PERFORMANCE OF A NOVEL TOPOLOGY CONTROL SCHEME FOR FUTURE WIRELESS MESH NETWORKS

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ABSTRACT

In this paper, we address the topology control issues for future wireless mesh networks (WMNs). In particular, we propose a novel topology control scheme that attempts to maximize the overall throughput in the network and that takes into account traffic patterns. The main idea of the scheme is to establish multiple semi-permanent wireless highways, *each of which can convey the traffic for nodes along the highway. To evaluate the performance of the proposed scheme, we conduct theoretical analysis, which demonstrates that viable solutions for highways do exist with high probability. The theoretical analysis also proves the optimality. Furthermore, we conduct simulations, and the simulation results verify our theoretical analysis.*

I. INTRODUCTION

Wireless mesh network (WMN) is a promising technology that can be applied to provide cost-effective wireless coverage for a large area [1]. Currently, many different working groups are developing wireless mesh support in a variety of networks, including *wireless personal area networks* (WPANs), *wireless local area networks* (WLANs), and *wireless metropolitan area networks* (WMANs) [2]. Despite the salient features, many challenges have not been fully understood and addressed.

One of the key issues is topology control problem, which has been studied extensively in the past twenty years, for the emerging *wireless ad hoc networks* and *wireless sensor networks* (WSNs). The main purpose of topology control is to identify a subset of possible wireless links to provide connectivity for wireless networks, with certain design criteria have been considered, including power consumption [3], interference [4], broadcast [5], quality-of-service (QoS) [6], antennas [7], and reliability [8].

In the literature, most studies on topology control focus on characteristics of the resulting graph and skip the detailed routing, scheduling schemes, which may affect throughput and delay performance in wireless networks. In addition, most studies have implied broadcast traffic [5], [9].

Compared to existing studies, our focus in this paper is to design a topology such that the overall throughput capacity can be maximized for a certain traffic pattern. In other words, our topology control scheme will consider all aspects in the network design, including connectivity, routing, and scheduling.

The main idea of the new scheme is to establish multiple *wireless highways*, on both the horizontal direction and the vertical direction. Moreover, on the same direction, multiple highways can operate simultaneously, without interfering with each other. In this manner, near optimal throughput capacity can be obtained. To evaluate the performance of the proposed scheme, we conduct a theoretical study, which demonstrates that viable solutions exist with arbitrarily high probability in random wireless networks. Our theoretical study also proves the optimality.

The rest of this paper is organized as the following. In Section II, we discuss the new topology control scheme. To demonstrate the advantages of the proposed framework, we present theoretical and simulation analysis in Section III. Finally, we conclude this paper in Section IV.

II. A NOVEL TOPOLOGY CONTROL SCHEME

A. THE MAIN IDEA AND DESIGN CRITERIA

The key idea of our topology control scheme is to identify a set of semi-permanent highways, such that the best throughput capacity of the network can be obtained. Particularly, we envision that the wireless highways will be rather similar to the highway system in public transportation system, which can efficiently provide connectivity in reality. In our framework, we consider that the highways can be partitioned into two groups, horizontal and vertical. Highways in each group can operate simultaneously because they are mutually parallel and can be placed away enough to reduce interference below a certain threshold. Consequently, horizontal and vertical highways will partition the whole geographical area into grids, in which nodes will try to forward their traffic to the nodes on neighboring highways.

In our framework, the combination of the following parameters can be considered

1) Transmission range: transmission range of each node in the network is traditionally an important design parameter in topology control. In general, a smaller transmission range will improve the channel reuse but may compromise the connectivity. A larger transmission range will improve the connectivity but reduce the channel reuse. Therefore, choosing an appropriate range is a trade-off between connectivity and channel reuse.
2) Type of antenna: Clearly, using directional antenna or beam forming may improve the capacity of the network by reducing the interference and improve the transmission quality.
3) Traffic pattern: Traffic pattern is very important to the topology. For instance, most studies in the literature are based on the implication that the traffic is broadcast. With such an assumption, the problem is formulated in a way such that the overall transmission for each message is minimized. However, broadcast traffic may only be a special case in the future service-oriented WMN, in which a variety of patterns may appear, from one-to-one to many-to-many [10].
4) Quality of service (QoS): For the successful of future WMNs, a crucial issue is to enable services with certain QoS requirements, such as bandwidth, delay, security, and reliability.

As a first step of our study, we consider only omnidirectional antenna and we consider purely random unicast traffic pattern. Moreover, we elaborate on the random wireless network to gain insights for our future investigation.

B. NETWORK MODEL

In our study, we consider an arbitrary area that is covered by a set of equal-sized hexagonal cells, which is similar to the typical scenario in cellular networks. One of the reasons for choosing such a method is that the traveling distance is bounded to approximately $\sqrt{4/3}$ to the shortest distance.

We now let the length of the edge of any hexagon be equal to R. In this system, the communication range is selected such that any two nodes in two adjacent cells can communicate with each other. As shown in Fig. 1, we can derive that the communication range r can be expressed by $r = \sqrt{13}R$. Moreover, we only consider that communications between two nodes are in adjacent cells and the transmission will be omnidirectional in our study.

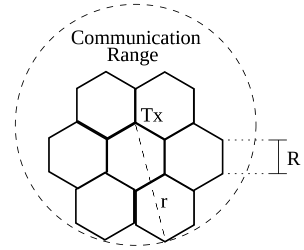


Figure 1. The communication model.

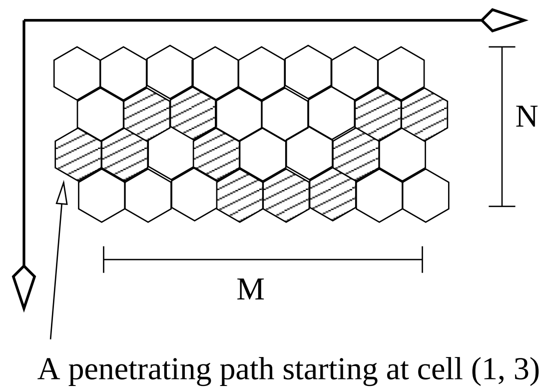


Figure 2. An example of the band model.

A set of cells can form a *band*, as shown in Fig. 2. In particular, a band is constructed by N rows of cells, and each row consists of M horizontal cells. Note that, all cells are placed in a way such that there is no overlap or gap between adjacent cells. In this manner, we can specify a cell in the band by (i, j), which represents the cell on the i-th column and j-th row.

To provide connectivity in this network, we consider two types of band, horizonal and vertical. In *horizontal band*, a penetrating path is defined as a sequence of connected cells, each of which contains at least a node, that pass through the band horizontally, i.e., connecting the leftmost column and the rightmost column. An example for horizontal band is illustrated in Fig. 2, which starts at cell (1, 3).

Similarly, in *vertical band*, a penetrating path is defined as a sequence of connected cells, with at least one node in each cell, that passes through the band vertically.

C. A TOPOLOGY FORMATION METHOD

To simplify the discussion, we assume that all nodes are randomly located in a unit square area, and let the transmission range r be the parameter.

- Step 1: For a given transmission range r, determine R, the length of the edge of any hexagon.
- Step 2: Calculate the probability that one or more nodes are located inside a cell.
- Step 3: Derive parameter M in the horizontal band.
- Step 4: Find parameter N that will lead to a high probability that a horizontal path exists in the band.
- Step 5: Partition the area using M by N bands, and find the horizontal highway within each horizontal band.
- Step 6: Find vertical highways using the same method described in Step 3-5.

In the following section, we conduct theoretical analysis to evaluate the performance of the topology control scheme in random wireless networks.

III. PERFORMANCE OF THE TOPOLOGY CONTROL SCHEME IN RANDOM WIRELESS NETWORKS

To conduct our analysis, we assume that each cell contains one or more nodes with probability p. Our analysis is based on the percolation theoretical approach [11]. The objective of the analysis is to investigate the probability that a penetrating path exists in a band, given the parameters M, N, and p.

Since the horizontal band is similar to a vertical one, we focus on the horizontal band hereafter. To facilitate the analysis, we define the following events:

- $P(M, N, p)$: the event that at least one penetrating path exists in a band with parameters M, N, and p.
- $P(M, N, p, K^+)$: the event that at least K penetrating paths exist in a band with parameters M, N, and p.
- $P(M, N, p, K^-)$: the event that less than K penetrating paths exist in a band with parameters M, N, and p.

From Fig. 2, we can also observe that, if there is no penetrating path in the band, then we can find at least one partition method that separates the band into two disconnected parts. To represent such a phenomenon, here, we define a *vacant path* as a sequence of cells, each of which is empty and contains no nodes, that starts from the first row to the last row in the band. An example of such vacant path can be found in Fig. 3.

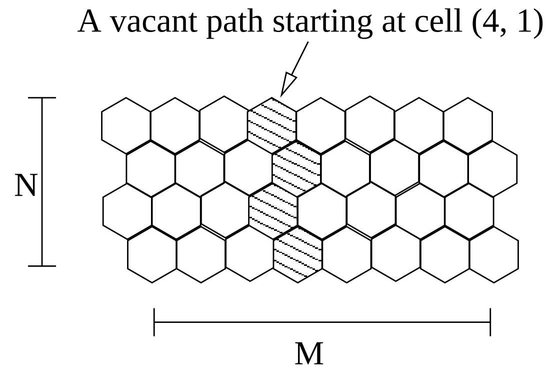


Figure 3. An example of the vacant path.

Similar to the event defined above, we can also define the following events:

- $V(M, N, p)$: the event that at least one vacant path exists in a band with parameters M, N, and p.
- $V(M, N, p, i)$: the event that at least one vacant path exists in a band with parameters M, N, and p, given that the path starts at cell $(i, 1)$.
- $W(M, N, p, i)$: the event that at least one N-hop path starts at cell $(i, 1)$ in a horizontal band with parameters M, N, and p, given that the path consists of only empty cells.

With the definition above, we can first derive that

$$
\begin{aligned}
Pr[W(M, 1, p, i)] &= (1-p) && (1)\\
Pr[W(M, 2, p, i)] &\leq 4(1-p)^2 && (2)\\
Pr[W(M, N, p, i)] &\leq \frac{4}{25}[5(1-p)]^N, \text{ if } N > 2 && (3)
\end{aligned}
$$

In Eq. (2), the coefficient 4 means that cell $(i, 1)$ has at most 4 neighboring cells if $i \neq 1$ and $i \neq M$. Note that in Eqs. (2)-(3), we have applied the inclusion-exclusion principle. In addition, to guarantee that the right-hand-side of inequality Eqs. (2)-(3) is less than 1, we need

$$p \geq 1 - \frac{1}{5}(\frac{25}{4})^{\frac{1}{N}} \quad (4)$$

To simplify the notations below, we let $N \geq 2$. Since the length of a vacant path is at least N, we have

$$Pr[V(M, N, p, i)] \leq Pr[W(M, N, p, i)]. \quad (5)$$

By using the inclusion-exclusion principle again, we can prove that

$$
\begin{aligned}
Pr[V(M, N, p)] &\leq M \times Pr[V(M, N, p, i)]\\
Pr[V(M, N, p)] &\leq \frac{4M}{25}[5(1-p)]^N. && (6)
\end{aligned}
$$

From Eq. (6) we can see that, for arbitrarily small value $\epsilon > 0$, $Pr[V(M,N,p)] < \epsilon$, if we can choose p such that

$$p > 1 - \frac{1}{5}(\frac{25\epsilon}{4M})^{\frac{1}{N}}. \quad (7)$$

Since event $P(M,N,p)$ and $V(M,N,p)$ are compliment to each other, we can see that

$$Pr[P(M,N,p)] \geq 1 - \frac{4M}{25}[5(1-p)]^N. \quad (8)$$

From Eqs. (7)-(8), we can prove the following theorom.

Theorem 1: For a M by N horizontal band, a penetrating path exists with arbitrarily high probability with appropriate p.

A. THE SCALABILITY ANALYSIS

We now consider the scenario in which the number of cells tends to infinity[1]. In such a case, we can use the hexagon model to cover the whole area without any gap. We assume that the area is rectangular that can be covered by a M by N band, as in Fig. 2. In other words, we assume that M tends to infinity in this scenario. To analyze the scalability of the scheme, we assume that N is an integer that divides M and we can partition the whole area with N non-overlap horizontal bands.

From the definition of event $P(M,N,p)$, we can see that it is an increasing event in the sense that adding a new penetrating path to this event is still the same event. Therefore, we can apply the percolation theory (Lemma 6 in [11]).

$$Pr[P(M,N,q,(\delta N)^-)] \leq (\frac{q}{q-p})^{\delta N}\frac{4M}{25}[5(1-p)]^N, \quad (9)$$

where $\delta < 1$ is a constant, δN is an integer, and $q > p$.

We now let $M = e^{\xi N}$, where ξ is a constant. In this manner, we can guarantee that $Pr[P(M,N,p)]$ (in Eq. (8)) approaches 1, by choosing ξ such that

$$5e^{\xi}(1-p) < 1, \quad (10)$$

for any $p > \frac{4}{5}$. Consequently, we can further derive that

$$Pr[P(M,N,q,(\delta N)^-)] \leq \frac{4}{25}\left[5\left(\frac{q}{q-p}\right)^{\delta} e^{\xi}(1-p)\right]^N. \quad (11)$$

Clearly, Eq. (11) expresses that the probability the number of penetrating path is less than δN in one horizontal band.

[1]The conclusion for this scenario is the same as that for another well-known scaling scenario, in which the number of nodes tends to infinity in a unit area.

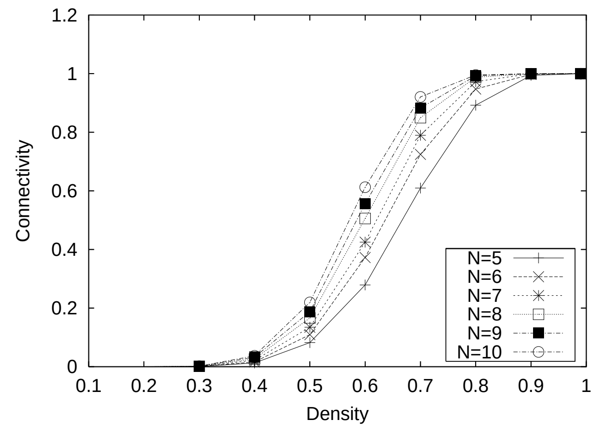


Figure 4. Connectivity vs. density for different N when $M = 10$.

Therefore, the probability that the number of a penetrating path is less than δN in every horizontal band can be obtained by Eq. (12).

In Eq. (12), we can see that the first component $(\frac{4}{25})^{\frac{M}{N}}$ of the right hand side tends to zero because $\frac{4}{5} < 1$ and $\frac{e^{\xi N}}{N} \to \infty$. To make sure that the second component also tends to zero, we just need to make sure that

$$5e^{\xi}(1-p)\left(\frac{q}{q-p}\right)^{\delta} < 1. \quad (13)$$

Since parameters p, q, ξ, and δ are independent to M and N, we can see that, when M goes to infinity, then we can guarantee that each horizontal band contains at least δN penetrating paths. Therefore, we can guarantee that there are at least δM horizontal penetrating paths for the whole area, provided that a constant portion of these penetrating paths can operate simultaneously, without affecting (via interference) each other.

Since the above proof can also be applied for the vertical penetrating paths. We can prove the following theorem.

Theorem 2: The throughput capacity of two-dimensional random wireless network tends to $\Theta(M)$.

Proof: Skipped. ■

Note that the above theorem is in agreement with the conclusion in [11], in which the authors assumed the physical model in their discussion, while we assume the protocol model.

B. CONNECTIVITY IN A BAND

In this subsection, we present simulation and analysis results for our theoretical analysis formulated in the previous section. Figs. 4 and 5 show simulation results, where we assume $M = 10$. We show the connectivity versus density

$$Pr[P(M,N,q,(\delta N)^{-})]^{\frac{M}{N}} \leq \left(\frac{4}{25}\right)^{\frac{M}{N}} \left[5\left(\frac{q}{q-p}\right)^{\delta} e^{\xi}(1-p)\right]^{M} \quad (12)$$

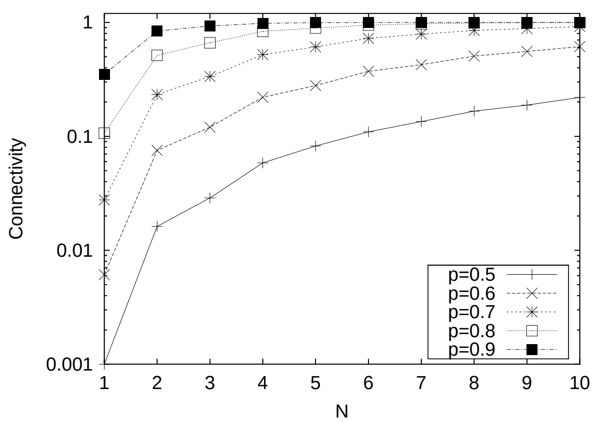


Figure 5. Connectivity vs. N for different density when $M = 10$.

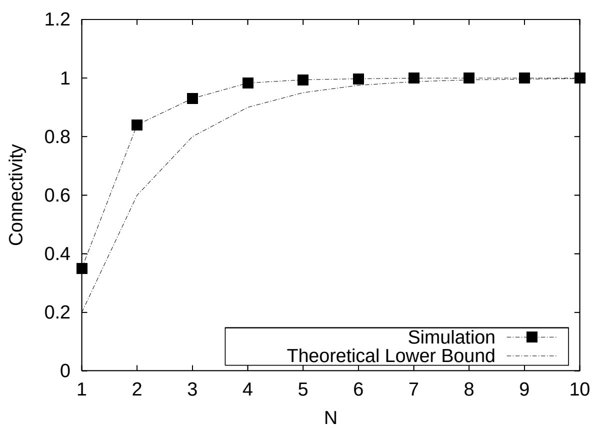


Figure 6. Simulation and theoretical lower bound for connectivity vs. N when $M = 10$ and density $p = 0.9$.

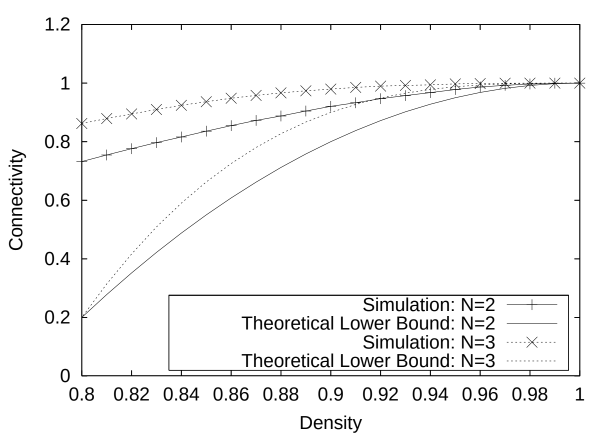


Figure 7. Simulation and theoretical lower bound for connectivity vs. density when $M = 5$.

for different vertical size N in Fig. 4. We can see that the connectivity keep increasing with the increased density. The connectivity is approaching 1 when the density is greater than 0.7. We can also observe that the connectivity is getting larger if we increase the vertical size N. However, the degree of the improvement is small. It implies that the connectivity reaches near optimal value when density is great than 0.7 and N is greater than 7 in this simulation setting. Fig. 5 depicts connectivity versus vertical size under different density. We note that the connectivity for different density is very close, especially, when the density is greater than 0.7.

Figs. 6 and 7 compare simulation and theoretical analysis. As an example, we present the simulation and theoretical lower bound for connectivity versus N when $M = 10$ and density $p = 0.9$ in Figure 6. It shows that the theoretical lower bound generated by Eq. (8) provides a relatively tight lower bound for the connectivity analysis. Fig. 7 shows simulation results and theoretical lower bounds for connectivity versus density when $M = 5$ and $N = 2, 3$, respectively. Our investigation indicates that both simulation and analysis can guide the practical deployment of the cells.

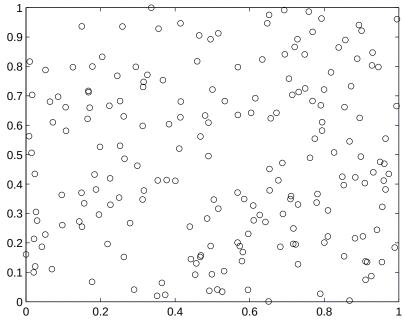
C. TOPOLOGY FORMATION

In this subsection, we illustrate and discuss the outcome of the topology formation method presented in Section II-C. In Fig. 8, we consider that 200 nodes are randomly deployed in the unit square. Fig. 8(a) shows the location of each node. In Fig. 8(b), we present the outcome of the method proposed in Section II-C. Here, the range is assumed to be 0.08 and then p is estimated by

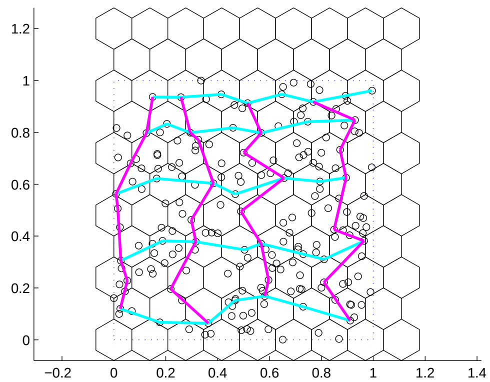
$$p = (1 - \frac{3\sqrt{3}}{2} R^3)^{N_{nodes}}. \quad (14)$$

Note that some hexagons are placed at the boundary of the unit square such that all the area of the unit square is covered. Consequently, the probability that one or more nodes locate in an edge hexagon is less than that of a hexagon in the middle of the unit square. Nevertheless, we can observe that the proposed method can still successfully establish horizontal and vertical paths. In Fig. 8(b), we can also observe that all the nodes can access the highway in one hop because N is 2.

In Fig. 9(a), we show another scenario in which 500 nodes are located in the unit square. Fig. 9(b) illustrates the topology formed by using the proposed method.

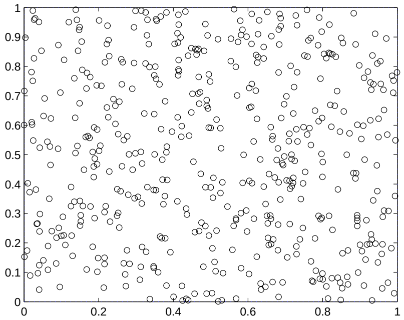


(a) The node distribution.

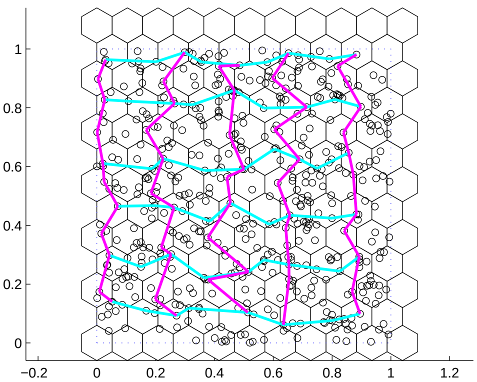


(b) The topology with $R = 0.08$ and $N = 2$.

Figure 8. The topology formation for a 200-node random network.



(a) The node distribution.



(b) The topology with $R = 0.06$ and $N = 2$.

Figure 9. The topology formation for a 500-node random network.

IV. CONCLUSION

In this paper, we have proposed a novel topology control scheme for future wireless mesh networks (WMNs). The objective of the scheme is to maximize the overall throughput in the network according to a certain traffic pattern. The main idea of the scheme is to establish semi-permanent multiple *wireless highways*, each of which can convey the traffic for nodes along the highway. To evaluate the performance of the proposed framework, we have conduct theoretical and simulation analysis.

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