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Neutron Self-Shielding Factors for Simple Geometries, Revisited★

by Richard M. Lindstrom[1]* and Ronald F. Fleming[2]

[1] *Analytical Chemistry Division, National Institute of Standards and Technology Gaithersburg MD 20899-8395, USA*
[2] *Department of Nuclear Engineering and Radiological Sciences, The University of Michigan Ann Arbor MI 48109, USA*

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To assure quality measurements, the algorithms used in data analysis need to be demonstrably correct. In practice, however, some less transparent or more complicated algorithms may be difficult to trace back to their original derivation. We point out that a commonly cited publication in neutron self-shielding is in need of correction in some details.

Zapewnienie jakości pomiarowej wymaga aby algorytmy stosowane do analizy danych były w oczywisty sposób wolne od błędów. Jednakże w praktyce bywa, że trudno jest dotrzeć do pierwotnego wyprowadzenie niektórych mniej przejrzystych lub bardziej skomplikowanych algorytmów. W niniejszej pracy wskazujemy, że pewna powszechnie cytowana publikacja na temat efektu samoosłaniania neutronów wymaga poprawienia w niektórych szczegółach.

* Corresponding author. E-mail: richard.lindstrom@nist.gov; fax +001-301-208-9279
★ Dedicated to Professor Rajmund Dybczyński on the occasion of his 75th birthday.

If an absorbing sample is immersed in a neutron field, the interior of the sample will be exposed to a smaller neutron fluence rate than the exterior. The neutron self-shielding factor f is the ratio of the mean fluence rate inside the sample volume to the fluence rate incident on the sample. In a purely absorbing sample the value of f is always less than unity, unless the sample contains fissile materials. A strong absorber also leads to a depression in the neutron field outside the sample, a separate (and less important) phenomenon that depends on the scattering power of the surrounding medium.

A complete treatment of self-shielding requires consideration of the neutron energy spectrum inside and outside the sample, and the absorption and scattering cross sections as a function of energy. Although Monte Carlo methods have become routine in calculating self-shielding, analytic expressions for f as a function of the scalar cross sections and sample composition have been derived for a few specific geometries. These expressions are convenient in establishing the size of the effect to be expected in applied fields such as neutron activation analysis and thermoluminescence dating. A frequently cited summary paper is that of Fleming [1] who collected the equations for a sphere, an infinite slab, and an infinite cylinder, irradiated in either an isotropic monoenergetic neutron field or a monoenergetic parallel neutron beam. By employing a geometrical argument [2], these equations are extendable to finite cylinders, a convenient configuration for many irradiation experiments. In light of recent insights, three subtle errors in Fleming's publication require correction.

The infinite cylinder equation

For an infinite cylinder of radius R cm and macroscopic absorption cross section Σ_a cm^{-1} in an isotropic neutron field, the exact expression is said to be

$$f = \frac{2x}{3}\left\{ \begin{array}{l} 2[x \cdot K_1(x) \cdot I_1(x) + K_0(x) \cdot I_0(x) - 1] + \\ \left[\frac{K_1(x) \cdot I_1(x)}{x}\right] - K_0(x) \cdot I_1(x) + K_1(x) \cdot I_0(x) \end{array} \right\} \tag{1}$$

where $x = R\Sigma_a$ and K and I are the modified Bessel functions of the first and second kind, respectively. An approximation to the self-shielding factor for nearly transparent samples (where x is small) is

$$\lim_{x \to 0} f = 1 - \frac{4x}{3} + \frac{x^2}{2}\left[\ln(2/x) + \frac{5}{4} - \gamma\right] \tag{2}$$

where $\gamma = 0.577216...$ These equations were taken from the often-cited original work of Case *et al.* [3]

For use with a modern spreadsheet with an extensive set of built-in functions, both formulas are nearly equally convenient, but a numerical comparison reveals that the exact expression is incorrect as given. As shown in Figure 1, the calculated value of f is greater than unity, a physical impossibility, for values of $x < 0.42$. The approximate equation shows the expected behavior, but the two equations are similar only at very small values of x, where they approach unity from opposite directions.

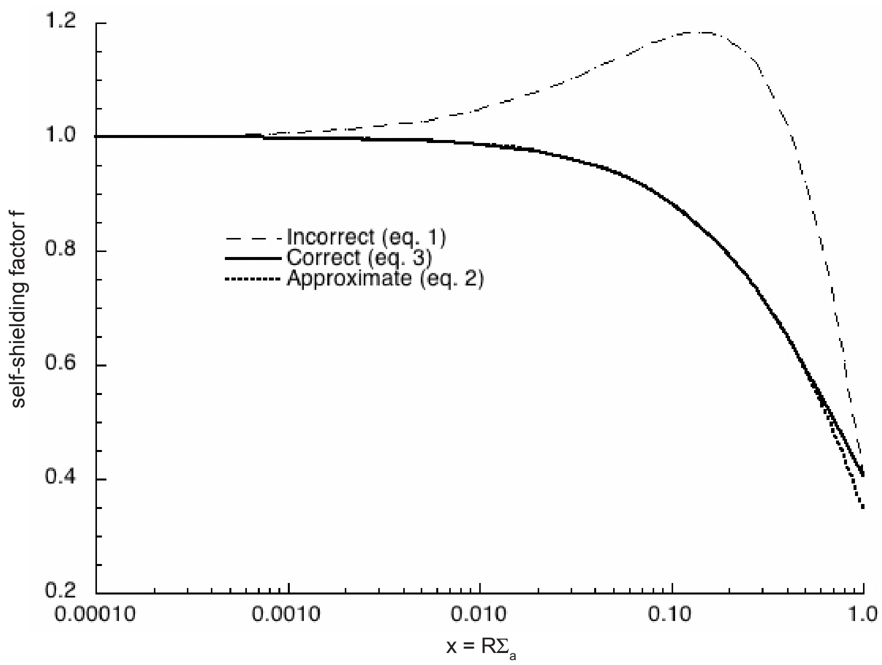

Figure 1. Behavior of three equations for self-shielding of an infinite cylinder. The non-physical behavior of the incorrect expression is clearly evident

Equation 1 was first published in typescript lecture notes from a series of 1949 lectures by Placzek [4] and as a classic Los Alamos report in 1953. [3] Its ultimate origin is cited as „D. Inglis – Los Alamos classified report LA-26, page 8". We have found, as doubtless others have found, that a pair of braces is missing from the first term of the equation. The Bessel function terms appear correctly in a related expression in the text by Beckurts and Wirtz [5; eq. 11.2.27], from which the infinite-cylinder self-shielding function is readily derived. Presumably the error in Case *et al.* occurred in typesetting, but the resolution of this issue has consequences for the traceability of algorithms. [This equation has an additional obvious typographical error in the 1950 typescript, and yet another in the 1953 report.] The correct expression is

$$f = \frac{2x}{3}\left\{\begin{array}{l} 2[x\{K_1(x)\cdot I_1(x) + K_0(x)\cdot I_0(x)\} - 1 + \\ \left[\frac{K_1(x)\cdot I_1(x)}{x}\right] - K_0(x)\cdot I_1(x) + K_1(x)\cdot I_0(x) \end{array}\right\} \quad (3)$$

The correct, incorrect, and approximate expressions are plotted in the following figure. The approximation is good to better than 0.1% for $x < 0.3$.

Correction for scattering

An additional small correction is necessary when the sample scatters neutrons, which may change the neutron's path length and hence the probability of absorption within the sample [2, 6, 7–9]. Scattering usually increases the path length and thus increases f, except for a highly scattering and weakly absorbing sample, in which scattering can decrease f by removing the neutron from the sample before it can be absorbed.

A correction formulae for this effect, generally ascribed to Stewart and Zweifel, [6] is given in the Fleming paper. It has been shown by Wachspress [2, 10] and later explained in detail by Blaauw [9] that the proper way to account for this effect is to calculate an initial self-shielding factor f_0 using the total cross section $\Sigma_t = \Sigma_a + \Sigma_s$ in the analytical expressions, and then to adjust for scattering according to equation 4:

$$f = \frac{f_0}{1-(\Sigma_s/\Sigma_a)(1-f_0)} \quad (4)$$

where Σ_a and Σ_s are the macroscopic absorption and scattering cross sections, respectively.

Correction for neutron spectrum

The preceding expressions are valid for monoenergetic neutrons with a velocity v, conventionally 2200 m s^{-1}, for which thermal cross absorption sections are tabulated. In reality the incident energy spectrum is broad, and changes as it penetrates the sample due to absorption and scattering. For an isotropic neutron field with a Maxwellian distribution and a cross section which varies as 1/v, the macroscopic cross section should be adjusted by [5, 11, 12]

$$\Sigma_a = \frac{2}{\sqrt{\pi}}\sqrt{\frac{T_0}{T}}\cdot\Sigma_a(\nu_0) \quad (5)$$

where T_0 = 293.6 K and v_0 = 2200 m s^{-1}. The corresponding equation in the earlier Fleming paper

$$\Sigma_a = \frac{\sqrt{\pi}}{2}\sqrt{\frac{T_0}{T}} \cdot \Sigma_a(\nu_0) \tag{6}$$

is correct only for a neutron beam with a Maxwellian distribution of velocities [13].

DISCUSSION

A reexamination of a collection of formulae for neutron self-shielding has led to correction of a historical misprint and clarification of several points in the original paper that have been recently addressed in the literature.

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