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# A model of transport of fuel gases in a charring solid and its application to opposed-flow flame spread

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# Abstract

This paper outlines the development of a mathematical model for the transport of gases through the char matrix of a burning solid. Two basic assumptions are made. First, the gases evolved by the degradation of the virgin material are transported by pressure differences through a network of narrow passageways created in the char by the conversion of material from the solid to the gas phase. This process is treated as flow through a porous medium, with the mass flux related to the pressure gradient by Darcy's law. Second, the gas temperature is the same as the local char temperature. This model is first used to study the time-dependent thermal degradation of a semi-infinite charring solid heated above the charring temperature. Then, the opposed flow flame spread treated by Atreya and Baum [Atreya, A., Baum, H.R., Proc. Combust. Inst., 29 (2002) 227-236] is revisited. It was found that the solution to the condensed phase flame spread problem is identical to the initial transient problem. Weak dependence of the solution on the accumulation parameter 'β' validates the assumption made in [Atreya, A., Baum, H.R., Proc. Combust. Inst., 29 (2002) 227–236] and completes the flame spread solution. Fuel mass flux follows the heat flux lines and is normal to the isobars. Calculations using representative values for wood show that the pressure generation at the char–virgin material interface is considerable and equal to 13.27 kPa. Finally, in view of the fact that the nonlinear pressure equation poses considerable numerical difficulties, this analytical solution may help in determining the stability and accuracy of the numerical scheme used for more complicated problems. © 2006 The Combustion Institute. Published by Elsevier Inc. All rights reserved.

Keywords: Charring solids; Fuel transport; Pressure generation; Flame spread

## 1. Introduction

An understanding of the burning of charring materials is essential for developing models of fire growth in buildings. The transient growth of the char layer during fuel production complicates both the heat and the mass transfer analysis. It introduces an insulating char layer of unknown thickness between the heat transfer from the gasphase flames and the virgin material that generates fuel gases by solid-phase pyrolysis. Further, the combustible gases produced at the char–virgin material interface must be expelled by generation of high pressures. Initially, when the interface is at or near the surface of the solid being heated, it is often assumed that the gases are instantly expelled from the solid material into the adjacent oxidizing atmosphere permitting combustion to take place in the gas phase. However, if the

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process goes on long enough, the interface will no longer be adjacent to the heated surfaces and the spatial distribution of gases over the surfaces of the solid will depend on the internal pressure generation and the char porosity. Since these fuel gases are responsible for fire growth, it is necessary to account for their transport through the char matrix to any surface exposed to the oxidizer. Figure 1 shows how the fuel gases generated at one location and transported to another can affect fire growth. Significant asymmetry is observed due different permeability across and along the grain direction. This is discussed in greater detail in Refs. [6,7].

Several models of pyrolysis of charring materials have been developed in the past motivated either by fires or biomass pyrolysis. Di Blasi [2] summarizes the state of the art of transport models for charring solid degradation. However, the primary focus of many of these models was often not pressure driven gas transport because the onedimensional assumption usually made obviates the need. Some models [3-5] have considered volatile products transport using Darcy's law with isotropic permeability despite being one-dimensional. Fredlund [9] has considered a two-dimensional model and even measured the internal pressure generation. All these models were numerically solved to obtain the progress of the char front and gas generation. In this work a physicsbased model of fuel gas generation and transport is developed, analytically solved, and applied to the flame spread model of [1].

First, basic equations and boundary conditions controlling the gas transport are derived and used



Fig. 1. Axis-symmetric flame spread on horizontal surface of Douglas fir ignited at the center in quiescent atmosphere (taken from Ref. [6]). The remaining char pattern shows the effect of different permeability across and along the grain direction. Clearly, transport of fuel gases generated in one location to another can significantly affect the flame spread.

to study the time-dependent thermal degradation of a semi-infinite charring material heated above the charring temperature. Then, the opposed-flow flame spread treated by [1] is revisited. The question how the evolved fuel gas is transported from the interface where it is generated to the surface is addressed.

#### 2. Gas transport model

Basic assumptions made in the model are: (i) The gases evolved by thermal degradation are transported by pressure differences through the porous char. This process is treated as flow through a porous medium, with the mass flux related to the pressure gradient by Darcy's law. (ii) The charring material is assumed to be initially moisture-free and non-porous and the porosity develops as a result of charring. This assumption eliminates the condensation and re-evaporation of pyrolysis gases and moisture that are forced by high pressure into the virgin interior. (iii) The temperature in the gas is the same as the local char temperature and the convective heat transfer between the gas and the char is ignored. Thus, the model is reduced to an energy equation for the virgin material and char, together with mass conservation for the transport of gaseous degradation products through the char. (iv) Thermal decomposition occurs at a well-defined pyrolysis temperature  $T_{\rm p}$ . The corresponding equations are supplemented by boundary conditions at the char-virgin material interface, and at the char-oxidizer interface.

Let  $\rho_v$  denote the density of the virgin material, and let  $\rho_c$  be the density of the char, where  $\rho_c < \rho_v$ .  $\rho_v$  and  $\rho_c$  are assumed to be constants with abrupt change occurring at a well-defined pyrolysis temperature  $T_p$ . The lower char density is assumed to be caused by the creation of small void spaces through which the evolved gases flow. Shrinkage of char is ignored. The void fraction ' $\varepsilon$ ' is then given by the expression

$$\varepsilon = (\rho_{\rm v} - \rho_{\rm c})/\rho_{\rm v}.\tag{1}$$

To characterize the thermodynamic state of the gas in terms of the usual density, temperature, and pressure,  $\rho$  must denote the gas density per unit volume of space *occupied by the gas*. The velocity  $\vec{u}$  must be defined such that  $\vec{m} = \rho \vec{u}$  is the local mass flux of gas through a macroscopic surface element in the char. Here, char is treated as a continuum, with gas and solid matter coexisting in the volume occupied by the char. Also, since the generation of decomposition products occur at a predefined temperature ( $T_p$ ) that separates char from the virgin material, there are no gas-generation terms in the porous char matrix. Thus the equation expressing conservation of mass for the evolved gas takes the form:

$$\frac{\partial(\rho\varepsilon)}{\partial t} + \nabla .(\rho\vec{u}) = 0.$$
<sup>(2)</sup>

Darcy's Law is used to relate the local mass flux and pressure gradient in char. For gas percolating through char Landau's [10] approach is used. Accordingly, for an elliptical pipe of crosssection with semi-axes a and b, the mass flux vector is related to the pressure gradient through the formula:

$$\vec{m} = -\frac{(ab)^2}{4v(a^2 + b^2)} \nabla p.$$
 (3)

Here, v is the kinematic viscosity, related to density and gas viscosity  $\mu(T)$  by  $v = \mu/\rho$ . The pressure in the gas is denoted as p, while the temperature of both gas and adjacent char is T. Since the model is based on the actual thermodynamic properties in the gas, the perfect gas law can be used to relate density to pressure and temperature.

$$p = \rho RT, \tag{4}$$

where, *R* is the gas constant for the evolved gases. Finally, using the fact that the cross-sectional area *A* of the pipe is given by the expression  $A = \pi ab$ , the relation between the mass flux and pressure gradient takes the form:

$$\vec{m} = -\frac{(ab)A}{4\pi(a^2 + b^2)} \left(\frac{p\nabla p}{\mu(T)RT}\right).$$
(5)

Equation (5) shows that the local mass flux is composed of two parts; a shape dependent factor proportional to the cross-sectional area of the pipe, multiplied by a term which is proportional to the gradient of pressure *squared*, and which depends only on the properties of the gas. The shape factor will have to be replaced by an empirical factor, since there is no way of knowing the actual shape of the passages created by the gasification processes.

The final form of Eq. (5) comes from noting that the mass flux averaged over an area that is large compared with any individual pore passage must account for the fact that only a fraction of the area of order  $\varepsilon^{2/3}$  contains voids. Thus, the form of Darcy's law that will actually be used is:

$$\vec{m} = \rho \vec{u} = -\varepsilon^{(2/3)} K \left( \frac{p \nabla p}{\mu(T) RT} \right). \tag{6}$$

The permeability K has dimensions of area. It is an empirical parameter that characterizes the char as a porous medium. The mass conservation equation, which controls the transport of gases through the char, then takes the final form:

$$\frac{\partial}{\partial t} \left( \frac{p}{RT} \varepsilon \right) = \nabla \cdot \left( \frac{\varepsilon^{(2/3)} K p}{\mu(T) RT} \nabla p \right). \tag{7}$$

Note that Eq. (7) is parabolic. However, the domain through which the evolved gases can percolate is not known in advance, but must be determined as part of the solution for the associated problem for the evolution of the temperature field.

At surfaces exposed to the surrounding atmosphere, the pressure is ambient,  $p_a$ . The boundary condition at the interface between the char and the virgin material is as follows: Let  $\vec{V}$  be the local velocity of the interface and  $\vec{n}$  be the unit normal to the surface defined with positive  $\vec{n}$  pointing into the virgin material. Then:

$$-\rho \vec{u} \cdot \vec{n} = \varepsilon^{(2/3)} K \left( \frac{p \nabla p \cdot \vec{n}}{\mu(T) R T} \right)$$
$$= (\rho_{\rm v} - \rho_{\rm c}) \vec{V} \cdot \vec{n}. \tag{8}$$

Physically, Eq. (8) states that the rate at which evolved gases are created is equal to the product of the velocity of the char front normal to itself multiplied by the density difference between the char and the virgin material. Thus, all the mass lost by the solid phase is taken up by the gas, with the pressure adjusting accordingly. Finally, at an impermeable boundary between the char and an inert solid material, the gas cannot penetrate into the inert solid. Thus, if  $\vec{v}$  denotes a unit normal to the impermeable boundary:

$$\nabla p \cdot \vec{v} = 0. \tag{9}$$

# 3. Transient thermal model

Consider an idealized scenario in which the temperature at the surface of a semi-infinite solid is instantaneously raised from an initial ambient temperature  $T_{\infty}$  to a surface temperature  $T_{\rm s} > T_{\rm p}$ . Since the pressure field is determined by the temperature distribution in the char and the virgin material, it is considered first. Let  $T_{\rm v}(x,t)$  be the temperature,  $\lambda_{\rm v}$  be the thermal conductivity, and  $C_{\rm pv}$  be the heat capacity of the virgin material, with an analogous notation in the char. Then:

$$\rho_{\rm c} C_{\rm pc} \frac{\partial T_{\rm c}}{\partial t} = \lambda_{\rm c} \frac{\partial^2 T_{\rm c}}{\partial x^2} \quad 0 \leqslant x \leqslant X(t);$$
  
$$\rho_{\rm v} C_{\rm pv} \frac{\partial T_{\rm v}}{\partial t} = \lambda_{\rm v} \frac{\partial^2 T_{\rm v}}{\partial x^2} \quad X(t) \leqslant x \leqslant \infty.$$
(10)

Here, x = X(t) is the position of the interface between materials, with x measured from the heated surface.

The boundary conditions are: At the surface x = 0,  $T_c = T_s$ . Far from the surface, the virgin material is at the ambient temperature. Thus;  $T_v = T_\infty$  as  $x \to \infty$ . Finally, at the interface x = X(t), three conditions must be satisfied.

$$T_{\rm v} = T_{\rm c} = T_{\rm p} \ ; \ \lambda_{\rm v} \frac{\partial T_{\rm v}}{\partial x} = \lambda_{\rm c} \frac{\partial T_{\rm c}}{\partial x} + \rho_{\rm v} X'(t) Q.$$
(11)

The first two conditions require that the temperature at the interface is continuous and equal to the char formation temperature. The final condition states that the heat flux transmitted from the char must supply an energy Q per unit mass of virgin material to liberate the gas at the char front, with the excess conducted into the interior of the solid. Solutions satisfying the temperature boundary conditions at the surface, interface, and in the interior can be obtained by noting that the absence of any independent length or time scale implies that all physical quantities depend only on a similarity variable  $\eta$ , defined as:

$$\eta = x/\sqrt{\alpha_{\rm v} t} \; ; \; \alpha_{\rm v} = \lambda_{\rm v}/(\rho_{\rm v} C_{\rm pv}). \tag{12}$$

This implies that the interface position corresponds to a constant value of  $\eta$ . Denoting this value as  $\eta = C$ , the interface position can be expressed as:

$$X(t) = C\sqrt{(\alpha_v t)}.$$
(13)

The solutions satisfying the first two of Eq. (11) can be written in the form:

$$T_{\rm v} = T_{\rm p} F(\eta)$$
  
$$F(\eta) = \frac{T_{\infty}}{T_{\rm p}} + \left(1 - \frac{T_{\infty}}{T_{\rm p}}\right) \frac{\operatorname{erfc}(\eta/2)}{\operatorname{erfc}(C/2)}, \qquad (14)$$

$$T_{\rm c} = T_{\rm p}G(\eta) G(\eta) = \frac{T_{\rm S}}{T_{\rm p}} - \left(\frac{T_{\rm S}}{T_{\rm p}} - 1\right) \frac{{\rm erf}(\eta^*/2)}{{\rm erf}(C^*/2)},$$
(15)

$$C^* = \sqrt{(\alpha_v/\alpha_c)}C \qquad \eta^* = \sqrt{(\alpha_v/\alpha_c)}\eta.$$
 (16)

The parameter C that determines the location of the interface is found by requiring that the last of Eq. (11) be satisfied. The result is identical with that obtained for the interface condition in the flame spread problem considered in [1]. In slightly different notation it can be expressed as follows:

$$\frac{Q}{C_{\rm pv}(T_{\rm p}-T_{\infty})} = \frac{\lambda_{\rm c}}{\lambda_{\rm v}} \cdot \frac{(T_{\rm S}-T_{\rm p})}{(T_{\rm p}-T_{\infty})} \cdot f_1(C^*) - f_2(C),$$
(17)

where

$$f_1(C^*) = \frac{2}{\sqrt{\pi}C^*} \cdot \frac{\exp(-C^{*2}/4)}{\operatorname{erf}(C^*/2)};$$
  
$$f_2(C) = \frac{2}{\sqrt{\pi}C} \cdot \frac{\exp(-C^2/4)}{\operatorname{erfc}(C/2)}.$$
 (18)

Figure 2 shows the variation of the interface location parameter *C* with the dimensionless endothermic energy parameter  $Q/C_{pv}((T_p - T_{\infty}))$ . The other parameters are fixed at the values indicated in the figure caption, which are chosen to be consistent with those used in [1]. For these values, when the interface is no longer endothermic, C = 1.15392. In principle, an exothermic interface is possible, but the maximum possible energy release corresponds to  $Q/C_{pv}((T_p - T_{\infty})) = -1$ no matter what the thermal properties of either



Fig. 2. Interface location parameter C as function of dimensionless endothermic energy  $Q/(C_{\rm pv} (T_{\rm p} - T_{\infty}))$ . The remaining parameters are fixed at the values  $\lambda_c/\lambda_v = 1/3$ ,  $(T_{\rm s} - T_{\rm p})/(T_{\rm p} - T_{\infty}) = 3$ ,  $(\alpha_v/\alpha_c) = 3/4$ .

the char or the virgin material. As this value is approached, the char layer depth approaches infinity. Beyond this point, there are no solutions to Eq. (17), and thus no solutions to the energy equations of the postulated form.

### 4. Mass transfer in char

For the calculation of the mass transport through the char, it will be assumed that the interface is endothermic. The mass transport and hence the pressure distribution is confined to a domain bounded by the interface  $x = X(t) = C^* \sqrt{\alpha_c t}$  and the char surface x = 0. The coefficients in Eq. (7) depend only on the variable  $\eta^*$ , while the char interface and surface correspond to the fixed values  $\eta^* = C^*$  and  $\eta^* = 0$ , respectively. Thus, it makes sense to assume that the pressure is a function of  $\eta^*$  only. Denoting the pressure at the char surface by  $p_s$ , the pressure in the evolved gas is written as:

$$p(x,t) = p_s P(\eta^*). \tag{19}$$

Moreover, since the evolved gas viscosity is a function of temperature only, a reasonable assumption is that  $\mu/\mu_p = (T/T_p)^n$ . Using the solution for the temperature field obtained above, Eq. (7) becomes:

$$\beta \frac{\eta^*}{2} \frac{d}{d\eta^*} (P(\eta^*) / G(\eta^*)) + \frac{d}{d\eta^*} \left( P(\eta^*) / (G(\eta^*))^{n+1} \frac{dP}{d\eta^*} \right) = 0,$$
(20)

$$\beta = \varepsilon^{1/3} \alpha_{\rm c} \mu_{\rm p} / (p_{\rm s} K). \tag{21}$$

The boundary conditions at the interface and the char surface respectively are:

$$P\frac{dP}{d\eta^*}(\eta^* = C^*) = \frac{C^*}{2}\beta(\rho_v T_p)/(\rho_s T_s) \equiv M;$$
  

$$P(0) = 1.$$
(22)

Equations (20)-(22) reveal some important physics. Clearly, the parameter  $\beta$  plays a major role in the mass transport. Small values of  $\beta$ correspond to nearly instantaneous transport of the evolved gases from the interface to the surface, with little accumulation in the interior of the char. Thus, models that postulate instant surface emission of gases inherently assume that the material in question has small values of  $\beta$ . Conversely, if the material has a value of  $\beta$  that is not small, then the local accumulation in the interior cannot be neglected, and the mass flux at the surface is not the same as that evolved in the interior. Moreover, the spatial distribution of the evolved gases on the surface is no longer simply related to the spatial generation pattern in the interior.

The magnitude of the pressure rise can be inferred from Eq. (22). Note that even for small values of  $\beta$  the pressure rise can be considerable. The right-hand side of the first of Eq. (22) is proportional to  $\beta(\rho_v/\rho_s)$ . Here,  $\rho_s$  is the evolved gas density evaluated at the char surface. Even if the first factor is small, the density ratio of virgin solid to gas is quite large. The remaining terms in the right hand side of this equation are typically of order unity. This implies that M is typically of order unity, even if  $\beta$  is small. Thus, the pressure rise will typically not be small compared with the ambient pressure in the gas at the surface of the char. Indeed, if the internal pressure is high enough, it can rupture the char surface producing a crackling sound – a familiar experience of a wood log in a fire place.

A natural starting point for the analysis of Eqs. (20)–(22) is the observation that when  $\beta = 0$ , an analytical solution can be found for any value of *M*. Denoting the resulting pressure distribution by  $P = P^{(0)}$ , the solution takes the form:

$$P^{(0)}(\eta^*) = \left(1 + 2M \int_0^{\eta^*} [G(x)]^{(1+n)} \,\mathrm{d}x\right)^{1/2}.$$
 (23)

The integral in Eq. (23) represents the effect of the temperature dependence of the viscosity of the evolved gases. If it is further assumed that  $\mu \sim T$  so that n = 1, the integral can be evaluated explicitly. Denoting the integral as  $I(\eta^*)$ , the result is:

$$I(\eta^*) = \frac{T_s}{T_p} \eta^* - 2\frac{T_s}{T_p} \left(\frac{T_s}{T_p} - 1\right) \frac{g_1(\eta^*)}{\operatorname{erf}(C^*/2)} + \left[ \left(\frac{T_s}{T_p} - 1\right) / \operatorname{erf}(C^*/2) \right]^2 g_2(\eta^*), \quad (24)$$

$$g_{1}(\eta^{*}) = \eta^{*} \operatorname{erf}(\eta^{*}/2) - \frac{2}{\sqrt{\pi}} \left(1 - \exp\left(-(\eta^{*})^{2}/4\right)\right), \quad (25)$$

$$g_{2}(\eta^{*}) = \eta^{*} [\operatorname{erf}(\eta^{*}/2)]^{2} + \frac{4}{\sqrt{\pi}} \operatorname{erf}(\eta^{*}/2) \exp\left(-(\eta^{*})^{2}/4\right) - 2\sqrt{\frac{2}{\pi}} \operatorname{erf}(\eta^{*}/\sqrt{2}).$$
(26)

Figure 3 shows how the pressure distribution of the evolved gases in the char changes as the mass flux parameter increases, assuming that the accumulation parameter  $\beta = 0$ . Note that since  $P^{(0)}$ represents a pressure normalized with respect to the ambient pressure at the char surface, the pressure rise is quite significant. Thus, a linearized treatment of this equation would be very inaccurate. This nonlinear pressure equation poses considerable difficulties in numerical solution. The above analytical solution is helpful in determining stable and accurate numerical schemes.

The above solution, although derived for  $\beta = 0$ and M fixed, has a much wider range of validity. Indeed, if we write  $M = \beta \tilde{M}$  and treat  $\tilde{M}$  as a large parameter with  $\beta$  fixed, then it is easy to see that the same solution holds with an error  $O(1/\sqrt{\tilde{M}})$ . The accuracy of the analytical solution has been confirmed by testing it against numerical results computed for  $\beta = 1$ , M = 10, 100, and 1000. The errors are actually much smaller for the cases investigated than the error estimate. This is in fact the most realistic case for this particular problem, since  $\tilde{M}$  takes the form:

$$\tilde{M} = \frac{C^*}{2} (\rho_{\rm v} T_{\rm p}) / (\rho_{\rm s} T_{\rm s}).$$
<sup>(27)</sup>

In general,  $C^*$  is a number of order one (see Fig. 2), as is the temperature ratio  $T_p/T_s$ . However, the density ratio of virgin solid to gas at the char surface will almost always be large. Finally, as shown below, the solution is also valid for the pressure distribution in the opposed flow flame spread problem studied in [1].



Fig. 3. Evolved gas pressure distribution  $p/p_s = P^{(0)}(\eta^*)$ in char for different values of mass flux parameter M. The solutions are valid for values of the transport parameter  $\beta = 0$ . The remaining parameters are  $T_s/T_p = 2$ , and  $C^* = 0.5$ .

## 5. Opposed flow flame spread

Analysis of the flame spread problem requires the solution of a coupled problem involving both the gas and condensed phases. Most previous research has focused on the gas phase. The thermal degradation of the condensed-phase material is typically treated using a surface pyrolysis model coupled to a simple heat conduction analysis in the interior of the solid. There has been almost no work on charring materials like wood, where the gaseous "fuel" is liberated at an interior surface whose location must be found as part of the solution.

Figure 4 shows the overall geometry of the flame spread problem in a coordinate system moving with the flame. While this paper is concerned only with the condensed phase gas transport model, it is important to explain certain features of the overall problem. To begin, it is assumed that the problem can be regarded steady in a frame of reference moving with the flame speed V. The objective of the analysis is to determine V as a function of the material properties of the solid fuel, the gaseous oxidizer, and the ambient speed  $U_{\infty}$  of the opposed flow.

Two simplifications are introduced that permit the gas phase and condensed phase analyses to be considered separately, with the results of each analysis combined to produce the desired results. First, it is assumed that there is no heat or mass transfer between the gas and condensed phase upstream of the flame front. Note that this does *not* mean that there is no preheating of the gas or solid. While the gas phase dynamics is not of interest here, it should be noted that both the flow and temperature distributions depart from their ambient values ahead of the flame. The second assumption is that the surface temperature of the char is uniform downstream of the flame front. This was experimentally found to be approximately constant [8]. This assumption yields an internally consistent coupled solution to the heat transfer problem in both phases. Specification of a value for this temperature, together with all the material properties and upstream flow conditions, then uniquely determines the flame speed.

Up to this point, all assumptions and simplifications are contained in the solutions described in [1]. However, that analysis required a third major assumption that relates the spatial distribution of the mass flux of gas liberated at the char–virgin material interface to that at the surface. The analysis presented below removes that assumption, and replaces it with a physics-based model for the evolved gas transport. It will be demonstrated that the results obtained using this model are consistent with the earlier analysis, and thus complete the solution for the opposed flow flame spread over charring materials presented in [1].

The starting point for the analysis is the representation of the steady state version of Eq. (7) in the parabolic coordinate  $\tau^*$ ,  $\omega^*$  system shown in Fig. 4 (which is similar to Fig. 5 of Ref. [1]).



Fig. 4. A composite flame spread figure similar to Fig. 5 of Ref. [1] showing computed isobars in char and streamlines in the gas phase. The property values are the same as those in [1]. However, instead of isotherms, isobars are shown. Both isotherms and isobars follow lines of constant  $\omega^*$ . Fuel gases are driven along lines of constant  $\tau^*$ , as shown. Property values are:  $Q/C_{\rm pv}$   $(T_{\rm p} - T_{\infty}) = 1$ ;  $\alpha_{\rm v}/\alpha_{\rm c} = 0.75$ ;  $(T_{\rm p} - T_{\infty})/(T_{\rm s} - T_{\infty}) = 0.275$ ;  $\lambda_{\rm c}/\lambda_{\rm v} = 1/3$ ; c = 0.5018;  $\mu_{\rm p} = 2.3 \times 10^{-5}$  kg/ms;  $\alpha_{\rm c} = 6.164 \times 10^{-7}$ . Also, char porosity = 0.76; fuel gas molecular weight = 0.08 kg/mol; permeability of char =  $10^{-13}$  m<sup>2</sup>;  $\beta = 1.063 \times 10^{-03}$ ;  $T_{\rm s} = 850$  K;  $T_{\rm p} = 451$  K;  $T_{\infty} = 300$  K.

$$V \frac{\partial}{\partial x} \left( \frac{p}{RT} \varepsilon \right) = \frac{\partial}{\partial x} \left( \frac{\varepsilon^{(2/3)} K p}{\mu(T) RT} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\varepsilon^{(2/3)} K p}{\mu(T) RT} \frac{\partial p}{\partial y} \right).$$
(28)

Equation (28) must be solved subject to the following boundary conditions at the gas-solid interface y = 0.

$$p(x,0) = p_{s} \text{ for } x \ge 0 \quad \frac{\partial p}{\partial y}(x,0) = 0 \text{ for } x < 0.$$
(29)

The first of Eq. (29) states that at the surface of the solid, downstream of the flame front, the pressure must be the ambient pressure. The second of these equations states that there is no mass flux of gasified fuel through the surface upstream of the flame front. If this were not true, the flame would begin further upstream, violating the geometry on which the analysis is based.

Similarly, the temperature  $T_v$  in the virgin material and  $T_c$  in the char are solutions to the heat conduction equations in each material.

$$V \frac{\partial T_{\rm c}}{\partial x} = \alpha_{\rm c} \left( \frac{\partial^2 T_{\rm c}}{\partial x^2} + \frac{\partial^2 T_{\rm c}}{\partial y^2} \right);$$
$$V \frac{\partial T_{\rm v}}{\partial x} = \alpha_{\rm v} \left( \frac{\partial^2 T_{\rm v}}{\partial x^2} + \frac{\partial^2 T_{\rm v}}{\partial y^2} \right). \tag{30}$$

At the char-gas interface, temperature satisfies boundary conditions analogous to pressure.

$$T_{\rm c}(x,0) = T_{\rm s} \text{ for } x \ge 0 \quad \frac{\partial T}{\partial y}(x,0) = 0 \text{ for } x < 0.$$
  
(31)

Again, for  $x \ge 0$ , the temperature takes on the constant value  $T_s$ , while for x < 0 there is no interphase heat transfer. At the char–virgin material interface, continuity of temperature requires that  $T_c = T_v = T_p$ . As in the earlier problem, the location of this interface must be determined as part of the solution. Finally, far from the gas–solid interface, the temperature in the virgin material retains its ambient value;  $T_v = T_{\infty}$ .

Solutions are obtained in parabolic coordinates defined as (see Fig. 4):

$$\tau^* + \mathrm{i}\omega^* = \sqrt{\frac{\alpha_v}{\alpha_c}}(\tau + \mathrm{i}\omega) = \sqrt{\frac{V}{\alpha_c}(x + \mathrm{i}y)};$$

where V is the flame speed and  $\alpha_{\rm c} = \frac{\lambda_{\rm c}}{\rho_{\rm c} C_{\rm pc}}$ . (32)

The isotherms shown in Fig. 5 of Ref. [1] are similar to isobars shown in Fig. 4. These are lines of constant  $\omega^*$  (or constant  $\omega$ ). The transformation is defined so that the branch cut is taken to be the positive x axis. Thus,  $\tau^* = 0$ on the negative x axis, and  $\tau^* = -\sqrt{\frac{Vx}{x_c}}$  on the positive x axis in the char. The positive x axis corresponds to  $\omega^* = 0$ . Assuming all physical quantities to be functions of  $\omega^*$  ensures that the gradient of all physical quantities in the solid is perpendicular to the surface for x < 0. Since the temperature and pressure are constant for  $x \ge 0$ , assigning these values at  $\omega^* = 0$  specifies the boundary conditions at the gas-solid interface.

To transform the equations to parabolic coordinates, note that the pressure and temperature depend only on  $\omega^*$ . Introducing the dimensionless variables:

 $T = T_p G(\omega^*), \ \mu = \mu_p (T/T_p)^n$ , and  $p = p_s P(\omega^*)$ , the conservation of mass for the evolved gases becomes:

$$2\beta\omega^* \frac{\mathrm{d}}{\mathrm{d}\omega^*} (P(\omega^*)/G(\omega^*)) + \frac{\mathrm{d}}{\mathrm{d}\omega^*} \left( P(\omega^*)/(G(\omega^*))^{n+1} \frac{\mathrm{d}P}{\mathrm{d}\omega^*} \right) = 0$$
  
Where,  $\beta = \varepsilon^{1/3} \alpha_c \mu_p / (p_s K).$  (33)

Here,  $\beta$  is the accumulation parameter identified in Eq. (21). A similar analysis yields the analogous equations for the char temperature function  $G(\omega^*)$ and virgin material temperature  $T_v = T_p F(\omega)$ .

$$2\omega^* \frac{\mathrm{d}G}{\mathrm{d}\omega^*} + \frac{\mathrm{d}}{\mathrm{d}\omega^*} \left(\frac{\mathrm{d}G}{\mathrm{d}\omega^*}\right) = 0$$
  
$$2\omega \frac{\mathrm{d}F}{\mathrm{d}\omega} + \frac{\mathrm{d}}{\mathrm{d}\omega} \left(\frac{\mathrm{d}F}{\mathrm{d}\omega}\right) = 0. \tag{34}$$

It is clear from the discussion of the temperature boundary conditions that by identifying  $\omega^* = \eta^*/2$  and  $\omega = \eta/2$ , the solutions for the temperature field given in Eqs. (14) and (15) are also the solutions for the flame spread problem. Since we also require that P(0) = 1 to enforce the surface pressure boundary condition, if we can demonstrate that the interface condition for the flame spread problem is identical to Eq. (22), then the equivalence of the two problems is complete.

The mass flux  $\dot{m}$  of evolved gas at the interface is given by the expression:

$$\dot{m} = -(\rho_{\rm v} - \rho_{\rm c})V\vec{i}\cdot\vec{n},\tag{35}$$

where,  $\vec{i}$  is a unit vector in the flow direction,  $\vec{n}$  is a unit normal to the interface pointing *into* the virgin material and V is the flame spread velocity. Since the pressure and temperature are only functions of  $\omega^*$ , the interface is a surface defined by  $\omega^* = C^*$ . Using this we obtain:

$$\dot{m} = (\rho_{\rm v} - \rho_{\rm c}) 2\alpha_{\rm c} C^* \frac{|\nabla \omega^*|^2}{|\nabla \omega^*|}.$$
(36)

The mass flux can also be written in the form:

$$\dot{m} = \varepsilon^{2/3} K \frac{p \nabla p \cdot \vec{n}}{\mu(T) RT} \nabla p \cdot \vec{n} = \frac{\mathrm{d}p}{\mathrm{d}\omega^*} |\nabla \omega^*|$$
(37)

Introducing the dimensionless pressure *P* and using the equation of state for the evolved gas,  $p_s = \rho_s RT_s$ , the two expressions for  $\dot{m}$  can be combined to yield the following condition at the interface:

$$P\frac{\mathrm{d}P}{\mathrm{d}\omega^*} = 2C^* \frac{\rho_{\rm v} T_{\rm p}}{\rho_{\rm s} T_{\rm s}} \beta.$$
(38)

This result is identical with that obtained in Eq. (22) if we again make the identification  $\omega^* = \eta^*/2$ and note that the value of  $C^*$  obtained here must satisfy the same condition. Thus, the solution of the condensed phase portion of the opposed flow flame spread problem is identical to the solution of the one-dimensional impulsively heated problem. The flame spread solution obtained in Ref. [1] assumed that the mass flux of gaseous fuel emerging at the interface moves along the curves of constant  $\tau^*$  without change until it reaches the surface of the char layer where it is oxidized by air to produce the flame. The present analysis shows that it is equivalent to assuming that  $\beta \ll 1$ . Thus, the analytical solution for *P* obtained in Eq. (23) completes the solution in [1], and provides a self-consistent physics-based model of opposed-flow flame spread over charring materials.

Figure 4 shows the isobars (lines of constant dimensionless pressure *P*) during flame spread over a charring material. It is similar to Ref. [1] where isotherms were shown. It is computed with the same property values (see Fig. 4 caption). Computed streamlines in the gas phase from Ref. [1] are also shown for completeness. Both isotherms and isobars follow lines of constant  $\omega^*$  and fuel gases are driven along lines of constant  $\tau^*$ .

## 6. Summary and conclusions

The solution to the condensed phase flame spread problem is identical to the initial transient problem where the surface temperature of a semiinfinite solid is instantaneously raised from an

#### Comments

*Jose Torero, The University of Edinburgh, UK.* What properties were used for comparing with the experiments?

*Reply*. This apparently simple question is very important because it inquires about the fundamental usefulness of the model. The experimental data on measured flame spread rates and char depths for wood correlated well according to the parabolic char-material interface, (Vy/ $\alpha_v)^2 = c^2 + 2c(Vx/\alpha_v)$ , given by the model. The data from four experiments on poplar collapsed onto one curve and the parabola that passed through this data had a value of initial ambient temperature  $T_{\infty}$  to  $T_{\rm s} > T_{\rm p}$  under the transformation  $\eta^* = 2\omega^*$ . Weak dependence of the solution on the accumulation parameter  $\beta$ validates the assumption made in Ref. [1] and completes the flame spread solution. The fuel mass flux follows the heat flux lines and is normal to the isobars. Calculations using representative values for wood [1,5] show that the pressure generation at the char–virgin material interface is considerable and equal to 13.27 kPa. While problems with more complicated geometry will have to be numerically solved, the analytical solution presented here helps in determining the stability and accuracy of potential numerical schemes.

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c = 0.45. For the correlation only  $\alpha_v$  for poplar was needed (=0.1 mm<sup>2</sup>/s, obtained from the literature).

The value of the charring constant 'c' can also be determined from wood and char properties, as shown in Fig. 2, and the model can be used in a predictive manner. Using the literature values for poplar and its char,:

$$\begin{split} \lambda_c / \lambda_{\rm v} &= 1.4; \alpha_{\rm v} / \alpha_{\rm c} = 0.248; (T_p - T_\infty) / (T_{\rm s} - T_\infty) \\ &= 0.618; Q / (C_{\rm pv} (T_{\rm p} - T_\infty)) = 1 \end{split}$$

we get c = 0.502. This is not too different from that obtained by the correlation, given the large uncertainty in the value of the heat of pyrolysis of wood and properties of char.

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*Michael Delichatsios, University of Ulster, UK.* Great effort, needing some comments:

(1). The surface temperature is constant along the fuel bed following ignition. This is surpassing and its value at the leading edge of 650°C seems too high. (2). Wood can form cracks so that pressure build up may not occur for all species of wood.

*Reply.* (1). The measured char surface temperature behind the flame foot was found to be approximately constant for various experiments under different external radiation conditions. The data showed considerable fluctuations in the measurements. This is unavoidable because there is no way of assuring that the thermocouple will stay on the char surface after pyrolysis.

However, within the measurement accuracy, the char temperature was roughly constant for a given experiment but depending on the experimental condition, the value of the constant char temperature changed from 550°C to 750°C with the average being 650°C. While it may seem high, it is not unusual. Similar measurements are available in the literature. The constant char temperature used for numerical calculations and data correlation was 850 K. This was also measured during the experiment. Fortunately, the constant surface temperature boundary condition (see Fig. 4) is actually applied some distance behind the flame foot as required by the parabolic coordinates. This leaves some room for the temperature to rise from the pyrolysis temperature to the char surface temperature.

(2). Yes indeed, if the wood cracks there is no pressure build up. Interestingly, a long time ago, Professor Emmons mentioned a method of making uncracked wood charcoal. The secret was to pyrolyze wood in its own volatile atmosphere without oxygen. Thus, it is possible that the char underneath the flame does not crack. We will have to look at the experiments more carefully next time.