Reducing the Effects of Record Truncation Discontinuities in Waveform Reconstructions

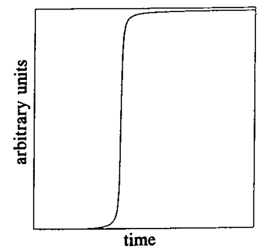
Nicholas G. Paulter, Jr. and Robert B. Stafford

***Abstract*—Record truncation discontinuities (RTD's) are artifacts in recorded data caused by the difference between the values of the data at the two ends of the record. The RTD causes errors in waveform reconstructions, in particular, in digital reconstructions that use a deconvolution process. Consequently, we examine the effects of these RTD's on reconstructions of discrete-time waveforms. Four previously proposed methods for reducing the effects of the RTD on the spectra of step-like waveforms are examined for application in deconvolution, and a comparison of their effects in deconvolution is given. An analysis of the errors is given for each case.**

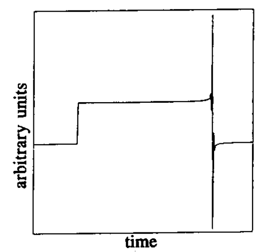
I. Introduction

DATA acquired from the measurement of a given signal are affected by the measurement instrument (such as measuring an electrical pulse with an oscilloscope). These data represent the signal as recorded within the limitations of the measurement instrument and, therefore, can be described by the convolution of the instrument's impulse response with the signal. Consequently, it is necessary to remove the effects of the instrument on the data to obtain a more accurate representation of the original signal; this is done by deconvolution. In addition, the data may have different values at the ends of the record because of the finite record length. This difference is the record truncation discontinuity (RTD), and it will cause errors in a reconstruction that uses waveform deconvolution.

The deconvolution technique we use here uses the discrete Fourier transformation (DFT's) of the data and the instrument response and, therefore, requires periodicity of the data. Consequently, if the data is step-like (that is, the waveform has zero or near-zero slope at either end of the record and that the values at the ends of the record are not equal), the RTD's will cause oscillations in the record of the deconvolved data. These oscillations are an undesirable artifact caused by the abrupt transition and the periodicity assumed by the DFT (see Fig. 1). Therefore, to perform deconvolutions using step-like waveforms, the effects of RTD's must be removed. In this paper, we provide derivations of the techniques [1]–[5] proposed for reducing the effects of RTD's in the spectra of step-like waveforms and derive operator equations to facilitate the analysis of the effects of these techniques in deconvolution. This information will be useful for those who have a need for deconvolving step-like waveforms and who may, in addition, desire a mathematical description of these techniques as used in waveform deconvolution.



(a)



(b)

Fig. 1. Effect of RTD on deconvolution result. (a) The deconvolution input waveform (b) The deconvolution result when the RTD is ignored. Oscillations on the right side of (b) are caused by the RTD.

II. Background

The techniques examined include time-domain windowing [4], the Nicolson ramp-subtraction method [1], the Nahman–Gans (NG) record-extension method [2], and using first differences [5]. The waveforms considered here are real-valued and either step-like or impulse-like (that is, the waveforms have zero or near-zero slope and the same value at both ends of the record). The analysis of the effects of the RTD on a reconstructed waveform is more easily calculated using the frequency-domain representation of the waveforms than the time-domain representation; therefore, we begin the analysis with the Fourier transform of the time-domain convolution.

Manuscript received December 9, 1991; revised August 4, 1992.
The authors are with the Electromagnetic Fields Division, National Institute of Standards and Technology, Boulder, CO 80303.
IEEE Log Number 9207676.

0018-9456/93$03.00 © 1993 IEEE

The discrete-time convolution process is described by

$$f_\tau = \sum_{m=0}^{N-1} g_m h_{\tau-m}, \qquad 0 \leq \tau \leq 2N - 1 \tag{1a}$$

and its frequency-domain equivalent is

$$F_k = G_k H_k, \qquad 0 \leq k \leq N - 1 \tag{1b}$$

where, for the purpose of illustration, g_m is the measured event; h_m is the impulse response of the measurement system; f_τ is the acquired signal; m is the time index; N is the number of points in the time record; τ is the delay; k is the frequency index; and G_k, H_k, and F_k are the DFT's of g_m, h_m, and f_m. We assume h_m is known so that g_m can be obtained by deconvolving h_m from f_m. In the frequency domain, the deconvolution becomes a division of the frequency components,

$$G_k = \frac{F_k}{H_k} \tag{2}$$

and g_m is recovered by an inverse DFT of G_k. We will assume for the purpose of this paper that, for the functions used and for sufficiently long epochs, (2) would give the correct reconstruction of g_m. However, here we will be concerned with the case when the epoch is too short, that is, for a step-like f_m.

It should be pointed out that (1a) represents a linear convolution process whereas (1b) represents a cyclic convolution process. These two processes provide the same result as long as time-aliasing in the cyclic process is prevented; this will be assumed here. Furthermore, if g_m and h_m are both N-point records, the result of (1a) is a $(2N - 1)$-point record. For the purpose of this paper the convolution result will be of length $2N$ because the record length of the acquired data, f_m, can be any length desired.

This study will be performed with a single-step deconvolution (division of spectra) and will assume complete knowledge of h_m. A noise-free f_m is used to determine and compare the errors introduced into the reconstructions by the different techniques. In Section III we examine the effects of RTD-reducing techniques on waveform reconstructions and develop operator expressions to describe these techniques. These expressions are used to analyze the effect of these techniques on waveform deconvolutions. We will consider noise in f_m in Section IV.

III. The Effects of Record Truncation Discontinuities

A. Time Windowing

Windowing techniques have been used to reduce the effects of RTD's on either the time or frequency representations of data [4]. The problem with windowing is that two mutually nonassociative mathematical processes are being used. Consider windowing f_m by w_m; the subsequent deconvolution is given by

$$G_k' = \frac{(G_k H_k) * W_k}{H_k} \tag{3}$$

where $*$ denotes a convolution process, and G_k' is an approximation to G_k. The desired signal G_k will not be recovered because $G_k H_k$ remains convolved with W_k. Therefore, any technique that uses windowing to remove the RTD of a waveform should be used cautiously in a reconstruction. Table I shows the mean and standard deviation of the differences between the values of the original waveform and the waveforms reconstructed using windowing or one of the other techniques examined here.

B. Nicolson Ramp-Subtraction Method

1) Derivation: The Nicolson method subtracts a ramp from the acquired data [Fig. 2(a)]; the result is a waveform where the two ends of the record have the same value [Fig. 2(b)]. The Nicolson-modified record is

$$f_m' = f_m - r_m \tag{4a}$$

and its frequency-domain equivalent is

$$F_k' = F_k - R_k. \tag{4b}$$

r_m is the ramp function and R_k its spectrum,

$$R_k = \begin{cases} f_\beta \dfrac{(N-1)}{2}; & k = 0 \\[2ex] \dfrac{-f_\beta}{1 - \exp(-i2\pi k/N)}; & k = 1, 2, \cdots, N-1 \end{cases} \tag{5}$$

with

$$f_\beta = \left[\frac{N}{N-1}\right](f_{N-1} - f_0) \tag{6}$$

where f_{N-1} and f_0 are the values of the last and first points, respectively, of the original record. Using (5) in (4b), F_k' becomes

$$F_k' = \begin{cases} \displaystyle\sum_{m=0}^{n-1} f_m - f_\beta \frac{(n-1)}{2}; & k = 0 \\[2ex] \displaystyle\sum_{m=0}^{N-1} f_m \exp(-i2\pi km/N) + \frac{f_\beta}{1 - \exp(-i2\pi k/N)}; & k = 1, 2, \cdots, N-1 \end{cases} \tag{7}$$

where the summation gives the DFT of f_m.

To develop an operator equivalent of (7), first consider r_m. Let r_m be the result of a convolution of two step functions with a mutual delay of one sample, the first, s_m, having unit height, and the second, $f_{\beta,m}$, having height f_β/N. Let $f_{\beta,m}$ be the N-point truncated record of the convolution of $g_{\beta,m}$ with h_m such that $f_\beta = g_\beta \Sigma h_m$, where $g_\beta = g_{N-1} - g_0$, and g_{N-1} and g_0 are the values of the last and first points of g_m. Convolving $f_{\beta,m}$ with s_m gives r_m; its frequency representation is

$$R_k = G_{\beta,k} H_k S_k \tag{8}$$

where S_k is the Fourier transform of s_m. Using (8) in (7) yields

$$F_k' = G_k H_k + G_{\beta,k} H_k S_k. \tag{9}$$

Equation (9) will be used in the deconvolution analysis.

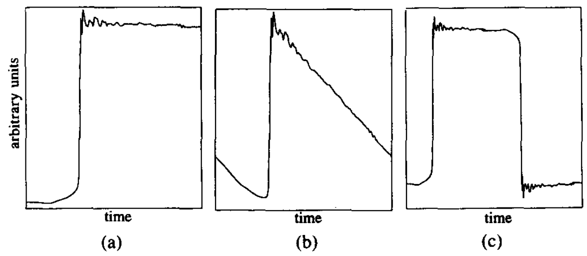


Fig. 2. The effect of the Nicolson ramp-subtraction technique and the Nahman–Gans record-extension technique on a waveform. (a) The original record. (b) The Nicolson-modified record. (c) The Nahman–Gans-modified record.

TABLE I

MEAN VALUE AND STANDARD DEVIATION OF ERRORS IN THE RECONSTRUCTED WAVEFORMS FOR DECONVOLUTIONS USING THE FOUR METHODS INDICATED. THE $p_{i,m}$ ARE STEP-LIKE WAVEFORMS, AND THE $q_{i,m}$ ARE IMPULSE-LIKE WAVEFORMS. THE FUNCTIONS DESCRIBING THEM ARE: $q_{1,m} = 1/[1 + m/\tau]$, $q_{2,m} = \exp[-(m/\tau)^2]$, $q_{3,m} = \exp(-m/\tau)$, $p_{1,m} = u(m - \tau)$ (UNIT STEP FUNCTION), AND $p_{2,m} = \exp(-m/\tau_1)[1 - \exp(m/\tau_2)]$, WHERE τ IS A TIME CONSTANT. THE VALUES OF ALL WAVEFORMS RANGE FROM 0 TO 1 AND ARE 1024 POINTS LONG. THE "*" INDICATES A CONVOLUTION AND "/" A DECONVOLUTION. THE MEAN ERROR VALUE IS THE TOP LISTING OF EACH PAIR. THE FIRST FIVE ENTRIES FOR THE NICOLSON METHOD ACTUALLY REPRESENT ERRORS IN A DECONVOLUTION WHERE NO RECORD MODIFICATIONS WERE PERFORMED (THE RAMPS WERE ZERO-VALUED); THEREFORE, THIS IS A MEASURE OF THE COMPUTER-INDUCED ERROR. THE OFFSET CORRECTION FOR THE NICOLSON AND DIFFERENCE TECHNIQUES WAS PERFORMED BEFORE THE MEAN ERROR WAS CALCULATED.

	Time-Window	Nicolson	Nahman–Gans	Difference
$q_{1,m} * q_{2,m}/q_{1,m}$	−1.26e-01	4.16e-10	4.46e-09	9.24e-13
	4.35e-01	1.26e-07	1.27e-07	1.36e-03
$q_{1,m} * q_{3,m}/q_{1,m}$	−1.56e-02	3.75e-05	3.75e-05	1.34e-08
	1.26e-01	5.61e-05	3.75e-05	5.25e-02
$q_{1,m} * q_{3,m}/q_{3,m}$	−1.64e-02	4.12e-05	9.52e-06	3.52e-09
	1.83e-01	8.78e-06	9.64e-06	6.84e-02
$q_{3,m} * q_{2,m}/q_{3,m}$	−8.24e-02	−9.76e-05	2.95e-05	4.52e-09
	6.93e-01	4.28e-05	2.96e-05	1.36e-03
$q_{3,m} * q_{3,m}/q_{3,m}$	−9.52e-03	4.53e-05	3.75e-05	1.34e-08
	1.27e-01	4.50e-06	3.75e-05	5.25e-02
$p_{1,m} * q_{2,m}/p_{1,m}$	−1.18e-01	−9.50e-04	−5.66e-09	8.40e-10
	4.89e-01	1.37e-04	1.44e-06	1.44e-06
$p_{1,m} * q_{3,m}/p_{1,m}$	−1.43e-02	−9.36e-04	1.87e-05	3.75e-05
	1.23e-01	4.81e-05	1.87e-05	3.73e-07
$p_{1,m} * q_{3,m}/q_{3,m}$	−4.44e-02	−1.22e-03	3.21e-06	1.95e-03
	4.95e-00	5.75e-04	3.31e-06	6.25e-02
$p_{2,m} * q_{1,m}/p_{2,m}$	−3.12e-02	4.44e-05	1.88e-05	3.75e-10
	2.28e-01	4.33e-04	2.00e-05	6.82e-06
Mean	−5.09e-02	−3.46e-04	1.72e-05	2.25e-04
	4.35e-02	4.95e-04	1.42e-05	6.10e-04
Standard	9.17e-01	1.45e-04	1.75e-05	2.65e-02
Deviation	1.43e-00	1.99e-04	1.41e-05	2.94e-02

2) Deconvolution Effects: The deconvolution of h_m from the Nicolson-modified record of f_m is, in the frequency domain, given by

$$G_k' = \frac{F_k'}{H_k} = \frac{G_kH_k + G_{\beta,k}H_kS_k}{H_k} = G_k + G_{\beta,k}S_k. \quad (10)$$

The g_m is recovered by an inverse transform of G_k' and then a subtraction of a ramp. This ramp is obtained by dividing r_m by Σh_m for all m. An offset correction is also necessary because f_β and g_β do not contain information on the absolute position. The offset correction is done by subtracting g_0' (g_m' is the inverse DFT of G_k' from g_m and then adding $f_0/\Sigma h_m$ to g_m, for all m. The next equation shows the results for the deconvolution of one step-like waveform by another:

$$G_k' = \frac{G_kH_k + G_kH_{\beta,k}S_k}{H_k + H_{\beta,k}S_k} = G_k. \quad (11)$$

Only the offset correction is required in this case.

C. Nahman–Gans Method

1) Derivation: The NG operation consists of negating f_m ($f_m = 0$ for $m < 0$ and $m > N - 1$), delaying this negated version of f_m by N points, adding it to the original record, and then adding a square pulse to force the end values to be the same [Fig. 2(c)]. Call this new function f_m'. The square pulse is zero-valued for $m < N$ and for m

$\geq N$ has a value equal to the sum of the values of the first and last points of the original record. The DFT of f'_m is

$$\begin{aligned} F'_k &= \sum_{m=0}^{N-1} f_m \exp(-i2\pi km/2N) \\ &\quad + \sum_{m=N}^{2N-1} [f_{N-1} + f_0 - f_{m-N}] \exp(-i2\pi km/2N) \\ &= \sum_{m=0}^{N-1} f_m \exp(-i\pi km/N) \\ &\quad + \sum_{m=0}^{N-1} [f_{N-1} + f_0] \exp(-i\pi km/N) \exp(-i\pi k) \\ &\quad - \sum_{m=0}^{N-1} f_m \exp(-i\pi km/N) \exp(-i\pi k). \end{aligned} \tag{12a}$$

Letting $f_\alpha = f_{N-1} + f_0$ and rewriting (12a):

$$F'_k = \begin{cases} Nf_\alpha; \quad k = 0 \\ 0; \quad k = 2, 4, \cdots, 2N-2 \\ 2\sum_{m=0}^{N-1} f_m \exp(-i\pi km/N) \\ \qquad - f_\alpha \sum_{m=0}^{N-1} \exp(-i\pi km/N) \\ \qquad = 2\sum_{m=0}^{N-1} f_m \exp(-i\pi km/N) \\ \qquad - \dfrac{2f_\alpha}{1 - \exp(-i\pi k/N)}; \\ \qquad\qquad k = 1, 3, \cdots, 2N-1. \end{cases} \tag{12b}$$

We now develop an operator expression to describe the NG technique that can be used to analyze the reconstruction results. The NG extension process can be described by a convolution of f_m and the step-like function with another function, d_m, and then adding these two new waveforms. Let the function for the convolution process, d_m, be equal to 1 for $m = 0$, -1 for $m \geq N$, and zero-valued for all other m. We want to examine

$$f'_m = f_m * d_m + f_{\alpha,m} * d_m + \frac{f_\alpha}{2} = f_{1,m} + f_{2,m} + \frac{f_\alpha}{2}, \tag{13}$$

where $f_{\alpha,m}$ is the step-like function. The frequency representation of $f_{1,m}$ is

$$\begin{aligned} F_{1,k} &= F_k D_k \\ &= \left(\sum_{m=0}^{2N-1} f_m \exp(-i\pi km/N)\right) \\ &\quad \cdot \left(\sum_{m=0}^{2N-1} d_m \exp(-i\pi km/N)\right) \\ &= \left(\sum_{m=0}^{N-1} f_m \exp(-i\pi km/N)\right) \{1 - \exp(-i\pi k)\} \\ &= \begin{cases} 0; \quad k = 0, 2, 4, \cdots, 2N-2, \\ 2\sum_{m=0}^{N-1} f_m \exp(-i\pi km/N); \\ \qquad k = 1, 3, \cdots, 2N-1. \end{cases} \end{aligned} \tag{14}$$

The change of the upper limit from $2N - 1$ to $N - 1$ is possible because f_m is zero-valued for $m \geq N$. Now consider $f_{2,m}$. The step-like function $f_{\alpha,m} = f_\alpha/2$ for $0 \leq m \leq N - 1$, 0 for all other m, and arises from the convolution of $g_{\alpha,m}$ (a function with value g_α over some m so that $f_\alpha = g_\alpha \Sigma h_m$, where $g_\alpha = g_{N-1} + g_0$) and h_m. Convolving $f_{\alpha,m}$ with d_m, we get

$$\begin{aligned} F_{2,k} &= F_{\alpha,k} D_k, \\ &= \begin{cases} 0; \quad k = 0, 2, 4, \cdots, 2N-2, \\ 2\,\tfrac{1}{2}\sum_{m=0}^{N-1} f_{\alpha,m} \exp(-i\pi km/N); \\ \qquad k = 1, 3, \cdots, 2N-1, \end{cases} \end{aligned} \tag{15}$$

where $F_{\alpha,k}$ is the DFT of $f_{\alpha,m}$. Adding an N-point time-shifted version of $F_{2,k}$ to $F_{1,k}$ we get for F'_k,

$$\begin{aligned} F'_k &= F_{1,k} - F_{2,k} + Nf_\alpha \delta(k) \\ &= F_k D_k - F_{\alpha,k} D_k + Nf_\alpha \delta(k), \end{aligned} \tag{16}$$

where $\delta(0) = 1$. Using (14) and (15) in (16), we have

$$F'_k = \begin{cases} Nf_\alpha; \quad k = 0, \\ 0; \quad k = 2, 4, \cdots, 2N-2 \\ 2\sum_{m=0}^{N-1} f_m \exp(-i\pi km/N) \\ - \sum_{m=0}^{N-1} f_{\alpha,m} \exp(-i\pi km/N) \\ \qquad k = 1, 3, \cdots, 2N-1, \end{cases} \tag{17a}$$

or

$$F'_k = G_k H_k D_k - \frac{G_{\alpha,k} H_k D_k}{2} + Ng_\alpha \delta(k) \sum_m h_m. \tag{17b}$$

Equation (17a) is identical to (12b); therefore, (17b) will be used to examine the errors associated with the NG-extended records on waveform reconstructions.

2) Deconvolution Effects: First consider the deconvolution where h_m is impulse-like,

$$\begin{aligned} G'_k &= \frac{\left[G_k H_k D_k - \dfrac{G_{\alpha,k} H_k D_k}{2} + Ng_\alpha \delta(k) \sum_m h_m\right]}{H_k} \\ &= G_k D_k - \frac{G_{\alpha,k} D_k}{2} + Ng_\alpha \delta(k). \end{aligned} \tag{18}$$

This is the spectrum for an NG-extended record, and g_m is recovered by simply ignoring the reconstructed record for $m \geq N$. The reconstruction error statistics are listed in Table I.

Now consider the case for h_m being step-like. Let $H'_k = 2H_k - H_{\alpha,k}$ be the Fourier transform of the NG-extended h_m. Because F'_k is described by (12a), f'_m should have been produced by a convolution of one NG-extended waveform and one nonextended waveform. Therefore, either g_m or h_m should be NG-extended. Choosing h_m to be NG-extended gives the resultant deconvolution,

$$G'_k = \frac{G_k H_k D_k - \dfrac{H_{\alpha,k} G_k D_k}{2} + N g_\alpha \delta(k) \sum_m h_m}{H_k D_k - \dfrac{H_{\alpha,k} D_k}{2} + \delta(k) \sum_m h_m}$$

$$= \begin{cases} \dfrac{2G_k H_k - H_{\alpha,k} G_k}{H'_k}; & k = 1, 3, \cdots, 2N-1 \\ N g_\alpha \delta(k); & k = 0 \end{cases}$$

$$= G''_k \neq G_k. \tag{19}$$

The $G''_k \neq G_k$ because G'_k is undefined where $H'_k = 0$, that is, for $k = 2, 4, \cdots, 2N-2$. We can set $G''_k = 0$ for $k = 2, 4, \cdots, 2N-2$, but in doing so g_m becomes NG-extended (to get g'_m). Consequently, f'_m is the result of the convolution of two NG-extended records, g'_m and h'_m. Therefore, we need to examine how this apparent convolution of two NG-extended records came about.

The linear-time convolution of g'_m and h'_m, both of length $2N$, produces a $4N$-point record, and its frequency representation is

$$F''_j = \sum_{m=0}^{4N-1} f'_M \exp(-i2\pi jm/4N), \qquad 0 \leq j \leq 4N-1. \tag{20a}$$

The cyclic convolution [7] (1b), on the other hand, produces a $4N$-point record from the multiplication of the spectra of two $4N$-point records where each record is zero-valued for a least $2N$ points. For the cyclic convolution, the two $4N$-point records are the replicas of NG-extended waveforms from 0 to $2N - 1$ and are zero-padded from $2N$ to $4N - 1$. Consequently, the convolution record is also zero-padded from $2N$ to $4N - 1$. Equation (12a) with $f_\alpha = 0$ shows that for any NG-extended impulse-like (including an already NG-extended) waveform, each frequency component will be two times that of a zero-padded waveform. Therefore, for this case, the spectrum of the cyclic convolution should be half that of the linear convolution, so that (20a) becomes

$$F''_j = \frac{1}{2} \sum_{m=0}^{4N-1} f'_m \exp(-i2\pi jm/4N)$$
$$0 \leq j \leq 4N-1. \tag{20b}$$

A subsequent deconvolution of h'_m (h_0-padded from $2N$ to $4N - 1$) from (20b), followed by an inverse DFT, then ignoring the g'_0-padded portion of this time record ($2N$ to $4N - 1$), and finally performing a DFT on the new $2N$-point time record gives

$$G''_k = \frac{1}{2} \sum_{m=0}^{2N-1} g'_m \exp(-i\pi km/N)$$
$$0 \leq k \leq 2N-1 \tag{21}$$

where g'_m is the NG-extended g_m. Comparing (21) to (12a) shows that G''_k is the spectrum of an NG-extended g_m multiplied by 1/2. Therefore, we get g_m by multiplying (21) by two, inverse transforming, and ignoring the reconstructed record for $m \geq N$.

At this point it is worthwhile to rectify a few erroneous claims regarding the NG method. The conclusion that the Nicolson and NG techniques provide identical frequency information [7] is not correct. We have obtained analytic expressions for both the Nicolson and NG modified records and tested the accuracy of these expressions. The equations describing the two processes do not give identical spectra, even when ignoring the zeroes for the NG spectrum. The difference can also be verified by deconvolution. The dc component ($k = 0$) is not zero [2] nor is it twice that of the unextended data's spectrum [8]. The dc component is Nf_α, as shown in (12b). Another claim pertains to the power of the NG-extended waveform. The nonzero frequency components of the NG spectrum are not always related to the DFT spectrum of the original data record by a factor of two [8] as can be seen from (12b). The power of an NG-extended record, in terms of the power of an unextended record, is

$$P_{NG} = \sum_{m=0}^{N-1} f_m^2 + \sum_{m=N}^{2N-1} (f_\alpha - f_{m-N})^2$$
$$= \sum_{m=0}^{N-1} (2f_m^2 + f_\alpha^2 - 2f_\alpha f_m)$$
$$= 2P + Nf_\alpha^2 - 2f_\alpha \sum_{m=0}^{N-1} f_m \tag{22}$$

where P is the power of the unextended waveform.

D. First Difference

The first-difference technique maintains the original record length and increases the noise power of the data. Let the first difference of the data be given by the convolution of f_m with the first-difference operator v_m,

$$f_m^{(1)} = f_m * v_m \tag{23a}$$

where the superscript "(1)" indicates the first difference and $v_0 = 1$, $v_1 = -1$, and $v_m = 0$ otherwise. The frequency-domain equivalent of (23a) is

$$F_k^{(1)} = F_k V_k = G_k H_k V_k = G_k H_k^{(1)} \text{ or } G_k^{(1)} H_k \tag{23b}$$

where V_k is the Fourier transform of v_m and is given by

$$V_k = 1 - \exp(-i2\pi k/N) \tag{24}$$

TABLE II
MEAN AND STANDARD DEVIATION OF ERRORS FOR DECONVOLUTIONS USING THE THREE METHODS INDICATED AND FOR VARYING LEVELS OF PRECONVOLUTION ADDITIVE NOISE A_m. THE CONSTANT c VARIES THE RELATIVE CONTRIBUTION OF A_m TO q_m. SEE TABLE I FOR AN EXPLANATION OF THE VARIABLES. THE MEAN ERROR VALUE IS THE TOP LISTING OF EACH PAIR. BOTH THE MEAN AND STANDARD DEVIATION VALUES PRESENTED ARE THE AVERAGE OF SIX MEASURED VALUES.

	Nicolson	Nahman–Gans	Difference
$q_{1,m} * (q_{2,m} + cA_m)/q_{1,m}$	−4.96e-5	4.60e-5	1.14e-7
$c = 0.0001$	1.23e-7	4.96e-5	1.36e-3
$c = 0.001$	−4.97e-4	4.97e-4	1.14e-6
	1.22e-7	4.97e-4	1.53e-3
$c = 0.01$	−4.95e-3	4.13e-3	9.46e-6
	1.31e-7	5.30e-3	7.33e-3
$c = 0.1$	−4.96e-2	4.96e-2	5.27e-5
	2.56e-6	4.96e-2	7.01e-2
$p_{1,m} * (q_{3,m} + cA_m)/p_{1,m}$	−9.88e-4	3.87e-6	−7.75e-6
$c = 0.0001$	8.22e-5	1.70e-5	1.66e-5
$c = 0.001$	−1.44e-4	−2.27e-4	−4.54e-4
	1.33e-4	2.46e-4	9.45e-5
$c = 0.01$	−5.91e-3	−2.46e-3	−4.93e-3
	9.49e-4	2.63e-3	9.30e-4
$c = 0.1$	−5.08e-2	2.48e-2	−4.98e-2
	9.46e-3	2.66e-2	9.43e-3

and $H_k^{(1)}$ and $G_k^{(1)}$ are the Fourier transforms of the successive differences of h_m and g_m, respectively. If $H_k^{(1)}$ is used in (23b), G_k is obtained by dividing $F_k^{(1)}$ by the difference of H_k. A subsequent inverse Fourier transform of G_k yields g_m. On the other hand, if $H_k^{(1)}$ is used in (23b), we first divide $F_k^{(1)}$ by H_k and then perform an inverse transform on $G_k^{(1)}$ to get $g_m^{(1)}$. The desired signal g_m is recovered by integrating (running sum) $g_m^{(1)}$. An offset error may occur, and it is corrected the same way as done for the Nicolson method. Deconvolution error statistics are shown in Table I.

IV. NOISE

The effects of additive noise on a waveform reconstruction can be described by

$$F_k^{\mathrm{Nic}} = (G_k + A_{1,k})H_k + (G_{\alpha,k} + A_{1,\beta,k})H_k S_k + A_{2,k}$$

$$F_k^{\mathrm{NG}} = (G_k + A_{1,k})H_k D_k - (G_{\alpha,k} + A_{1,\alpha,k})\frac{H_k D_k}{2} + A_{2,k}$$

$$F_k^{\mathrm{Dif}} = (G_k + A_{1,k})H_k V_k + A_{2,k} \tag{25}$$

where $A_{1,k}$ and $A_{2,k}$ are pre- and postconvolution additive noise, and the superscripts on F_k refer to the given process. H_k is assumed to be noise free. Division of the expressions in (25) by H_k causes amplification of the high-frequency components of $A_{2,k}$. Consequently, the deconvolution is unstable, and the reconstructed waveform is not representative of the original input. Techniques are available for dealing with this problem, but this is not a topic of this paper. If $A_{2,k} = 0$, however, the reconstruction may resemble the input. Table II shows statistics of the results of reconstructions with varying levels of preconvolution additive noise. The uniformly distributed noise A_m was generated using a pseudorandom number generator and has an amplitude range of −0.5–0.5.

V. CONCLUSIONS

We have examined techniques that reduce the effects of record truncation discontinuities in the spectra of step-like waveforms for application in waveform reconstructions. These techniques include time-domain windowing, the Nicolson ramp-subtraction technique, the Nahman–Gans record-extension method, and taking first differences of the data. We have shown that some of these techniques can be used as a predeconvolution process to facilitate the return of an accurate waveform reconstruction.

The Nicolson, Nahman–Gans, and difference methods do not produce significant error in the reconstructed waveform, whereas windowing may. Our observations show that an offset error may occur for all techniques and, neglecting this offset error, the errors in the reconstructed waveforms for deconvolutions using the Nahman–Gans technique are typically less than the errors caused by the other techniques. The trends displayed in Table I were also observed using other test waveforms, the data of which is not shown. The Nicolson method, however, was the least affected by preconvolution additive noise. Mathematical operations were introduced to describe the Nicolson, Nahman–Gans, and first-difference techniques and, in using these, we have been able to accurately determine the nonmachine-induced errors in the reconstructed waveform.

ACKNOWLEDGMENT

We would like to thank W. L. Gans and W. A. Kissick for administrative support and J. Baker-Jarvis, A. Ondrejka, and D. S. Friday for their technical comments on this paper.

REFERENCES

[1] A. M. Nicolson, "Forming the fast Fourier transform of a step response in time-domain metrology," *Elec. Tran. Lett.*, vol. 9, p. 317, 1973.
[2] W. L. Gans and N. S. Nahman, "Continuous and discrete Fourier transforms of steplike waveforms," *IEEE Trans. Instrum. Meas.*, vol. IM-31, p. 97, 1982.
[3] Y. Balcou, "The problem of calculating the fast-Fourier transform of s step-like sampled function," *IEEE Trans. Instrum. Meas.*, vol. IM-28, p. 97, 1979.
[4] For example see A. Papoulis, *Signal Analysis*. New York: McGraw-Hill, 1977.
[5] H. A. Samulon, "Spectrum analysis of transient response curves," *Proc. I.R.E.*, p. 175, 1951.
[6] A. V. Oppenheim and R. W. Schafer, *Discrete Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
[7] J. Waldmeyer, "Fast Fourier transformations for step-like functions: The synthesis of three apparently different methods," *IEEE Trans. Instrum. Meas.*, vol. IM-29, p. 36, 1980.
[8] A. Shaarawi and S. M. Riad, "Computing the complete FFT of a step-like waveform," *IEEE Trans. Instrum. Meas.*, vol. IM-35, p. 91, 1986.