

Spatial Frequency Response Function Estimation Program

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Abstract

Measurements of optical scatter are often employed in production line diagnostics for surface roughness of silicon wafers. However, the geometry of the optical scatter instrumentation lacks universal standardization, making it difficult to compare values obtained by instruments made by different manufacturers. The bidirectional reflectance distribution function (BRDF), on the other hand, is a well-defined quantity, and under conditions usually met with bare silicon wafers, can be related to the power spectral density (PSD) of the surface roughness. In a separate paper,¹ we presented an approach for characterizing low level optical scatter instrumentation using a spatial frequency response function. Methods for calculating or measuring the response function were presented. We have written and made publicly available a computer program, called SFRF, which implements that approach. This document is a user's manual for SFRF, intended to aid manufacturers of light scattering instruments in evaluating the spatial frequency response functions for those instruments.

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1. Introduction

As the critical dimensions of integrated circuit device structures become smaller, the tolerance for surface micro-roughness becomes tighter. Scanning surface inspection systems (SSIS) are currently employed to measure levels of optical scatter and to detect particulate contamination. These instruments rely upon integrated light scatter, treating the background signal on the wafer as resulting from surface microroughness, and treating localized signals as resulting from particulate contamination or other localized defects.

Recent efforts have focused on the characterization of optical scatter instrumentation. An important step was made when it was recognized that different instruments measure roughness on different spatial frequency scales, which results from measurements being carried out with differing optical geometries, wavelengths, polarizations, etc. Instrument manufacturers often report a bandwidth by specifying minimum and maximum spatial frequencies for which their instruments are sensitive, thus making an implicit assumption that the response is uniform over that bandwidth. In Ref. 1, we expanded upon the notion of a bandwidth, by defining the spatial frequency response function $r(f)$, and provided a means for calculating or measuring that function. Knowledge of this function will allow absolute calibration of low-level optical scatter instruments by enabling an appropriate integration of the power spectral density (PSD) function of the surface microroughness that could then be correlated to SSIS data. Furthermore, manufacturers of optical scatter instrumentation will have a well-defined methodology for specifying their products.

The spatial frequency response function $r(f)$ is the relationship between the 2-d power spectral density function $PSD(f)$ of the surface microroughness and the signal H measured by the integrated scatter instrument:

$$H = \int_0^{\infty} df \ 2\pi f \ r(f) \ PSD(f) \quad (1)$$

By applying Eq. (1), a sample with a known PSD can be used to calibrate a particular instrument. Multiple samples with different PSDs can be used to test the validity of the response function $r(f)$.

In order to determine the response function, it is necessary to ascertain the geometry of the scattered light collection system for the instrument. The program SFRF is designed to take a description of a scattered light collection system, together with a specified incident light condition, and to return an isotropic spatial frequency response function as described in Ref. 1. Since the polarization dependence of the BRDF was not written out explicitly in Ref. 1, that dependence is given in Appendix A for completeness.

2. System Requirements

SFRF is an MS-DOS application, requiring less than 600K of free memory. A numerical coprocessor is required. It is recommended that the user have software available, such as a spreadsheet or a scientific graphing package to view the output of SFRF and to apply Eq. (1).

3. Executing SFRF

The program is executed by typing `SFRF` at the MS-DOS command prompt, or by double-clicking on `SFRF.EXE` in Explorer (Windows 95) or Program Manager (Windows 3.1). The user is prompted for the name of a system description file, the format of which must conform to the specifications described in Section 4 of this document. The user is also prompted for the name of an output file. The program will create this output file. If it already exists, the program will overwrite it. Both of these prompts have defaults; the user need only press "Return" to accept the default.

The output of SFRF is an ASCII text file containing two tab-delimited columns:

1. The spatial frequency f (in $1/\mu\text{m}$), and
2. The isotropic spatial frequency response function $r(f)$ (in \AA^{-2}).

The user may then view the results by importing the output file into a spreadsheet or scientific graphing package. The results may then be applied to the integration of PSD data to yield the amount of scattered light that will result from such a PSD.

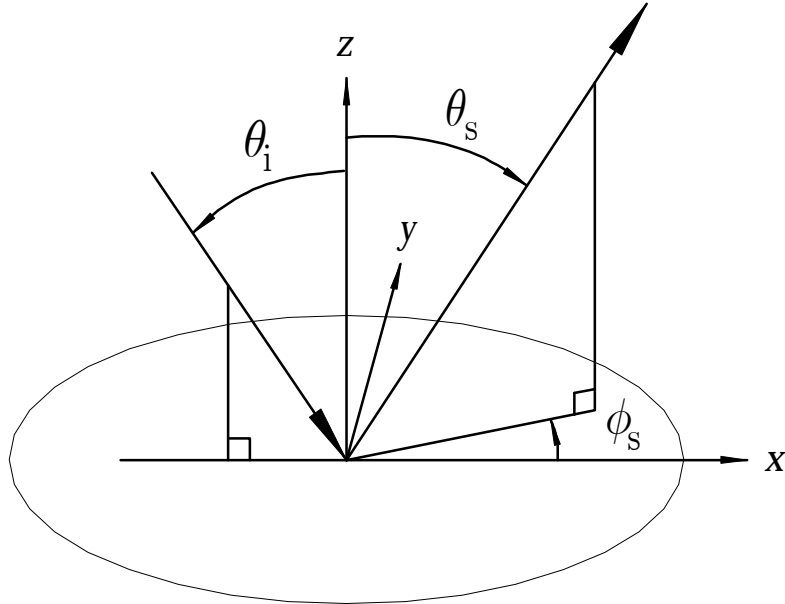


Figure 1 Coordinate system used throughout this document. Light is incident at an angle θ_i in the xz plane, where the z -axis corresponds to the surface normal. The scattered light is parameterized by a polar scattering angle θ_s and azimuthal scattering angle ϕ_s .

4. Format of the System Description File

To begin, it is important to establish the coordinate system that is used throughout this discussion (see Figure 1). Light is incident at an angle θ_i in the xz plane, where the z -axis corresponds to the sample surface normal. The scattered light is parameterized by a polar scattering angle θ_s and azimuthal scattering angle ϕ_s . The incident light is s -polarized (p -polarized) when the electric field is perpendicular (parallel) to the plane of incidence (defined by the incident direction and the sample normal). Likewise, the scattered light is considered s -polarized (p -polarized) when its electric field is perpendicular (parallel) to the plane of exitance (defined by the scattering direction and the sample normal).

The system description file contains a description of the optical system and can be edited with a text editor such as Notepad in Windows or EDIT in MSDOS. The following is an example of a system specification file:

```

number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 0.0001
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end

```

The first nine lines of the system description file each consist of a label and a value. The label reminds the user of the interpretation of the value that follows it. All of the values can either be single numbers or arithmetic expressions. The format of arithmetic expressions is outlined in Appendix B.

4.A. Header Information

The interpretations of the nine parameters in the system specification file are as follows:

- **Number_of_spatial_frequencies** is the number of spatial frequencies over which the results will be tabulated. The spatial frequency will be evenly divided between zero and the maximum allowable spatial frequency f_{\max} for the incident angle and wavelength, where $f_{\max} = [1 + \sin(\theta_i)]/\lambda$.
- **Wavelength** is the wavelength λ of the incident light in μm . The measurement is assumed to be carried out at a specified wavelength. If the incident light is not monochromatic, then an appropriate average over the range of wavelengths may have to be performed.
- **Incident_angle** is the angle of incidence θ_i of the incoming light in degrees. The measurement is assumed to be carried out at a specified incident angle. If the incident laser is strongly focused onto the sample, then an appropriate average over the range of different incident angles may have to be performed. Also note that the incident angle should not be zero, since the polarization of the light is not well defined for this orientation; for light incident along the surface normal, use a small value for θ_i such as 0.0001° .
- **Solid_angle** is a loose approximation of the solid angle $d\Omega$ used by the program during integration. Changing this value to a lower value will increase the accuracy of the program's output, but at a cost of computation time. It is recommended that the user adjust this value to assure that the results are invariant upon changing this number. If the results of the program look "noisy," reduce the value of `solid_angle`.
- **Material_n** and
- **Material_k** are the real and imaginary parts, respectively, of the complex optical constant $\tilde{n} = n + ik$ of the substrate material. The convention is that `Material_k` should be positive for an absorbing medium.
- **Input_lightQ**,
- **Input_lightU**, and
- **Input_lightV** are the second, third, and fourth components of the normalized Stokes vector of the incident light, respectively. These components are defined by the relationships

$$Q = (E_s^2 - E_p^2) / (E_s^2 + E_p^2)$$

$$U = 2 \operatorname{Re}(E_s E_p) / (E_s^2 + E_p^2)$$

$$V = 2 \operatorname{Im}(E_s E_p) / (E_s^2 + E_p^2)$$

where E_s and E_p are the components of the incident light fields in the s and p directions, respectively. By being normalized, the value of the first element of the Stokes vector, I , is assumed to be unity. Examples of common polarized Stokes vectors (I, Q, U, V) are given below:

(1, 1, 0, 0)	s -polarized
(1, -1, 0, 0)	p -polarized
(1, 0, 1, 0)	\mathbf{E} rotated 45° ; counterclockwise from s (when looking into beam)
(1, 0, -1, 0)	\mathbf{E} rotated 45° ; clockwise from s (when looking into beam)
(1, 0, 0, 1)	left circularly polarized light
(1, 0, 0, -1)	right circularly polarized light
(1, $\cos 30^\circ$, $\sin 30^\circ$, 0)	\mathbf{E} rotated 15° ; counterclockwise from s (when looking into beam)

4.B. Collection System Description

The remainder of the system specification file contains descriptions of the collection elements. Each line of the file corresponds to a single collection element and begins with four numbers describing the Stokes vector (I , Q , U , and V) for the detection efficiency. The four components must be normalized so that the first component represents the detection efficiency for unpolarized light. Examples of collection Stokes vectors are as follows:

(1, 0, 0, 0)	Collects all light entering specified optic.
(0.5, 0.5, 0, 0)	Collects only s -polarized light.
(0.5, -0.5, 0, 0)	Collects only p -polarized light.
(0.5, $\cos 30^\circ$, $\sin 30^\circ$, 0)	Collects only light polarized 15° counterclockwise from s (looking into the beam).
(0.5, 0, 0, 0.5)	Collects only left-hand circularly polarized light.
(0.5, 0, 0, 0)	Collects all light, but with an efficiency of 50%.

The rest of each line describes the collection optic by specifying the solid angle of collection. A number of different shapes are provided to allow the user to specify any shaped collection system. If the particular shape is not included in the list, the user may piece together a number of different elements to make up the whole of the system. Subtractive elements may also be used to specify “holes” in optics. In that case, all four elements of the collection Stokes vector for the hole should be of the opposite sign and of equal magnitude as those for the region in which the hole is located.

After all the elements are specified, a final line consisting of

```
0 0 0 0 END
```

will complete the system specification file. This line is required and signals the program to begin the calculation. Any lines following this line will be ignored. The solid angle shapes that are supplied are as follows:

Hemispherical collection

Syntax:

```
i q u v HEMI
```

Description:

Collect light over the entire hemisphere.

Example:

The following collects all light over the entire hemisphere:

```
1 0 0 0 HEMI
```

Right circular conical collection

Syntax:

```
i q u v CONE theta_center phi_center half_angle
```

Description:

Collect light that enters a right circular cone with a specified center direction and a specified opening half-angle.

Example:

The following collects all light entering a cone centered at $q_s = 45^\circ$ and $f_s = 90^\circ$, and with a 30° half-angle of collection:

```
1 0 0 0 CONE 45 90 30
```

Annular collection

Syntax:

```
i q u v ANNULUS theta_center phi_center begin_angle end_angle
```

Description:

Collect light in a right circular annulus with a specified center direction and with a specified starting and ending opening angles.

Example:

The following collects all scattered light that has a scattering angle q_s between 45° and 80° :

```
1 0 0 0 ANNULUS 0 0 45 80
```

Triangular Collection

Syntax:

```
i q u v TRIANGLE t1 p1 t2 p2 t3 p3 {G,P} {G,P} {G,P}
```

Description:

Collects light in a triangular region defined by three directions. The sides of the triangle can be one of two different types:

G Geodesic sides. These sides correspond to the intersection of the unit sphere with a plane defined by the two directions and the origin. An optic having a flat edge will have a geodesic side when projected onto the unit scattering hemisphere.

P Straight lines when viewed as a projection of the hemisphere onto the xy plane.

Each of the three corners are defined by $\{q,f\}$ pairs.

Examples:

The following specifies a triangular region bounded by the three directions $q_1 = 10^\circ$, $f_1 = 20^\circ$, $q_2 = 30^\circ$, $f_2 = 40^\circ$, $q_3 = 50^\circ$, $f_3 = 60^\circ$, where two sides (1-2 and 2-3) of the triangle are geodesics, and the third (3-1) is the intersection of a vertical plane with the unit hemisphere:

```
1 0 0 0 TRIANGLE 10 20 30 40 50 60 G G P
```

The following two lines specify a rectangular region by piecing two different triangles which share an edge:

```
1 0 0 0 TRIANGLE 45 45 70.6 32.0 70.6 -32.0 P P P  
1 0 0 0 TRIANGLE 45 45 45 -45 70.6 -32.0 P P P
```

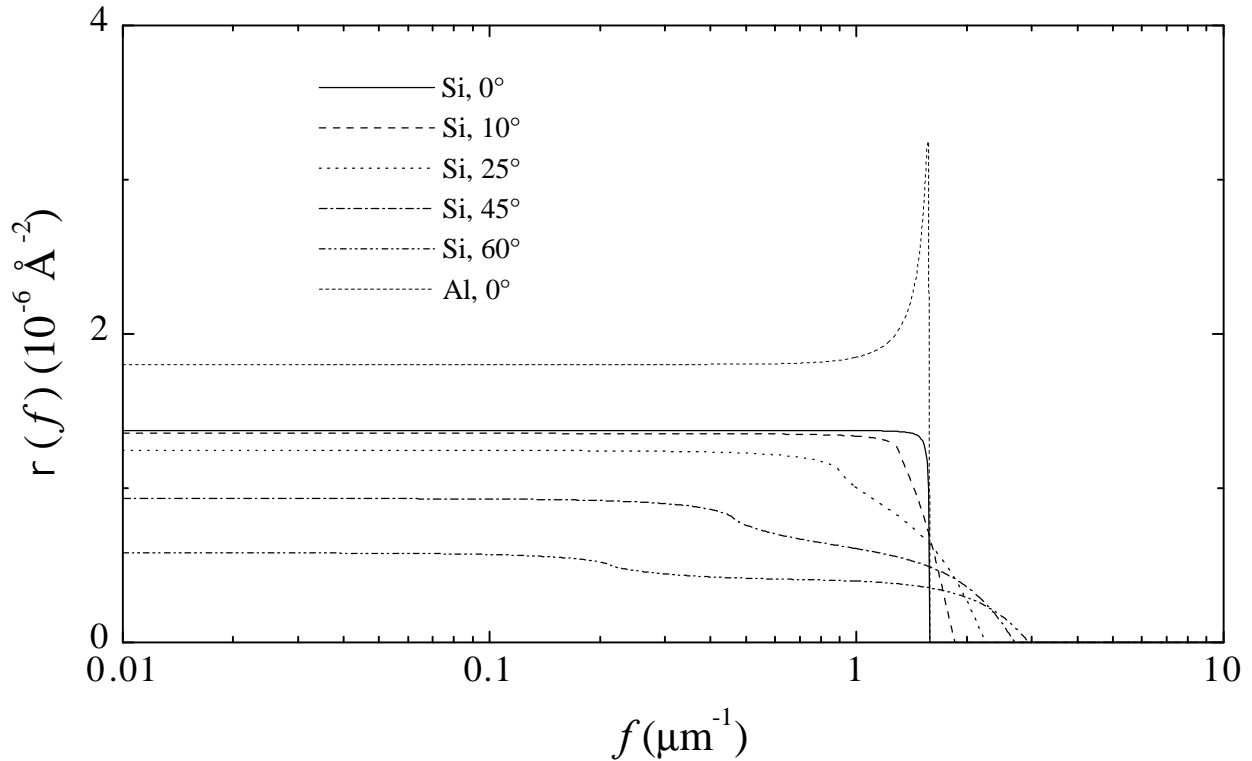


Figure 2 The spatial frequency response function calculated for the ideal hemispherical detection system with a variety of incident angles. Except where noted, the material is assumed to be silicon, and the wavelength of the incident light is 632.8 nm.

5. Further Examples

This section will show the system specification files for each of the systems discussed in Ref. 1. The examples are each given as a separate system specification file distributed with the program.

5.A. Hemispherical Collection Systems

Figure 2 shows a hemispherical collection system with different incident angles and two materials (silicon and aluminum). The input data for the six curves shown in Figure 2 follow:

The input file that will generate the data for a hemispherical collection system with an incident angle of 0° and with silicon as the material is found in file FIG2A.SSF:

```

number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 0.0001
solidangle 5e-5
material_n 3.882

```

```
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

The input file that will generate the data for a hemispherical collection system with an incident angle of 10° and with silicon as the material is found in file FIG2B.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 10
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

The input file that will generate the data for a hemispherical collection system with an incident angle of 25° and with silicon as the material is found in file FIG2C.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 25
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

The input file that will generate the data for a hemispherical collection system with an incident angle of 45° and with silicon as the material is found in file FIG2D.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 45
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

The input file that will generate the data for a hemispherical collection system with an incident angle of 60° and with silicon as the material is found in file FIG2E.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 60
```



```
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

The input file that will generate the data for a hemispherical collection system with an incident angle of 0° and with aluminum as the material is found in file FIG2F.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 0.0001
solidangle 5e-5
material_n 1.39
material_k 7.65
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
0 0 0 0 end
```

Note that aluminum has not been found to “wavelength-scale” in the visible part of the spectrum, implying that the scattering is not entirely due to topography. The use of aluminum here is shown only as an illustrative example. The use of the analysis in this program for a system that analyzes the scattered light from aluminum might be inappropriate.

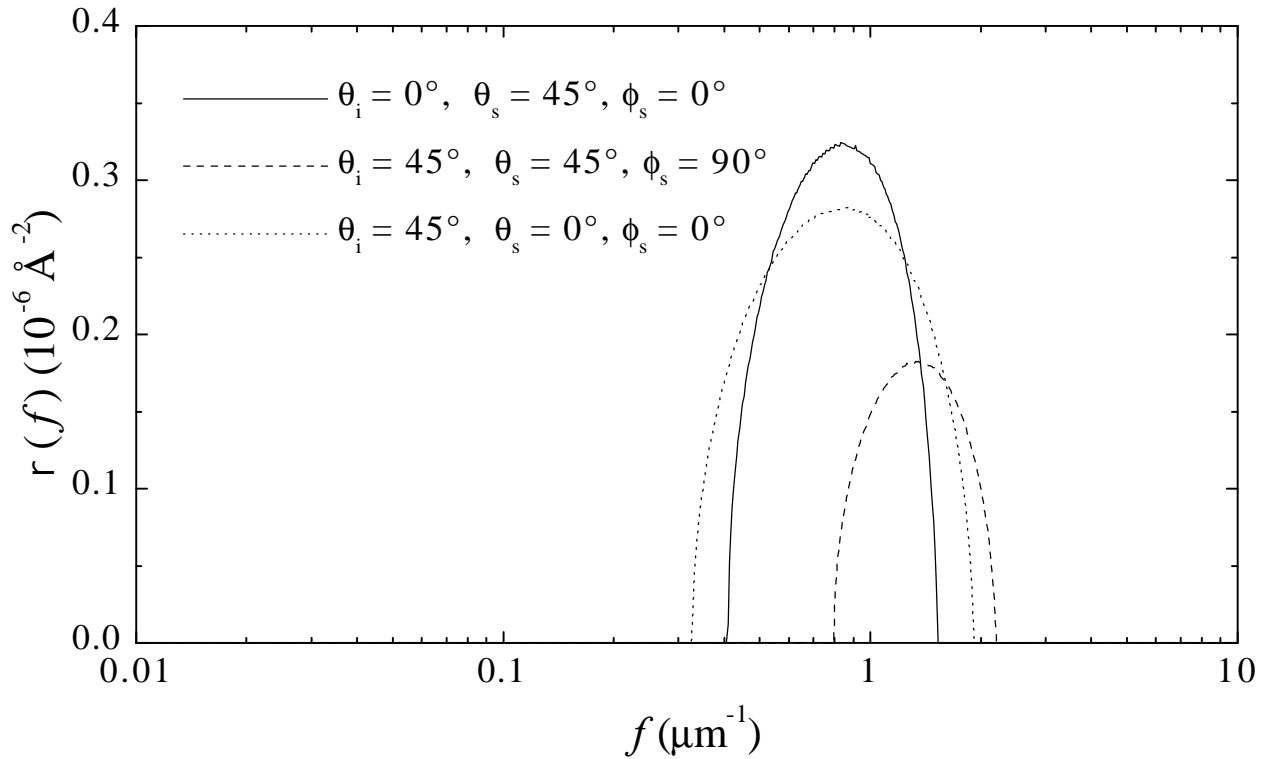


Figure 3 The spatial frequency response function calculated for a conical collection system for a variety of central incident/exitant angle combinations. The conical half-angle of collection is 30° for each curve. The material is assumed to be silicon, and the wavelength of the incident light is 632.8 nm.

5.B. Conical Collection Systems

Figure 3 shows the spatial frequency response function for three conical collection systems. The system specifications for those three systems follow:

The input file that will generate the data for a system collecting light scattered into a right circular cone whose center axis is tilted by 45° with respect to the surface normal (toward the x -axis) and whose opening half-angle is 30° , with light incident along the surface normal and polarized along the y -axis, and with silicon as the material is found in file FIG3A.SSF:

```

number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 0.0001
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 cone 45 0 30
0 0 0 0 end

```

The input file that will generate the data for a system collecting light scattered into a right circular cone whose center axis is tilted by 45° with respect to the surface normal (toward the y-axis) and whose opening half-angle is 30° , with light incident with s-polarization onto the sample at 45° (along the x-axis), and with silicon as the material is found in file FIG3B.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 45
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 cone 45 90 30
0 0 0 0 end
```

The input file that will generate the data for a system collecting light scattered into a right circular cone whose center axis is aligned with the surface normal and whose opening half-angle is 30° , with light incident with s-polarization onto the sample at 45° (along the x-axis), and with silicon as the material is found in file FIG3C.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 45
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 cone 0 0 30
0 0 0 0 end
```

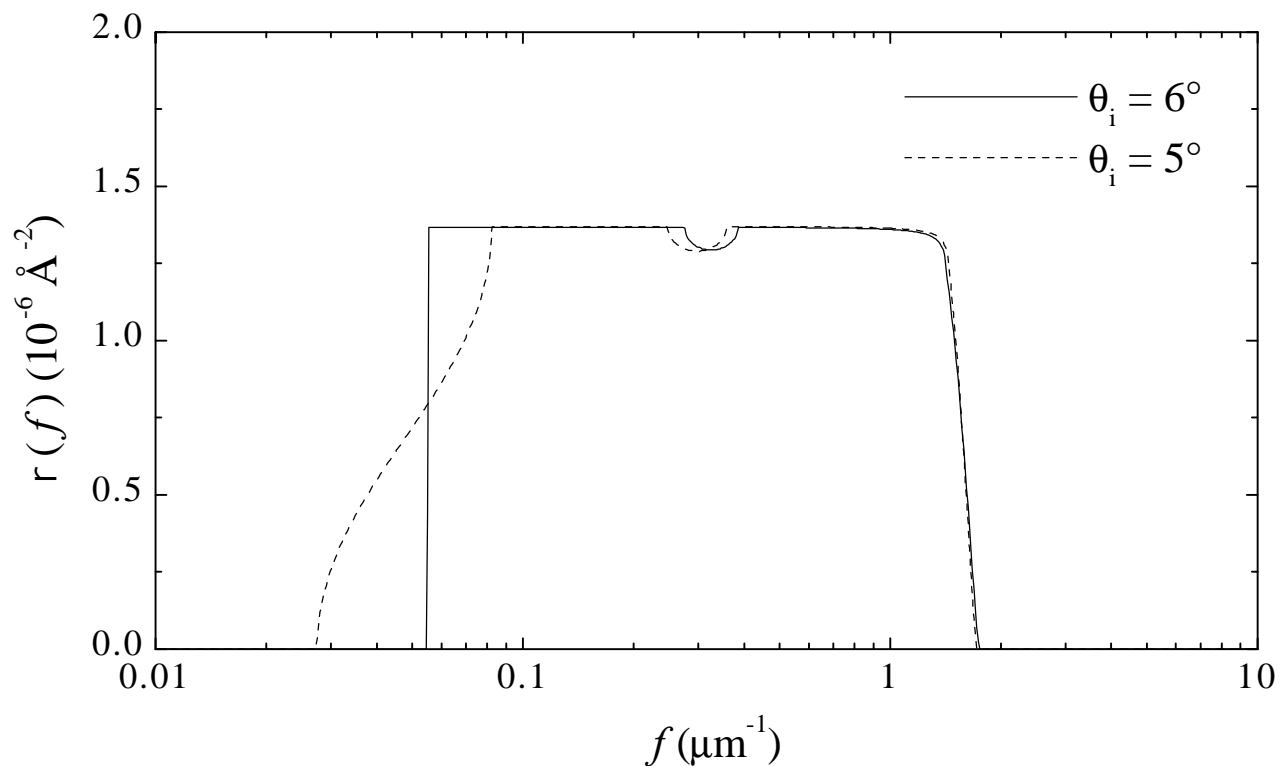


Figure 4 The spatial frequency response function calculated for a total integrated scatter collection system with input and output ports aligned at polar angles of 6° and having opening half-angles of 2° . The curves are shown for incident angles of 6° and 5° , illustrating the effects of misalignment. The material is assumed to be silicon, and the wavelength of the incident light is 632.8 nm.

5.C. Total Integrated Scatter (TIS) Collection Systems

Figure 4 shows two total integrated scatter configurations. These configurations are similar to the hemispherical collection systems, except that they have two ports allowing the incident light to enter and the specular light to escape. The two systems below have ports that subtend half angles of 2° with axes tilted 6° from the surface normal. The two systems differ by having slightly different incident angles, one aligned properly through the centers of the ports, the other “misaligned” by 1° toward the surface normal.

The input file that will generate the data for a total integrated scatter collection system as described above with light incident with s-polarization at an angle of 6° and with silicon as the material is found in file FIG4A.SSF:

```

number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 6
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
-1 0 0 0 cone 6 0 2
-1 0 0 0 cone 6 180 2
0 0 0 0 end

```

The input file that will generate the data for a total integrated scatter collection system as described above with light incident with *s*-polarization at an angle of 5° and with silicon as the material is found in file FIG4B.SSF:

```
number_of_spatial_frequencies 200
wavelength 0.6328
incident_angle 5
solidangle 5e-5
material_n 3.882
material_k 0.019
inputlightQ 1
inputlightU 0
inputlightV 0
1 0 0 0 hemi
-1 0 0 0 cone 6 0 2
-1 0 0 0 cone 6 180 2
0 0 0 0 end
```

6. Miscellaneous

6.A. Using finite-sized elements and polarized collection

Care must be taken to ensure accuracy of the results of this program when determining the response function for polarization sensitive instruments. Due to the definition of the scattered light polarization vectors being associated with each scattering direction, specifying a detection Stokes vector for a large collection solid angle can be misleading. Few polarizers exist which exhibit polarization pass directions that vary in a controlled fashion over their surfaces. This fact may create some uncertainties that have not been appropriately taken into account by SFRF. It is therefore inadvisable to use a response function generated by SFRF for a polarizing collection system for which large solid angles are accepted. Subdividing the system into many smaller connected elements should improve the specification of such a system.

Take for example the total hemispherical collection system. If one were to specify

```
0.5 -0.5 0 0 HEMI
```

for a detection system, then the program will assume that only light of p -polarization will be detected. A realization of such a system will have a polarizer whose polarization pass direction consists of lines distributed radially outward from the surface normal. Such a system would be better represented by a series of triangular regions sharing the surface normal as one of their corners. The SFRF user must assess the errors that will result from using finite solid angles in a polarizing collection system.

6.B. Systems which vary their optical geometry

The program assumes that the optical geometry of the scattered light collection system is fixed. However, systems exist which scan the incident beam over the sample surface. This type of system has an optical geometry that changes with position on the wafer surface, which is not accounted for in the SFRF program. A separate $r(f)$ should be calculated for each point on the wafer, and the results presented in such a way that it is clear where $r(f)$ was evaluated and how it changes from point to point.

6.C. Source of scatter

The user must recognize that the program is limited to the analysis of a system designed to measure light that is scattered by microroughness. Only sources of scatter that originate from modulations of the surface height and can be treated using first-order vector perturbation theory (Rayleigh-Rice theory) are treated by this analysis. Any use of SFRF to determine the response function for other sources of scatter will yield misleading results.

7. Support

Support for SFRF may be obtained and comments may be made by contacting:

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Optical Technology Division
National Institute of Standards and Technology
Gaithersburg, MD 20899-0001 U.S.A.

Telephone: (301) 975-2876
Electronic Mail: tgermer@nist.gov

8. Downloading the Software

All the required files necessary to use SFRF are contained in a self-extracting ZIP file named "SFRFZIP.EXE". Place the file in its own directory (perhaps C:\SFRF), and then execute SFRFZIP.EXE. This file contains

- The program SFRF.EXE,
- System Specification Files for the examples given in Sec. 5,
- The PDF files containing Ref. 1 and this document.

Appendix A: Rayleigh-Rice Theory

The scattering is assumed to result from variations in the surface height function $z(x,y)$ which are sufficiently small that first-order vector perturbation theory (Rayleigh-Rice theory) is valid.² Light of wavelength λ is assumed to be incident at an angle q_i with a polarization state represented by S_i . The complex index of refraction is $\tilde{n} = n+ik$. Scattered light is parameterized by a polar angle q_s , azimuthal angle f_s , and polarization state S_s . Rayleigh-Rice theory predicts a bidirectional reflectance distribution function given by

$$\text{BRDF} = \frac{16\rho^2}{\lambda^4} \cos q_i \cos q_s Q(q_i, S_i, q_s, f_s, S_s) S(f_x, f_y),$$

where $S(f_x, f_y)$ is the power spectral density (PSD) function of the surface height $z(x,y)$, and f_x and f_y are the x and y components of the two-dimensional spatial frequency, respectively. The components of the spatial frequency are related to the incident and scatter angles by

$$\lambda f_x = \sin q_s \cos f_s - \sin q_i$$

$$\lambda f_y = \sin q_s \sin f_s.$$

The polarizations, S_i and S_s , are characterized by the projections e_{nm} of each unit Jones vector onto the s and p directions. The $Q(q_i, S_i, q_s, f_s, S_s)$ are then determined by

$$Q(q_i, S_i, q_s, f_s, S_s) = \left| \sum_j \sum_k q_{jk} e_{ij} e_{sk} \right|^2.$$

The q_{jk} are given by

$$q_{ss} = \frac{(\tilde{n}^2 - 1) \cos f_s}{\left(\cos q_i + \sqrt{\tilde{n}^2 - \sin^2 q_i} \right) \left(\cos q_s + \sqrt{\tilde{n}^2 - \sin^2 q_s} \right)}$$

$$q_{sp} = \frac{- (\tilde{n}^2 - 1) \sin f_s \sqrt{\tilde{n}^2 - \sin^2 q_s}}{\left(\cos q_i + \sqrt{\tilde{n}^2 - \sin^2 q_i} \right) \left(\tilde{n}^2 \cos q_s + \sqrt{\tilde{n}^2 - \sin^2 q_s} \right)}$$

$$q_{ps} = \frac{- (\tilde{n}^2 - 1) \cos f_s \sqrt{\tilde{n}^2 - \sin^2 q_i}}{\left(\tilde{n}^2 \cos q_i + \sqrt{\tilde{n}^2 - \sin^2 q_i} \right) \left(\cos q_s + \sqrt{\tilde{n}^2 - \sin^2 q_s} \right)}$$

$$q_{pp} = \frac{(\tilde{n}^2 - 1) \left(\tilde{n}^2 \sin q_i \sin q_s - \sqrt{\tilde{n}^2 - \sin^2 q_i} \sqrt{\tilde{n}^2 - \sin^2 q_s} \cos f_s \right)}{\left(\tilde{n}^2 \cos q_i + \sqrt{\tilde{n}^2 - \sin^2 q_i} \right) \left(\tilde{n}^2 \cos q_s + \sqrt{\tilde{n}^2 - \sin^2 q_s} \right)}$$

Appendix B: Arithmetic Expressions

Any numerical element of the system specification file can be replaced by an arithmetic expression. If an expression is used, it may contain no spaces and may use a variety of operators, functions, and constants:

Exponential/Logarithmic Functions:

EXP(), LOG(), and LOG10()

Trigonometric Functions (with arguments in radians):

SIN(), COS(), and TAN()

Trigonometric Functions (with arguments in degrees):

SIND(), COSD(), and TAND()

Inverse Trigonometric Functions (returning values in radians):

ASIN(), ACOS(), and ATAN()

Inverse Trigonometric Functions (returning values in degrees):

ASIND(), ACOSD(), and ATAND()

Hyperbolic Trigonometric Functions:

SINH(), COSH(), TANH(), ASINH(), ACOSH(), and ATANH()

Power Functions:

SQRT(), SQR(), and CUBE()

Binary Operators:

+, -, *, /, ^ (power), > (greater than), < (less than), = (equal), & (and), and | (or)

Constants:

PI (=3.1415...), E (=2.7183...), and DEG (= 0.0175...)

References

¹ T. A. Germer and C. C. Asmail, "Proposed methodology for characterization of microroughness-induced optical scatter instrumentation," in *Flatness, Roughness, and Discrete Defect Characterization for Computer Disks, Wafers, and Flat Panel Displays*, John C. Stover, Editor, Proc. SPIE vol. 2862, pp. 12–17 (1996).

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