Assessment of Sandwich Beam in Three-Point Bending for Measuring Adhesive Shear Modulus

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A finite element analysis (FEA) was conducted to examine the feasibility of determining the shear modulus of an adhesive in a bonded geometry using a three-point bending test on a sandwich beam specimen. The FEA results were compared with the predictions from two analytical solutions for the geometry used to determine the impact of the assumptions that were made in these analyses. The analytical results showed significantly different to the values obtained from other experiments on bulk samples of the adhesive in the glassy region. Although there were some agreements in rubbery region, the negligible sensitivity of the beam stiffness to the presence of adhesive layer makes the agreements very questionable. To examine the possible explanations for these differences in glassy adhesives, sensitivity analysis was conducted to explore the effects of experimental variables. Some possible reasons for the differences are discussed, but none of these reasons taken alone satisfactorily account for the discrepancies. Until an explanation is found, the three-point bending test using a sandwich beam specimen to determine the adhesive shear modulus might not be a desirable test method, at least for the range of geometry examined in this study. [DOI: 10.1115/1.1375159]

Keywords: Adhesives, Finite Element Analysis, 3-Point Bending Sandwich Beam, Shear Modulus, Sensitivity Analysis, Contact Problem

Introduction

There is considerable interest in measuring the shear properties of an adhesive when it is in a bonded joint. Such measurements allow one to assess the state of the adhesive and monitor it as a function of time. For example, changes in properties during bond formation can be monitored, information on the quality of the bond can be obtained, and degradation that occurs during environmental exposure can be followed. Many test methods for determining the shear modulus of adhesive materials have been reported in the literatures [1–6]. Commonly used test methods, such as the napkin ring or thick adherend test, require costly machining and fabricating of the test specimens as well as very careful alignment and testing during the measurement. In 1987, Moussiaux et al. [1] proposed a test method that is simple in both sample preparation and measurement procedure. It involves a three-point bending test of a specimen made by bonding together two substrate bars with the adhesive layer (sandwich beam). The experiment involves measuring the bending stiffness of the sandwich beam, and combining the result with an appropriate analysis to determine the shear modulus of the adhesive. Moussiaux et al. [1] provided a strength-of-materials solution to deduce the shear modulus. Their analysis depends on an assumption that the adhesive is constrained to a thin layer in the core of a thick bonded structure.

Spigel and Roy [7] compared the adhesive shear modulus obtained from a tensile test on bulk material with the calculated adhesive shear modulus obtained from the three-point bending test of a sandwich beam using the analytical solution provided by Moussiaux et al. The results showed an inconsistency of up to one order of magnitude between these two values. The authors replicated the three-point bending result by performing a finite element simulation to validate their experimental measurement. They briefly mentioned some possible causes of the inconsistency between the shear moduli measured, but they did not provide detailed mechanistic analyses.

In addition to Moussiaux et al. [1], a number of other authors have addressed the problem of bending in a sandwich beam under a variety of different boundary and loading conditions (e.g., [8–13]). The analysis of Adams and Weinstein [10] is particularly relevant to the test under discussion here since it also addresses the three-point bending of a sandwich beam. In their solution, the adherends were assumed to be thin enough that the induced axial stress can be approximated as constant along the cross section. The assumptions in this analysis differ from those made in the derivation of Moussiaux et al. [1], but both formulations are limited to linear elastic materials.

One particularly attractive feature of the sandwich beam test is that the standard viscoelastic test equipment can utilize the 3-point bend geometry. Consequently, extension of the method to viscoelastic characterization of the adhesive might be possible. One constraint, however, is that the range of geometry that can be used in the viscoelastic characterization equipment is limited. A recent study by Miyagi et al. [6] investigated this potential and found that the behavior of sandwich beam was liner viscoelastic. Unfortunately, at low temperatures where the adhesive behavior was elastic, calculations with the analyses discussed above produced values for the adhesive shear modulus that were not realistic. One possible explanation is that the geometry required by the test apparatus violates the assumptions in the analyses. The purpose of this paper is to examine this question by conducting finite element analyses (FEA) for the geometries used in the viscoelastic study as well as those outside this range. In addition, comparisons are made with the two analytical solutions to see the effects of the assumptions made in each. Finally, a series of sensitivity studies are conducted to determine the effects of both controlled variables

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(bond thickness, adherend stiffness etc.) and uncontrolled experimental variables (e.g., interfacial debonding, or variation of the bond thickness).

Analytical Solutions

The test geometry is shown in Fig. 1. The stiffness of the sandwich beam loaded in three-point bending (Fig. 1) is called K_s, which is equal to P/δ, where P is the concentrated load at the center of the specimen, and δ is the corresponding deflection at the center under the loading. The two analytical solutions discussed above relate this stiffness to the various elastic and geometric variables of the specimen including the shear modulus of the adhesive, G_a. With the measured stiffness and the elastic and geometric variables other than G_a known, an iterative process can be used to determine the appropriate value of G_a.

Analytical Solution 1. The solution by Moussiaux et al. [1] for the stiffness K_{s1} of an adhesively bonded sandwich beam loaded in three-point bending assumes a state of pure shear in the adhesive layer and linear elasticity for the sandwich beam. This gives a result of:

$$K_{s1}=\frac{P}{\delta}=\frac{32E_f}{\beta}\frac{(t+h)^3}{L^3} \tag{1}$$

with

$$\beta=\left(1+\frac{t}{h}\right)^3\left[4-\frac{4}{\gamma^2}+\frac{3E_f}{2G_f}\left(\frac{h}{L}\right)^2+\frac{12}{\gamma^2}\left(\frac{1}{(\alpha\gamma)^2}-\frac{\tanh(\alpha\gamma)}{(\alpha\gamma)^3}\right)\right] \tag{2}$$

and

$$\alpha^2=\frac{3G_a}{E_f}\left(\frac{L}{h}\right)^2\frac{(1+2t/h)^2}{(t/h)}, \quad \gamma^2=1+\frac{1}{3(1+2t/h)^2} \tag{3}$$

where L $(=2l$ in Fig. 1) is the loading span for the sandwich beam; h and t are the thickness of the adherend and half thickness of the adhesive layer, respectively; E_f is the Young's modulus of the adherends; G_f and G_a are the shear modulus of the adherends and adhesive layer, respectively. In this analytical approach, the assumption of the adhesive layer being in a state of pure shear means that the adhesive layer should be very thin compared with the thickness of the adherend and the length of the beam.

Analytical Approach 2. The analysis by Adams and Weinstein [10] was developed for a structure with thin adherend sheets on a core that is taken as the adhesive layer. Their result for the stiffness of the sandwich beam in three-point bending, K_{s2}, is given by:

$$K_{s2}=\frac{P}{\delta}=\frac{6K_T}{l^3}\left[1+\frac{3(K_T-K)}{l^3K\alpha^2}\left(l-\frac{\tanh\alpha l}{\alpha}\right)\right]^{-1} \tag{4}$$

with

$$K=E_aI_a+2E_fI_f, \quad K_T=E_aI_a+2E_fI_{fa},$$
$$\alpha^2=G_a\left(\frac{2K+(2t+h)^2E_fbh}{KE_fh(2t)}\right) \tag{5}$$

and

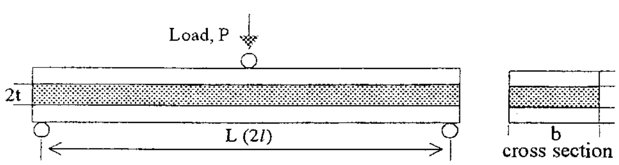

Fig. 1 The 3-point bending sandwich beam, the gray adhesive layer between two adherends

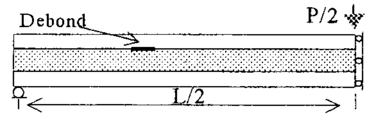

Fig. 2 The model of FEA for the 3-point bending sandwich beam ($L=2l$)

$$I_f=\frac{bh^3}{12}, \quad I_a=\frac{b(2t)^3}{12}, \quad I_{fa}=\frac{bh^3}{12}+\frac{bh(2t+h)^2}{4} \tag{6}$$

where E_a is the Young's modulus of the adhesive ($E_a=2(1+\nu_a)G_a$, with ν_a the Poisson's ratio of adhesive), K is the sum of the bending rigidity of three separate homogenous beams (two adherend bars and one adhesive layer) bending about their own axes, K_T is the pure bending rigidity of the sandwich beam without the shear contribution, and b is the width of the sandwich beam. In this approach, no special restriction was made on the thickness of adhesive layer, as long as the adherends were thin enough to satisfy the requirement that the induced axial stresses in the adherends can be approximated as constant across the section. Contrary to the previous analytical approach, this solution includes not only the contribution of adhesive shear deformation, but also the contribution of adhesive bending to the total beam stiffness.

Finite Element Analysis

For the analysis conducted here, a commercial finite element program, ABAQUS [14] was used. Only a half-length sandwich beam (Fig. 2) was needed for the FEA because the geometry and loading conditions are symmetric. For the best accuracy, the sandwich beam specimen should be modeled using a three-dimensional (3D) finite element analysis. We compared the two-dimensional approach and 3D analyses, however, and found that the added cost and complexity of 3D analysis were not warranted since the 2D (plane stress) approach provided information accurate enough for this study. Therefore, all the analyses performed here used the 2D approach. The deflection of the beam was assumed to be much smaller than the beam thickness, so a linear analysis was adopted. Also, in order to mimic possible debonding during the fabrication of the specimen, an analysis was performed using the symmetric model and assuming symmetric debonding areas along one of the two adherend/adhesive interfaces (Fig. 2). Four-node isoparametric elements were used to model the sandwich beam, with the element dimensions continuously decreasing towards the loading and stress concentration points. The effect of adhesive-adherend contact that may develop in the debonded portion of the interface was studied using a built-in contact element in ABAQUS. A Coulomb-type friction was assumed for the debonding interfaces between the adhesive and adherends [15]. A parametric study showed that the friction has only a marginal effect on the data reported in this work.

Geometry and Material Properties

All of the analyses (FEA and two analytical approaches) were applied to the geometries and material system used in the experiments by Miyagi et al. [6]. They determined the modulus of the adherends by measuring a single adherend bar (steel) in the three-point bend apparatus, and this produced a value typical for steel, (200.9 ± 10) GPa ($\pm$ indicates the standard uncertainty in the data here and in the tables). The Poisson's ratio of the steel adherends was taken as 0.3 [1] so the shear modulus of the adherend, G_f, is (77.3 ± 3.9) GPa. The loading span, L, was (40.88 ± 0.05) mm, the width of the beam, b, was (12.70 ± 0.05) mm, and the thickness of each adherend bar, h, was (0.508 ± 0.01) mm. This results in an L/h ratio of approximately 80. Three different adhesive thicknesses, $2t$, were examined: (0.152 ± 0.008) mm, (0.573 ± 0.017) mm, and (1.297 ± 0.066) mm. Poisson's ratio for the adhesive was assumed to be 0.35 [6]. For the analyses, the shear modulus of the adhesive was varied from 10^0 MPa–10^5 MPa. In addition to the adhesive thicknesses used in the experiments, two other values corresponding to $h/t=10$ and $h/t=100$ were examined in the FEA studies. Finally, a second value of the aspect ratio, $L/h=40$, was examined by FEA to assess the importance of this parameter.

Results and Discussion

Comparison of Different Analyses. Figure 3 shows the relationship between the adhesive shear modulus and the sandwich beam stiffness obtained from FEA, K_{FEA}, for various adhesive layers. If sensitivity in the experiment is defined as the change in stiffness K_s needed to produce a fractional change in G_a, then the sensitivity is given by $d \ln K_s / d \ln G_a$. This quantity is directly related to the slope in the semi-log plot of stiffness versus adhesive shear modulus (Fig. 3). As seen in this figure, regardless of the adhesive shear modulus, the sensitivity is lower when adherend to adhesive thickness ratio, h/t, is high (thin adhesive layer). In addition, the sensitivity becomes higher for most thick bond thicknesses when the adhesive shear modulus G_a is larger than 10 MPa. Below 10 MPa the change in stiffness is very small for most bond thicknesses, it means that the sensitivity of the beam stiffness to the presence of adhesive layer is very low. Unfortunately, this covers most of the rubber range that is of importance for a polymer adhesive.

Figure 4 shows the variation of the total stiffness (K_{FEA}) and the pure bending stiffness (K_B) of the sandwich beam as a function of the adhesive shear modulus. K_{FEA} was calculated from FEA. K_B was obtained from the following equation through the equivalent section method from Timoshenko [16], without including the contribution of the shear deformation:

$$K_B = \frac{6K_T}{l^3} \tag{7}$$

where K_T is defined in Eq. (5). The results in this figure correspond to two bond configurations, the thin ($h/t=6.68$) and thick ($h/t=0.783$) bonds tested by Miyagi et al. [6].

The difference between the pure bending stiffness and the total stiffness of the sandwich beam shown in Fig. 4 is defined as the shear-reduced stiffness and is attributed to the shear deformation of the bond. For the thin and thick bonds, the pure bending stiffness is virtually constant with respect to the change of adhesive modulus, except when the adhesive modulus is comparable to that of the adherend in the thick bond. The stiffness of the sandwich beam with a thin adhesive layer differs from that of the thick bonded sandwich beam primarily by the less pronounced shear deformation of the bond interlayer with softer adhesive material. In addition, one can notice from this figure that, regardless of the bond thickness, when the adhesive modulus is very low (lower than 10 MPa in rubber range), the sandwich beam can be treated as three separate homogenous beams bending about their own axes. Consequently, the total stiffness of sandwich beam can be approximated very well by the sum of the three individually derived stiffnesses for each beam. Since the individual stiffness of the adhesive beam is much less than that of adherend beams, this implies that the sensitivity of the sandwich beam stiffness to the presence of the adhesive layer is diminished at the range of thickness ratio considered here. When the adhesive modulus is comparable to that of the adherend, the total beam stiffness converges to the pure bending stiffness of the beam, since the shear deformation of the bonded layer becomes negligible.

Figures 5 and 6 show comparisons of the sandwich beam stiffness obtained from Eqs. (1) and (4), respectively, to the results obtained from FEA. As shown in Figs. 5 and 6, both analytical approaches, especially Eq. (1), typically overestimate the beam stiffness for a given adhesive shear modulus. This implies that the calculated adhesive modulus from an experimental measurement of the beam stiffness using either of these two analytical solutions is probably lower than the actual value of the adhesive shear modulus. One also can see that Eq. (4) compares much better with the FEA results in evaluating the beam stiffness over the ranges of the adhesive shear modulus and bond thickness considered. The

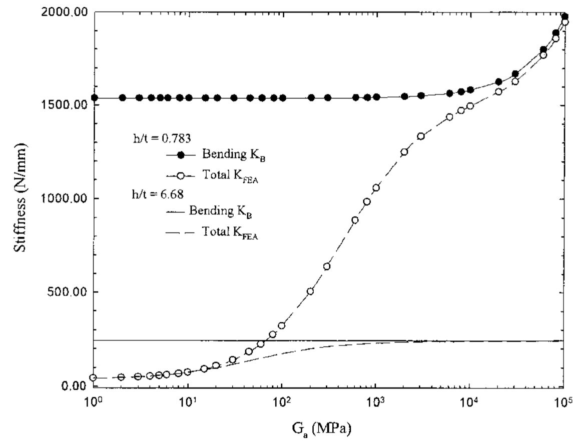

Fig. 4 The variation of the total stiffness and bending stiffness with respect to adhesive shear modulus

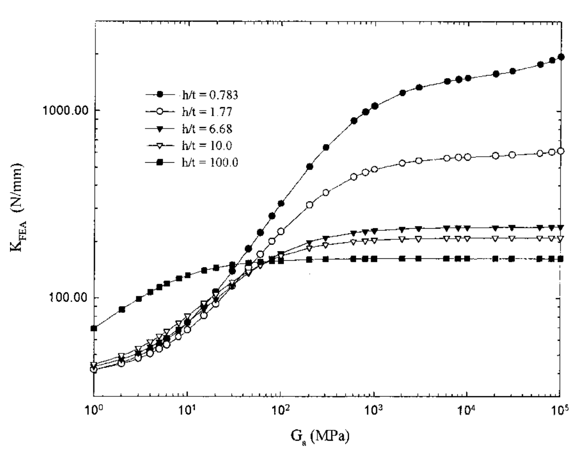

Fig. 3 The stiffness of the sandwich beam K_{FEA} obtained from the FEA

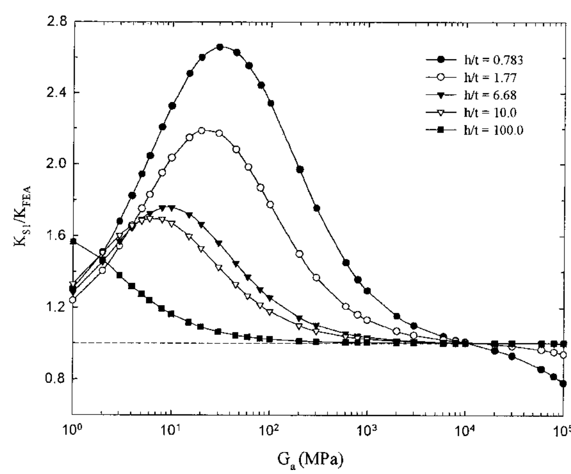

Fig. 5 Comparison of the analytical stiffness based on Eq. (1), K_{s1}, to the stiffness K_{FEA} from FEA

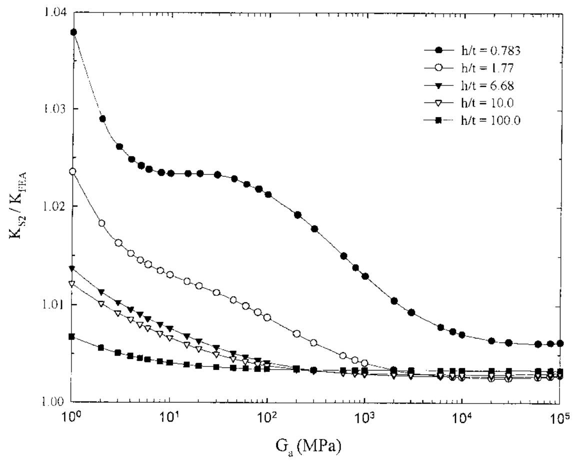

Fig. 6 Comparison of the analytical stiffness based on Eq. (4), K_{s2}, to the stiffness K_{FEA} from FEA

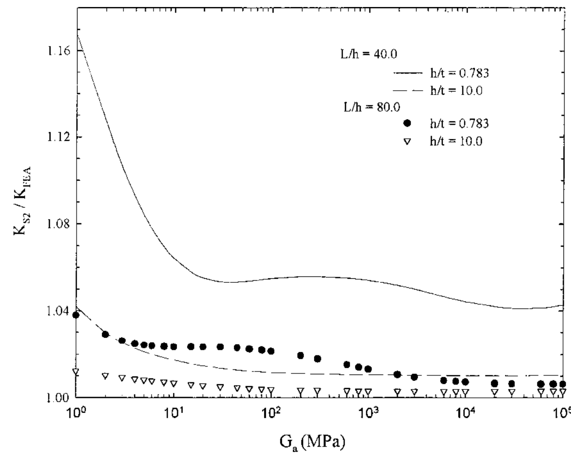

Fig. 8 Comparison of the analytical stiffness based on Eq. (4), K_{s2}, to the stiffness (K_{FEA}) from FEA under different aspect ratio L/h

derivation of Eq. (4) is based on the assumption that the adherends were thin compared to the adhesive layer. However, note from Fig. 6 that Eq. (4) actually compares better with the FEA results when the adherends were relatively thick. This is because the sensitivity of the sandwich beam stiffness to the presence of the adhesive layer becomes negligible when the adherend is relatively thick (see Fig. 3). In the case of thick adherends, for stiffer adhesives, the sandwich beam can be treated as a solid homogeneous beam, while for softer adhesives, the sandwich beam behaves as three individual beams bending with their own axes. In either case, Eq. (4) can pick up the nature of these bending behaviors.

Sensitivity Study. One way to increase the sensitivity of the sandwich beam deflection to the presence of the adhesive is to decrease the aspect ratio of the adherend (L/h) from approximately 80, which is the case for the samples analyzed above, to 40. This is seen in Fig. 7 where the stiffness of the sandwich beam is plotted against the log of the adhesive shear modulus for two samples with the same thickness dimensions but different span lengths, L. The effects of decreasing L on the predictions from Eqs. (1) and (4) were also examined. From Eq. (1) the shape of the curves for $L/h=40$ are nearly identical to those for the longer beam ($L/h=80.0$) except the curves for the short beam are shifted to the right (toward higher shear moduli). This can be understood by noting that the lower aspect ratio will redistribute the dominance of shear deformation due to the adhesive modulus, and the analytical approach of Eq. (1) assumes a pure shear state in the adhesive layer. Figure 8 shows the comparison of Eq. (4) with FEA, and the results show that decreasing the aspect ratio from 80 to 40 will increase the overestimate of the beam stiffness with Eq. (4) for all the adhesive shear moduli considered. This can be explained by recognizing that Eq. (4) assumes the deformation of sandwich beam is represented as three separate beams bending about their own axes (without shear deformation) plus the shear stress induced by the bond between their interfaces. However, for a shorter beam the shear deformation in each individual layer should be included.

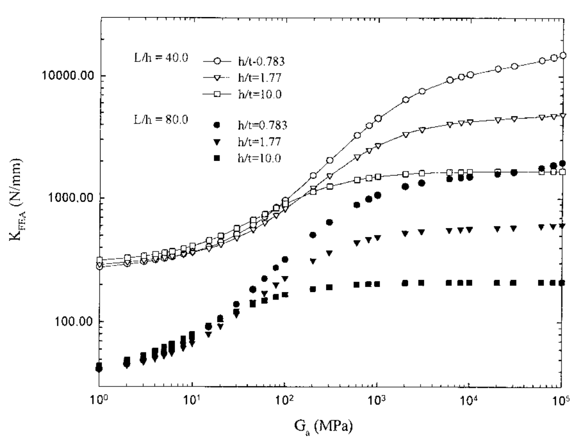

Fig. 7 Beam stiffness from finite element analysis, K_{FEA}, as a function of adhesive shear modulus, G_a, for samples with 2 different span lengths, L

Comparisons With Experimental Results. As indicated above, the specimen geometry was based on the experimental study by Miyagi et al. [6]. Although that work looked at viscoelasticity, measurements were also made in regions where the adhesive's behavior should be elastic. Tests were run at 55°C, which is well above the T_g of the adhesive (≈30°C), so the behavior is rubbery, and at 5°C, which is well below the T_g of the adhesive, so the behavior is glassy. Tables 1 and 2 give the experimentally determined stiffnesses (± indicates standard uncertainty in the data) for beams with three different adhesive thicknesses, along with the adhesive shear moduli calculated from these stiffnesses using Eqs. (1) and (4), and FEA. As noted in the study by Miyagi et al. [6], the calculated values seem unreasonably low, particularly in the glassy range. For comparison, they determined values for the shear modulus of the adhesive in the glassy range (at 5°C) using measurements on bulk specimens. Because the adhesive layers in the test specimens were not very thin compared to molecular dimensions, the modulus of the adhesive in the sandwich was expected to be similar to that for a bulk sample.

The results from the bulk samples are shown in Table 3 (± indicates standard uncertainty in the data) and support the hypothesis that Eqs. (1) and (4) do not give the correct value. In more recent work, the adhesive shear moduli of bulk adhesive

Table 1 Shear modulus deduced from three-point bending of sandwich beam at 5°C

Sample thickness ratio (h/t)	Exp. stiffness (N/mm)	Calculated adhesive shear modulus (MPa)		
		Eq. (1)	Eq. (4)	FEA
6.68 ±0.35	218.4± 3.2	232 ±72	438±199	452±205
1.77 ±0.05	445.3± 7.4	160 ±34	606±130	620±133
0.783±0.040	882.2±34.3	146.2±23	574± 88	596± 91

Table 2 Shear modulus deduced from three-point bending of sandwich beam at 55°C

Sample thickness ratio (h/t)	Exp. stiffness (N/mm)	Calculated adhesive shear modulus (MPa)		
		Eq. (1)	Eq. (4)	FEA
6.68 ±0.35	50.96±0.89	0.71±0.13	2.83±0.50	2.96±0.52
1.77 ±0.05	48.45±0.72	0.75±0.15	2.99±0.61	3.25±0.66
0.783±0.040	50.97±0.40	0.79±0.12	3.14±0.47	3.49±0.52

samples were measured in our laboratory using dynamic mechanical torsion tests over the temperature range from 0°C–60°C. This provides data at both the temperatures of interest here: rubbery (55°C) and glassy (5°C) behavior. These results are also included in Table 3, and a comparison shows that Eq. (1) gives results that are too low for both the glassy and rubbery moduli. Equation (4), on the other hand, gives good agreement in the rubbery range and is much closer to the correct value in the glassy region, but is still too low. The FEA results in Tables 1 and 2 are found to be only slightly larger than the predictions from Eq. (4). This leads to two important conclusions. First, the lack of agreement for adhesive shear modulus in the glassy region (Table 1) indicates that the assumptions made in the analytical solutions can't provide an explanation for these discrepancies, and without such an explanation, the sandwich beam can't be used to characterize glassy adhesives. The tables also show another problem with using the sandwich beam test for characterizing glassy adhesives. The uncertainties in the calculated values are very high; that is, small uncertainties in the measurement of test parameters generate large uncertainties in the calculated values for shear modulus. This is undesirable in a test method.

Second, for soft adhesives (rubbery region), note that the measurements of beam stiffness are insensitive to the change of bond thickness (Table 2). Also, one can see from that table that a 1 percent relative uncertainty in the measurement (very small uncertainty) can cause a 15 percent relative uncertainty in the calculated modulus. Therefore, although there is agreement between the shear modulus values measured on bulk samples and those calculated from the results of the sandwich beam test (Table 2), the lack of the sensitivity of beam stiffness to the presence of the adhesive layer still makes this test method questionable, even for adhesives in the rubbery state.

Possible Explanations for the Discrepancy. Since the lack of sensitivity discussed above is a fundamental problem for using the sandwich beam test to determine the adhesive shear modulus in the rubbery state, we will focus on the discrepancies observed between the shear modulus measured on bulk samples and those calculated from the results of the sandwich beam in the glassy region. Two possible sources for the discrepancy are considered here, first the uncertainties in the measurement of the experimental parameters, and second the effects of undetected voids and/or debonds at the interfaces between the adherends and adhesive.

Sensitivity Analyses. To examine the first question, a sensitivity analysis was performed using Eq. (4) since it agrees closely with the FEA analysis. The uncertainty in calculating the shear modulus δG_a was considered primarily due to the uncertainty in the measurements of the bond thickness $\delta(2t)$ and/or the adherend stiffness $\delta(E_f)$. Theoretically, δG_a can be calculated from the following equation:

$$\delta K_s=\frac{\partial K_s}{\partial(2t)}\,\delta(2t)+\frac{\partial K_s}{\partial(E_f)}\,\delta(E_f)+\frac{\partial K_s}{\partial(G_a)}\,\delta(G_a)+\frac{1}{2!}\,\frac{\partial^2 K_s}{\partial(2t)^2}\,(\delta(2t))^2+\ \dots \tag{8}$$

with the assumption that δK_s equals to zero.

First, we assessed the effect of the variation in adherend stiffness $\delta(E_f)$ to the calculated adhesive shear modulus $\delta(G_a)$ using Eq. (4) and FEA. We found that for the glassy adhesive, small variation (about ±5 percent) in the adherend stiffness will result in very large variation (up to 100 percent) in the calculated value for the shear modulus when the adhesive layer was very thin ($h/t=6.68$ or more). The easy way to explain this result is to consider the pure bending Eq. (7), because for very thin layer the shear contribution to the beam stiffness becomes smaller and negligible with the adhesive in glassy range (see Fig. 4). Based on Eqs. (5) and (7) with the assumption that δK_B equals zero, we have

$$\delta(E_a)=-(I_{fa}/I_a)\,\delta(E_f) \tag{9}$$

For very thin adhesive layer, the ratio of I_{fa}/I_a will approach 200 with $h/t=6.68$. Since the uncertainty in the calculated shear modulus $(\delta(G_a)/G_a=\delta(E_a)/E_a)$ is magnified by the ratio of I_{fa}/I_a, the uncertainty will become very large for small variation of adherend stiffness. For thicker adhesive layer, this effect is reduced so uncertainty in the calculated adhesive shear modulus is smaller (about 10 percent in relative uncertainty at $h/t=0.78$) for similar variations of adherend stiffness. Although this effect is large and limited to the glassy range, it is also dependent on adhesive layer thickness. Consequently, it does not provide a satisfactory explanation for the discrepancy discussed above since that is approximately independent of adhesive thickness (see Table 1).

The second parameter studied was the adhesive layer thickness that is the hardest parameter to control and measure. The effect of an uncertainty in the bond thickness, denoted by $\delta(2t)$, can be approximately related to the uncertainty in the calculated adhesive modulus, δG_a, as follows:

$$\delta(G_a)=-\left(\frac{\partial K_s}{\partial(2t)}\Big/\frac{\partial K_s}{\partial(G_a)}\right)\delta(2t) \tag{10}$$

For a stiffer adhesive shear modulus in glassy range, Eq. (10) can be used to approximate δG_a only with very small $\delta(2t)$ since the sensitivity to the bond thickness is high. For thin adhesive layers, the uncertainty of the bond thickness can be large so the sensitivity calculates are made using numerical methods with Eq. (4).

Figure 9 shows the numerically calculated variations of δG_a normalized by the adhesive shear modulus (G_a) with respect to $\delta(2t)$ normalized by the adhesive thickness $(2t)$. Three different bond configurations ($h/t=6.68$, 1.77, and 0.783) were evaluated based on the experimentally determined beam stiffness shown in Table 1. We adopted the adhesive shear modulus 1.06 GPa at 5°C obtained by dynamic testing on bulk adhesive (Table 3). From the results in this figure, the potential error in calculating the shear modulus due to the error in measuring the bond thickness can be seen to increase dramatically when the adhesive layer becomes stiffer (high G_a). If the bond thickness used in the calculation is measured larger than the actual value $(\delta(2t)>0)$, the adhesive shear modulus in the glassy region (5°C) will be significantly underestimated $(\delta G_a<0)$. This is exactly what is needed to explain the discrepancy discussed above. On the other hand, the over estimation of bond thickness would need to be significant (by 10 percent–15 percent of the thickness). Table 1 shows the discrepancies in shear modulus to be the same for all bond thickness. Thus the difference between the assumed bond thickness and ac-

Table 3 Shear modulus from measurements on bulk specimens (details in ref. [1])

Test method	Shear modulus (MPa)	
	5°C	55°C
Dynamic mechanical test	1160± 64	2.88±0.17
Double lap test	1400±300	⋯
Bending test	1060± 70	⋯

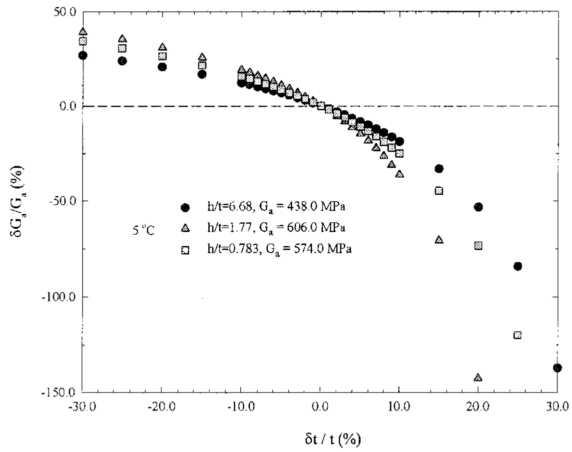

Fig. 9 The uncertainty analysis results based on Eq. (4)

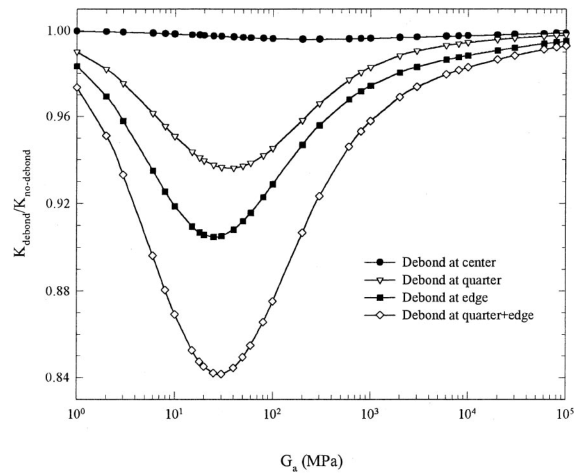

Fig. 10 The comparison of results between debonding and no-debonding with respect to adhesive shear modulus G_a ($h/t = 6.68$, $a/l = 0.1$, a: crack length)

tual bond thickness may need to vary with bond thickness in a specific way to get an adhesive shear modulus that was independent of the bond thickness. Consequently, although this explanation is encouraging, it is not completely satisfactory.

Contact Problem. The second area examined, as a possible explanation for the discrepancies is the presence of an undetected debond region in the adhesive or along the interfaces between adhesive and adherends. This could result from improper wetting or formation of a void during fabrication. Moreover, the mismatch of the material properties in the bonded interfaces can produce thermally induced stresses during fabrication, and this might produce an interface debonding. Such flaws in the adhesive bond would lower the stiffness of the beam and a calculation of the adhesive shear modulus would yield an artificially low value. Consequently, the effect of possible pre-existing debonding areas on the stiffness of the sandwich beam loaded in three-point loading was also examined by FEA. In these studies, the debonds were assumed to reside along the upper adhesive-adherend interface (Fig. 2) and be symmetrically located relative to the centerline so that the analysis can be performed with half of the beam (Fig. 2). Specimens were analyzed with debonding at three different locations: the free edge, the quarter point, and center of the sandwich beam along the upper interface (Fig. 2). A separate study indicated that placing the debonding area at the upper or lower interface does not make much difference in the reduction of beam stiffness, since the shear stress does not vary significantly over the range of adhesive thickness used in the experiments.

Figures 10 and 11 present the change of the beam stiffness with respect to adhesive shear modulus for two bond configurations with one debonding area. The ratio of debond length to beam length (a/l) was equal to 0.1 here, and h/t was varied between 6.68 and 0.783 (thin bond and thick bond). From the results shown in the Figs. 10 and 11, the different debond locations were found to influence the beam stiffness to different degrees. The influence becomes more pronounced when the debond location moves from the center of the sandwich beam to the free edge, irrespective of the bond thickness. This is because the shear stress in the interlayer along the length of the bonded beam increases from the center to the edge. Consequently, strain energy lost due to the existence of the debonding area is higher when the debond is at the edge than when it is at the center. These figures also show the FEA result for the change of beam stiffness when more than two existing debonding areas are present at different locations (at the quarter length and the edge of the beam). The resulting reduction in the beam stiffness is equal to the superposition of two single debonding areas because of the linearity. As indicated in Figs. 10 and 11, the reduction of beam stiffness due to the possible existence of interfacial debond is by a fraction of about 4 percent at maximum when the adhesive shear modulus is 1.06 GPa. This reduction will correspond to a 10 percent increase of the calculated shear modulus from Eq. (4). Even with this increase, the shear modulus obtained from Eq. (4) using three-point bending test is still lower than the expected value (1.06 GPa). That means the reduction of the beam stiffness caused by the interfacial debonding is likely not the primary source for the discrepancies in shear modulus measurements.

The data in Figs. 10 and 11 also indicate that the reduction of the beam stiffness is limited when the adhesive modulus is low. This is because the sandwich beam could be treated as three separate homogenous beams bending about their own axes (as mentioned previously), and the existence of the debonding area becomes immaterial. When the adhesive modulus becomes comparable to the adherend modulus, the specimen can be treated as a homogeneous beam under pure bending deformation only (see Fig. 4). The shear deformation of the bond becomes negligible and the local perturbation due to the existence of the debonding will not affect this global mechanical response. Figure 12 displays the reduction of the beam stiffness due to the change of the debond size for the adhesive modulus of 1.06 GPa and two bond configurations. The result in this figure demonstrates that the

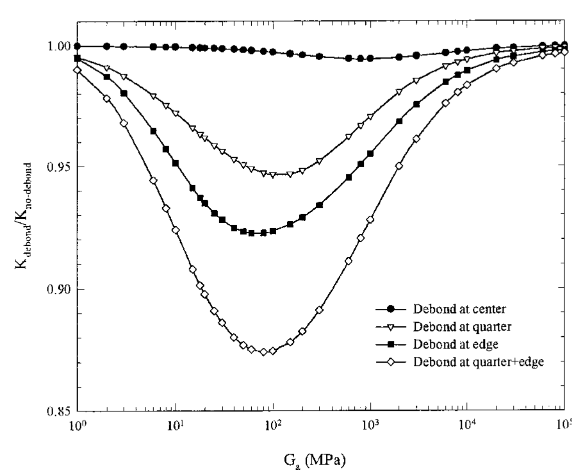

Fig. 11 The comparison of results between debonding and no-debonding with respect to adhesive shear modulus G_a ($h/t = 0.783$, $a/l = 0.1$, a: crack length)

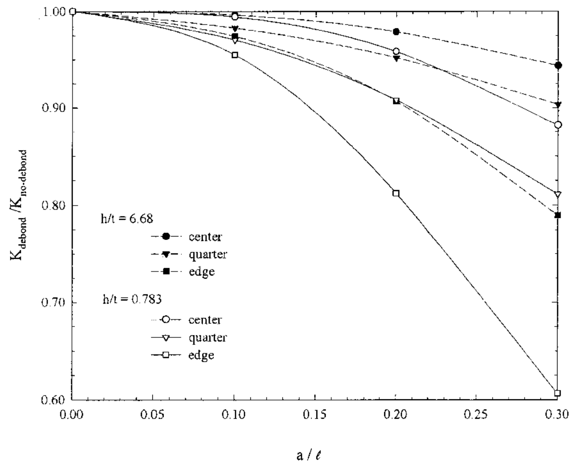

Fig. 12 The comparison of results between debonding and no-debonding with respect to the length of debonds (G_a = 1.06 GPa)

thicker bond has more reduction in the beam stiffness than that of the thinner bond. This is because there are higher shear stresses developed in the thicker bond.

In terms of debonds as explanations for the discrepancy discussed above, however, there are several problems. The most obvious is that the debonging areas would need to be very large to explain the differences observed in the glassy region. Moreover, for some thicknesses the effect is as large in the rubbery region as it is in the glassy range. Finally, thick bonds show a bigger effect than thin bonds, and this is the reverse of what is seen in Tables 1–3. Consequently, this does not appear to be a factor contributing to the discrepancies in the sandwich bond experiment.

Conclusions

This paper utilizes a FEA analysis to assess the feasibility of the three-point bending test on a sandwich beam specimen to characterize the shear modulus of an adhesive in a bonded geometry. The results are compared with two analytical solutions for this configuration, and the comparison indicates that the solution provided by Adams and Weinstein [4] is more appropriate for the combinations of the adhesive modulus and bond thickness that were used in the experiments of Miyagi et al. [6]. For soft adhesives (rubbery behavior) although there is agreement between values for the shear modulus of the adhesive measured on bulk samples and calculated from the sandwich beam results using either FEA or the Adams and Weinstein equation, the poor sensitivity of the sandwich beam stiffness to the presence of the adhesive layer (the second column of Table 2) makes this agreement very questionable. Often, one might be misled to believing the test results due to this agreement. For glassy adhesives, the agreement was not good. Moreover, the uncertainties in the calculated shear modulus from the bending tests were high because the results are very sensitive to adhesive thickness and adherend modulus for glassy adhesives. The study also examined possible causes for discrepancy found in glassy adhesives. Factors studied included the uncertainty in the adherend stiffness, adhesive bond thickness and the presence of a nonbonded region along the interfaces between adhesive and adherends. Unfortunately, none of the possible causes taken singly can provide a satisfactory explanation for the discrepancies. Consequently, the sandwich beam test is not a desirable method to characterize the adhesive shear modulus. Additional studies are needed before the sandwich beam can become an acceptable method for evaluating adhesives, at least with the geometry range studied here.

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