

The relationship between the formation factor and the diffusion coefficient of porous materials saturated with concentrated electrolytes: theoretical and experimental considerations

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ABSTRACT

It has been proposed previously that the formation factor, in conjunction with the self-diffusion coefficient, can be used to determine the apparent diffusion coefficient. Strictly speaking, this application is incorrect. The formation factor is equal to the ratio of the self-diffusion coefficient to the *microstructural* diffusion coefficient, which is a quantity that characterizes the pore structure and is independent of the pore solution electrochemistry. The origin of this relationship will be shown using both the electro-diffusion transport equation and the definition of the formation factor. In practice, service life models that solve the electro-diffusion transport equation as a function of time require the formation factor in order to calculate the *microstructural* diffusion coefficient; the effects of the pore solution chemistry are then calculated independently. A method is needed to estimate the formation factor from either diffusion or conductivity data so that service life models can be applied to a particular material. An experiment on a model porous material is used to demonstrate one method for determining the formation factor from divided cell diffusion data. The estimated formation factor is then compared to results from conductivity measurements. Differences among the self-diffusion coefficients of the various diffusing species accentuates the difference between the microstructural and the apparent diffusion coefficients. The significance of this result to cementitious systems is discussed.

1. INTRODUCTION

Electrical measurements hold great promise for estimating the diffusion coefficients of saturated porous materials. The two most commonly used conduction techniques are the electrical migration test (driven diffusion) and the conductivity measurement. Migration tests are used to determine the ionic mobility, which can be related

to diffusivity. Conduction tests are used to estimate the formation factor, which can also be related to diffusivity. While there have been many reports on the use of the electrical migration tests to determine the apparent diffusion coefficient of cementitious systems [1], there have been fewer reports on the use of the formation factor. This is unfortunate since the formation factor characterizes the material pore structure. The relationship between formation factor and diffusivity, however, has subtleties that must be considered when making estimates of one from the other.

The formation factor \mathcal{F} has its origin in geological research on saturated porous materials. For a nonconductive porous solid saturated with a conductive pore solution, the formation factor is the ratio of the pore solution conductivity σ_p to the bulk conductivity (solid microstructure plus pore solution) σ_b [2]:

$$\mathcal{F} = \frac{\sigma_p}{\sigma_b} \quad (1)$$

This quantity characterizes the solid microstructure since the only difference between the conductivities is due to the restricted pathways through which the current is constrained in the bulk conductivity measurement.

The motivation for using conduction tests to estimate the diffusion coefficient is the relationship between conductivity and diffusivity. In an electrolytic solution, the contribution an ionic species makes to the overall conductivity can be expressed as a function of its conventional (electrochemical) mobility u , amount-of-substance concentration c , and valence z [3]:

$$\sigma = |z| c F u \quad (2)$$

The quantity F is the Faraday constant. The mobility can then be related to diffusivity through the Einstein relation [3]:

$$z F D = R T u \quad (3)$$

The quantity R is the universal gas constant and T is the absolute temperature. Using these equations, one could estimate the diffusion coefficient from either measurements of ion mobility (driven diffusion) or from the conductivity contribution of a particular ion (conductivity). The application of Equation (3) should be performed with some caution since it is exact only at infinite dilution and becomes less accurate with increasing concentration.

Regardless of the electrolyte concentration, there is a direct relationship between an ion's diffusion coefficient and its mobility. Because of this relationship, if a porous material is saturated with a dilute electrolytic solution, the ratio of the pore solution conductivity σ_p to the bulk conductivity σ_b would be equal to the ratio of the diffusion coefficient of an ion in the pore solution D_p to the bulk diffusion coefficient D_b of that ion:

$$\mathcal{F} = \frac{\sigma_p}{\sigma_b} = \frac{D_p}{D_b} \quad (4)$$

This relationship holds even if the Einstein relation Equation (3) is incorrect at high values of concentration because the Einstein relation would be incorrect by some unknown multiplicative error that would apply to both D_p and D_b , and would cancel. Since the self-diffusion coefficient of ions can be found in books, this is a provocative approach to determining the bulk diffusion coefficient D_b [4].

An experimental difficulty is that the pore solution of typical cementitious systems can have a large ionic strength (0.5 mol.kg⁻¹ to 1.0 mol.kg⁻¹), and so care must be exercised when predicting the bulk diffusion coefficient from the formation factor. At these large ionic strengths the pore solution diffusion coefficient D_p of an ion is different from the self-diffusion coefficient reported in tables [5]. Equation (4) can still be used, given that one can correctly determine the diffusion coefficient D_p of a particular ion in the pore solution. Unfortunately, given the constraints of most experimental diffusion apparatus, and given that the pore solution typically changes during the test, an exact value for D_p can be difficult to define.

An alternative approach is to determine the formation factor from the diffusion data by separating the microstructural dependence from the concentration dependence in the diffusion coefficient. It will be shown that the formation factor can be alternatively expressed as a function of a *microstructural* diffusion coefficient of the porous material that is independent of changes in the pore solution chemistry, and a self-diffusion coefficient that is independent of the experimental procedure. The

resulting formation factor could then be used to predict the diffusive transport of any ion within the studied system since it uniquely characterizes the solid pore structure; the chemical effects would be calculated independently.

The origin of the relationship between formation factor and diffusion coefficient will be studied in detail. The analysis will elucidate the proper use of the formation factor for estimating either the microstructural or the concentration dependent diffusion coefficient. Conversely, determining the formation factor from diffusion measurements will also be discussed. The method proposed here will use a computer program to solve the electro-diffusion equation. To demonstrate this method, an experiment will be performed on a commercial sintered alumina ceramic specimen, saturated with various electrolytes. The estimate of the formation factor from the diffusion test, after accounting for ion-ion interactions, is compared to the value determined from the conductivity measurement.

2. THEORY

The proper application of the formation factor to porous systems containing concentrated electrolytes can be best studied by starting with the appropriate transport equation. For the case of diffusing charged species in concentrated electrolytes, the transport is governed by coupled electrical and diffusive transport, and ion-ion interactions must be also be considered. To facilitate its application to experimental systems, the discussion begins with the phenomenological Fick's equation.

2.1 Diffusion coefficient

The typical approach to characterizing diffusive transport of ionic species in porous materials begins with Fick's law of diffusion that relates the bulk diffusion flux \mathbf{j} to an apparent bulk diffusion coefficient D_b for a species with concentration c [7]:

$$\mathbf{j}_b = D_b \nabla c \quad (5)$$

This equation is applied to each diffusing species. The typical approach is to then "fit" experimental data to this equation, yielding an apparent bulk diffusion coefficient D_b for the particular species being investigated. Fick's law does not account for interactions known to exist between, and among, diffusion ionic species. This has led some researchers to characterize the diffusion of ionic

species through the use of Fick's law and a diffusion coefficient tensor [13]. This approach, however, yields a diffusion coefficient tensor that characterizes a particular experiment and cannot, in general, be applied to different scenarios.

The diffusion of ionic species in an electrolyte is better characterized by the electro-diffusion equation. For the i -th ionic species, the electro-diffusion equation relates the bulk flux \mathbf{j} to the concentration c , the diffusion potential ψ , the bulk microstructural diffusion coefficient D_μ , and the bulk conventional mobility u [6]:

$$\mathbf{j} = -D_\mu \left(1 + \frac{\partial \ln \gamma}{\partial \ln c} \right) \nabla c - z F u c \nabla \psi \quad (6)$$

The quantity γ is the activity coefficient for the species. Although this equation neglects adsorption effects, it is otherwise a complete equilibrium thermodynamic description of nonreactive diffusive transport of charged species in concentrated electrolytes. Equation [6] still bears a resemblance to Fick's law of diffusion [7]. The quantity $D_\mu [1 + \partial \ln \gamma / \partial \ln c]$ is an *agglomerated* diffusion coefficient that includes the effects of both the microstructure and ion-ion interactions at high concentration.

It should be noted that the *agglomerated* diffusion coefficient is not the *apparent* bulk diffusion coefficient D_b . Strictly speaking, the apparent diffusion coefficient also includes the effects of the diffusion potential ψ , which is related to the electrostatic interactions of the ions. In those cases where the self-diffusion coefficients of all the diffusing species are nearly identical, the diffusion potential will be nearly zero, and the apparent bulk diffusion coefficient will be nearly equal to the agglomerated diffusion coefficient. For cementitious systems, however, there are many species present with varying self-diffusion coefficients.

Ideally, one would like to distinguish between the effects due to microstructural changes and the effects due to changes in the pore solution electro-chemistry. By observation of Equation (6), the microstructural diffusion coefficient D_μ characterizes the solid microstructure, the quantity in parenthesis characterizes the thermodynamics, and the last term characterizes electrostatic interactions. The microstructural diffusion coefficient is itself independent of the pore solution chemistry. Demonstrating how to extract the value of microstructural diffusion coefficient from a diffusion experiment is accomplished through a detailed discussion of the formation factor.

2.2 Formation factor

Consider an electrical measurement of the formation factor \mathcal{F} performed on a porous specimen saturated with

a concentrated electrolyte. The conductivity of the pore solution σ_p is a function of the ionic strength I (for N ionic species, each with valence z_i and molality m_i):

$$I = \frac{1}{2} \sum_{i=1}^N z_i^2 m_i \quad (7)$$

Based on Equation (2), the conductivity is proportional to the ionic strength so that as the ionic strength approaches a value of zero, the conductivity also approaches a value of zero. Similarly, the bulk conductivity should have the same dependence on the pore solution conductivity. If the pore solution conductivity doubles, the bulk conductivity will also double; surface conduction contributions will be ignored until later in the discussion.

The functional dependence that the diffusion coefficient has on concentration is different from that of conductivity. In the limit that the ionic strength I goes to zero, the diffusion coefficient of each ion in the solution remains finite. It is these finite values D_∞ , in the limit of infinite dilution, that are reported in tables, and commonly referred to as the self-diffusion coefficient. For increasing concentration, the effect of the thermodynamic and electrostatic interactions on the pore solution diffusion coefficient D_p can be characterized by some arbitrarily complex analytical function g that is a function of the ionic strength I , the activity coefficient γ , and the diffusion potential ψ :

$$D_p = D_\infty [1 + g(I, \gamma, \psi)] \quad (8)$$

Effectively, D_∞ characterizes the diffusion of an ionic species in "free space," in the absence of interactions with ions of the same, or of different, species. An analytical expression for the function g can be formulated for a symmetrical binary electrolyte from a combination of the Debye-Hückel [8] and the Onsager-Fuoss [9] theories.

A similar relation would hold for the apparent bulk diffusion coefficient for an ion within a porous medium saturated with the same electrolyte as was discussed above. Within the electrolyte, the ion would experience the same ion-ion interactions. In addition, the ion would also be traversing the microstructure of the porous solid. Therefore, the bulk diffusion coefficient D_b would have the same chemical dependence as the pore diffusion coefficient:

$$D_b = D_\mu [1 + g(I, \gamma, \psi)] \quad (9)$$

Since the function g encapsulates all the ion-ion interactions, and since interactions between diffusing species and the solid are ignored here, the quantity D_μ would characterize the "free diffusion" of the ionic

species. This would include the self-diffusion of the species within the electrolyte and the constraint of the solid microstructure, and is identical to D_μ in Equation (6).

Using these relations, the formation factor can be written as a function of quantities that are themselves functions of the ionic strength I , activity coefficient γ , and the diffusion potential ψ_p :

$$\mathcal{F} = \frac{\sigma_p(I)}{\sigma_b(I)} = \frac{D_\infty [1 + g(I, \gamma, \psi_p)]}{D_\mu [1 + g(I, \gamma, \psi_p)]} = \frac{D_\infty}{D_\mu} \quad (10)$$

The resulting formation factor \mathcal{F} is independent of both the ionic strength and the ionic species; a fact that is exploited by numerical calculations of formation factor on model microstructures [10].

The advantage of using the ratio D_∞/D_μ is not immediately obvious. In Equation (4), D_b is observable, but D_p is difficult to determine precisely in typical diffusion apparatus. In Equation (10) above, D_∞ can be found in tables, but the quantity D_μ cannot be observed in electrolytes because the effects of the diffusion potential cannot be eliminated. One can, however, exploit the relationship among D_μ , D_∞ , and \mathcal{F} in the solution to the electro-diffusion equation.

The formation factor is extremely useful in models of ionic transport. Accurate models for service life predictions must account for independent changes in either the solid microstructure or the pore solution chemistry. A suitable computer program could solve the electro-diffusion transport equation by using estimates of the ion activity coefficient γ from published empirical relations and using the local electro-neutrality condition to solve for the diffusion potential. The microstructural diffusion coefficient D_μ in the electro-diffusion equation would be calculated from the formation factor \mathcal{F} and the dilute limit self diffusion coefficient D_∞ :

$$D_\mu = \frac{D_\infty}{\mathcal{F}} \quad (11)$$

From this and the boundary conditions, the electro-diffusion equation (Equation (6)) is completely specified.

Similarly, one can determine the formation factor from experimental diffusion data by using a computer program that implements Equations (6) and (11). Since the formation factor characterizes the microstructure, it is a constant, independent of ionic species. The formation factor and porosity are then the only two adjustable parameters required to model time-dependent diffusive transport, regardless of the number of different ionic species present. To estimate the formation factor from

experimental data, one adjusts the formation factor in the computer program until the computed output matches the experimentally observed quantities. The experimental program described subsequently used this approach to determine the formation factor from divided cell diffusion measurements.

3. EXPERIMENT

The specimens for this experiment were a sintered alumina ceramic frit typically used for filtration, with an advertised pore size less than $0.5 \mu\text{m}$. Mercury intrusion porosimetry (MIP) measurements confirmed this value, and also gave an estimated total porosity of 26%. The cylindrical specimens were approximately 50 mm in diameter and 6.4 mm thick. Each specimen was mounted into an acrylic annulus using an epoxy adhesive. After saturating with a known electrolyte, the specimens were clamped between glass vessels, each filled with approximately 250 mL of electrolyte. The setup for both the conductivity and the diffusion measurements is shown schematically in Fig. 1.

After the specimens were prepared, they were placed into an environmental chamber maintained at a temperature of 25°C . Both the conductivity and the diffusivity experiments were conducted in this chamber, with all measurements and sampling performed within the chamber.

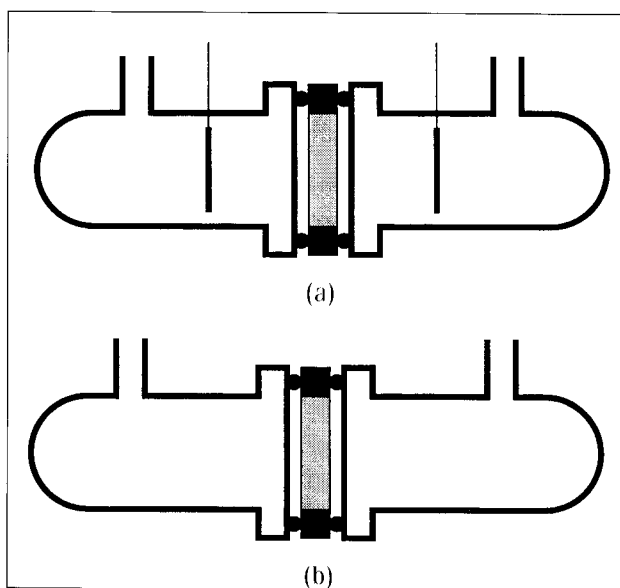


Fig. 1 – Cross section of both the conductivity cell apparatus (a) and the diffusion divide cell apparatus (b). The schematics depicts the configuration of the two cylindrical glass vessels on either side of a mounted specimen. The system is sealed using rubber o-rings; the clamps are not shown. The apparatus differ only in the vertical platinum electrodes in the conductivity cell. The diameter of the specimen, the glass vessels, and the platinum electrodes are approximately 50 mm.

3.1 Formation factor

The formation factor measurements were performed using an aqueous solution of potassium chloride as the pore solution because the conductivity of a few standard concentrations of this electrolyte are known to a high precision [11]. A range of concentrations were used to ensure that the surface conduction component was properly accounted for. The specimens were first vacuum saturated with one of the KCl electrolyte concentrations and then mounted, using rubber o-rings and clamps, between two glass vessels, each containing a platinum electrode. The setup is shown schematically in Fig. 1(a). The apparatus, in the absence of a sample and holder, had a conductivity cell constant of 0.3567 cm^{-1} .

To determine the formation factor of the saturated specimen, both sides of the cell were filled with the same solution as was used to saturate the specimen and then the entire system was allowed to thermally equilibrate in the environmental chamber. The direct current (dc) resistance of the sample and cell was then determined using a commercial impedance spectrometer that sampled frequencies between 10 Hz and 1 MHz. The bulk conductivity of each specimen was calculated from the cell constant and the specimen geometry. The formation factor was calculated from the ratio of the known pore solution conductivity to the calculated bulk conductivity. The procedure was repeated for the other KCl concentrations to investigate the effect of surface conduction.

Potassium chloride pore solution concentrations of 0.01 mol.kg^{-1} , 0.10 mol.kg^{-1} , and 1.00 mol.kg^{-1} were used to determine the specimen conductivity and to assess the contribution from surface conduction. Due to surface conduction contributions, the formation factor increases with increasing pore solution conductivity, converging to the correct value as the concentration increases. The specimen conductivity measured using the 0.01 mol.kg^{-1} solution was approximately 85% of the value using the 1.0 mol.kg^{-1} solution, and the specimen conductivity using the 0.10 mol.kg^{-1} solution was approximately 98% of the value using the 1.0 mol.kg^{-1} solution. Therefore, the formation factor calculated at 1.0 mol.kg^{-1} was used as the best estimate. An estimate of the formation factor at infinite conductivity can be estimated from a Padé approximation [12]. Unfortunately, the approximation contains four parameters, and only three conductivity measurements were taken. Nonetheless, the reported result represents a lower bound to the true formation factor. Since the change in formation factor was only 2% for a ten fold increase in concentration, the true value is probably not

more than a fraction of percent larger than the values reported.

Since the ceramic specimens are the result of a controlled commercial process, they have relatively little variation among specimens. The four specimens used in this experiment had formation factors ranging from 10.6 to 10.9. These values were determined after each sample was allowed sufficient time to reach thermal equilibrium, and then the values of the dc resistance varied by less than 1%. The corresponding calculated formation factor varied by approximately 2%.

3.2 Diffusion

Upon completion of the conductivity measurements, the specimens were removed from the conductivity cell, saturated with distilled water, and then saturated with a test solution for use in the divided cell apparatus. The geometry of the divided cell apparatus was, with the exception of the platinum electrodes, otherwise similar to the conductivity cell. A schematic of the setup is shown in Fig. 1(b).

Four test solutions were chosen for this experiment and are shown in Table 1. The test solution was first used to saturate the specimen and then added to the vessel on one side of the specimen. Potassium iodide was then added to the opposite chamber. The concentration of the potassium iodide was the same as that for the test solution.

Table 1 – Test solutions used in the divided cell experiment. Specimens are initially saturated with the test solution. The opposite vessel contains potassium iodide at the same concentration as the test solution.

Test Solution	Concentration (mol.L^{-1})
KCl	0.1
NaCl	0.1
NaOH	0.1
KCl	1.0

Transport through the specimen was monitored by periodically measuring the iodide concentration in both vessels as a function of time. For each concentration measurement, a one milliliter sample was taken from each of the two vessels and diluted in distilled water. The iodide concentration was determined using a commercial ion selective iodide combination electrode. Reference solutions were used to standardize the probe each day the concentrations were recorded. Previous experiments conducted on nearly identical specimens and apparatus demonstrated that the use of magnetic stirrers has no observable effect on measured results.

3.3 Analysis

The analysis of the data can be performed using any one of many possible methods. In this experiment, the concentration of iodide on both sides of the specimen is changing with time. Rather than try to "fit" the time-dependent concentration data on both sides of the cell, the analysis was based upon a linear idealization of Fick's law so that behavior that cannot be sufficiently modeled using Fick's law will be apparent.

Assuming ideal diffusive behavior, for a sufficiently low diffusivity sample and sufficiently large vessels, the concentration profile across the specimen should become practically linear after some initial induction period. At this point, the flux of iodide would be constant across the sample, and the corresponding concentration gradient would also be constant. This behavior is referred to here as the constant gradient approximation (CGA), and has been used elsewhere to analyze diffusion data [13]. The schematic shown in Fig. 2 depicts a system in the constant gradient state. The thin line depicts the concentration of iodide throughout the system; constant in each vessel and a straight line with constant slope across the sample. The specimen apparent bulk diffusivity is D_b , the thickness is L , the area is A , and the volume of each vessel is v_1 and v_2 , each with iodide concentration c_1 and c_2 , respectively. Under the CGA, the flux is constant and the rate of change in iodide concentration in each vessel is also a constant:

$$\frac{\partial c_1}{\partial t} = \frac{AD_b}{v_1} \frac{c_2 - c_1}{L} = -\frac{v_2}{v_1} \frac{\partial c_2}{\partial t} \quad (12)$$

Upon making the following substitution for the concentration difference, $\Delta = c_2 - c_1$, the time dependent behavior for Δ can be expressed as an exponential [13]:

$$\Delta = \Delta_0 \exp \left[-\frac{AD_b}{L} \left(\frac{1}{v_1} + \frac{1}{v_2} \right) t \right] \quad (13)$$

The quantity Δ_0 is the concentration difference at the onset of the constant gradient. Based on Equation (13), a semi-logarithmic plot of Δ versus time data would be a straight line, with a slope that is proportional to the apparent bulk diffusion coefficient D_b .

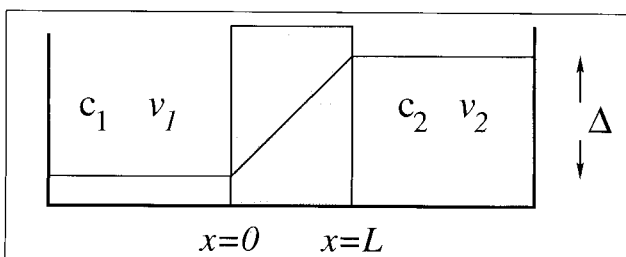


Fig. 2 – Schematic of the constant gradient approximation (CGA) for a sample with thickness L . Vessels 1 and 2 contain ionic species with concentrations c_1 and c_2 , and have volumes v_1 and v_2 , respectively.

The formation factor is estimated from the diffusion data through the use of a computer program that simulates the diffusion experiment by implementing the electro-diffusion equation (Equation (6)). A similar computer program has been described previously [14]. The computer program performs an electro-diffusion transport calculation using Equation (6) and knowledge of the sample porosity and the pore solution chemistry. The microstructural diffusion coefficient D_μ is calculated from the formation factor using Equation (11). The formation factor is varied (porosity is held constant) until the slope of the calculated values of Δ equals the slope of the experimental values.

The computer program calculated the solution to Equation (6) using a finite difference scheme. The system was represented by a one-dimensional mesh composed of 21 nodes. The differencing algorithm was fully explicit, but the stability criterion was satisfied by a factor of five. The computer program calculated the activity coefficients using an implementation of the Pitzer equations [15] that was based on the PHRQPITZ [16] computer program. The diffusion potential was calculated using the local electro-neutral (zero current) hypothesis [6]. For the species flux j_i , as given in Equation (6), the total current I_T is the sum over the individual fluxes, each proportional to the species charge z_i :

$$I_T = \sum_i z_i j_i = 0 \quad (14)$$

The diffusion potential gradient is chosen so that this relation is satisfied at the boundary of each computational element, assuring both local and global charge neutrality.

As a test of the computer program, the diffusion

Table 2 – Comparison of diffusion coefficients D from the computer program (CP) and handbook (HB) values for some 1:1 valence salts. The handbook values are from the CRC Handbook of Chemistry and Physics.

Salt	conc. mol.L ⁻¹	D_{HB} 10 ⁻⁹ m ² .s ⁻¹	D_{CP} 10 ⁻⁹ m ² .s ⁻¹
KCl	0.01	1.917	1.902
	0.10	1.844	1.807
	1.00	1.892	1.801
NaCl	0.01	1.545	1.539
	0.10	1.483	1.476
	1.00	1.484	1.571
KI	0.10	1.865	1.829
	1.00	2.065	1.911

coefficient of 1:1 valence salts in bulk liquid are calculated by the computer program (CP) and the values

are compared to values reported in a chemistry handbook (HB) [17]; the computer program was executed with both the formation factor and the porosity fixed at a value of one. The calculations are performed over a range of salt concentrations and the results are shown in Table 2. Generally, the computed results agree quite favorably with reported values, with the worst case being a difference of approximately 10%. Having thus demonstrated the ability and accuracy of the computer program, one can make direct comparisons between its prediction of a particular scenario and the corresponding experimental data.

Based on Equation (13), a semi-logarithmic plot of Δ versus time data would be a straight line. The slope of this line is determined first for the experimental data. To determine the formation factor \mathcal{F} from these data, values for \mathcal{F} are input to the computer program and are varied until a linear least squares regression of the calculated values of Δ versus time, on a semi-logarithmic plot, gives the same slope as for the corresponding experimental data. Deviations from linearity would indicate behavior that cannot be characterized using Fick's law.

4. RESULTS

Both the experimental and calculated values are shown in Fig. 3. The measured values of Δ for the iodide concentration are shown as symbols, and the calculated results are shown as curves. The estimated uncertainties in the experimental values are approximately the size of the symbols, and are not shown as error bars for

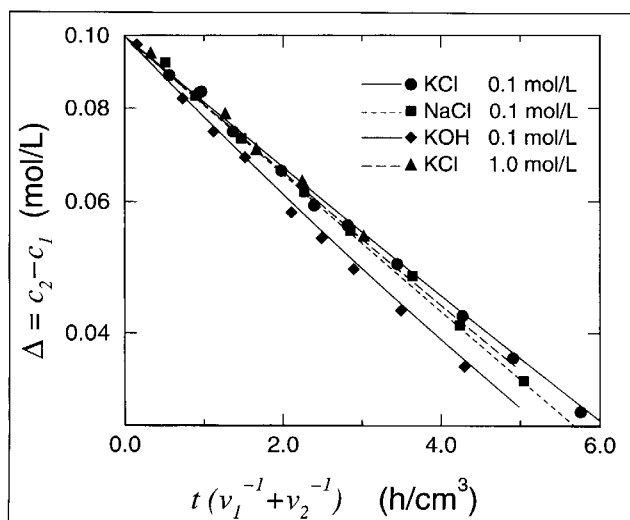


Fig. 3 – Concentration difference Δ across each sample as a function of time. The experimental values are shown as filled symbols, the calculated values are shown as solid curves. The measurement uncertainties would appear as the same size as the symbols, so are omitted for visual clarity. The value of Δ for the 1.0 mol/L KCl system is divided by ten in order to appear on the same scale as the other data.

reasons of visual clarity. The values of Δ for the 1.0 mol/L KCl system were divided by ten so that they could be included on the same plot.

The KCl and the NaCl systems behaved similarly. The data for the KCl systems are nearly collinear, demonstrating nearly ideal diffusive behavior that can be accurately characterized by Fick's law. There is virtually no concentration dependence in the results for the KCl/KI systems because the self diffusion coefficient D_∞ for K^+ , Cl^- , and I^- are nearly equal to one another. The experimental data for the NaCl/KI system was also practically linear, also indicating nearly ideal diffusive behavior. The estimated diffusion potential for both of the KCl systems was less than 1 mV, and was less than 5 mV for the NaCl system.

The KOH system showed a noticeable difference in behavior. The curvature in the experimental data indicates behavior that cannot be characterized by Fick's law of diffusion. The calculated values, based on the electrodiffusion equation (Equation (6)), also exhibit the same curvature as was observed in the experimental data. One reason for this is that the self diffusion coefficient D_∞ for OH^- is significantly greater than that of the other ions present, resulting in a calculated diffusion potential of approximately 16 mV.

Table 3 – The values for the slope (AD_b/L) of the experimental data shown in Fig. 3. Also shown is the ratio D_∞/D_b , using D_∞ for iodide. The uncertainties shown for the slopes are the estimated standard deviation reported by the statistical software, and also characterize the uncertainty in the ratio D_∞/D_b reported.

System	AD_b/L ($cm^3 \cdot h^{-1}$)	D_∞/D_b
KCl – 0.1 mol/L	0.2004 ± 0.0032	11.1
NaCl – 0.1 mol/L	0.2118 ± 0.0032	10.3
KOH – 0.1 mol/L	0.2381 ± 0.0047	9.3
KCl – 1.0 mol/L	0.2079 ± 0.0050	10.7

The measured slopes of the experimental data, on semi-logarithmic axes, are shown in Table 3. The estimated standard deviations shown are typically less than 3% of the slope value, suggesting that the CGA is reasonable approximation for these systems. The systems reached a constant gradient state at a relatively early age. Output from the computer program suggested that a nearly constant gradient is achieved in less than 12 h.

Also shown in Table 3 are the values for the ratio D_∞/D_b , using the iodide value for D_∞ ($2.045 \times 10^{-5} cm^2 s^{-1}$ [5]). This ratio represents an incorrect application of using the formation factor to determine the apparent bulk diffusion coefficient. Since the quantity D_∞ in this ratio is a constant, the ratios are simply proportional to

the apparent bulk diffusivity D_b . This ratio, however, does not reflect the actual formation factor, because the iodide self diffusion coefficient within the pore solutions is not equal to D_∞ . Also, arbitrary changes in the pore solution will lead to changes in the apparent bulk diffusion coefficient of iodide in these systems, while the formation factor is nearly equal for all systems.

The correct values of the formation factor \mathcal{F} were determined from the experimental data using the aforementioned computer program, and are shown in Table 4, labelled \mathcal{F}_{sim} . Also shown in the table are the values of the formation factor calculated from the impedance spectroscopy measurements, labelled \mathcal{F}_{IS} . The uncertainty in \mathcal{F}_{IS} reflects the variation in the dc resistance measurement as mentioned previously. The values of \mathcal{F}_{sim} shown in Table 4 were used to calculate values for Δ , and these values of Δ are plotted in Fig. 3, denoted by the curves. The experimental data and the calculated values for the KCl and the NaCl systems were all nearly linear. The values of Δ for the KOH system are easily distinguished from the other systems.

The calculated values of \mathcal{F}_{sim} shown in Table 4 were consistent with the measured values \mathcal{F}_{IS} . The values of \mathcal{F}_{sim} varied by approximately 7%, compared to the 18% variation in the values of D_∞/D_b . The differences between the values of \mathcal{F}_{sim} and \mathcal{F}_{IS} were less than 3 % for the KCl and the NaCl system, and less than 8 % for the KOH system.

The calculated values \mathcal{F}_{sim} were generally greater than the measured \mathcal{F}_{IS} values. A partial explanation for this can be found in the data in Table 2. In that table, the estimated diffusion coefficients were consistently less than the handbook values. This suggests that results from the computer program yield a bulk diffusion coefficient that is smaller than it is in reality. Therefore, for the computer program to agree with experimental data, the formation factor \mathcal{F}_{sim} must be made smaller than its true value, which is consistent with the data in Table 4.

Sighting along the KOH data reveals that these data have some curvature. It is interesting to note that the computed output (solid curve) also exhibits this nonlinear behavior. This suggests that this nonlinear behavior is due to effects of the pore solution chemistry since output from the computer program indicates that the iodide concentration profile across the sample is stable within 12 h. The calculated concentration profile of iodide, however, is not linear due to the diffusion potential.

Table 4 – Measured and calculated formation factors from impedance spectroscopy (\mathcal{F}_{IS}), computer simulation (\mathcal{F}_{sim}), and apparent diffusivity (D_∞/D_b) using D_∞ for iodide.

System	\mathcal{F}_{IS}	\mathcal{F}_{sim}	D_∞/D_b
KCl – 0.1 mol/L	10.7±0.2	10.9	11.1
NaCl – 0.1 mol/L	10.9±0.2	11.2	10.5
KOH – 0.1 mol/L	10.6±0.2	11.4	9.3
KCl – 1.0 mol/L	10.7±0.2	10.6	10.7

This electro-chemical effect of the KOH system is revealed in Table 4. The apparent diffusion coefficient of iodide in this system is considerably greater than that for the other systems, even though the computer calculation reveals that the calculated formation factors \mathcal{F}_{sim} are all nearly identical to the electrical values \mathcal{F}_{IS} . This fact demonstrates the effect of using the apparent diffusion coefficient D_b to characterize a microstructure. For that particular test solution, the apparent diffusion coefficient describes how the iodide ion behaves in the presence of KOH, but does not characterize its behavior in the presence of other test solutions. Similarly, it does not necessarily characterize how other ions behave in the same, or similar, microstructure.

Since the pore solution of cementitious systems is typically alkaline, the results for the KOH system have direct relevance to the prediction of ion transport in portland cement systems. The pore solution ionic strength in cementitious systems can be nearly ten times greater than the 0.1 mol/L KOH system studied here. Further, there will be number of additional ions present, with a corresponding number of different self diffusion coefficients. This raises the question of the correct method for characterizing the microstructure of these systems. Either predicting the formation factor from diffusion data or predicting the diffusion coefficient from a formation factor measurement will require a knowledge of the pore solution chemistry. For a formation factor measurement that implements pore solution extraction, a chemical analysis of the extracted pore fluid would be a logical extension of the measurement procedure. Typical diffusion experiments do include these measurements, and so additional analysis may be required for experimental programs that use diffusion measurements to characterize the microstructure of porous cementitious systems.

5. CONCLUSION

Experiments performed using ceramic frits yield evidence for the equivalence between the formation factor and the *microstructural* diffusion coefficient, which is a characterization of the porous microstructure. Due to the complexity of accounting for the chemical behavior of the pore solution, extracting the microstructural diffusion coefficient from diffusion data requires a numerical calculation. While the apparent diffusion coefficient depended upon the chemical makeup of the pore solution, the procedure outlined here was able to extract the microstructural diffusion coefficient for each system, yielding a similar value for the systems studied. The presence of KOH in the pore solution had a noticeable affect on the apparent diffusion coefficient of iodide. Due to the similarity between the self diffusion coefficient of iodide and chloride, one would expect similar effects on chloride ions in cementitious systems. The ability to extract the microstructural diffusion coefficient from observed data has a direct influence on service life modeling that can independently account for changes in either the pore structure or the pore solution chemistry. This is particularly important in cementitious systems containing pore solutions with large ionic strengths.

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