]

A subsequent deconvolution of \( h'_m \) (\( k \)-padded from \( 2N \) to \( 4N - 1 \)) from (20b), followed by an inverse DFT, then ignoring the \( g'_m \)-padded portion of this time record (\( 2N \) to \( 4N - 1 \)), and finally performing a DFT on the new \( 2N \)-point time record gives

\[
G_k^* = \frac{1}{2} \sum_{m = 0}^{4N - 1} g_m \exp(-\text{i}\pi km/N)
\]

\[
0 \leq k \leq 2N - 1
\]

where \( g_m^* \) is the NG-extended \( g_m \). Comparing (21) to (12a) shows that \( G_k^* \) is the spectrum of an NG-extended \( g_m \) multiplied by \( 1/2 \). Therefore, we get \( g_m \) by multiplying (21) by two, inverse transforming, and ignoring the reconstructed record for \( m \geq N \).

At this point it is worthwhile to rectify a few erroneous claims regarding the NG method. The conclusion that the Nicolson and NG techniques provide identical frequency information [7] is not correct. We have obtained analytic expressions for both the Nicolson and NG modified records and tested the accuracy of these expressions. The equations describing the two processes do not give identical spectra, even when ignoring the zeroes for the NG spectrum. The difference can also be verified by deconvolution. The dc component (\( k = 0 \)) is not zero [2] nor is it twice that of the unextended data’s spectrum [8]. The dc component is \( N f_u \), as shown in (12b). Another claim pertains to the power of the NG-extended waveform. The nonzero frequency components of the NG spectrum are not always related to the DFT spectrum of the original data record by a factor of two [8] as can be seen from (12b). The power of an NG-extended record, in terms of the power of an unextended record, is

\[
P_{NG} = \sum_{m = 0}^{N - 1} f_m^2 + \sum_{m = N}^{2N - 1} (f_m - f_{m+N})^2
\]

\[
= \sum_{m = 0}^{N - 1} (2f_m^2 + f_0^2 - 2f_m f_{m+N})
\]

\[
= 2P + Nf_u^2 - 2f_u \sum_{m = 0}^{N - 1} f_m
\]

where \( P \) is the power of the unextended waveform.

D. First Difference

The first-difference technique maintains the original record length and increases the noise power of the data. Let the first difference of the data be given by the convolution of \( f_m \) with the first-difference operator \( v_m \),

\[
f_m^{(1)} = f_m * v_m
\]

where the superscript “(1)” indicates the first difference and \( v_0 = 1, v_1 = -1 \), and \( v_m = 0 \) otherwise. The frequency-domain equivalent of (23a) is

\[
F^{(1)} = F_k V_k = G_k H_k V_k = G_k \text{H}_k^{(1)} \quad \text{or} \quad G_k^{(1)} \text{H}_k
\]

where \( V_k \) is the Fourier transform of \( v_m \) and is given by

\[
V_k = 1 - \exp(-\text{i}2\pi k/N)
\]
and $H_k^{(1)}$ and $G_k^{(1)}$ are the Fourier transforms of the successive differences of $h_m$ and $g_m$, respectively. If $H_k^{(1)}$ is used in (23b), $G_i$ is obtained by dividing $F_k^{(1)}$ by the difference of $H_k$. A subsequent inverse Fourier transform of $G_i$ yields $g_m$. On the other hand, if $H_k^{(1)}$ is used in (23b), we first divide $F_k^{(1)}$ by $H_k$ and then perform an inverse transform on $G_k^{(1)}$ to get $g_m^{(1)}$. The desired signal $g_m$ is recovered by integrating (running sum) $g_m^{(1)}$. An offset error may occur, and it is corrected the same way as done for the Nicolson method. Deconvolution error statistics are shown in Table I.

IV. NOISE

The effects of additive noise on a waveform reconstruction can be described by

\[ F_k^{(\text{Nc})} = (G_k + A_{1,k})H_k + (G_{o,k} + A_{1,\delta,k})H_k S_k + A_{2,k} \]
\[ F_k^{(\text{NG})} = (G_k + A_{1,k})H_k D_k - (G_{o,k} + A_{1,\alpha,k})H_k D_k \]
\[ F_k^{(\text{DF})} = (G_k + A_{1,k})H_k V_k + A_{2,k} \]

(25)

where $A_{1,k}$ and $A_{2,k}$ are pre- and postconvolution additive noise, and the superscripts on $F_k$ refer to the given process. $H_k$ is assumed to be noise free. Division of the expressions in (25) by $H_k$ causes amplification of the high-frequency components of $A_{2,k}$. Consequently, the deconvolution is unstable, and the reconstructed waveform is not representative of the original input. Techniques are available for dealing with this problem, but this is not a topic of this paper. If $A_{2,k} = 0$, however, the reconstruction may resemble the input. Table II shows statistics of the results of reconstructions with varying levels of preconvolution additive noise. The uniformly distributed noise $A_m$ was generated using a pseudorandom number generator and has an amplitude range of $-0.5$ to $0.5$.

V. CONCLUSIONS

We have examined techniques that reduce the effects of record truncation discontinuities in the spectra of step-like waveforms for application in waveform reconstructions. These techniques include time-domain windowing, the Nicolson ramp-subtraction technique, the Nahman-Gans record-extension method, and taking first differences of the data. We have shown that some of these techniques can be used as a predeconvolution process to facilitate the return of an accurate waveform reconstruction.

The Nicolson, Nahman-Gans, and difference methods do not produce significant error in the reconstructed waveform, whereas windowing may. Our observations show that an offset error may occur for all techniques and, neglecting this offset error, the errors in the reconstructed waveforms for deconvolutions using the Nahman-Gans technique are typically less than the errors caused by the other techniques. The trends displayed in Table I were also observed using other test waveforms, the data of which is not shown. The Nicolson method, however, was the least affected by preconvolution additive noise. Mathematical operations were introduced to describe the Nicolson, Nahman-Gans, and first-difference techniques, and, in using these, we have been able to accurately determine the nonmachine-induced errors in the reconstructed waveform.

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