A Novel Topology Control Scheme for Future Wireless Mesh Networks

Kejie Lu[1], Tao Zhang[2], Yi Qian[3], and Shengli Fu[4]
[1] Department of Electrical and Computer Engineering
University of Puerto Rico at Mayagüez, Mayagüez, PR 00681, USA.
[2] School of Engineering and Computing Sciences
New York Institute of Technology, Old Westbury, NY 11568, USA.
[3] National Institute of Standards and Technology
[4] Department of Electrical Engineering
University of North Texas, Denton, Texas 76207.

***Abstract*—In this paper, we address the topology control issue for future wireless mesh networks (WMNs) based on the principle of service-oriented design. In particular, we propose a novel topology control scheme that attempts to maximize the overall throughput in the network and that takes into account the traffic pattern. The main idea of the scheme is to establish multiple semi-permanent wireless highways, each of which can convey the traffic for nodes along the highway. To evaluate the performance of the proposed scheme, we conduct theoretical analysis, which demonstrate that viable solutions for highways do exist with high probability. The theoretical analysis also proves the optimality. Within the new topology control framework, we demonstrate that advance technologies, including network coding and physical-layer network coding (PLNC), can be applied to substantially improve the throughput capacity and confidentiality of the communications.**

***Index Terms*—Wireless mesh networks, topology control, network coding.**

I. Introduction

In the past few years, *Wireless mesh network* (WMN) is a promising technology that can be applied to provide cost-effective wireless coverage for a large area [1]. Currently, many different working groups are developing wireless mesh support in a variety of networks, including *wireless personal area networks* (WPANs), *wireless personal area networks* (WPANs), and *wireless metropolitan area networks* (WMANs) [2]. Despite the salient features, many challenges have not been fully understood and addressed.

One of the major issues is how to design WMN solutions that can help the service providers to gain more revenue, while enabling and providing good support for a variety of applications, which may have diverse requirements. In our recent studies [3], [4], we investigated the features of existing and potential services in future wireless networks, and we concluded that these services may express diverse requirements, in terms of 1) communication pattern (one-to-one, one-to-many, many-to-one, and many-to-many), 2) delay (real-time, non-real-time, and delay-tolerant), 3) service availability (centralized, distributed, and location-aware), 4) security, and 5) reliability.

To fulfill such demands, we proposed to develop "service-oriented" network layer for future WMN [3]. Compared to existing approaches, the new framework has the following major features:

- Availability of service: The network layer is aware of the availability of different services, such as Internet access, real-time communications, content distribution, interactive gaming, medical applications, and vehicular safety applications.
- Communication pattern: A service can have a specific communication pattern, including one-to-one, one-to-many, many-to-one, and many-to-many.
- Service requirements: A service can also be associated with a variety of service requirements, including bandwidth, delay (real-time, non-real-time, and delay-tolerant), security, and reliability.
- Service-orientation: To obtain a service, a customer only needs to request the service (with certain specifications) from the network layer. The network layer will determine the availability of service and will route the traffic accordingly.

In this paper, we will address the topology control problem within the service-oriented framework for the design of future WMNs. Particularly, we propose a novel topology control scheme that can efficiently provide connectivity for the network, and can take into account the traffic patterns in the network. The main idea of the new scheme is to establish multiple *wireless highways*, on both the horizontal direction and the vertical direction. Moreover, on the same direction, multiple highways can operate simultaneously, without interferencing with each other. In this manner, near optimal throughput capacity can be obtained. To evaluate the performance of the proposed scheme, we conduct theoretical study, which demonstrates that viable solutions exist with arbitrarily high probability in random wireless network. Our theoretical study also proves the optimality. Based on the topology control scheme, we also provide efficient scheduling schemes that exploit advance technologies, including network coding and physical-layer network coding.

The rest of this paper is organized as the following. In Section II, we briefly review the background and related works in topology control and network coding. We then elaborate on the new topology control scheme in Section III. To demonstrate the advantages of the proposed framework, we present efficient scheduling schemes based on network coding and physical-layer network coding in Section IV. Finally, we conclude this paper in Section V.

II. Background and Related Works

A. Topology Control

Topology control problem has been studied extensively in the past twenty years, for the emerging *wireless ad hoc networks* and *wireless sensor networks* (WSNs). The main purpose of topology control is to identify a subset of possible wireless links to provide connectivity for wireless networks, with certain design criteria have been considered, including power consumption [5], interference [6], broadcast [7], quality-of-service (QoS) [8], antennas [9], and reliability [10].

In the literature, most studies on topology control focus on the characteristics of the resulting graph and skip the detail routing, scheduling schemes, which may affect the throughput and delay performance in wireless networks. In addition, most studies have implied broadcast traffic [7], [11].

Compared to existing studies, our focus in this paper is to design a topology such that the overall throughput capacity can be maximized for a certain traffic pattern. In other words, our topology control scheme will consider all aspects in the network design, including connectivity, routing, and scheduling.

B. Network Coding and Physical-Layer Network Coding

Network coding is an emerging technology that can significantly improve the performance of communication networks [12]. The main idea behind network coding is that it allows nodes (switches and routers) in the network to forward coded data based on previously received data. By contrast, the nodes in existing communication networks can be considered as having "copy" or "store-and-forward" functionality.

In the literature, most existing studies on network coding have focused on how to efficiently support a single multicast in a wired network [13]. Several interesting schemes have been proposed recently for multiple unicast flows in typical wireless ad hoc networks [14]. The key advantage of such schemes is that they utilize the broadcast nature of wireless communications.

Recently, several groups have extended the idea of network coding to the physical layer [15]–[18]. The key innovation of *physical-layer network coding* (PLNC) is that it combines the transmitted signals directly in the time domain, at the receiver. In PLNC, a relay node (or nodes [18]) can decode the combined signals and forward new signals at a later time (decode-and-forward) [15], or just simply amplify the received signals [16] and then forward them (amplify-and-forward) later.

TABLE I
Comparison of Transmission Schemes

	Traditional	Network coding	PLNC
Trans. 1	$x : A \to R$	$x : A \to R$	$x : A \to R$ $y : B \to R$
Trans. 2	$y : B \to R$	$y : B \to R$	$x + y : R \to A$ and B
Trans. 3	$x : R \to B$	$x + y : R \to A$ and B	
Trans. 4	$y : R \to A$		

To illustrate the difference between the traditional transmission scheme, the network coding scheme, and the PLNC scheme, let us consider a simple network that consists of only three nodes: A, R, and B. In this network, nodes A and B will generate traffic designated to nodes B and A, respectively. Their communication will be relay by node R. In Table I, we show simple scheduling schemes for the three communication mechanism, where x and y represent message from A to B and from B to A, respectively. We can observe that, the network coding scheme can improve the capacity of the network by reducing the number of transmission from 4 to 3. And the PLNC scheme can improve the capacity even more because it requires only two transmissions to exchange the two messages.

Nevertheless, the studies in previous works also suggest that the gain of network coding in wireless network may be limited due to coordination overheads [14]. Compared to network coding, PLNC may require even more coordinations among nodes in wireless networks because of the nature of wireless channel and wireless communications [17].

In our topology control framework, we will consider both network coding and PLNC schemes. Moreover, we will consider how to reduce the control and coordination overhead in the design.

III. A Novel Topology Control Scheme

A. The Topology Control Scheme

The key idea of our topology control scheme is to identify a set of semi-permanent highways, such that the best throughput capacity of the network can be obtained. Particularly, we envision that the wireless highways will be rather similar to the highway system in public transportation system, which can efficiently provide connectivity in reality. In our framework, we consider that the highways can be partitioned into two groups, horizontal and vertical. Highways in each group can operate simultaneously because they are mutually parallel and can be placed away enough to reduce interference below a certain threshold. Consequently, horizontal and vertical highways will partition the whole geographical area into grids, in which nodes will try to forward their traffic to the nodes on neighboring highways.

In our framework, the combination of the following parameters can be considered

1) Transmission range: transmission range of each node in the network is traditionally an important design parameter in topology control. In general, a smaller transmission range will improve the channel reuse but may compromise the connectivity. A larger transmission

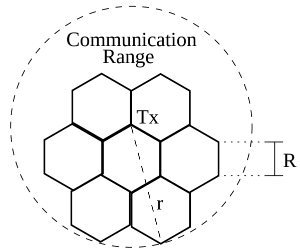

Fig. 1. The communication model.

range will improve the connectivity but reduce the channel reuse. Therefore, choose an appropriate range is a trade-off between connectivity and channel reuse.

2) Type of antenna: Clearly, using directional antenna or beam forming may improve the capacity of the network by reducing the interference and improve the transmission quality.
3) Traffic pattern: Traffic patter is very important to the topology. For instance, most studies in the literature are based on the implication that the traffic is broadcast. With such an assumption, the problem is formulated in a way such that the overall transmission for each message is minimized. However, broadcast traffic may only be a special case in the future service-oriented WMN, in which a variety of patterns may appear, from one-to-one to many-to-many [4].
4) Quality of service (QoS): For the successful of future WMNs, a crucial issue is to enable services with certain QoS requirements, such as bandwidth, delay, security, and reliability.

As a first step of our study, we will consider only omnidirectional antenna and we will consider purely random unicast traffic pattern. Moreover, we will elaborate on the random wireless network to gain insights for our future investigation.

B. Network Model

In our study, we consider an arbitrary area that is covered by a set of equal-sized hexagonal cells, which is similar to the typical scenario in cellular networks. One of the reasons for choosing such a method is that the traveling distance is bounded to approximately $\sqrt{4/3}$ to the shortest distance.

We now let the length of the edge of any hexagon be equal to R. In this system, the communication range is selected such that any two nodes in two adjacent cells can communicate with each other. As shown in Fig. 1, we can derive that the communication range r can be expressed by $r = \sqrt{13}R$. Moreover, is our study, we only consider that communications between two nodes in adjacent cells, and the transmission will be omnidirectional.

A set of cells can form a *band*, as shown in Fig. 2. In particular, a band is constructed by N rows of cells, and each row consists of M horizontal cells. Note that, all cells are placed in a way such that there is no overlap or gap between adjacent cells. In this manner, we can specify a cell in the

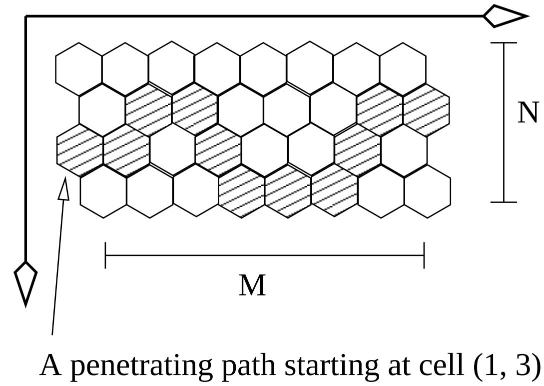

Fig. 2. An example of the band model.

band by (i, j), which represents the cell on the i-th column and j-th row.

To provide connectivity in this network, we consider two types of band, horizonal and vertical. In *horizontal band*, a penetrating path is defined as a sequence of connected cells, each of which contains at least a node, that pass through the band horizontally, i.e., connecting the leftmost column and the rightmost column. An example for horizontal band is illustrated in Fig. 2, which starts at cell (1, 3).

Similarly, in *vertical band*, a penetrating path is defined as a sequence of connected cells, with at least one node in each cell, that pass through the band vertically.

In the following subsections, we conduct theoretical analysis to evaluate the performance of the topology control scheme in random wireless network.

C. Performance the Scheme in Random Wireless Networks

To conduct our analysis, we assume that each cell will contain one or more nodes with probability p. Our analysis will be based on the percolation theoretical approach applied in [19]. The objective of the analysis is to investigate the probability that a penetrating path exist in a band, given the parameter M, N, and p.

Since the horizontal band is similar to a vertical one, we will focus on the horizontal band hereafter. To facilitate the analysis, we define the following events:

- $P(M, N, p)$: the event that at least one penetrating path exists in a band with parameter M, N, and p.
- $P(M, N, p, K^{+})$: the event that at least K penetrating path exist in a band with parameter M, N, and p.
- $P(M, N, p, K^{-})$: the event that less than K penetrating path exist in a band with parameter M, N, and p.

From Fig. 2 we can also observe that, if there is no penetrating path in the band, then we can find at least one partition method that separate the band into two disconnected parts. To represent such a phenomenon, here we define a *vacant path* as a sequence of cells, each of which is empty and contains no nodes, that starts from the first row to the last row in the band. An example of such vacant path can be found in Fig. 3.

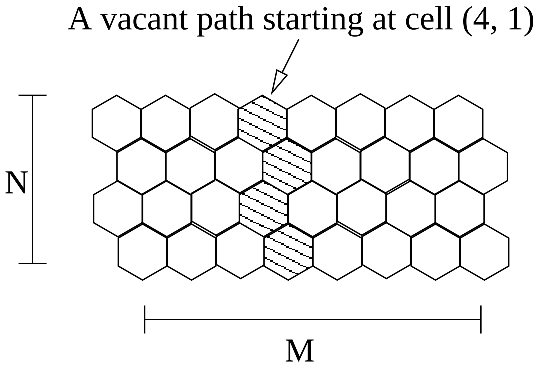

Fig. 3. An example of the vacant path.

Similar to the event defined above, we can also define the following events:

- $V(M,N,p)$: the event that at least one vacant path exists in a band with parameter M, N, and p.
- $V(M,N,p,i)$: the event that at least one vacant path exist in a band with parameter M, N, and p, given that the path starts at cell $(i,1)$.
- $W(M,N,p,i)$: the event that at least one N-hop path starts at cell $(i,1)$ in a horizontal band with parameter M, N, and p, given that the path consists of only empty cells.

With the definition above, we can first derive that

$$\begin{aligned} Pr[W(M,1,p,i)] &= (1-p) && (1)\\ Pr[W(M,2,p,i)] &\leq 4(1-p)^2 && (2)\\ Pr[W(M,N,p,i)] &\leq \frac{4}{25}[5(1-p)]^N, \text{ if } N>2 && (3)\end{aligned}$$

In Eq. (2), parameter 4 means that cell $(i,1)$ has at most 4 neighboring cells if $i \neq 1$ and $i \neq M$. Note that in Eqs. (2)-(3), we have applied the inclusion-exclusion principle. In addition, to guarantee that the right-hand-side of inequality Eqs. (2)-(3) is less than 1, we need

$$p \geq 1 - \frac{1}{5}(\frac{25}{4})^{\frac{1}{N}} \tag{4}$$

To simplify the notations below, we let $N \geq 2$. Since the length of a vacant path is at least N, we have

$$Pr[V(M,N,p,i)] \leq Pr[W(M,N,p,i)]. \tag{5}$$

By using the inclusion-exclusion principle again, we can prove that

$$\begin{aligned} Pr[V(M,N,p)] &\leq M \times Pr[V(M,N,p,i)]\\ Pr[V(M,N,p)] &\leq \frac{4M}{25}[5(1-p)]^N. && (6)\end{aligned}$$

From Eq. (6) we can see that, for arbitrarily small value $\epsilon > 0$, $Pr[V(M,N,p)] < \epsilon$, if we can choose p such that

$$p > 1 - \frac{1}{5}(\frac{25\epsilon}{4M})^{\frac{1}{N}}. \tag{7}$$

Since event $P(M,N,p)$ and $V(M,N,p)$ are compliment to each other, we can see that

$$Pr[P(M,N,p)] \geq 1 - \frac{4M}{25}[5(1-p)]^N. \tag{8}$$

From Eqs. (7)-(8), we can prove the following theorom.

Theorem 1: For M by N horizontal band, a penetrating path exists with arbitrarily high probability with appropriate p.

D. The Scalability Analysis

We now consider the scenario in which the number of cells tends to infinity[1]. In such a case, we can use the hexagon model to cover the whole area without any gap. We assume that the area is rectangular that can be covered by an M by M band, as in Fig. 2. In other words, we assume in this scenario that M tends to infinity. To analyze the scalability of the scheme, we assume that N is an integer that divides M and we can partition the whole area with N non-overlap horizontal bands.

From the definition of event $P(M,N,p)$, we can see that it is an increasing event in the sense that adding a new penetrating path to this event is still then same event. Therefore, we can apply the percolation theory (Lemma 6 in [19]).

$$Pr[P(M,N,q,(\delta N)^-)] \leq (\frac{q}{q-p})^{\delta N}\frac{4M}{25}[5(1-p)]^N, \tag{9}$$

where $\delta < 1$ is a constant, δN is an integer, and $q > q$.

We now let $M = e^{\xi N}$, where ξ is a constant. In this manner, we can guarantee that $Pr[P(M,N,p)]$ (in Eq. (8)) approaches 1, by choosing ξ such that

$$5e^{\xi}(1-p) < 1, \tag{10}$$

for any $p > \frac{4}{5}$. Consequently, we can further derive that

$$Pr[P(M,N,q,(\delta N)^-)] \leq \frac{4}{25}\left[5\left(\frac{q}{q-p}\right)^{\delta} e^{\xi}(1-p)\right]^N. \tag{11}$$

Clearly, Eq. (11) expresses that the probability the number of penetrating path is less than δN in one horizontal band. Therefore, the probability the number of penetrating path is less than δN in every horizontal band is

$$Pr[P(M,N,q,(\delta N)^-)]^{\frac{M}{N}} \leq \left(\frac{4}{25}\right)^{\frac{M}{N}}\left[5\left(\frac{q}{q-p}\right)^{\delta} e^{\xi}(1-p)\right]^M \tag{12}$$

In Eq. (12), we can see that the first component $(\frac{4}{25})^{\frac{M}{N}}$ of the right hand side tends to zero because $\frac{4}{5} < 1$ and $\frac{e^{\xi N}}{N} \to \infty$. To make sure that the second component also tends to zero, we just need to make sure that

$$5e^{\xi}(1-p)\left(\frac{q}{q-p}\right)^{\delta} < 1. \tag{13}$$

[1]The conclusion for this scenario is the same as that for another well-known scaling scenario, in which the number of nodes tends to infinity in a unit area.

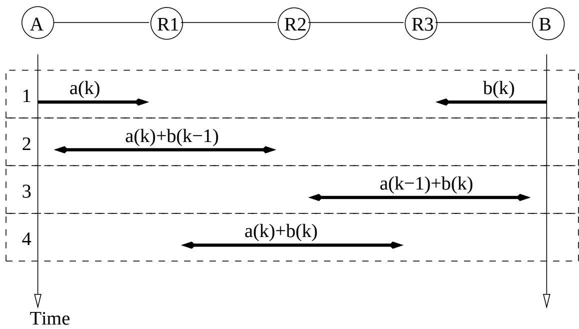

Fig. 4. An example of the scheduling scheme based on network coding.

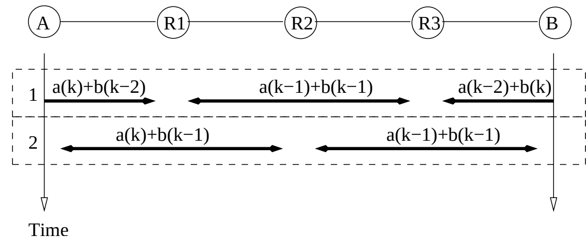

Fig. 5. An example of the scheduling scheme based on PLNC.

Since parameters p, q, ξ, and δ are independent to M and N, we can see that, when M goes to infinity, then we can guarantee that each horizontal band contains at least δN penetrating paths. Therefore, we can guarantee that there are at least δM horizontal penetrating paths for the whole area, provided that a constant portion of these penetrating paths can operate simultaneously, without affecting (via interference) each other.

Since the above proof can also be applied for the vertical penetrating paths. We can prove the following theorem.

Theorem 2: The throughput capacity of two-dimensional random wireless network tends to $\Theta(M)$.

Proof: Skipped. ■

Note that the above theorem is in agreement with the conclusion in [19], in which the authors assume the physical model in their discussion, while we assume the protocol model.

IV. Scheduling Schemes

Based on the topology control scheme we discussed in Section III, we have seen that the basic component for providing connectivity in the network is chain-based highway. Since the highways will be established and maintain indefinitely, we can develop simple but efficient scheduling schemes that exploit advance communication technologies such as network coding and PLNC. To simplify the discussion, we assume a *time-division multiplexing* (TDM) scenario.

Fig. 4 illustrates an example of the scheduling scheme based on network coding, in which we assume that distance between adjacent nodes are equal and the interference range is greater than 2 hops but less than 3 hops. From this example, we can observe that a simple scheduling scheme can be developed for bi-directional symmetric traffic with three relay nodes. Specifically, the schedules in four consecutive time slots can be simply repeated. Consequently, we can derive that the traffic rate per source node is 0.25 per time slot. By comparison, a traditional transmission scheme without network coding is only 0.17 per time slot.

We show the scheduling scheme for the PLNC in the same scenario in Fig. 5. We can observe that, the PLNC can improve the traffic rate per source node to 0.5 per time slot. From the figure we can also see that the end-to-end delay has been minimized. More interestingly, with the scheduling scheme, each transmission is the combination of signals from two sources, which improve the confidentiality in a wireless environment because a passive eavesdropper may not be able to decode individual messages.

V. Conclusion

In this paper, we have proposed a novel topology control scheme for future wireless mesh networks (WMNs) based on the principle of service-oriented design. In the proposed scheme, we have tried to maximize the overall throughput in the network according to a certain traffic pattern. The main idea of the scheme is to establish semi-permanent multiple wireless highways, each of which can convey the traffic for nodes along the highway. Within the new topology control framework, we have also demonstrated that advance technologies, including network coding and physical-layer network coding (PLNC), can be applied to substantially improve the throughput capacity. Furthermore, we observe that a proposed PLNC scheme can provided confidentiality of the communications in wireless network, which has not been reported in the literature.

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