MODIFIED ALLARD METHOD FOR EFFECTIVE INTENSITY OF FLASHING LIGHTS

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ABSTRACT

For the measurement of effective intensity of flashing lights, the Blondel-Rey, Blondel-Rey-Douglas, Form Factor, and Allard methods are adopted in various applications. These methods can produce significantly different results depending on the pulse waveforms. These methods have been studied and compared by computation using ten different pulse waveforms including a train of pulses and a modulated pulse at varied duration from 0.001 s to 10 s. The results indicate failures of the Blondel-Rey-Douglas and Form Factor methods for a train of pulses and some problems with other forms of pulses. The Allard method shows reasonable results for all forms of pulses except that it shows results considerably higher than the Blondel-Rey results for rectangular pulses at 0.1 s to 1 s duration. To solve this problem of the Allard Method, its visual impulse response function is modified so that the results for rectangular pulses match the results by the Blondel-Rey equation. This modified method produces results equivalent to the Blondel-Rey equation for rectangular pulses, yet, solving all the problems identified with the other methods for non-rectangular pulses.

Keywords: effective intensity, flashing light, photometry, pulsed light, visual range

1. INTRODUCTION

Flashing lights are widely used in many signaling applications in aviation, marine navigation, and land transportation. The visibility or conspicuity of flashing lights varies depending on the duration and waveform of flashes for the same physical energy and spectrum of the flashes. To take into account such visual effects, the term, *effective intensity,* is used to specify the intensity of flashing lights for signaling applications. Effective intensity is defined as the luminous intensity (cd) of a steady light source that would have the same luminous range (or visual range in aviation terminology) as the flashing lights in question.

The three well-known methods to determine effective intensity are the Allard method (1876) [1], the Blondel-Rey method (1911) [2], and the Form Factor method (1968) by Schmidt-Clausen [3]. The Blondel-Rey equation or its extended form, Blondel-Rey-Douglas equation [4], has been most widely accepted in many application areas [5,6,7]. The Form Factor method is recently gaining acceptance [8]. The Allard method did not prevail probably due to lack of publicity and difficulty of calculation in the past. These three methods can produce significantly different results depending on the pulse waveforms. The past comparison studies [9, 10], however, have not recognized remarkable differences between these methods. The International Association of Lighthouse Authorities (IALA) published a recommendation in 1977 [11], which put all three methods on equal footing except for trains of pulses (for which the Allard method is recommended), and left users to select a method. It is desired that one agreed method is used universally for all forms of pulses in all applications.

To address such needs, a task has been undertaken in the Commission Internationale de l'Éclairage (CIE) to develop an international recommendation on photometry of flashing lights [12], and in the American Society for Testing and Materials (ASTM) to possibly standardize the definition of the effective intensity [13]. In these committees, Couzin reported a failure of the Form Factor method for a certain form of pulse, and, to solve the problem, proposed a method based on convolution of the flash pulse with a certain visual impulse response function [14], similar to the Allard method.

To examine the differences between the three methods mentioned above, to identify problems, and to provide directions to the committees, analyses have been made by calculation on ten different theoretical waveforms of pulses including a train of pulses and modulated pulses. The results have identified failures of the Blondel-Rey and Form Factor methods for a train of narrow pulses and some problems in other pulses. The Allard method showed reasonable results for all forms of pulses except that it showed considerably higher (20 % to 30 %) results than the Blondel-Rey results for a rectangular pulse at the 0.1 s to 1 s duration. The authors developed a solution for this problem by modifying the Allard method. This modified method produces results equivalent to the Blondel-Rey equation for rectangular pulses, yet, solving all the problems identified with the other methods for a variety of pulses. The results of the analysis and the modified Allard method are presented in this paper.

2. DEFINITIONS OF THE THREE CONVENTIONAL METHODS

2.1 Blondel-Rey Equation

In 1911, based on visual experiments for threshold detection of flashing lights, Blondel-Rey proposed that the effective intensity I_{eff} of flashing lights is described by the equation

$$I_{eff} = \frac{\int_{t_1}^{t_2} I(t)\mathrm{d}t}{a+(t_2-t_1)} , \qquad (1)$$

where $I(t)$ is the instantaneous luminous intensity of the flash, (t_2-t_1) is the duration of the flash, and a is a visual time constant, 0.2 s, known as the Blondel-Rey constant [2]. The numerator of the equation is the time-integral of $I(t)$, which is given in the unit of cd·s. This equation was straightforward for rectangular pulses, but they soon faced a question as to how t_1 and t_2 should be determined for non-rectangular pulses rising and diminishing slowly. They proposed also in 1911 that, for any pulse waveforms, t_1 and t_2 should be determined in such a way that

$$I_{eff}=I(t_1)=I(t_2) \qquad (2)$$

is satisfied in Eq. (1), resulting in an integral equation,

$$\int_{t_1}^{t_2} (I(t)-I_{eff})\mathrm{d}t = a\ I_{eff} . \qquad (3)$$

This equation, as depicted in Fig. 1, can be solved only by iterative calculation. This must have been a difficulty until the advent of computers in 1950s.

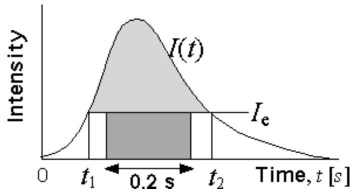

Figure 1. Graphical solution of the Blondel-Rey equation for a non-rectangular pulse.

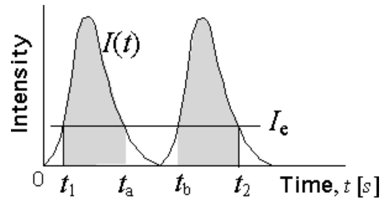

Figure 2. An example of a train of two pulses.

In 1957, Douglas proved that the condition given in Eq. (2) is achieved when I_{eff} is maximized [4]. He also proposed that, for a train of pulses as shown in Fig. 2, the effective intensity I_{eff} is determined by

$$I_{eff} = \frac{\int_{t_1}^{t_a} I(t)\mathrm{d}t + \int_{t_b}^{t_2} I(t)\mathrm{d}t}{a+(t_2-t_1)} , \qquad (4)$$

and $I_{eff} = I(t_1) = I(t_2)$.

This formula was accepted in a recommendation in the USA in 1964 [5], referred to as the Blondel-Rey-Douglas method.

2.2 Form Factor Method

In 1968, Schmidt-Clausen introduced a concept of Form Factor, and proposed a method that simplified the calculation of effective intensity for non-rectangular pulses [3]. The effective intensity I_{eff} of a flash pulse $I(t)$ is given by

$$I_{eff} = \frac{I_{max}}{1+\frac{a}{F\cdot T}} ; \quad F = \frac{\int_0^T I(t)\mathrm{d}t}{I_{max}\cdot T} \qquad (5)$$

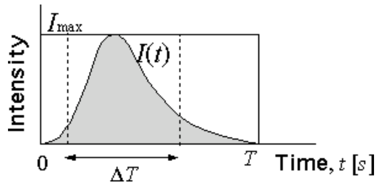

Figure 3. Schematic of the Form Factor method.

where F is called Form Factor, and I_{max} is the maximum of the instantaneous effective intensity $I(t)$. This equation can be transformed into a form

$$I_{\text{eff}} = \frac{\int_0^T I(t)\mathrm{d}t}{a + \Delta T}; \qquad \Delta T = \frac{\int_0^T I(t)\mathrm{d}t}{I_{\max}} = F \cdot T \tag{6}$$

which gives an interpretation that this method is an extension of the Blondel-Rey equation with a new way of determining the duration of the flash. Figure 3 illustrates the concept of this method.

The effective intensity is determined by the time integral and the instantaneous maximum of the flash pulse, both of which can be directly measured with a detector and analog circuitry. The measurement of I_{eff} does not necessarily require the pulse waveform.

2.3 Allard Method

Allard proposed in 1876 [1] that the visual sensation $i(t)$ in the eyes for flashing light with instantaneous intensity $I(t)$ is given by

$$\frac{\mathrm{d}i(t)}{\mathrm{d}t} = \frac{I(t) - i(t)}{a}. \tag{7}$$

This differential equation, as depicted in Fig.4, indicates an exponential decay of $i(t)$ with a visual time constant a, and is solved as a mathematical convolution of $I(t)$ with a visual impulse function $q(t)$ as given by

$$i(t) = I(t) * q(t); \qquad q(t) = \frac{1}{a}\, \mathrm{e}^{-\frac{t}{a}}. \tag{8}$$

(*convolution)

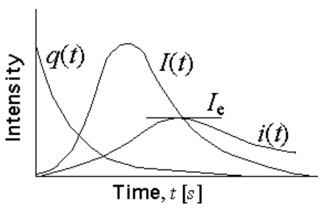

Figure 4. Schematic of the Allard method.

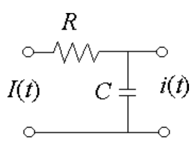

Figure 5. R-C low-pass filter circuit to achieve the convolution in Eq. (8)

The effective intensity I_{eff} is given as the maximum value of $i(t)$. This convolution is achieved electronically by a simple resistor-capacitor filter circuit as shown in Fig. 5. The effective intensity can be directly measured with a detector and simple analog circuitry.

3. ANALYSIS ON THE THREE CONVENTIONAL METHODS

To evaluate the differences between the three methods, analyses were made by calculating the effective intensity of 10 different pulse waveforms shown in Fig. 6 with varied duration. Pulse #2 is a 1:2 trapezoid, #6 is a sine-squared function, #7 is 20 cycles of sine-wave oscillation with amplitude from 1/7 to 1 (peak to peak) modulated by the sine-squared function (simulating a discharge lamp emission), #8 is a sine-square function with a peak height of 0.3, on top of which a narrow peak (1/500 width) with a height of 0.7 is added. #9 is a train of four short pulses of real xenon flash with a half-maximum width of ~1 ms at intervals of 0.1 s. #10 is a train of four sine-squared pulses. The duration of these pulses (the time of the first non-zero value to the last non-zero value) are varied from 0.001 s to 10 s. The effective intensity of these pulses using the Blondel-Rey (Blondel-Rey-Douglas for multiple pulses), Form Factor, and Allard methods was calculated.

The results for the rectangular pulse are presented in Fig. 7 (a) and (b). The two graphs are different presentations of the same results. The upper figure (a) is the plots of $I_{\text{eff}}/I_{\max}$, which is the effective intensity I_{eff} for the pulses with constant height ($I_{\max}$=1) with its duration varied. As the duration increases and exceeds 1 s, I_{eff} saturates to 1, where I_{eff} equals the luminous intensity of a steady light. In the lower figure (b) are the plots of I_{eff}/J, which

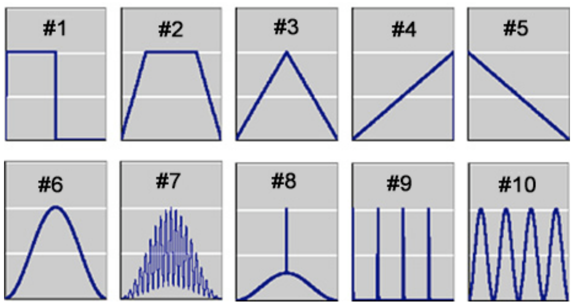

Figure 6. Ten different pulse waveforms used in the analysis.

is the effective intensity normalized for constant energy or integral J of the pulse, where

$$J = \int_t I(t)\,\mathrm{d}t\,. \qquad (9)$$

When the pulse duration is much shorter than the time constant (0.2 s), the effective intensity I_{eff} is 5 times (reciprocal of the 0.2 s time constant) of J. As the duration becomes longer, I_{eff}/J decreases and diminishes at longer duration.

The plots of $I_{\mathrm{eff}}/I_{\max}$ present characteristics at longer duration well and I_{eff}/J does well for shorter duration. Both figures show four curves for the results by the different methods. The Modified Allard results are discussed in the next section.

Fig. 7 (a) and (b) show that Allard results are 20 % to 30 % higher than the Blondel-Rey results at durations of 0.1 s to 1 s. This tendency is observed for most other pulses, and is confirmed as a problem of the Allard method, since the Blondel-Rey equation is believed to be accurate for rectangular pulses [3].

Fig. 8 shows the results in I_{eff}/J for a train of four short pulses (#9 in Fig.6). It should be noted that the duration ΔT on the horizontal axis shows the time interval from the first pulse to the last pulse, thus the interval of the pulses are 1/3 of ΔT. Each pulse width is ~1 ms (real xenon flash data) when ΔT=0.5 s, and it changes proportionally as ΔT varies. In this figure, Form Factor shows significantly higher values than the others at durations longer than 0.1 s. At a duration of 0.3 s, where the pulse interval is 0.1 s (e.g., found in some aircraft anticollision lights), the Form Factor result is 2.5 times higher than the Blondel-Rey-Douglas result. It is often said that, around this length of pulse interval (0.1 s) and longer, the eyes start seeing individual flashes, and thus the effective intensity should be calculated for a single pulse (only one of the pulses in the group) [4]. The effective intensity values for such cases (calculated for only one of the four pulses) are plotted also in Fig.8 for the Form Factor method and Blondel-Rey-Douglas method, labeled as “FF (single)” and “B-R-D (single)”. Both of these curves are nearly flat at a level around 1.2. If we make a transition to these flat curves at 0.1 s interval (ΔT=0.3 s), the Form Factor result would sharply drop from 4.9 to 1.2, and Blondel-Rey-Douglas from 2.0 to 1.2 (indicated by an arrow in Fig. 8). We presume that the eye response will not have such an abrupt change but should have a gradual change. The Allard result shows such a gradual transition and naturally merges into the single pulse curve at longer durations (1 s to 10 s), which implies a physiological validity of the Allard method. If the transition to the single pulse is not applied to the Form Factor and Blondel-Rey methods, both results (calculated from the four pulses) at

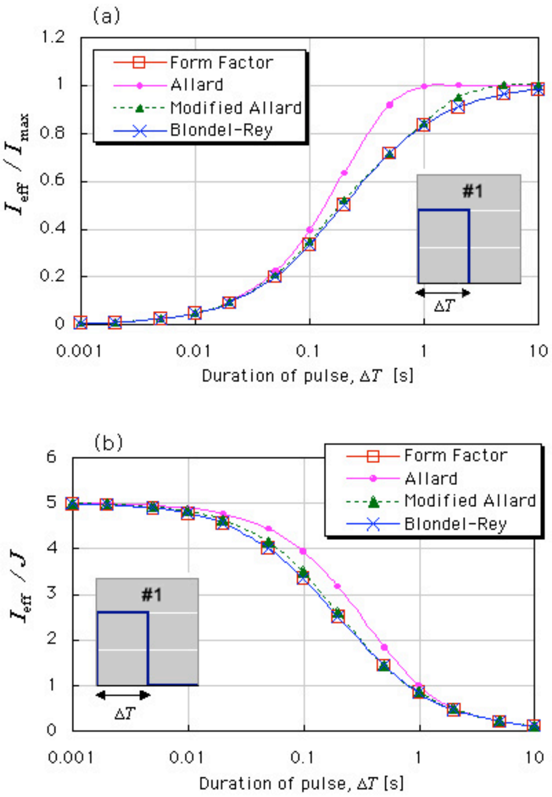

Figure 7. Results for rectangular pulses.

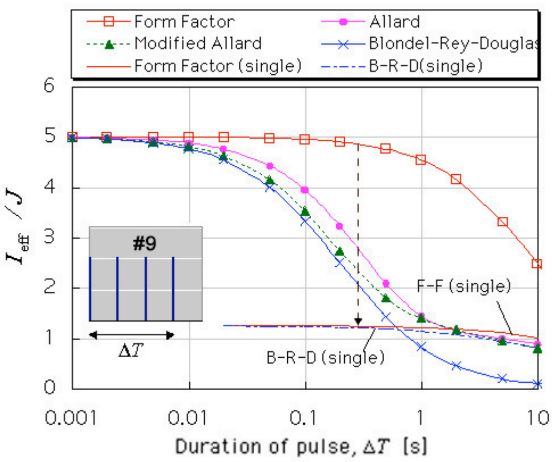

Figure 8. Results for a train of short pulses.

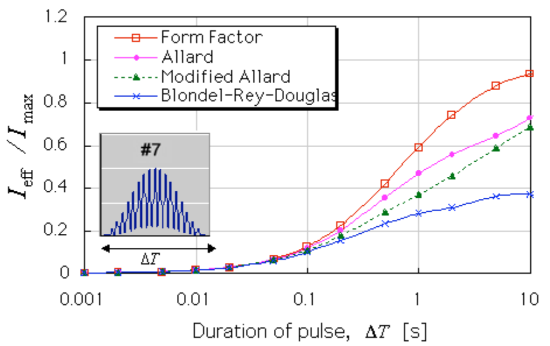

Figure 9. Results for a modulated pulse.

longer durations show serious deviations from the single pulse curves (Form Factor values go too high and Blondel-Rey-Douglas too low by several folds.) These observations clearly indicate that the Form Factor and Blondel-Rey-Douglas methods fail to work properly for a train of pulses, in spite of the report that suggested that the Form Factor method could be applied to multiple pulses without restrictions [15]. The failure of the Form Factor method for a train of narrow pulses is explained from its equation, which ignores the interval between the pulses, and its results only depend on the duration of each short pulse.

Fig. 9 shows the results in I_{eff}/I_{max} for the modulated pulse (#7). Significantly large differences (by a factor of two) are shown between the Form Factor and Blondel-Ray-Douglas methods at longer durations. Allard and Modified Allard (described later) show values between the two. Pulse #10 showed similar results.

The results for pulse #8 (a narrow pulse added on a sine-squared pulse) clearly showed a failure of the Form Factor method, which produces I_{eff} that is higher by a factor of two than Blondel-Rey or Allard at a duration of 0.5 s and longer.

For all triangular pulses (#3, #4, #5), the differences between Form Factor and Blondel-Ray are fairly large, with Form Factor showing about 30 % higher I_{eff} in the 0.2 s to 2 s duration. Allard results vary depending on the form of the triangle. The three different triangular pulses are measured with no difference by Form Factor or Blondel-Ray, while Allard gives different results for the different triangles. At the moment, no past data is available to infer which result is more accurate than the others.

4. MODIFICATION OF ALLARD METHOD

The results presented in the last section indicated that the Allard method has no problems with any forms of pulses at any duration, except that it gives values too high in the 0.1 s to 1 s duration. To solve this problem, attempts were made to modify the visual impulse response function to match the Allard's results to Blondel-Rey for rectangular pulses. Fig. 7 indicated that there would be a much longer time constant component in the Blondel-Rey results. The Allard's visual impulse response function $q(t)$ has been modified to be composed of two exponential functions with different time constants. The two time constants have been optimized in such a way that the difference of the response function to rectangular pulses from that of Blondel-Rey equation was minimized, while keeping the total time constant to be 0.2 s and the I_{eff} for steady light to be equal to I_{max}. These requirements are met when the maximum value of $q(t)$ is $1/a$ (a=0.2 s) and the integral of $q(t)$ is 1. The modified impulse response function is given by

$$q(t) = \frac{w_1}{a_1} e^{-\frac{t}{a_1}} + \frac{w_2}{a_2} e^{-\frac{t}{a_2}}, \qquad (10)$$

where

$$\frac{w_1}{a_1} + \frac{w_2}{a_2} = \frac{1}{a},$$

$$w_1 + w_2 = 1, \quad a = 0.2 \text{ s},$$

and

$q(t) = 0$, when $t < 0$.

One of the solutions for the optimization was a_1=0.113, a_2=0.869, w_1=0.5, w_2=0.5. With these parameters, the results agreed with Blondel-Rey for rectangular pulses to within 5 %. These results are shown as the curves labeled "Modified Allard" in Figs. 7 to 9. Fig. 7 shows that Modified Allard's results are practically equivalent to Blondel-Rey for rectangular pulses, yet showing no problems in Figs. 8 and 9. This modified Allard method can be easily realized into a photometer by two R-C filter circuits and a peak-hold circuit. Further details of the benefits in circuit design are discussed in Ref. [16]

Further theoretical analysis discovered a $q(t)$ function that perfectly matches the results of Blondel-Rey for rectangular pulses, as

$$q(t) = a/(a+t)^2, \text{ when } t \geq 0$$

$$q(t) = 0, \text{ when } t < 0. \qquad (11)$$

No analog circuits have been yet found to achieve the convolution with this function perfectly, however, this function may be useful to define the method, or to fit the method to experimental data in different conditions by changing only one time constant.

5. CONCLUSIONS

The analysis revealed failures of the Blondel-Rey-Douglas and Form Factor methods for a train of short pulses and other problems in some other pulses. The modified Allard method proposed by the authors gives practically equivalent results to Blondel-Rey for single rectangular pulses, yet solving all the problems identified with the other methods. In addition, this method has a practical advantage in simple realization of practical photometers. It is considered that this method can be used for all forms of pulses (including a train of pulses) at any duration, with sufficient accuracy, though it needs to be verified experimentally.

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