MODEL BASED PREDICTION OF PERMEABILITY IN PREFORM MATERIALS

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Abstract

Knowledge of the permeability tensor in liquid composite molding is important for process modeling and optimization. However, experimental determination of the permeability is difficult and time consuming. In this work, a lattice Boltzmann simulation which has been modified for flow in porous media is used to predict permeability as a function of yarn location, orientation, and fiber fraction. Calculated permeabilities are compared with experimental measurements for a variety of systems. Good agreement is achieved as long as the mesh size is greater than the size of the smallest throats in the porous medium.

Introduction

Fluid flow in the fiber porous media used in composites processing operations is generally modeled using Darcy's law, given by

$$\vec{v} = -\frac{\underline{\underline{K}}}{\mu}\times \nabla P \qquad (1)$$

where $\vec{v}$ is the average (superficial) velocity in the medium, P is the pressure, $\underline{\underline{K}}$ is a symmetric, second order tensor known as the permeability and μ is the fluid viscosity.

The development of tools for predicting the permeability of fibrous porous media as a function of structure is of practical importance in a number of composites manufacturing processes. This capability would speed process design by helping to reduce the large number of experimental measurements currently required to determine such data, and help towards establishing processing-performance relations. This is especially true in light of the advances being made in the development of computational textile modeling tools, which enable rapid construction and evaluation of new textile designs (1,2), as well as imaging methods for determining the structure of the fiber material (3,4).

Computational prediction of permeability (4-9) involves developing a detailed model for the flow geometry, imposing a pressure drop across the media, solving the appropriate transport equations for the detailed flow field, and then back-calculating the permeability by applying Darcy's law. Selection of the appropriate transport equations for modeling flow in fibrous porous media is complicated by the fact that this problem involves both an open region around the fiber tows that make up the media, and the porous region inside the tows. One approach is to model the flow in the open media using the Stokes equation given by

$$\nabla P = \mu \nabla^2 \vec{v} \qquad (2)$$

and to model the flow inside the tows using the Brinkman equation, which has the form

$$\nabla P = \mu \nabla^2 \vec{v} - \mu \underline{\underline{K}}^{-1} \times \vec{v} \qquad (3)$$

Equation (3) has the form of a superposition of Darcy's law and the Stokes equation. Thus, it is able to properly describe Darcy flow in regions where velocity gradients are low, and also, due to the second order terms, to satisfy the proper boundary conditions (i.e., continuity of velocity and stress) at the tow-fluid interface separating the discontinuous media.

Previous studies using the "Brinkman approach" fall into two categories. In the first case, the model equations are solved using "unit cell" flow geometries which due to imperfections and packing anomalies, are only approximations of the actual fiber structure. In the second case (such as that obtained by imaging data), detailed information on the flow geometry is available, but it is difficult to be sure that all the statistical variations present in the full porous media are accounted for in the model microstructure. Thus, the lack of one-to-one correspondence between the flow geometry used in the calculations and the actual porous media used to obtain experimental measurements, makes evaluation of the accuracy of the Brinkman approach difficult to interpret.

In this study, a model porous media is constructed. Experimental measurements are compared with calculations using a Lattice Boltzmann model modified for flow in fibrous porous media. This controlled study is undertaken in order to try and accurately evaluate the applicability of the Brinkman approach to flow in such heterogeneous media. Results for saturated permeability computations are reported here.

Experimental

Model Porous Media

The model porous media was constructed using an array of porous disks thermoformed from continuous strand mat (CSM). The 7.62 cm disks were formed by compression molding layers of CSM (Uniflo 750 by Vetrotex Certainteed) in a hydraulic press. The resulting preform was placed in a pneumatic press and the disks were stamped out using a 7.62 cm circular punch. The CSM disks, representing the tows in a fiber preform, were arranged in either a square or hexagonal array in a window frame type mold. These arrays were typically 4 unit cells wide and 4 unit cells in length. The spacing between the circles was varied from 0 cm to 0.08 cm. A typical configuration is shown in ***Figure 1***.

The mold is a window frame type mold with spreading gates at both the inlet and outlet ports. The top platen of the mold has a 30.48 cm x 35.56 cm x 3.81 cm thick glass window for monitoring the flow process. The bottom platen has an array of pins that can be projected into the mold cavity to secure the circles or recessed depending on the desired arrangement of the preform circles. The pins can also be removed to serve as ports for pressure transducers. The mold platens were clamped together with bolts around the perimeter. Two 103 kPa pressure transducers (Omega, model PX605) were used to monitor the pressure in the mold. One pressure transducer was 3.81 cm from the spreading gate and was in-line with the inlet and outlet ports. The second pressure transducer was 15.24 cm from the first pressure transducer and was also in-line with the ports. The molding experiments were carried out in a vertical arrangement resulting in a hydrostatic head. The hydrostatic head ranged from 5.5 kPa to 6.2 kPa depending on the arrangement of circles.

Permeability Measurements

Permeability measurements were conducted on pure both pure CSM mat alone (in order to characterize the permeability of the model tows) and on the model porous media in a number of different configurations. Corn syrup solution was used as the test fluid. Distilled water was added to corn syrup in a ratio of 2/5. This ratio produced a corn syrup/water solution with a viscosity of 0.125 Pa-s at 23° C.

The corn syrup solution was injected into the mold under constant pressure conditions using compressed nitrogen. The flow rate into the mold was monitored using calibrated capillary tubes and a differential pressure transducer across the ends of the tube. The flow rate exiting the mold was determined by measuring the volumetric flow rate of the fluid using a graduated cylinder and a stopwatch. The flow rate was adjusted using a control valve between the calibrated tubes and the inlet port of the mold.

During the experiments on the model porous media, as the initial shot of resin passed around the tows, a pocket of air was entrapped (***Figure 2***). The corn syrup solution was continually pumped through the system until all the voids were eliminating. After saturation occurred, the volumetric flow rate and pressures were measured. Typically six measurements were recorded at each pressure and the flow rate was cycled through the maximum pressure of the pressure transducers or the control valve range. This was done to assure there were no changes in the model porous media characteristics as a result of the flow.

Numerical Modeling

Lattice Boltzmann Method

Solutions to model Equations (2-3), together with a conservation of mass equation, were obtained using a lattice Boltzmann method previously described in detail elsewhere (5,6). The method involves the solution of the discrete Boltzmann equation for the particle velocity distribution function $n_a(\underline{x},t)$, where traditional fluid flow quantities such as density and velocity are obtained through the moment sums

$$r = m \sum_{a=1}^{N} n_a(\underline{x},t) \qquad (4)$$

$$u = \frac{m}{r(\underline{x},t)} \sum_{a=1}^{N} \underline{v}_a n_a(\underline{x},t) \qquad (5)$$

where $r(\underline{x},t)$ and $\underline{u}(\underline{x},t)$ are the macroscopic fluid density and velocity, m is the mass of fluid, $\underline{v}_a$ are components of the discrete velocity space, and N is the number of velocities comprising the velocity space. The particle distribution function $n_a(\underline{x},t)$ is governed by the discrete Boltzmann equation given by

$$n_a(\underline{x}+v_a, t+1) = n_a(\underline{x},t) + d_a(\underline{x},t) \qquad (6)$$

where $d_a(\underline{x},t)$ is the collision operator which couples the set of velocity states $\underline{v}_a$. Most LB formulations employ the linear "BGK" form (5,6,10) of the collision operator in which the distribution function is expanded about its equilibrium value

$$d_a(\underline{x},t) = \frac{n_a(\underline{x},t) \quad n_a^{eq}(\underline{x},t)}{t} \qquad (7)$$

where $n_a^{eq}(\underline{x},t)$ is called the equilibrium distribution function and t is a relaxation time for collisions controlling the rate of approach to equilibrium. The form of the equilibrium distribution function depends on the particular lattice model chosen. The three-dimensional, "d3q15" model (10) which resides on a cubic lattice is used here (d3 indicates the model is three-dimensional, q15 refers to the number of components in the velocity space).

To model flow in reinforcement materials that have both open and porous regions, the velocity in porous regions is defined as

$$\underline{U} = \underline{u}(\underline{x},t) + \frac{t\,\underline{F}(\underline{x},t)}{r(\underline{x},t)} \tag{8}$$

where the function $\underline{F}$ is given by

$$\underline{F}(\underline{x},t) = \quad m\underline{\underline{K}}_{tow}^{\;\;1}\, r(\underline{x},t)\underline{u}(\underline{x},t) \tag{9}$$

where $\underline{u}$ is the Navier-Stokes velocity, and $\underline{\underline{K}}_{tow}$ is the micro-permeability tensor of the reinforcement material in the porous region.

Permeability Computation

Permeability for different flow directions was computed by imposing a constant pressure along opposite faces of the lattice in the desired direction (see ***Figure 3***) and integrating the system of equations above to steady-state. Estimates for the intra-tow permeability values were obtained from the formulas given in (9). The steady-state velocity field at the inlet was integrated over the surface to obtain the flow rate, Q, and this was used in the formula

$$K_{eff} = \frac{mQL}{A\quad P} \tag{10}$$

to obtain the effective permeability, K_{eff}, for the desired direction.

Results and Discussion

In all, four cases were investigated. These are detailed in ***Table 1*** which shows the packing arrangement, inter-tow spacing, intra-tow permeability and the measured and computed permeabilities of the model media. In the first two cases, the tows were arranged such that there was no space between them, but with different intra-tow permeabilities. In the second two cases, the tows were arranged with a 0.08 cm spacing between them, but with square and hexaganol packing arrangements, respectively. An example velocity field from the flow calculations is shown in ***Figure 4***.

The table shows that the permeability values obtained for the case of 0.08 cm spacing between the tows, is in excellent agreement with the experimental measurements. The computed values is about 8% lower for the case of the square packing arrangement, and about 12% higher for the case of the hexagonal packing arrangement. Since permeability measurements themselves have a 20% error associated with them, these results must be considered to be excellent.

Calculated results for the case of zero packing spacing between the model tows are in poor agreement with the experimental measurements. The results for both cases are higher than the experimentally measured values by over an order of magnitude. We have observed the problem previously in purely numerical works (5,7). The problem stems from the inability to accurately mesh the gap between the tows when the tows get extremely close. In this case, the length scale for the diffusion of momentum is on the order of the square root of the tow permeability, which is extremely low. Thus, there is not sufficient resolution to resolve the flow. The consequences of this numerical problem on permeability computation are that we expect the Brinkman approach to be accurate provided our mesh size is finer than the smallest pore size in the medium. Whether this problem has any impact on calculations in woven (or like) materials in which tows touch each other, but there are at the same time large pore spaces for flow, will be the subject of future studies.

Conclusion

A model porous media was constructed for the purpose of testing the "Brinkman" approach to modeling flow in fibrous porous media. Modeling and experimental measurements were compared for flow through an array of porous circular cylinders. Excellent agreement (on the order of 10%) was found between measured and experimental permeabilities when the gap spacing between the tows was of the same order of magnitude as the flow mesh size. However, theory and experiment diverge when the gap spacing is reduced to zero.

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Array Type	**Array Spacing (cm)**	**Tow Permeability (cm^2)**	**Array Permeability (cm^2) Experimental**	**Array Permeability (cm^2) Model**
Square	0	6.17e-7	4.66e-6	6.5e-5
Square	0	1.55e-7	9.06e-7	2.04e-5
Square	0.08	2.68e-7	7.97e-5	7.4e-5
Hexagonal	0.08	2.95e-7	2.61e-5	2.97e-5

Table 1. -- Comparison of experimental cell permeabilities with the LB model for flow through arrays of permeable circles.

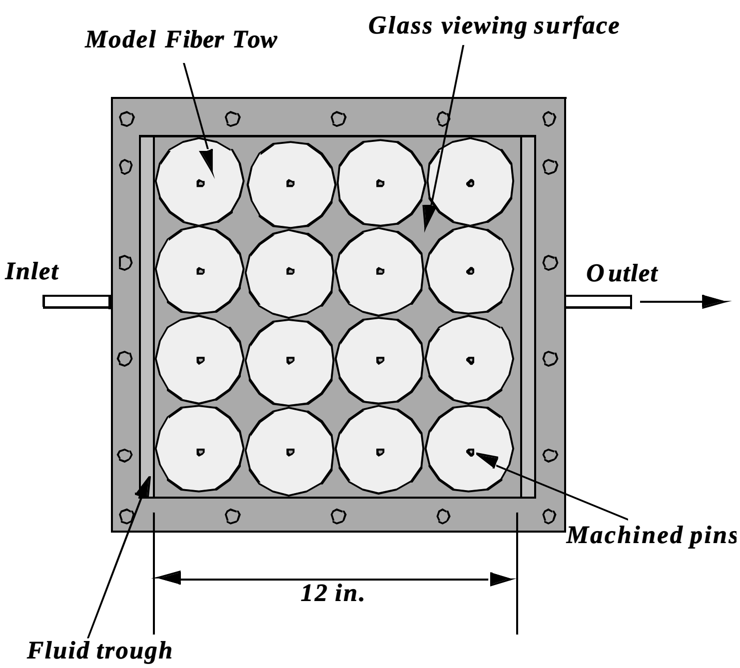

Figure 1. Schematic of the model porous media.

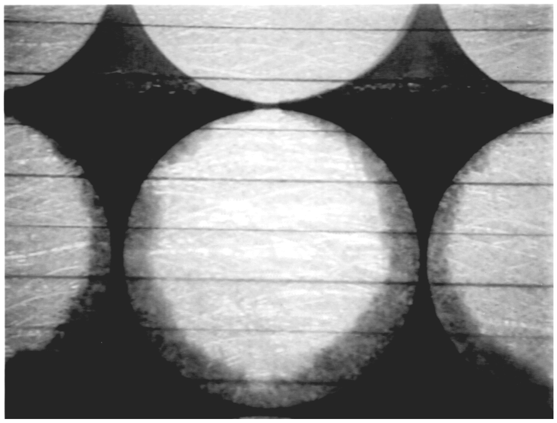

Figure 2. Void formation during the injection of the model porous media.

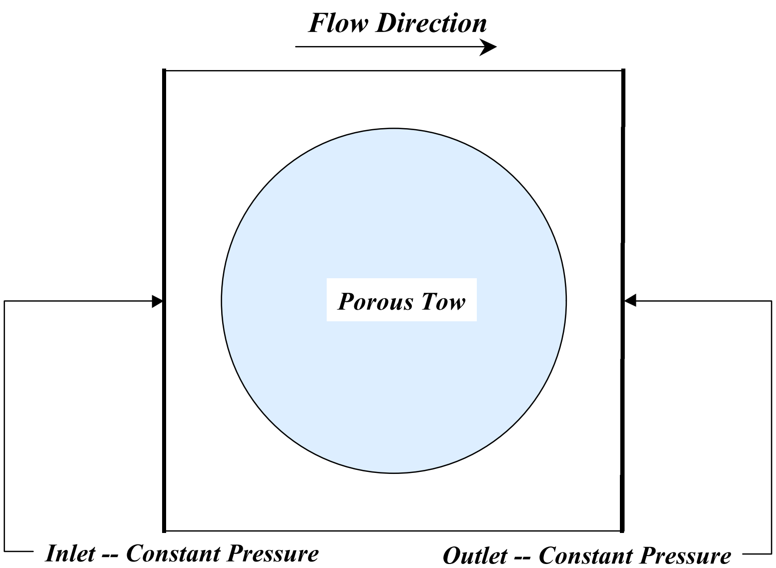

Figure 3. Boundary conditions for the computational geometry.

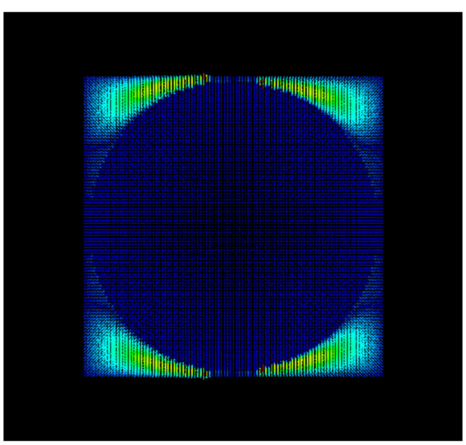

Figure 4. Computed velocity profile for flow through the model porous media.

Keywords: Permeability Tensor, Liquid Composite Molding, Lattice Boltzmann Method, Darcy's Law