H.-J. Kretzschmar[2]
Department of Technical Thermodynamics,
Zittau/Goerlitz University of Applied Sciences,
P.O. Box 1455,
D-02754 Zittau, Germany
e-mail: hj.kretzschmar@hs-zigr.de

A. H. Harvey
National Institute of Standards and Technology,
Physical and Chemical Properties Division,
Boulder, CO 80305

K. Knobloch
Department of Technical Thermodynamics,
Zittau/Goerlitz University of Applied Sciences,
P.O. Box 1455,
D-02754 Zittau, Germany

R. Mareš
Department of Technical Thermodynamics,
University of West Bohemia,
CZ 306 14 Plzeň, Czech Republic

K. Miyagawa
4-12-11-628 Nishiogu, Arakawa-ku,
Tokyo 116-0011, Japan

N. Okita
Thermal Plant Systems Project Department,
Toshiba Corporation,
Yokohama 230-0045, Japan

R. Span
Chair of Thermodynamics
Ruhr-University Bochum,
D-44780 Bochum, Germany

I. Stöcker
Department of Technical Thermodynamics,
Zittau/Goerlitz University of Applied Sciences,
P.O. Box 1455,
D-02754 Zittau, Germany

W. Wagner
Chair of Thermodynamics
Ruhr-University Bochum,
D-44780 Bochum, Germany

I. Weber
Siemens AG,
Fossil Power Generation,
D-91050 Erlangen, Germany

Supplementary Backward Equations $v(p,T)$ for the Critical and Supercritical Regions (Region 3) of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam[1]

When steam power cycles are modeled, thermodynamic properties as functions of pressure and temperature are required in the critical and supercritical regions (region 3 of IAPWS-IF97). With IAPWS-IF97, such calculations require cumbersome iterative calculations, because temperature and volume are the independent variables in the formulation for this region. In order to reduce the computing time, the International Association for the Properties of Water and Steam (IAPWS) adopted a set of backward equations for volume as a function of pressure and temperature in region 3. The necessary numerical consistency is achieved by dividing the region into 20 subregions, plus auxiliary subregions near the critical point in which the consistency requirements are relaxed due to the singular behavior at the critical point. In this work, we provide complete documentation of these equations, along with a discussion of their numerical consistency and the savings in computer time. The numerical consistency of these equations should be sufficient for most applications in heat-cycle, boiler, and steam-turbine calculations; if even higher consistency is required, the equations may be used to generate guesses for iterative procedures. [DOI: 10.1115/1.3028630]

1 Introduction

The International Association for the Properties of Water and Steam (IAPWS) adopted the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam (IAPWS-IF97) [1–3] as the standard for calculation of thermodynamic properties of water and steam in the power industry in 1997 and extended it in 2007. It contains basic equations, saturation equations, and equations for the commonly used "backward" functions $T(p,h)$ and $T(p,s)$ valid in the liquid region 1 and the vapor region 2; see Fig. 1.

In 2001, IAPWS-IF97 was supplemented by "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [4,5], referred to here as IAPWS-IF97-S01. These equations are valid in regions 1 and 2.

An additional supplementary release "Backward Equations for

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[2]Corresponding author.

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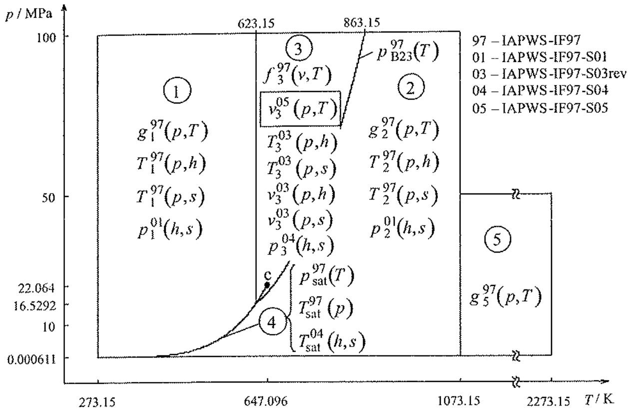

Fig. 1 Regions and equations of IAPWS-IF97, IAPWS-IF97-S01, IAPWS-IF97-S03rev, IAPWS-IF97-S04, and the equations $v_3(p,T)$ of this work adopted as IAPWS-IF97-S05

the Functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [6,7], referred to here as IAPWS-IF97-S03rev, was adopted by IAPWS in 2003 and extended in 2004.

In 2004, IAPWS-IF97 was supplemented by "Supplementary Release on Backward Equations $p(h,s)$ for Region 3, Equations as a Function of h and s for the Region Boundaries, and an Equation $T_{\text{sat}}(h,s)$ for Region 4 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [8,9], referred to here as IAPWS-IF97-S04.

The basic equation $f_3(\rho,T)$ is used in region 3 of IAPWS-IF97. This equation, along with the backward equations $v(p,h)$, $T(p,h)$, $v(p,s)$, $T(p,s)$, and $p(h,s)$, can be used to calculate all thermodynamic properties as a function of (p,h), (p,s), and (h,s) without iteration. However, in modeling modern steam power cycles, properties as a function of the variables (p,T) are required for region 3. Such calculations from the basic equation $f_3(\rho,T)$ are cumbersome, because they require iteration of v for given values of p and T using the relation $p(v,T)$ with $v=1/\rho$ derived from $f_3(\rho,T)$.

In order to avoid such iteration, this paper provides backward equations $v_3(p,T)$ for region 3 as shown in Fig. 1. With the specific volume v calculated from the backward equations $v_3(p,T)$, all other properties in region 3 can be calculated without iteration from the basic equation $f_3(\rho,T)$ with $\rho=1/v_3$.

For process calculations, the numerical consistency requirements for the backward equations $v_3(p,T)$ are very strict. Since the specific volume on the v-p-T surface has a complicated structure including an infinite slope at the critical point, region 3 had to be divided into 26 subregions. The first 20 subregions and their associated backward equations, described in Sec. 4, cover nearly all of region 3 and fully meet the consistency requirements given in Sec. 2. For a small area very near the critical point, it was not possible to meet the consistency requirements completely. This near-critical region is covered with reasonable consistency by six subregions with auxiliary equations that are described in Sec. 5.

This set of backward and auxiliary equations was adopted by IAPWS in 2005, referred to here as IAPWS-IF97-S05 [10]. The purpose of this paper is to fully document IAPWS-IF97-S05.

The entire system of supplementary backward equations adopted by IAPWS is summarized in Ref. [11] and described in detail in Ref. [12].

2 Numerical Consistency Requirements

In region 3, any property calculation from the basic equation $f_3(\rho,T)$ for given values of p and T requires the determination of the density by iteration. The permissible numerical consistency of the equations for specific volume with the IAPWS-IF97 fundamental equation was determined based on the required accuracy of the iteration otherwise used. The iteration accuracy depends on thermodynamic process calculations. To obtain specific enthalpy or entropy from pressure and temperature in region 3 with a maximum deviation of 0.001% from IAPWS-IF97, and isobaric heat capacity or speed of sound with a maximum deviation of 0.01%, the inconsistencies in v between the backward equations $v_3(p,T)$ and the basic equation $f_3(\rho,T)$ had to be less than 0.001%, and for some parts of region 3 even smaller. The consistency requirements for all of these properties are summarized in Table 1.

Table 1 Permissible numerical inconsistencies in the properties v, h, s, c_p, and w, when v is calculated first via iteration with the basic equation $f_3(\rho,T)$ for given inputs of p and T, and second directly from the backward equation $v_3(p,T)$. Based on these two (slightly different) v values, the properties h, s, c_p, and w are obtained from the basic equation $f_3(\rho,T)$. (The values for v are calculated from the backward equations $v_3(p,T)$.)

Permissible inconsistencies				
$\|\Delta v/v\|_{\text{perm}}$	$\|\Delta h/h\|_{\text{perm}}$	$\|\Delta s/s\|_{\text{perm}}$	$\|\Delta c_p/c_p\|_{\text{perm}}$	$\|\Delta w/w\|_{\text{perm}}$
0.001%	0.001%	0.001%	0.01%	0.01%

In the near-critical region, there are no defined numerical consistency requirements for the auxiliary equations, but the inconsistencies should be as small as possible.

3 Range of Validity of the Backward and Auxiliary Equations

The range of validity of the entire set of backward equations $v_3(p,T)$ corresponds to region 3 of IAPWS-IF97, which is defined by the following range of temperature and pressure:

$$623.15\ \text{K} < T < 863.15\ \text{K and } p_{\text{B23}}(T) < p \leq 100\ \text{MPa}$$

with $p_{\text{B23}}(T)$ defined by the B23-equation [1,2] as shown in Fig. 1.

The numerical consistency requirement of 0.001% for $v_3(p,T)$ proved to be infeasible to achieve with simple functional forms in the region

$$p_{\text{sat}}(643.15\ \text{K}) < p \leq 22.5\ \text{MPa}, \quad T_{3\text{qu}}(p) < T \leq T_{3\text{rx}}(p)$$

where $p_{\text{sat}}(643.15\ \text{K})=21.03436732$ MPa. This region is marked in gray in Fig. 2, which also shows the temperature and pressure range of the boundary equations $T_{3\text{qu}}(p)$ and $T_{3\text{rx}}(p)$. The boundary equations themselves are given in Sec. 4.2, and $p_{\text{sat}}(643.15\ \text{K})$ is calculated from the IAPWS-IF97 saturation-pressure equation.

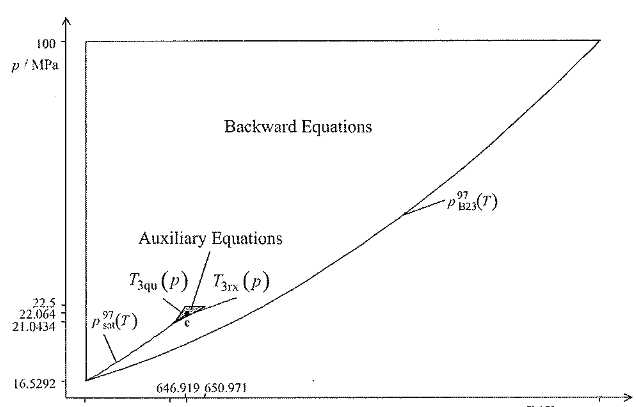

Fig. 2 Range of validity of the backward and auxiliary equations. The area in gray is not to scale but is enlarged to make the small area more visible.

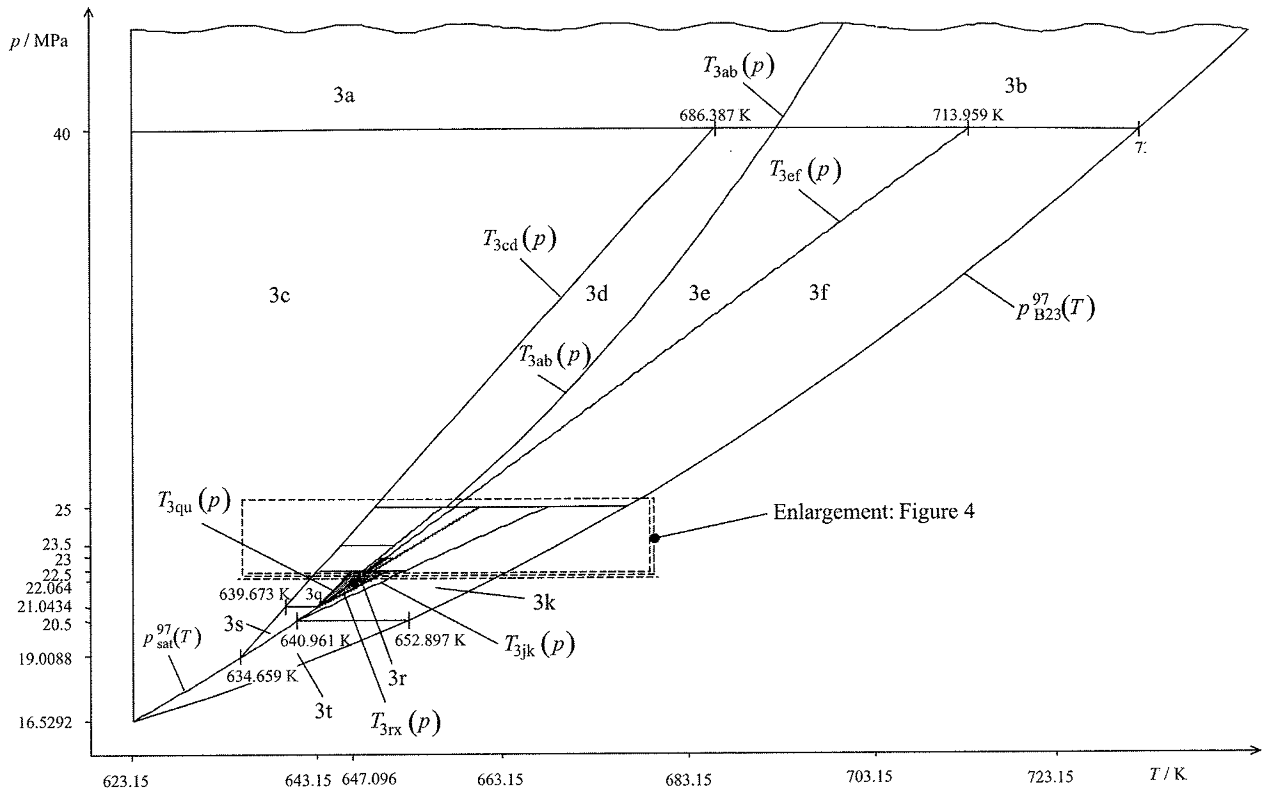

Fig. 3 Division of region 3 into subregions for the backward equations $v_3(p, T)$

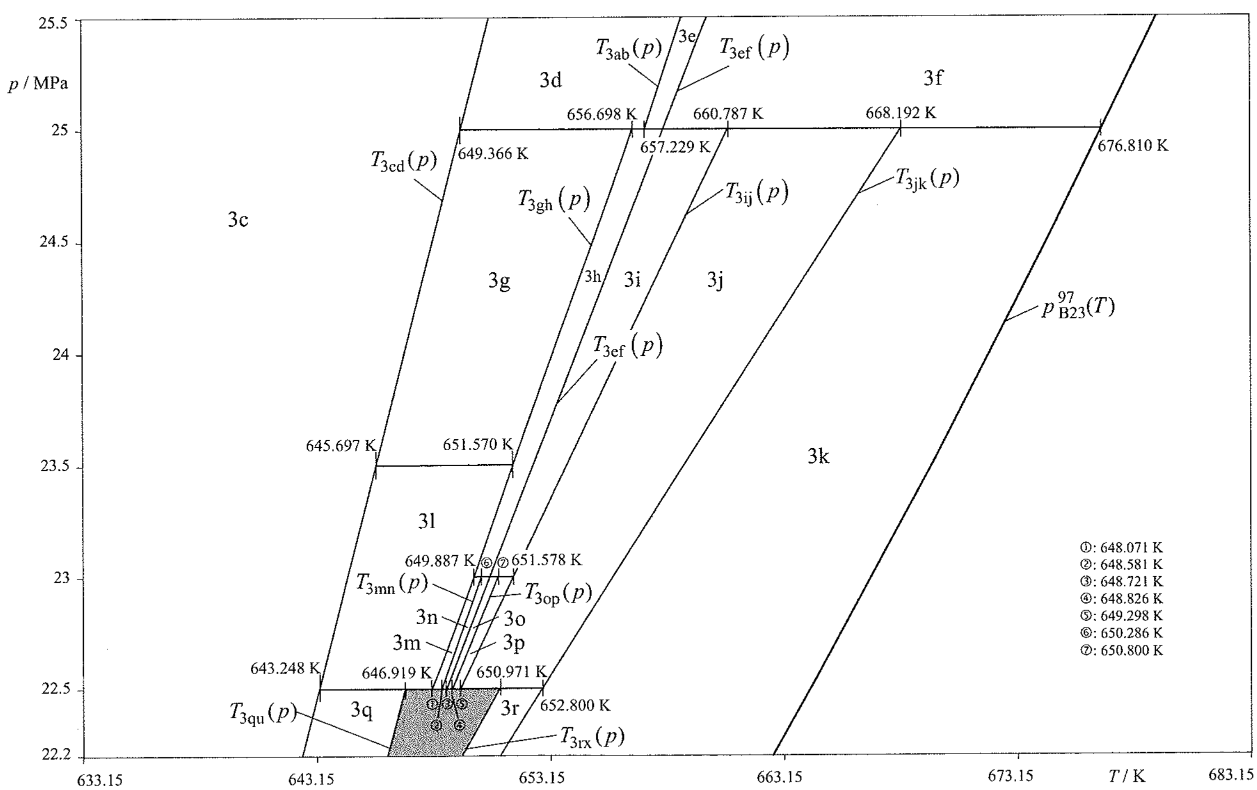

Fig. 4 Enlargement from Fig. 3 for the subregions 3c–3r for the backward equation $v(p, T)$

The reason for excluding the near-critical region (gray area in Fig. 2) from the range of validity of the backward equations $v_3(p,T)$ is based on the complex structure of this region on the v-p-T surface with the infinite slope $(\partial v/\partial p)_T$ at the critical point. In order to not exclude any region completely from the equations $v(p,T)$, Sec. 5 contains equations for this small region very close to the critical point. These equations exhibit larger inconsistencies with the basic equation $f_3(\rho,T)$ and are called auxiliary equations in this work.

4 Backward Equations $v(p,T)$ for Subregions 3a–3t

4.1 Development of the Equations $v(p,T)$. A major motivation for the development of IAPWS-IF97 and its supplementary backward equations was to reduce the time for computing thermodynamic properties. As shown previously [13], the following functional form is effective for this purpose:

$$\frac{Z(X,Y)}{Z^*} = \sum_i n_i \left(\frac{X}{X^*} + a\right)^{I_i} \left(\frac{Y}{Y^*} + b\right)^{J_i} \tag{1}$$

where the reducing parameters Z^*, X^*, and Y^* are typically maximum values of the corresponding property within the range of validity of the equation. The shifting parameters a and b were determined by nonlinear optimization. The exponents I_i and J_i and coefficients n_i were determined from the structure optimization method of Wagner [14] and Setzmann and Wagner [15], which chooses the optimal terms from a bank of terms with various

Table 2 Numerical values of the coefficients of the equations for subregion boundaries (except $T_{3ef}(p)$)

Equation	i	I_i	n_i
$T_{3ab}(p)$	1	0	$0.154793642129415\times10^{4}$
	2	1	$-0.187661219490113\times10^{3}$
	3	2	$0.213144632222113\times10^{2}$
	4	−1	$-0.191887498864292\times10^{4}$
	5	−2	$0.918419702359447\times10^{3}$
$T_{3cd}(p)$	1	0	$0.585276966696349\times10^{3}$
	2	1	$0.278233532206915\times10^{1}$
	3	2	$-0.127283549295878\times10^{-1}$
	4	3	$0.159090746562729\times10^{-3}$
$T_{3gh}(p)$	1	0	$-0.249284240900418\times10^{5}$
	2	1	$0.428143584791546\times10^{4}$
	3	2	$-0.269029173140130\times10^{3}$
	4	3	$0.751608051114157\times10^{1}$
	5	4	$-0.787105249910383\times10^{-1}$
$T_{3ij}(p)$	1	0	$0.584814781649163\times10^{3}$
	2	1	-0.616179320924617
	3	2	0.260763050899562
	4	3	$-0.587071076864459\times10^{-2}$
	5	4	$0.515308185433082\times10^{-4}$
$T_{3jk}(p)$	1	0	$0.617229772068439\times10^{3}$
	2	1	$-0.770600270141675\times10^{1}$
	3	2	0.697072596851896
	4	3	$-0.157391839848015\times10^{-1}$
	5	4	$0.137897492684194\times10^{-3}$
$T_{3mn}(p)$	1	0	$0.535339483742384\times10^{3}$
	2	1	$0.761978122720128\times10^{1}$
	3	2	-0.158365725441648
	4	3	$0.192871054508108\times10^{-2}$
$T_{3op}(p)$	1	0	$0.969461372400213\times10^{3}$
	2	1	$-0.332500170441278\times10^{3}$
	3	2	$0.642859598466067\times10^{2}$
	4	−1	$0.773845935768222\times10^{3}$
	5	−2	$-0.152313732937084\times10^{4}$
$T_{3qu}(p)$	1	0	$0.565603648239126\times10^{3}$
	2	1	$0.529062258221222\times10^{1}$
	3	2	-0.102020639611016
	4	3	$0.122240301070145\times10^{-2}$
$T_{3rx}(p)$	1	0	$0.584561202520006\times10^{3}$
	2	1	$-0.102961025163669\times10^{1}$
	3	2	0.243293362700452
	4	3	$-0.294905044740799\times10^{-2}$

Table 3 Pressure ranges and corresponding subregion-boundary equations for determining the correct subregion, 3a to 3t, for the backward equations $v_3(p,T)$

Pressure range	Subregion	Temperature range
$40\ \text{MPa} < p \le 100\ \text{MPa}$	3a	$T \le T_{3ab}(p)$
	3b	$T > T_{3ab}(p)$
$25\ \text{MPa} < p \le 40\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3d	$T_{3cd}(p) < T \le T_{3ab}(p)$
	3e	$T_{3ab}(p) < T \le T_{3ef}(p)$
	3f	$T > T_{3ef}(p)$
$23.5\ \text{MPa} < p \le 25\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3g	$T_{3cd}(p) < T \le T_{3gh}(p)$
	3h	$T_{3gh}(p) < T \le T_{3ef}(p)$
	3i	$T_{3ef}(p) < T \le T_{3ij}(p)$
	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3k	$T > T_{3jk}(p)$
$23\ \text{MPa} < p \le 23.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3l	$T_{3cd}(p) < T \le T_{3gh}(p)$
	3h	$T_{3gh}(p) < T \le T_{3ef}(p)$
	3i	$T_{3ef}(p) < T \le T_{3ij}(p)$
	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3k	$T > T_{3jk}(p)$
$22.5\ \text{MPa} < p \le 23\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3l	$T_{3cd}(p) < T \le T_{3gh}(p)$
	3m	$T_{3gh}(p) < T \le T_{3mm}(p)$
	3n	$T_{3mm}(p) < T \le T_{3ef}(p)$
	3o	$T_{3ef}(p) < T \le T_{3op}(p)$
	3p	$T_{3op}(p) < T \le T_{3ij}(p)$
	3j	$T_{3ij}(p) < T \le T_{3jk}(p)$
	3k	$T > T_{3jk}(p)$
$p_{sat}(643.15\ \text{K})^{a} < p \le 22.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3q	$T_{3cd}(p) < T \le T_{3qu}(p)$
	3r	$T_{3rx}(p) < T \le T_{3jk}(p)$
	3k	$T > T_{3jk}(p)$
$20.5\ \text{MPa} < p \le p_{sat}(643.15\ \text{K})^{a}$	3c	$T \le T_{3cd}(p)$
	3s	$T_{3cd}(p) < T \le T_{sat}(p)$
	3r	$T_{sat}(p) \le T \le T_{3jk}(p)$
	3k	$T > T_{3jk}(p)$
$p_{3cd}{}^{b} < p \le 20.5\ \text{MPa}$	3c	$T \le T_{3cd}(p)$
	3s	$T_{3cd}(p) < T \le T_{sat}(p)$
	3t	$T \ge T_{sat}(p)$
$p_{sat}(623.15\ \text{K})^{c} < p \le p_{3cd}{}^{b}$	3c	$T \le T_{sat}(p)$
	3t	$T \ge T_{sat}(p)$

[a] $p_{sat}(643.15\ \text{K}) = 21.04336732$ MPa.
[b] $p_{3cd} = 19.00881189$ MPa.
[c] $p_{sat}(623.15\ \text{K}) = 16.52916425$ MPa.

Table 4 Temperature values calculated from the subregion-boundary equations for selected pressures

Equation	p (MPa)	T (K)
$T_{3ab}(p)$	40	6.930341408×10^2
$T_{3cd}(p)$	25	6.493659208×10^2
$T_{3ef}(p)$	40	7.139593992×10^2
$T_{3gh}(p)$	23	6.498873759×10^2
$T_{3ij}(p)$	23	6.515778091×10^2
$T_{3jk}(p)$	23	6.558338344×10^2
$T_{3mn}(p)$	22.8	6.496054133×10^2
$T_{3op}(p)$	22.8	6.500106943×10^2
$T_{3qu}(p)$	22	6.456355027×10^2
$T_{3rx}(p)$	22	6.482622754×10^2

values of I_i and J_i. The final equations were developed using the approximation algorithm developed in previous work [16–19].

In the approximation process, the backward equations were fitted to v-p-T values, with p calculated from the IAPWS-IF97 basic equation $f_3(\rho,T)$ for values of $\rho=1/v$ and T distributed over the range of validity. The computing time is considered in the optimization. Details of the fitting processes are given in Refs. [11,16].

4.2 Division of Region 3 into Subregions and the Subregion-Boundary Equations. Preliminary investigations showed that it was not possible to meet the numerical consistency requirements with only a few subregions. Therefore, the main part of region 3 was divided into 20 subregions 3a–3t, as illustrated in Figs. 3 and 4.

The following subscripts mark the boundaries that separate the adjacent subregions:

3ab:	boundary between subregions	3a/3b and 3d/3e
3cd:	boundary between subregions	3c/3d, 3c/3g, and 3c/3l
3ef:	boundary between subregions	3e/3f, 3h/3i, and 3n/3o
3gh:	boundary between subregions	3g/3h and 3l/3m
3ij:	boundary between subregions	3i/3j and 3p/3j
3jk:	boundary between subregions	3j/3k and 3r/3k
3mn:	boundary between subregions	3m/3n
3op:	boundary between subregions	3o/3p
3qu:	boundary between subregions	3q/3u
3rx:	boundary between subregions	3r/3x
3uv:	boundary between subregions	3u/3v
3wx:	boundary between subregions	3w/3x
B23:	boundary between regions	2/3

These subregion boundaries are also shown in Figs. 3 and 4.

The subregion-boundary equations, except for the equations $T_{3ab}(p)$, $T_{3ef}(p)$, and $T_{3op}(p)$, have the dimensionless form

$$\frac{T(p)}{T^*} = \theta(\pi) = \sum_{i=1}^{N} n_i \pi^{I_i} \tag{2}$$

where $\theta=T/T^*$ and $\pi=p/p^*$ with $T^*=1$ K and $p^*=1$ MPa.

The equations $T_{3ab}(p)$ and $T_{3op}(p)$ have the form

Table 5 Reducing quantities v^*, p^*, and T^*, the number of coefficients N, the nonlinear parameters a and b, and exponents c, d, and e of the backward equations $v(p,T)$ of subregions 3a–3t

Subregion	v^* (m^3 kg^{-1})	p^* (MPa)	T^* (K)	N	a	b	c	d	e
3a	0.0024	100	760	30	0.085	0.817	1	1	1
3b	0.0041	100	860	32	0.280	0.779	1	1	1
3c	0.0022	40	690	35	0.259	0.903	1	1	1
3d	0.0029	40	690	38	0.559	0.939	1	1	4
3e	0.0032	40	710	29	0.587	0.918	1	1	1
3f	0.0064	40	730	42	0.587	0.891	0.5	1	4
3g	0.0027	25	660	38	0.872	0.971	1	1	4
3h	0.0032	25	660	29	0.898	0.983	1	1	4
3i	0.0041	25	660	42	0.910	0.984	0.5	1	4
3j	0.0054	25	670	29	0.875	0.964	0.5	1	4
3k	0.0077	25	680	34	0.802	0.935	1	1	1
3l	0.0026	24	650	43	0.908	0.989	1	1	4
3m	0.0028	23	650	40	1.000	0.997	1	0.25	1
3n	0.0031	23	650	39	0.976	0.997	—	—	—
3o	0.0034	23	650	24	0.974	0.996	0.5	1	1
3p	0.0041	23	650	27	0.972	0.997	0.5	1	1
3q	0.0022	23	650	24	0.848	0.983	1	1	4
3r	0.0054	23	650	27	0.874	0.982	1	1	1
3s	0.0022	21	640	29	0.886	0.990	1	1	4
3t	0.0088	20	650	33	0.803	1.020	1	1	1

Table 6 Values of the specific volume calculated from the backward equations $v(p,T)$ of subregions 3a–3t for selected values of pressure and temperature

Equation	p (MPa)	T (K)	v (m^3 kg^{-1})
$v_{3a}(p,T)$	50	630	$1.470853100 \times 10^{-3}$
	80	670	$1.503831359 \times 10^{-3}$
$v_{3b}(p,T)$	50	710	$2.204728587 \times 10^{-3}$
	80	750	$1.973692940 \times 10^{-3}$
$v_{3c}(p,T)$	20	630	$1.761696406 \times 10^{-3}$
	30	650	$1.819560617 \times 10^{-3}$
$v_{3d}(p,T)$	26	656	$2.245587720 \times 10^{-3}$
	30	670	$2.506897702 \times 10^{-3}$
$v_{3e}(p,T)$	26	661	$2.970225962 \times 10^{-3}$
	30	675	$3.004627086 \times 10^{-3}$
$v_{3f}(p,T)$	26	671	$5.019029401 \times 10^{-3}$
	30	690	$4.656470142 \times 10^{-3}$
$v_{3g}(p,T)$	23.6	649	$2.163198378 \times 10^{-3}$
	24	650	$2.166044161 \times 10^{-3}$
$v_{3h}(p,T)$	23.6	652	$2.651081407 \times 10^{-3}$
	24	654	$2.967802335 \times 10^{-3}$
$v_{3i}(p,T)$	23.6	653	$3.273916816 \times 10^{-3}$
	24	655	$3.550329864 \times 10^{-3}$
$v_{3j}(p,T)$	23.5	655	$4.545001142 \times 10^{-3}$
	24	660	$5.100267704 \times 10^{-3}$
$v_{3k}(p,T)$	23	660	$6.109525997 \times 10^{-3}$
	24	670	$6.427325645 \times 10^{-3}$
$v_{3l}(p,T)$	22.6	646	$2.117860851 \times 10^{-3}$
	23	646	$2.062374674 \times 10^{-3}$
$v_{3m}(p,T)$	22.6	648.6	$2.533063780 \times 10^{-3}$
	22.8	649.3	$2.572971781 \times 10^{-3}$
$v_{3n}(p,T)$	22.6	649.0	$2.923432711 \times 10^{-3}$
	22.8	649.7	$2.913311494 \times 10^{-3}$
$v_{3o}(p,T)$	22.6	649.1	$3.131208996 \times 10^{-3}$
	22.8	649.9	$3.221160278 \times 10^{-3}$
$v_{3p}(p,T)$	22.6	649.4	$3.715596186 \times 10^{-3}$
	22.8	650.2	$3.664754790 \times 10^{-3}$
$v_{3q}(p,T)$	21.1	640	$1.970999272 \times 10^{-3}$
	21.8	643	$2.043919161 \times 10^{-3}$
$v_{3r}(p,T)$	21.1	644	$5.251009921 \times 10^{-3}$
	21.8	648	$5.256844741 \times 10^{-3}$
$v_{3s}(p,T)$	19.1	635	$1.932829079 \times 10^{-3}$
	20	638	$1.985387227 \times 10^{-3}$
$v_{3t}(p,T)$	17	626	$8.483262001 \times 10^{-3}$
	20	640	$6.227528101 \times 10^{-3}$

$$\frac{T(p)}{T^*} = \theta(\pi) = \sum_{i=1}^{N} n_i (\ln \pi)^{I_i} \quad (3)$$

and $T_{3\mathrm{ef}}(p)$ has the form

$$\frac{T_{3\mathrm{ef}}(p)}{T^*} = \theta(\pi) = \left.\frac{\mathrm{d}\theta}{\mathrm{d}\pi}\right|_{\mathrm{c}} (\pi - 22.064) + 647.096 \quad (4)$$

where the derivative of the IAPWS-IF97 saturation-temperature equation at the critical point is $\mathrm{d}\theta/\mathrm{d}\pi|_{\mathrm{c}}=3.727888004$.

The coefficients n_i and exponents I_i of these subregion-boundary equations are listed in Table 2.

With the help of the ranges of pressure and temperature given in Table 3, any (p,T) point can be assigned to the corresponding subregions 3a–3t as given in Figs. 3 and 4; the subregion-boundary equations $T_{3\mathrm{ab}}(p)$ to $T_{3\mathrm{rx}}(p)$ are defined in Eqs. (2)–(4) in combination with Table 2.

The information given above completely documents the equations for these subregion boundaries. Some of the equations are designed to approximate specific physical relationships such as isentropes. This is discussed in more detail elsewhere [10,11].

To assist the user in computer-program verification of the equations for the subregion boundaries, Table 4 contains test values for calculated temperatures.

4.3 Backward Equations $v(p,T)$. The backward equations $v(p,T)$ for subregions 3a–3t, except for 3n, have the dimensionless form

$$\frac{v(p,T)}{v^*} = \omega(\pi,\theta) = \left[\sum_{i=1}^{N} n_i[(\pi-a)^c]^{I_i}[(\theta-b)^d]^{J_i}\right]^e \quad (5)$$

The equation for subregion 3n has the form

$$\frac{v_{3\mathrm{n}}(p,T)}{v^*} = \omega(\pi,\theta) = \exp\left[\sum_{i=1}^{N} n_i(\pi-a)^{I_i}(\theta-b)^{J_i}\right] \quad (6)$$

where $\omega=v/v^*$, $\pi=p/p^*$, and $\theta=T/T^*$. The reducing quantities v^*, p^*, and T^*, the number of coefficients N, the nonlinear parameters a and b, and the exponents c, d, and e are listed in Table 5. The coefficients n_i and exponents I_i and J_i of these equations are given in the Appendix.

To assist the user in computer-program verification of the backward equations $v(p,T)$, Eqs. (5) and (6), for subregions 3a–3t, Table 6 contains test values for calculated specific volumes.

4.4 Calculation of Properties Utilizing the Backward Equations $v(p,T)$. The backward equations $v_{3\mathrm{a}}(p,T)$–$v_{3\mathrm{t}}(p,T)$, described in Sec. 4.3, along with the IAPWS-IF97 basic equation $f_3(\rho,T)$, make it possible to determine all thermodynamic properties, e.g., specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound, for given values of pressure p and temperature T in region 3 without iteration.

The following steps should be made.

- Identify the subregion (3a–3t) for the given values of the pressure p and temperature T following the instructions in Sec. 4.2 in conjunction with Table 3 and Figs. 3 and 4. Then, calculate the specific volume v for the subregion with the corresponding backward equation $v(p,T)$, Eq. (5) or Eq. (6).
- Calculate the desired property for the previously calculated specific volume v and the given temperature T by use of the relation of this property to the basic equation $f_3(\rho,T)$, where $\rho=1/v$ is determined from the corresponding backward equation, Eq. (5) or Eq. (6).

Table 7 Maximum and root-mean-square inconsistencies in v, h, s, c_p, and w, when these properties are calculated from the basic equation $f_3(\rho,T)$ after ρ is determined by iteration and when ρ is calculated directly from the backward equations $v(p,T)$ of regions 3a–3t

	Inconsistencies in v, h, s, c_p, and w (%)									
	$\lvert\Delta v/v\rvert$		$\lvert\Delta h/h\rvert$		$\lvert\Delta s/s\rvert$		$\lvert\Delta c_p/c_p\rvert$		$\lvert\Delta w/w\rvert$	
Subregion	max	rms	max	rms	max	rms	max	rms	max	rms
3a	0.00061	0.00031	0.00018	0.00008	0.00026	0.00011	0.0016	0.0006	0.0015	0.0006
3b	0.00064	0.00035	0.00017	0.00008	0.00016	0.00008	0.0012	0.0003	0.0008	0.0003
3c	0.00080	0.00038	0.00026	0.00012	0.00025	0.00011	0.0059	0.0016	0.0023	0.0010
3d	0.00059	0.00025	0.00018	0.00008	0.00014	0.00006	0.0035	0.0010	0.0012	0.0004
3e	0.00072	0.00033	0.00018	0.00009	0.00014	0.00007	0.0017	0.0005	0.0006	0.0002
3f	0.00068	0.00020	0.00018	0.00005	0.00013	0.00004	0.0015	0.0003	0.0002	0.0001
3g	0.00047	0.00016	0.00014	0.00005	0.00011	0.00004	0.0032	0.0011	0.0010	0.0003
3h	0.00085	0.00044	0.00022	0.00012	0.00017	0.00009	0.0066	0.0018	0.0006	0.0002
3i	0.00067	0.00028	0.00018	0.00008	0.00013	0.00006	0.0019	0.0006	0.0002	0.0001
3j	0.00034	0.00019	0.00009	0.00005	0.00007	0.00004	0.0020	0.0006	0.0002	0.0001
3k	0.00034	0.00012	0.00008	0.00003	0.00007	0.00002	0.0018	0.0003	0.0002	0.0001
3l	0.00033	0.00019	0.00010	0.00006	0.00008	0.00005	0.0035	0.0015	0.0008	0.0004
3m	0.00057	0.00031	0.00015	0.00009	0.00011	0.00006	0.0062	0.0030	0.0006	0.0002
3n	0.00064	0.00029	0.00017	0.00008	0.00012	0.00006	0.0050	0.0013	0.0002	0.0001
3o	0.00031	0.00015	0.00008	0.00004	0.00006	0.00003	0.0007	0.0002	0.0001	0.0001
3p	0.00044	0.00022	0.00012	0.00006	0.00009	0.00005	0.0026	0.0010	0.0002	0.0001
3q	0.00036	0.00018	0.00012	0.00006	0.00009	0.00005	0.0040	0.0016	0.0010	0.0005
3r	0.00037	0.00007	0.00010	0.00002	0.00008	0.00002	0.0030	0.0004	0.0002	0.0001
3s	0.00030	0.00016	0.00010	0.00005	0.00007	0.00004	0.0033	0.0015	0.0009	0.0005
3t	0.00095	0.00045	0.00022	0.00010	0.00018	0.00008	0.0046	0.0015	0.0004	0.0002
Permissible values	0.001		0.001		0.001		0.01		0.01	

Table 8 Numerical values of the coefficients of the equations $T_{3uv}(p)$ and $T_{3wx}(p)$ for subregion boundaries

Equation	i	I_i	n_i
$T_{3uv}(p)$	1	0	$0.528199646263062\times10^{3}$
	2	1	$0.890579602135307\times10^{1}$
	3	2	-0.222814134903755
	4	3	$0.286791682263697\times10^{-2}$
$T_{3wx}(p)$	1	0	$0.728052609145380\times10^{1}$
	2	1	$0.973505869861952\times10^{2}$
	3	2	$0.147370491183191\times10^{2}$
	4	−1	$0.329196213998375\times10^{3}$
	5	−2	$0.873371668682417\times10^{3}$

4.5 Numerical Consistency. The numerical inconsistencies between the backward equations $v(p,T)$, Eqs. (5) and (6), and the basic equation $f_3(\rho,T)$ in comparison with the permissible inconsistencies given in Table 1 are listed in Table 7. In addition to the inconsistencies in specific volume v itself, the effects of these inconsistencies with regard to the inconsistencies in specific enthalpy, specific entropy, specific isobaric heat capacity, and speed of sound are also given; the calculation of these properties based on the calculation of v from the backward equations $v(p,T)$, Eqs. (5) and (6), is described in Sec. 4.4.

Table 7 shows that the inconsistencies in specific volume v, specific enthalpy h, and specific entropy s are less than 0.001% when v is calculated from the backward equations $v(p,T)$ given in Sec. 4.3 and from the basic equation $f_3(\rho,T)$. The corresponding inconsistencies in the specific isobaric heat capacity c_p and in the speed of sound w are less than 0.01%. Thus, all inconsistencies are less than the permissible values. The values for v, h, and s calculated only from the basic equation $f_3(\rho,T)$ (with iteration of $\rho=1/v$) are represented to within five significant figures by values determined from the basic equation $f_3(\rho,T)$, where $\rho=1/v$ is calculated from the backward equations $v(p,T)$ (i.e., without iteration). The corresponding values of c_p and w are represented to within four significant figures.

Comprehensive tests [11] have shown that the maximum inconsistencies between the backward equations $v(p,T)$ of adjacent subregions are less than 0.001%. Moreover, the inconsistencies in h, s, c_p, and w along subregion boundaries, when these properties are calculated one time with the help of the backward equations $v(p,T)$ and the other time with the basic equation $f_3(\rho,T)$ alone, are also less than the permissible values given in Table 1; this is valid for subregion boundaries, isobars, and lines defined by the subregion-boundary equations according to Eqs. (2)–(4).

5 Auxiliary Equations $v(p,T)$ for the Near-Critical Region

5.1 Range of Validity, Division Into Subregions, and Subregion-Boundary Equations. The auxiliary equations $v(p,T)$ for subregions 3u–3z are valid in the temperature and pressure range given by

$$T_{3qu}(p) < T \leq T_{3rx}(p) \text{ and } p_{sat}(643.15\text{ K}) < p \leq 22.5\text{ MPa}$$

where $p_{sat}(643.15\text{ K})=21.04336732$ MPa is calculated from the saturation-pressure equation of IAPWS-IF97.

The subregion-boundary equation $T_{3uv}(p)$ has the form of Eq. (2), and the equation $T_{3wx}(p)$ has the form of Eq. (3). The coefficients n_i and the exponents I_i of these two boundary equations are listed in Table 8. The numerical information on the subregion-boundary equation $T_{3ef}(p)$ is given in Sec. 4.2. The description of the use of the subregion-boundary equations is summarized in Table 9, where the subregion boundaries are shown in Fig. 5.

The information given above completely documents the equations for these subregion boundaries. Some of the equations are designed to approximate specific physical relationships such as isochores. This is discussed in more detail elsewhere [10,11].

To assist the user in computer-program verification of the equations for the subregion boundaries, Table 10 contains test values for calculated temperatures.

5.2 Auxiliary Equations $v(p,T)$ for Subregions 3u–3z. The auxiliary equations $v(p,T)$ for subregions 3u–3z have the dimensionless form of Eq. (5). The reducing quantities v^*, p^*, and T^*, the number of coefficients N, the nonlinear parameters a and b, and the exponents c, d, and e are listed in Table 11. The coefficients n_i and exponents I_i and J_i are given in the Appendix.

Table 9 Pressure ranges and corresponding-subregion boundary equations for determining the correct subregion, 3u–3z, for the auxiliary equations $v(p,T)$

Temperature range	Subcritical pressure region ($p\leq p_c$) Pressure range	Subregion	Temperature range
$T\leq T_{sat}(p)$ (liquid)	$p_{sat}(0.00264\text{ m}^3\text{ kg}^{-1})$[a] $<p\leq 22.064$ MPa	3u	$T_{3qu}(p)<T\leq T_{3uv}(p)$
		3y	$T_{3uv}(p)<T$
	$p_{sat}(643.15\text{ K})<p\leq p_{sat}(0.00264\text{ m}^3\text{ kg}^{-1})$[a]	3u	$T_{3qu}(p)<T$
$T\geq T_{sat}(p)$ (vapor)	$p_{sat}(0.00385\text{ m}^3\text{ kg}^{-1})$[b] $<p\leq 22.064$ MPa	3z	$T\leq T_{3wx}(p)$
		3x	$T_{3wx}(p)<T\leq T_{3rx}(p)$
	$p_{sat}(643.15\text{ K})<p\leq p_{sat}(0.00385\text{ m}^3\text{ kg}^{-1})$[b]	3x	$T\leq T_{3rx}(p)$

Pressure range	Supercritical pressure region ($p>p_c$) Subregion	Temperature range
$22.064\text{ MPa}<p\leq 22.11$ MPa	3u	$T_{3qu}(p)<T\leq T_{3uv}(p)$
	3z	$T_{3ef}(p)<T\leq T_{3wx}(p)$
	3y	$T_{3uv}(p)<T\leq T_{3ef}(p)$
	3x	$T_{3wx}(p)<T\leq T_{3rx}(p)$
$22.11\text{ MPa}<p\leq 22.5$ MPa	3u	$T_{3qu}(p)<T\leq T_{3uv}(p)$
	3w	$T_{3ef}(p)<T\leq T_{3wx}(p)$
	3v	$T_{3uv}(p)<T\leq T_{3ef}(p)$
	3x	$T_{3wx}(p)<T\leq T_{3rx}(p)$

[a] $p_{sat}(0.00264\text{ m}^3\text{ kg}^{-1})=21.93161551$ MPa.
[b] $p_{sat}(0.00385\text{ m}^3\text{ kg}^{-1})=21.90096265$ MPa.

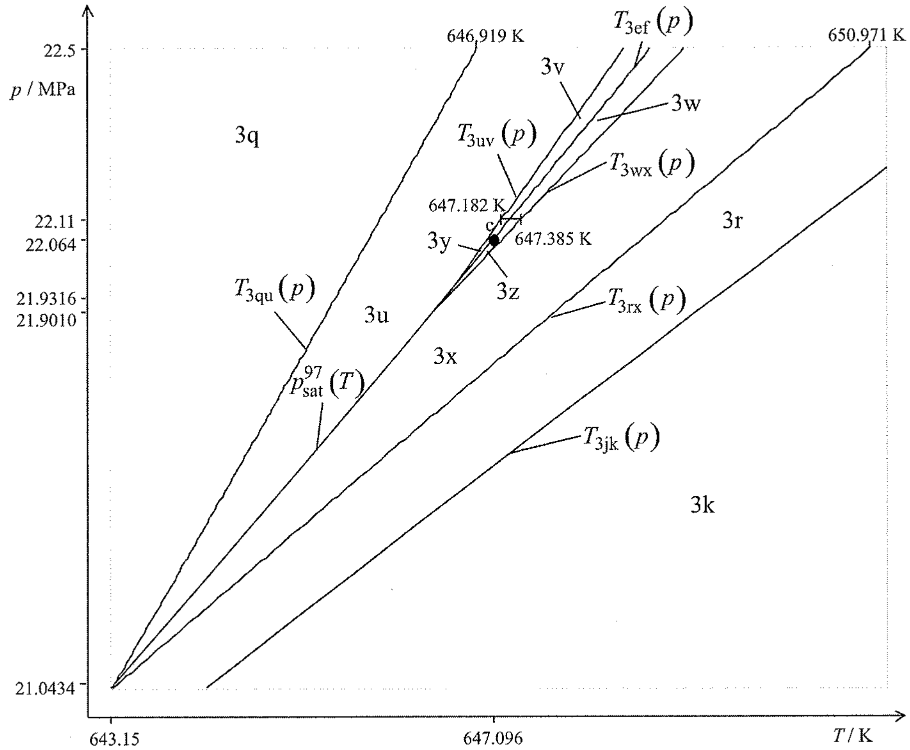

Fig. 5 Division of region 3 into subregions 3u–3z for the auxiliary equations

To assist the user in computer-program verification of these auxiliary equations, Table 12 contains test values for calculated specific volumes.

5.3 Calculation of Properties Utilizing the Auxiliary Equations $v(p,T)$. In order to calculate the thermodynamic properties in the range very close to the critical point with the help of the auxiliary equations $v(p,T)$ for regions 3u–3z, the description given in Sec. 4.4 must be applied analogously to the auxiliary equations $v(p,T)$.

5.4 Numerical Consistency. The numerical inconsistencies between the auxiliary equations $v(p,T)$ for subregions 3u–3z and the basic equation $f_3(\rho,T)$ are listed in Table 13. This table shows that the maximum inconsistencies in specific volume between these equations are less than 0.1%. Only in a small region at pressures less than 22.11 MPa, see Fig. 5, the maximum inconsistencies with the basic equation $f_3(\rho,T)$ approach 2%.

The maximum inconsistencies in specific volume between the auxiliary equations $v(p,T)$ of adjacent subregions along subregion

Table 10 Temperature values calculated from the subregion-boundary equations $T_{3uv}(p)$ and $T_{3wx}(p)$ for selected pressures

Equation	p (MPa)	T (K)
$T_{3uv}(p)$	22.3	6.477996121×10^{2}
$T_{3wx}(p)$	22.3	6.482049480×10^{2}

Table 11 Reducing quantities v^*, p^*, and T^*, number of coefficients N, nonlinear parameters a and b, and exponents c, d, and e of the auxiliary equations $v(p,T)$, Eq. (5), of subregions 3u–3z

Subregion	v^* (m^3 kg^{-1})	p^* (MPa)	T^* (K)	N	a	b	c	d	e
3u	0.0026	23	650	38	0.902	0.988	1	1	1
3v	0.0031	23	650	39	0.960	0.995	1	1	1
3w	0.0039	23	650	35	0.959	0.995	1	1	4
3x	0.0049	23	650	36	0.910	0.988	1	1	1
3y	0.0031	22	650	20	0.996	0.994	1	1	4
3z	0.0038	22	650	23	0.993	0.994	1	1	4

Table 12 Values of the specific volume calculated from the auxiliary equations $v(p,T)$ for subregions 3u–3z

Equation	p (MPa)	T (K)	v (m^3 kg^{-1})
$v_{3u}(p,T)$	21.5	644.6	2.268366647×10^{-3}
	22.0	646.1	2.296350553×10^{-3}
$v_{3v}(p,T)$	22.5	648.6	2.832373260×10^{-3}
	22.3	647.9	2.811424405×10^{-3}
$v_{3w}(p,T)$	22.15	647.5	3.694032281×10^{-3}
	22.3	648.1	3.622226305×10^{-3}
$v_{3x}(p,T)$	22.11	648.0	4.528072649×10^{-3}
	22.3	649.0	4.556905799×10^{-3}
$v_{3y}(p,T)$	22.0	646.84	2.698354719×10^{-3}
	22.064	647.05	2.717655648×10^{-3}
$v_{3z}(p,T)$	22.0	646.89	3.798732962×10^{-3}
	22.064	647.15	3.701940010×10^{-3}

Table 13 Maximum and root-mean-square inconsistencies in specific volume between the auxiliary equations $v(p,T)$ for subregions 3u–3z and the basic equation $f_3(\rho,T)$

Subregion	$\lvert\Delta v/v\rvert$ (%) max	rms
3u	0.097	0.058
3v	0.082	0.040
3w	0.065	0.023
3x	0.090	0.050
3y	1.77	1.04
3z	1.80	0.921

boundaries are as follows: Along subregion boundaries that are isobars, the inconsistencies are less than 0.1% for all subregions except for the boundaries between subregions 3v/3y and 3w/3z, where the inconsistencies amount to 1.7%. Along boundaries defined by the subregion-boundary equations given in Sec. 5.1, the inconsistencies are also less than 0.1%, except for the boundaries between subregions 3u/3v and 3u/3y ($T_{3uv}(p)$), 3y/3z ($T_{3ef}(p)$), and 3z/3x ($T_{3wx}(p)$), where the maximum inconsistencies are 0.14%, 1.8%, 3.5%, and 1.8%, respectively.

6 Computing Time Relative to IAPWS-IF97 Iteration

A very important motivation for the backward equations $v(p,T)$ was reducing the computing time to obtain thermodynamic properties for the given variables (p,T) in region 3. With the $v(p,T)$ equations, time-consuming iteration of IAPWS-IF97 can be avoided. The calculation speed is then about 17 times faster than that using only the basic equation. In this comparison, the basic equation is applied in combination with a one-dimensional Newton iteration with convergence tolerances corresponding to the consistency requirements for the backward equations given in Sec. 2.

7 Application of the Equations

The numerical consistency of the backward equations $v(p,T)$ presented in Sec. 4 with the IAPWS-IF97 basic equation is sufficient for most applications in heat-cycle and boiler calculations. For applications where the demands on numerical consistency are extremely high, iteration using the IAPWS-IF97 basic equation $f_3(\rho,T)$ may be necessary. In these cases, the backward and auxiliary equations $v(p,T)$ can be used to calculate very accurate starting values to reduce the time required for convergence of an iterative process.

In comparison with the backward equations $v(p,T)$, the corresponding numerical consistency of the auxiliary equations $v(p,T)$ for the range very close to the critical point is clearly worse. Nevertheless, for many applications, this consistency is satisfactory.

The backward and auxiliary equations $v(p,T)$ should only be used in their ranges of validity described in Sec. 4. They should not be used for determining any thermodynamic derivatives. They should also not be used together with the fundamental equation in iterative calculations of other backward functions such as $T(p,h)$ or $T(p,s)$. Iteration of backward functions can only be performed by using the fundamental equations.

8 Summary

Backward and auxiliary equations $v(p,T)$ for water in IAPWS-IF97 region 3 have been developed. The numerical consistency of specific volume calculated from the backward equations $v(p,T)$ with the IAPWS-IF97 basic equation are sufficient for most applications in heat-cycle and boiler calculations. The new backward equations are 17 times faster than iterative calculation from IAPWS-IF97. For applications where the demands on numerical consistency are extremely high, iteration using the IAPWS-IF97 equations may still be necessary. In these cases, the equations presented here can be used for calculating very accurate starting values.

Further details of the numerical consistency of all backward and region-boundary equations are in the dissertation of Knobloch [11] and the book of Wagner and Kretzschmar [12]. Computer code for the equations presented in this paper may be obtained from the corresponding author (H.-J.K.).

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Nomenclature

Thermodynamic Quantities

- a, b = shifting parameters
- c_p = specific isobaric heat capacity (kJ kg^{-1} K^{-1})
- f = specific Helmholtz free energy (kJ kg^{-1})
- h = specific enthalpy (kJ kg^{-1})
- I, J = exponent
- n = coefficient
- p = pressure (MPa)
- s = specific entropy (kJ kg^{-1} K^{-1})
- T = absolute temperature (K) on the International Temperature Scale of 1990 (ITS-90)
- v = specific volume (m^3 kg^{-1})
- w = speed of sound (m s^{-1})
- θ = reduced temperature $\theta=T/T^*$
- π = reduced pressure, $\pi=p/p^*$
- ω = reduced volume, $\omega=v/v^*$
- Δ = difference in any quantity

Subscripts

- 1–5 = regions 1–5
- 3a–3z = subregion 3a–3z
- 3ab = boundary between subregions 3a, 3d and 3b, 3e
- 3cd = boundary between subregions 3c and 3d, 3g, 3l, 3q, 3s
- 3ef = boundary between subregions 3e, 3h, 3n and 3f, 3i, 3o
- 3gh = boundary between subregions 3g, 3l and 3h, 3m
- 3ij = boundary between subregions 3i, 3p and 3j
- 3jk = boundary between subregions 3j, 3r and 3k
- 3mn = boundary between subregions 3m and 3n
- 3op = boundary between subregions 3o and 3p
- 3qu = boundary between subregions 3q and 3u
- 3rx = boundary between subregions 3r and 3x
- 3uv = boundary between subregions 3u and 3v
- 3wx = boundary between subregions 3w and 3x
- B23 = boundary between regions 2 and 3
- c = critical point
- max = maximum value of a quantity
- perm = permissible inconsistency
- rms = root-mean-square value of a quantity (see below)
- sat = saturation state
- root-mean-square value = $\Delta z_{\mathrm{rms}}=\sqrt{1/m\Sigma_{i=1}^{m}(\Delta z_i)^2}$, where Δz_i can be either absolute or percentage differences of the corresponding property z and m is the number of Δz_i values (depending on the property, between 10^7 and 10^8 points uniformly distributed over the range of validity)

Superscripts

- 97 = quantity or equation of IAPWS-IF97
- 01 = equation of IAPWS-IF97-S01
- 03 = equation of IAPWS-IF97-S03rev
- 04 = equation of IAPWS-IF97-S04
- 05 = equation of IAPWS-IF97-S05
- * = reducing quantity
- ′ = saturated liquid state
- ″ = saturated vapor state

Appendix

Coefficients for backward equations are shown in Tables 14–33. Coefficients for auxiliary equations are shown in Tables 34–39.

Table 14 Coefficients and exponents of the backward equation $v_{3a}(p,T)$ for subregion 3a

i	I_i	J_i	n_i
1	−12	5	$0.110879558823853 \times 10^{-2}$
2	−12	10	$0.572616740810616 \times 10^{3}$
3	−12	12	$-0.767051948380852 \times 10^{5}$
4	−10	5	$-0.253321069529674 \times 10^{-1}$
5	−10	10	$0.628008049345689 \times 10^{4}$
6	−10	12	$0.234105654131876 \times 10^{6}$
7	−8	5	0.216867826045856
8	−8	8	$-0.156237904341963 \times 10^{3}$
9	−8	10	$-0.269893956176613 \times 10^{5}$
10	−6	1	$-0.180407100085505 \times 10^{-3}$
11	−5	1	$0.116732227668261 \times 10^{-2}$
12	−5	5	$0.266987040856040 \times 10^{2}$
13	−5	10	$0.282776617243286 \times 10^{5}$
14	−4	8	$-0.242431520029523 \times 10^{4}$
15	−3	0	$0.435217323022733 \times 10^{-3}$
16	−3	1	$-0.122494831387441 \times 10^{-1}$
17	−3	3	$0.179357604019989 \times 10^{1}$
18	−3	6	$0.442729521058314 \times 10^{2}$
19	−2	0	$-0.593223489018342 \times 10^{-2}$
20	−2	2	0.453186261685774
21	−2	3	$0.135825703129140 \times 10^{1}$
22	−1	0	$0.408748415856745 \times 10^{-1}$
23	−1	1	0.474686397863312
24	−1	2	$0.118646814997915 \times 10^{1}$
25	0	0	0.546987265727549
26	0	1	0.195266770452643
27	1	0	$-0.502268790869663 \times 10^{-1}$
28	1	2	-0.369645308193377
29	2	0	$0.633828037528420 \times 10^{-2}$
30	2	2	$0.797441793901017 \times 10^{-1}$

Table 15 Coefficients and exponents of the backward equation $v_{3b}(p,T)$ for subregion 3b

i	I_i	J_i	n_i
1	−12	10	$-0.827670470003621 \times 10^{-1}$
2	−12	12	$0.416887126010565 \times 10^{2}$
3	−10	8	$0.483651982197059 \times 10^{-1}$
4	−10	14	$-0.291032084950276 \times 10^{5}$
5	−8	8	$-0.111422582236948 \times 10^{3}$
6	−6	5	$-0.202300083904014 \times 10^{-1}$
7	−6	6	$0.294002509338515 \times 10^{3}$
8	−6	8	$0.140244997609658 \times 10^{3}$
9	−5	5	$-0.344384158811459 \times 10^{3}$
10	−5	8	$0.361182452612149 \times 10^{3}$
11	−5	10	$-0.140699677420738 \times 10^{4}$
12	−4	2	$-0.202023902676481 \times 10^{-2}$
13	−4	4	$0.171346792457471 \times 10^{3}$
14	−4	5	$-0.425597804058632 \times 10^{1}$
15	−3	0	$0.691346085000334 \times 10^{-5}$
16	−3	1	$0.151140509678925 \times 10^{-2}$
17	−3	2	$-0.416375290166236 \times 10^{-1}$
18	−3	3	$-0.413754957011042 \times 10^{2}$
19	−3	5	$-0.506673295721637 \times 10^{2}$
20	−2	0	$-0.572212965569023 \times 10^{-3}$
21	−2	2	$0.608817368401785 \times 10^{1}$
22	−2	5	$0.239600660256161 \times 10^{2}$
23	−1	0	$0.122261479925384 \times 10^{-1}$
24	−1	2	$0.216356057692938 \times 10^{1}$
25	0	0	0.398198903368642
26	0	1	-0.116892827834085
27	1	0	-0.102845919373532
28	1	2	-0.492676637589284
29	2	0	$0.655540456406790 \times 10^{-1}$
30	3	2	-0.240462535078530
31	4	0	$-0.269798180310075 \times 10^{-1}$
32	4	1	0.128369435967012

Table 16 Coefficients and exponents of the backward equation $v_{3c}(p,T)$ for subregion 3c

i	I_i	J_i	n_i
1	−12	6	$0.311967788763030 \times 10^{1}$
2	−12	8	$0.276713458847564 \times 10^{5}$
3	−12	10	$0.322583103403269 \times 10^{8}$
4	−10	6	$-0.342416065095363 \times 10^{3}$
5	−10	8	$-0.899732529907377 \times 10^{6}$
6	−10	10	$-0.793892049821251 \times 10^{8}$
7	−8	5	$0.953193003217388 \times 10^{2}$
8	−8	6	$0.229784742345072 \times 10^{4}$
9	−8	7	$0.175336675322499 \times 10^{6}$
10	−6	8	$0.791214365222792 \times 10^{7}$
11	−5	1	$0.319933345844209 \times 10^{-4}$
12	−5	4	$-0.659508863555767 \times 10^{2}$
13	−5	7	$-0.833426563212851 \times 10^{6}$
14	−4	2	$0.645734680583292 \times 10^{-1}$
15	−4	8	$-0.382031020570813 \times 10^{7}$
16	−3	0	$0.406398848470079 \times 10^{-4}$
17	−3	3	$0.310327498492008 \times 10^{2}$
18	−2	0	$-0.892996718483724 \times 10^{-3}$
19	−2	4	$0.234604891591616 \times 10^{3}$
20	−2	5	$0.377515668966951 \times 10^{4}$
21	−1	0	$0.158646812591361 \times 10^{-1}$
22	−1	1	0.707906336241843
23	−1	2	$0.126016225146570 \times 10^{2}$
24	0	0	0.736143655772152
25	0	1	0.676544268999101
26	0	2	$-0.178100588189137 \times 10^{2}$
27	1	0	-0.156531975531713
28	1	2	$0.117707430048158 \times 10^{2}$
29	2	0	$0.840143653860447 \times 10^{-1}$
30	2	1	-0.186442467471949
31	2	3	$-0.440170203949645 \times 10^{2}$
32	2	7	$0.123290423502494 \times 10^{7}$
33	3	0	$-0.240650039730845 \times 10^{-1}$
34	3	7	$-0.107077716660869 \times 10^{7}$
35	8	1	$0.438319858566475 \times 10^{-1}$

Table 17 Coefficients and exponents of the backward equation $v_{3d}(p,T)$ for subregion 3d

i	I_i	J_i	n_i
1	−12	4	$-0.452484847171645 \times 10^{-9}$
2	−12	6	$0.315210389538801 \times 10^{-4}$
3	−12	7	$-0.214991352047545 \times 10^{-2}$
4	−12	10	$0.508058874808345 \times 10^{3}$
5	−12	12	$-0.127123036845932 \times 10^{8}$
6	−12	16	$0.115371133120497 \times 10^{13}$
7	−10	0	$-0.197805728776273 \times 10^{-15}$
8	−10	2	$0.241554806033972 \times 10^{-10}$
9	−10	4	$-0.156481703640525 \times 10^{-5}$
10	−10	6	$0.277211346836625 \times 10^{-2}$
11	−10	8	$-0.203578994462286 \times 10^{2}$
12	−10	10	$0.144369489909053 \times 10^{7}$
13	−10	14	$-0.411254217946539 \times 10^{11}$
14	−8	3	$0.623449786243773 \times 10^{-5}$
15	−8	7	$-0.221774281146038 \times 10^{2}$
16	−8	8	$-0.689315087933158 \times 10^{5}$
17	−8	10	$-0.195419525060713 \times 10^{8}$
18	−6	6	$0.316373510564015 \times 10^{4}$
19	−6	8	$0.224040754426988 \times 10^{7}$
20	−5	1	$-0.436701347922356 \times 10^{-5}$
21	−5	2	$-0.404213852833996 \times 10^{-3}$
22	−5	5	$-0.348153203414663 \times 10^{3}$
23	−5	7	$-0.385294213555289 \times 10^{6}$
24	−4	0	$0.135203700099403 \times 10^{-6}$
25	−4	1	$0.134648383271089 \times 10^{-3}$
26	−4	7	$0.125031835351736 \times 10^{6}$
27	−3	2	$0.968123678455841 \times 10^{-1}$
28	−3	4	$0.225660517512438 \times 10^{3}$
29	−2	0	$-0.190102435341872 \times 10^{-3}$
30	−2	1	$-0.299628410819229 \times 10^{-1}$
31	−1	0	$0.500833915372121 \times 10^{-2}$
32	−1	1	0.387842482998411
33	−1	5	$-0.138535367777182 \times 10^{4}$
34	0	0	0.870745245971773
35	0	2	$0.171946252068742 \times 10^{1}$
36	1	0	$-0.326650121426383 \times 10^{-1}$
37	1	6	$0.498044171727877 \times 10^{4}$
38	3	0	$0.551478022765087 \times 10^{-2}$

Table 18 Coefficients and exponents of the backward equation $v_{3e}(p,T)$ for subregion 3e

i	I_i	J_i	n_i
1	−12	14	$0.715815808404721\times10^{9}$
2	−12	16	$-0.114328360753449\times10^{12}$
3	−10	3	$0.376531002015720\times10^{-11}$
4	−10	6	$-0.903983668691157\times10^{-4}$
5	−10	10	$0.665695908836252\times10^{6}$
6	−10	14	$0.535364174960127\times10^{10}$
7	−10	16	$0.794977402335603\times10^{11}$
8	−8	7	$0.922230563421437\times10^{2}$
9	−8	8	$-0.142586073991215\times10^{6}$
10	−8	10	$-0.111796381424162\times10^{7}$
11	−6	6	$0.896121629640760\times10^{4}$
12	−5	6	$-0.669989239070491\times10^{4}$
13	−4	2	$0.451242538486834\times10^{-2}$
14	−4	4	$-0.339731325977713\times10^{2}$
15	−3	2	$-0.120523111552278\times10^{1}$
16	−3	6	$0.475992667717124\times10^{5}$
17	−3	7	$-0.266627750390341\times10^{6}$
18	−2	0	$-0.153314954386524\times10^{-3}$
19	−2	1	0.305638404828265
20	−2	3	$0.123654999499486\times10^{3}$
21	−2	4	$-0.104390794213011\times10^{4}$
22	−1	0	$-0.157496516174308\times10^{-1}$
23	0	0	0.685331118940253
24	0	1	$0.178373462873903\times10^{1}$
25	1	0	-0.544674124878910
26	1	4	$0.204529931318843\times10^{4}$
27	1	6	$-0.228342359328752\times10^{5}$
28	2	0	0.413197481515899
29	2	2	$-0.341931835910405\times10^{2}$

Table 19 Coefficients and exponents of the backward equation $v_{3f}(p,T)$ for subregion 3f

i	I_i	J_i	n_i
1	0	−3	$-0.251756547792325\times10^{-7}$
2	0	−2	$0.601307193668763\times10^{-5}$
3	0	−1	$-0.100615977450049\times10^{-2}$
4	0	0	0.999969140252192
5	0	1	$0.214107759236486\times10^{1}$
6	0	2	$-0.165175571959086\times10^{2}$
7	1	−1	$-0.141987303638727\times10^{-2}$
8	1	1	$0.269251915156554\times10^{1}$
9	1	2	$0.349741815858722\times10^{2}$
10	1	3	$-0.300208695771783\times10^{2}$
11	2	0	$-0.131546288252539\times10^{1}$
12	2	1	$-0.839091277286169\times10^{1}$
13	3	−5	$0.181545608337015\times10^{-9}$
14	3	−2	$-0.591099206478909\times10^{-3}$
15	3	0	$0.152115067087106\times10^{1}$
16	4	−3	$0.252956470663225\times10^{-4}$
17	5	−8	$0.100726265203786\times10^{-14}$
18	5	1	$-0.149774533860650\times10^{1}$
19	6	−6	$-0.793940970562969\times10^{-9}$
20	7	−4	$-0.150290891264717\times10^{-3}$
21	7	1	$0.151205531275133\times10^{1}$
22	10	−6	$0.470942606221652\times10^{-5}$
23	12	−10	$0.195049710391712\times10^{-12}$
24	12	−8	$-0.911627886266077\times10^{-8}$
25	12	−4	$0.604374640201265\times10^{-3}$
26	14	−12	$-0.225132933900136\times10^{-15}$
27	14	−10	$0.610916973582981\times10^{-11}$
28	14	−8	$-0.303063908043404\times10^{-6}$
29	14	−6	$-0.137796070798409\times10^{-4}$
30	14	−4	$-0.919296736666106\times10^{-3}$
31	16	−10	$0.639288223132545\times10^{-9}$
32	16	−8	$0.753259479898699\times10^{-6}$
33	18	−12	$-0.400321478682929\times10^{-12}$
34	18	−10	$0.756140294351614\times10^{-8}$
35	20	−12	$-0.912082054034891\times10^{-11}$
36	20	−10	$-0.237612381140539\times10^{-7}$
37	20	−6	$0.269586010591874\times10^{-4}$
38	22	−12	$-0.732828135157839\times10^{-10}$
39	24	−12	$0.241995578306660\times10^{-9}$
40	24	−4	$-0.405735532730322\times10^{-3}$
41	28	−12	$0.189424143498011\times10^{-9}$
42	32	−12	$-0.486632965074563\times10^{-9}$

Table 20 Coefficients and exponents of the backward equation $v_{3g}(p,T)$ for subregion 3g

i	I_i	J_i	n_i
1	−12	7	$0.412209020652996\times10^{-4}$
2	−12	12	$-0.114987238280587\times10^{7}$
3	−12	14	$0.948180885032080\times10^{10}$
4	−12	18	$-0.195788865718971\times10^{18}$
5	−12	22	$0.496250704871300\times10^{25}$
6	−12	24	$-0.105549884548496\times10^{29}$
7	−10	14	$-0.758642165988278\times10^{12}$
8	−10	20	$-0.922172769596101\times10^{23}$
9	−10	24	$0.725379072059348\times10^{30}$
10	−8	7	$-0.617718249205859\times10^{2}$
11	−8	8	$0.107555033344858\times10^{5}$
12	−8	10	$-0.379545802336487\times10^{8}$
13	−8	12	$0.228646846221831\times10^{12}$
14	−6	8	$-0.499741093010619\times10^{7}$
15	−6	22	$-0.280214310054101\times10^{31}$
16	−5	7	$0.104915406769586\times10^{7}$
17	−5	20	$0.613754229168619\times10^{28}$
18	−4	22	$0.802056715528378\times10^{32}$
19	−3	7	$-0.298617819828065\times10^{8}$
20	−2	3	$-0.910782540134681\times10^{2}$
21	−2	5	$0.135033227281565\times10^{6}$
22	−2	14	$-0.712949383408211\times10^{19}$
23	−2	24	$-0.104578785289542\times10^{37}$
24	−1	2	$0.304331584444093\times10^{2}$
25	−1	8	$0.593250797959445\times10^{10}$
26	−1	18	$-0.364174062110798\times10^{28}$
27	0	0	0.921791403532461
28	0	1	-0.337693609657471
29	0	2	$-0.724644143758508\times10^{2}$
30	1	0	-0.110480239272601
31	1	1	$0.536516031875059\times10^{1}$
32	1	3	$-0.291441872156205\times10^{4}$
33	3	24	$0.616338176535305\times10^{40}$
34	5	22	$-0.120889175861180\times10^{39}$
35	6	12	$0.818396024524612\times10^{23}$
36	8	3	$0.940781944835829\times10^{9}$
37	10	0	$-0.367279669545448\times10^{5}$
38	10	6	$-0.837513931798655\times10^{16}$

Table 21 Coefficients and exponents of the backward equation $v_{3h}(p,T)$ for subregion 3h

i	I_i	J_i	n_i
1	−12	8	$0.561379678887577\times10^{-1}$
2	−12	12	$0.774135421587083\times10^{10}$
3	−10	4	$0.111482975877938\times10^{-8}$
4	−10	6	$-0.143987128208183\times10^{-2}$
5	−10	8	$0.193696558764920\times10^{4}$
6	−10	10	$-0.605971823585005\times10^{9}$
7	−10	14	$0.171951568124337\times10^{14}$
8	−10	16	$-0.185461154985145\times10^{17}$
9	−8	0	$0.387851168078010\times10^{-16}$
10	−8	1	$-0.395464327846105\times10^{-13}$
11	−8	6	$-0.170875935679023\times10^{3}$
12	−8	7	$-0.212010620701220\times10^{4}$
13	−8	8	$0.177683337348191\times10^{8}$
14	−6	4	$0.110177443629575\times10^{2}$
15	−6	6	$-0.234396091693313\times10^{6}$
16	−6	8	$-0.656174421999594\times10^{7}$
17	−5	2	$0.156362212977396\times10^{-4}$
18	−5	3	$-0.212946257021400\times10^{1}$
19	−5	4	$0.135249306374858\times10^{2}$
20	−4	2	0.177189164145813
21	−4	4	$0.139499167345464\times10^{4}$
22	−3	1	$-0.703670932036388\times10^{-2}$
23	−3	2	-0.152011044389648
24	−2	0	$0.981916922991113\times10^{-4}$
25	−1	0	$0.147199658618076\times10^{-2}$
26	−1	2	$0.202618487025578\times10^{2}$
27	0	0	0.899345518944240
28	1	0	-0.211346402240858
29	1	2	$0.249971752957491\times10^{2}$

Table 22 Coefficients and exponents of the backward equation $v_{3i}(p,T)$ for subregion 3i

i	I_i	J_i	n_i
1	0	0	$0.106905684359136 \times 10^{1}$
2	0	1	$-0.148620857922333 \times 10^{1}$
3	0	10	$0.259862256980408 \times 10^{15}$
4	1	−4	$-0.446352055678749 \times 10^{-11}$
5	1	−2	$-0.566620757170032 \times 10^{-6}$
6	1	−1	$-0.235302885736849 \times 10^{-2}$
7	1	0	-0.269226321968839
8	2	0	$0.922024992944392 \times 10^{1}$
9	3	−5	$0.357633505503772 \times 10^{-11}$
10	3	0	$-0.173942565562222 \times 10^{2}$
11	4	−3	$0.700681785556229 \times 10^{-5}$
12	4	−2	$-0.267050351075768 \times 10^{-3}$
13	4	−1	$-0.231779669675624 \times 10^{1}$
14	5	−6	$-0.753533046979752 \times 10^{-12}$
15	5	−1	$0.481337131452891 \times 10^{1}$
16	5	12	$-0.223286270422356 \times 10^{22}$
17	7	−4	$-0.118746004987383 \times 10^{-4}$
18	7	−3	$0.646412934136496 \times 10^{-2}$
19	8	−6	$-0.410588536330937 \times 10^{-9}$
20	8	10	$0.422739537057241 \times 10^{20}$
21	10	−8	$0.313698180473812 \times 10^{-12}$
22	12	−12	$0.164395334345040 \times 10^{-23}$
23	12	−6	$-0.339823323754373 \times 10^{-5}$
24	12	−4	$-0.135268639905021 \times 10^{-1}$
25	14	−10	$-0.723252514211625 \times 10^{-14}$
26	14	−8	$0.184386437538366 \times 10^{-8}$
27	14	−4	$-0.463959533752385 \times 10^{-1}$
28	14	5	$-0.992263100376750 \times 10^{14}$
29	18	−12	$0.688169154439335 \times 10^{-16}$
30	18	−10	$-0.222620998452197 \times 10^{-10}$
31	18	−8	$-0.540843018624083 \times 10^{-7}$
32	18	−6	$0.345570606200257 \times 10^{-2}$
33	18	2	$0.422275800304086 \times 10^{11}$
34	20	−12	$-0.126974478770487 \times 10^{-14}$
35	20	−10	$0.927237985153679 \times 10^{-9}$
36	22	−12	$0.612670812016489 \times 10^{-13}$
37	24	−12	$-0.722693924063497 \times 10^{-11}$
38	24	−8	$-0.383669502636822 \times 10^{-3}$
39	32	−10	$0.374684572410204 \times 10^{-3}$
40	32	−5	$-0.931976897511086 \times 10^{5}$
41	36	−10	$-0.247690616026922 \times 10^{-1}$
42	36	−8	$0.658110546759474 \times 10^{2}$

Table 23 Coefficients and exponents of the backward equation $v_{3j}(p,T)$ for subregion 3j

i	I_i	J_i	n_i
1	0	−1	$-0.111371317395540 \times 10^{-3}$
2	0	0	$0.100342892423685 \times 10^{1}$
3	0	1	$0.530615581928979 \times 10^{1}$
4	1	−2	$0.179058760078792 \times 10^{-5}$
5	1	−1	$-0.728541958464774 \times 10^{-3}$
6	1	1	$-0.187576133371704 \times 10^{2}$
7	2	−1	$0.199060874071849 \times 10^{-2}$
8	2	1	$0.243574755377290 \times 10^{2}$
9	3	−2	$-0.177040785499444 \times 10^{-3}$
10	4	−2	$-0.259680385227130 \times 10^{-2}$
11	4	2	$-0.198704578406823 \times 10^{3}$
12	5	−3	$0.738627790224287 \times 10^{-4}$
13	5	−2	$-0.236264692844138 \times 10^{-2}$
14	5	0	$-0.161023121314333 \times 10^{1}$
15	6	3	$0.622322971786473 \times 10^{4}$
16	10	−6	$-0.960754116701669 \times 10^{-8}$
17	12	−8	$-0.510572269720488 \times 10^{-10}$
18	12	−3	$0.767373781404211 \times 10^{-2}$
19	14	−10	$0.663855469485254 \times 10^{-14}$
20	14	−8	$-0.717590735526745 \times 10^{-9}$
21	14	−5	$0.146564542926508 \times 10^{-4}$
22	16	−10	$0.309029474277013 \times 10^{-11}$
23	18	−12	$-0.464216300971708 \times 10^{-15}$
24	20	−12	$-0.390499637961161 \times 10^{-13}$
25	20	−10	$-0.236716126781431 \times 10^{-9}$
26	24	−12	$0.454652854268717 \times 10^{-11}$
27	24	−6	$-0.422271787482497 \times 10^{-2}$
28	28	−12	$0.283911742354706 \times 10^{-10}$
29	28	−5	$0.270929002720228 \times 10^{1}$

Table 24 Coefficients and exponents of the backward equation $v_{3k}(p,T)$ for subregion 3k

i	I_i	J_i	n_i
1	−2	10	$-0.401215699576099 \times 10^{9}$
2	−2	12	$0.484501478318406 \times 10^{11}$
3	−1	−5	$0.394721471363678 \times 10^{-14}$
4	−1	6	$0.372629967374147 \times 10^{5}$
5	0	−12	$-0.369794374168666 \times 10^{-29}$
6	0	−6	$-0.380436407012452 \times 10^{-14}$
7	0	−2	$0.475361629970233 \times 10^{-6}$
8	0	−1	$-0.879148916140706 \times 10^{-3}$
9	0	0	0.844317863844331
10	0	1	$0.122433162656600 \times 10^{2}$
11	0	2	$-0.104529634830279 \times 10^{3}$
12	0	3	$0.589702771277429 \times 10^{3}$
13	0	14	$-0.291026851164444 \times 10^{14}$
14	1	−3	$0.170343072841850 \times 10^{-5}$
15	1	−2	$-0.277617606975748 \times 10^{-3}$
16	1	0	$-0.344709605486686 \times 10^{1}$
17	1	1	$0.221333862447095 \times 10^{2}$
18	1	2	$-0.194646110037079 \times 10^{3}$
19	2	−8	$0.808354639772825 \times 10^{-15}$
20	2	−6	$-0.180845209145470 \times 10^{-10}$
21	2	−3	$-0.696664158132412 \times 10^{-5}$
22	2	−2	$-0.181057560300994 \times 10^{-2}$
23	2	0	$0.255830298579027 \times 10^{1}$
24	2	4	$0.328913873658481 \times 10^{4}$
25	5	−12	$-0.173270241249904 \times 10^{-18}$
26	5	−6	$-0.661876792558034 \times 10^{-6}$
27	5	−3	$-0.395688923421250 \times 10^{-2}$
28	6	−12	$0.604203299819132 \times 10^{-17}$
29	6	−10	$-0.400879935920517 \times 10^{-13}$
30	6	−8	$0.160751107464958 \times 10^{-8}$
31	6	−5	$0.383719409025556 \times 10^{-4}$
32	8	−12	$-0.649565446702457 \times 10^{-14}$
33	10	−12	$-0.149095328506000 \times 10^{-11}$
34	12	−10	$0.541449377329581 \times 10^{-8}$

Table 25 Coefficients and exponents of the backward equation $v_{3l}(p,T)$ for subregion 3l

i	I_i	J_i	n_i
1	−12	14	$0.260702058647537 \times 10^{10}$
2	−12	16	$-0.188277213604704 \times 10^{15}$
3	−12	18	$0.554923870289667 \times 10^{19}$
4	−12	20	$-0.758966946387758 \times 10^{23}$
5	−12	22	$0.413865186848908 \times 10^{27}$
6	−10	14	$-0.815038000738060 \times 10^{12}$
7	−10	24	$-0.381458260489955 \times 10^{33}$
8	−8	6	$-0.123239564600519 \times 10^{-1}$
9	−8	10	$0.226095631437174 \times 10^{8}$
10	−8	12	$-0.495017809506720 \times 10^{12}$
11	−8	14	$0.529482996422863 \times 10^{16}$
12	−8	18	$-0.444359478746295 \times 10^{23}$
13	−8	24	$0.521635864527315 \times 10^{35}$
14	−8	36	$-0.487095672740742 \times 10^{55}$
15	−6	8	$-0.714430209937547 \times 10^{6}$
16	−5	4	0.127868634615495
17	−5	5	$-0.100752127917598 \times 10^{2}$
18	−4	7	$0.777451437960990 \times 10^{7}$
19	−4	16	$-0.108105480796471 \times 10^{25}$
20	−3	1	$-0.357578581169659 \times 10^{-5}$
21	−3	3	$-0.212857169423484 \times 10^{1}$
22	−3	18	$0.270706111085238 \times 10^{30}$
23	−3	20	$-0.695953622348829 \times 10^{33}$
24	−2	2	0.110609027472280
25	−2	3	$0.721559163361354 \times 10^{2}$
26	−2	10	$-0.306367307532219 \times 10^{15}$
27	−1	0	$0.265839618885530 \times 10^{-4}$
28	−1	1	$0.253392392889754 \times 10^{-1}$
29	−1	3	$-0.214443041836579 \times 10^{3}$
30	0	0	0.937846601489667
31	0	1	$0.223184043101700 \times 10^{1}$
32	0	2	$0.338401222509191 \times 10^{2}$
33	0	12	$0.494237237179718 \times 10^{21}$
34	1	0	-0.198068404154428
35	1	16	$-0.141415349881140 \times 10^{31}$
36	2	1	$-0.993862421613651 \times 10^{2}$
37	4	0	$0.125070534142731 \times 10^{3}$
38	5	0	$-0.996473529004439 \times 10^{3}$
39	5	1	$0.473137909872765 \times 10^{5}$
40	6	14	$0.116662121219322 \times 10^{33}$
41	10	4	$-0.315874976271533 \times 10^{16}$
42	10	12	$-0.445703369196945 \times 10^{33}$
43	14	10	$0.642794932373694 \times 10^{33}$

Table 26 Coefficients and exponents of the backward equation $v_{3m}(p,T)$ for subregion 3m

i	I_i	J_i	n_i
1	0	0	0.811384363481847
2	3	0	$-0.568199310990094 \times 10^{4}$
3	8	0	$-0.178657198172556 \times 10^{11}$
4	20	2	$0.795537657613427 \times 10^{32}$
5	1	5	$-0.814568209346872 \times 10^{5}$
6	3	5	$-0.659774567602874 \times 10^{8}$
7	4	5	$-0.152861148659302 \times 10^{11}$
8	5	5	$-0.560165667510446 \times 10^{12}$
9	1	6	$0.458384828593949 \times 10^{6}$
10	6	6	$-0.385754000383848 \times 10^{14}$
11	2	7	$0.453735800004273 \times 10^{8}$
12	4	8	$0.939454935735563 \times 10^{12}$
13	14	8	$0.266572856432938 \times 10^{28}$
14	2	10	$-0.547578313899097 \times 10^{10}$
15	5	10	$0.200725701112386 \times 10^{15}$
16	3	12	$0.185007245563239 \times 10^{13}$
17	0	14	$0.185135446828337 \times 10^{9}$
18	1	14	$-0.170451090076385 \times 10^{12}$
19	1	18	$0.157890366037614 \times 10^{15}$
20	1	20	$-0.202530509748774 \times 10^{16}$
21	28	20	$0.368193926183570 \times 10^{60}$
22	2	22	$0.170215539458936 \times 10^{18}$
23	16	22	$0.639234909918741 \times 10^{42}$
24	0	24	$-0.821698160721956 \times 10^{15}$
25	5	24	$-0.795260241872306 \times 10^{24}$
26	0	28	$0.233415869478510 \times 10^{18}$
27	3	28	$-0.600079934586803 \times 10^{23}$
28	4	28	$0.594584382273384 \times 10^{25}$
29	12	28	$0.189461279349492 \times 10^{40}$
30	16	28	$-0.810093428842645 \times 10^{46}$
31	1	32	$0.188813911076809 \times 10^{22}$
32	8	32	$0.111052244098768 \times 10^{36}$
33	14	32	$0.291133958602503 \times 10^{46}$
34	0	36	$-0.329421923951460 \times 10^{22}$
35	2	36	$-0.137570282536696 \times 10^{26}$
36	3	36	$0.181508996303902 \times 10^{28}$
37	4	36	$-0.346865122768353 \times 10^{30}$
38	8	36	$-0.211961148774260 \times 10^{38}$
39	14	36	$-0.128617899887675 \times 10^{49}$
40	24	36	$0.479817895699239 \times 10^{65}$

Table 27 Coefficients and exponents of the backward equation $v_{3n}(p,T)$ for subregion 3n

i	I_i	J_i	n_i
1	0	−12	$0.280967799943151 \times 10^{-38}$
2	3	−12	$0.614869006573609 \times 10^{-30}$
3	4	−12	$0.582238667048942 \times 10^{-27}$
4	6	−12	$0.390628369238462 \times 10^{-22}$
5	7	−12	$0.821445758255119 \times 10^{-20}$
6	10	−12	$0.402137961842776 \times 10^{-14}$
7	12	−12	$0.651718171878301 \times 10^{-12}$
8	14	−12	$-0.211773355803058 \times 10^{-7}$
9	18	−12	$0.264953354380072 \times 10^{-2}$
10	0	−10	$-0.135031446451331 \times 10^{-31}$
11	3	−10	$-0.607246643970893 \times 10^{-23}$
12	5	−10	$-0.402352115234494 \times 10^{-18}$
13	6	−10	$-0.744938506925544 \times 10^{-16}$
14	8	−10	$0.189917206526237 \times 10^{-12}$
15	12	−10	$0.364975183508473 \times 10^{-5}$
16	0	−8	$0.177274872361946 \times 10^{-25}$
17	3	−8	$-0.334952758812999 \times 10^{-18}$
18	7	−8	$-0.421537726098389 \times 10^{-8}$
19	12	−8	$-0.391048167929649 \times 10^{-1}$
20	2	−6	$0.541276911564176 \times 10^{-13}$
21	3	−6	$0.705412100773699 \times 10^{-11}$
22	4	−6	$0.258585887897486 \times 10^{-8}$
23	2	−5	$-0.493111362030162 \times 10^{-10}$
24	4	−5	$-0.158649699894543 \times 10^{-5}$
25	7	−5	-0.525037427886100
26	4	−4	$0.220019901729615 \times 10^{-2}$
27	3	−3	$-0.643064132636925 \times 10^{-2}$
28	5	−3	$0.629154149015048 \times 10^{2}$
29	6	−3	$0.135147318617061 \times 10^{3}$
30	0	−2	$0.240560808321713 \times 10^{-6}$
31	0	−1	$-0.890763306701305 \times 10^{-3}$
32	3	−1	$-0.440209599407714 \times 10^{4}$
33	1	0	$-0.302807107747776 \times 10^{3}$
34	0	1	$0.159158748314599 \times 10^{4}$
35	1	1	$0.232534272709876 \times 10^{6}$
36	0	2	$-0.792681207132600 \times 10^{6}$
37	1	4	$-0.869871364662769 \times 10^{11}$
38	0	5	$0.354542769185671 \times 10^{12}$
39	1	6	$0.400849240129329 \times 10^{15}$

Table 28 Coefficients and exponents of the backward equation $v_{3o}(p,T)$ for subregion 3o

i	I_i	J_i	n_i
1	0	−12	$0.128746023979718 \times 10^{-34}$
2	0	−4	$-0.735234770382342 \times 10^{-11}$
3	0	−1	$0.289078692149150 \times 10^{-2}$
4	2	−1	0.244482731907223
5	3	−10	$0.141733492030985 \times 10^{-23}$
6	4	−12	$-0.354533853059476 \times 10^{-28}$
7	4	−8	$-0.594539202901431 \times 10^{-17}$
8	4	−5	$-0.585188401782779 \times 10^{-8}$
9	4	−4	$0.201377325411803 \times 10^{-5}$
10	4	−1	$0.138647388209306 \times 10^{1}$
11	5	−4	$-0.173959365084772 \times 10^{-4}$
12	5	−3	$0.137680878349369 \times 10^{-2}$
13	6	−8	$0.814897605805513 \times 10^{-14}$
14	7	−12	$0.425596631351839 \times 10^{-25}$
15	8	−10	$-0.387449113787755 \times 10^{-17}$
16	8	−8	$0.139814747930240 \times 10^{-12}$
17	8	−4	$-0.171849638951521 \times 10^{-2}$
18	10	−12	$0.641890529513296 \times 10^{-21}$
19	10	−8	$0.118960578072018 \times 10^{-10}$
20	14	−12	$-0.155282762571611 \times 10^{-17}$
21	14	−8	$0.233907907347507 \times 10^{-7}$
22	20	−12	$-0.174093247766213 \times 10^{-12}$
23	20	−10	$0.377682649089149 \times 10^{-8}$
24	24	−12	$-0.516720236575302 \times 10^{-10}$

Table 29 Coefficients and exponents of the backward equation $v_{3p}(p,T)$ for subregion 3p

i	I_i	J_i	n_i
1	0	−1	$-0.982825342010366 \times 10^{-4}$
2	0	0	$0.105145700850612 \times 10^{1}$
3	0	1	$0.116033094095084 \times 10^{3}$
4	0	2	$0.324664750281543 \times 10^{4}$
5	1	1	$-0.123592348610137 \times 10^{4}$
6	2	−1	$-0.561403450013495 \times 10^{-1}$
7	3	−3	$0.856677401640869 \times 10^{-7}$
8	3	0	$0.236313425393924 \times 10^{3}$
9	4	−2	$0.972503292350109 \times 10^{-2}$
10	6	−2	$-0.103001994531927 \times 10^{1}$
11	7	−5	$-0.149653706199162 \times 10^{-8}$
12	7	−4	$-0.215743778861592 \times 10^{-4}$
13	8	−2	$-0.834452198291445 \times 10^{1}$
14	10	−3	0.586602660564988
15	12	−12	$0.343480022104968 \times 10^{-25}$
16	12	−6	$0.816256095947021 \times 10^{-5}$
17	12	−5	$0.294985697916798 \times 10^{-2}$
18	14	−10	$0.711730466276584 \times 10^{-16}$
19	14	−8	$0.400954763806941 \times 10^{-9}$
20	14	−3	$0.107766027032853 \times 10^{2}$
21	16	−8	$-0.409449599138182 \times 10^{-6}$
22	18	−8	$-0.729121307758902 \times 10^{-5}$
23	20	−10	$0.677107970938909 \times 10^{-8}$
24	22	−10	$0.602745973022975 \times 10^{-7}$
25	24	−12	$-0.382323011855257 \times 10^{-10}$
26	24	−8	$0.179946628317437 \times 10^{-2}$
27	36	−12	$-0.345042834640005 \times 10^{-3}$

Table 30 Coefficients and exponents of the backward equation $v_{3q}(p,T)$ for subregion 3q

i	I_i	J_i	n_i
1	−12	10	$-0.820433843259950\times10^{5}$
2	−12	12	$0.473271518461586\times10^{11}$
3	−10	6	$-0.805950021005413\times10^{-1}$
4	−10	7	$0.328600025435980\times10^{2}$
5	−10	8	$-0.356617029982490\times10^{4}$
6	−10	10	$-0.172985781433335\times10^{10}$
7	−8	8	$0.351769232729192\times10^{8}$
8	−6	6	$-0.775489259985144\times10^{6}$
9	−5	2	$0.710346691966018\times10^{-4}$
10	−5	5	$0.993499883820274\times10^{5}$
11	−4	3	-0.642094171904570
12	−4	4	$-0.612842816820083\times10^{4}$
13	−3	3	$0.232808472983776\times10^{3}$
14	−2	0	$-0.142808220416837\times10^{-4}$
15	−2	1	$-0.643596060678456\times10^{-2}$
16	−2	2	$-0.428577227475614\times10^{1}$
17	−2	4	$0.225689939161918\times10^{4}$
18	−1	0	$0.100355651721510\times10^{-2}$
19	−1	1	0.333491455143516
20	−1	2	$0.109697576888873\times10^{1}$
21	0	0	0.961917379376452
22	1	0	$-0.838165632204598\times10^{-1}$
23	1	1	$0.247795908411492\times10^{1}$
24	1	3	$-0.319114969006533\times10^{4}$

Table 31 Coefficients and exponents of the backward equation $v_{3r}(p,T)$ for subregion 3r

i	I_i	J_i	n_i
1	−8	6	$0.144165955660863\times10^{-2}$
2	−8	14	$-0.701438599628258\times10^{13}$
3	−3	−3	$-0.830946716459219\times10^{-16}$
4	−3	3	0.261975135368109
5	−3	4	$0.393097214706245\times10^{3}$
6	−3	5	$-0.104334030654021\times10^{5}$
7	−3	8	$0.490112654154211\times10^{9}$
8	0	−1	$-0.147104222772069\times10^{-3}$
9	0	0	$0.103602748043408\times10^{1}$
10	0	1	$0.305308890065089\times10^{1}$
11	0	5	$-0.399745276971264\times10^{7}$
12	3	−6	$0.569233719593750\times10^{-11}$
13	3	−2	$-0.464923504407778\times10^{-1}$
14	8	−12	$-0.535400396512906\times10^{-17}$
15	8	−10	$0.399988795693162\times10^{-12}$
16	8	−8	$-0.536479560201811\times10^{-6}$
17	8	−5	$0.159536722411202\times10^{-1}$
18	10	−12	$0.270303248860217\times10^{-14}$
19	10	−10	$0.244247453858506\times10^{-7}$
20	10	−8	$-0.983430636716454\times10^{-5}$
21	10	−6	$0.663513144224454\times10^{-1}$
22	10	−5	$-0.993456957845006\times10^{1}$
23	10	−4	$0.546491323528491\times10^{3}$
24	10	−3	$-0.143365406393758\times10^{5}$
25	10	−2	$0.150764974125511\times10^{6}$
26	12	−12	$-0.337209709340105\times10^{-9}$
27	14	−12	$0.377501980025469\times10^{-8}$

Table 32 Coefficients and exponents of the backward equation $v_{3s}(p,T)$ for subregion 3s

i	I_i	J_i	n_i
1	−12	20	$-0.532466612140254\times10^{23}$
2	−12	24	$0.100415480000824\times10^{32}$
3	−10	22	$-0.191540001821367\times10^{30}$
4	−8	14	$0.105618377808847\times10^{17}$
5	−6	36	$0.202281884477061\times10^{59}$
6	−5	8	$0.884585472596134\times10^{8}$
7	−5	16	$0.166540181638363\times10^{23}$
8	−4	6	$-0.313563197669111\times10^{6}$
9	−4	32	$-0.185662327545324\times10^{54}$
10	−3	3	$-0.624942093918942\times10^{-1}$
11	−3	8	$-0.504160724132590\times10^{10}$
12	−2	4	$0.187514491833092\times10^{5}$
13	−1	1	$0.121399979993217\times10^{-2}$
14	−1	2	$0.188317043049455\times10^{1}$
15	−1	3	$-0.167073503962060\times10^{4}$
16	0	0	0.965961650599775
17	0	1	$0.294885696802488\times10^{1}$
18	0	4	$-0.653915627346115\times10^{5}$
19	0	28	$0.604012200163444\times10^{50}$
20	1	0	-0.198339358557937
21	1	32	$-0.175984090163501\times10^{58}$
22	3	0	$0.356314881403987\times10^{1}$
23	3	1	$-0.575991255144384\times10^{3}$
24	3	2	$0.456213415338071\times10^{5}$
25	4	3	$-0.109174044987829\times10^{8}$
26	4	18	$0.437796099975134\times10^{34}$
27	4	24	$-0.616552611135792\times10^{46}$
28	5	4	$0.193568768917797\times10^{10}$
29	14	24	$0.950898170425042\times10^{54}$

Table 33 Coefficients and exponents of the backward equation $v_{3t}(p,T)$ for subregion 3t

i	I_i	J_i	n_i
1	0	0	$0.155287249586268\times10^{1}$
2	0	1	$0.664235115009031\times10^{1}$
3	0	4	$-0.289366236727210\times10^{4}$
4	0	12	$-0.385923202309848\times10^{13}$
5	1	0	$-0.291002915783761\times10^{1}$
6	1	10	$-0.829088246858083\times10^{12}$
7	2	0	$0.176814899675218\times10^{1}$
8	2	6	$-0.534686695713469\times10^{9}$
9	2	14	$0.160464608687834\times10^{18}$
10	3	3	$0.196435366560186\times10^{6}$
11	3	8	$0.156637427541729\times10^{13}$
12	4	0	$-0.178154560260006\times10^{1}$
13	4	10	$-0.229746237623692\times10^{16}$
14	7	3	$0.385659001648006\times10^{8}$
15	7	4	$0.110554446790543\times10^{10}$
16	7	7	$-0.677073830687349\times10^{14}$
17	7	20	$-0.327910592086523\times10^{31}$
18	7	36	$-0.341552040860644\times10^{51}$
19	10	10	$-0.527251339709047\times10^{21}$
20	10	12	$0.245375640937055\times10^{24}$
21	10	14	$-0.168776617209269\times10^{27}$
22	10	16	$0.358958955867578\times10^{29}$
23	10	22	$-0.656475280339411\times10^{36}$
24	18	18	$0.355286045512301\times10^{39}$
25	20	32	$0.569021454413270\times10^{58}$
26	22	22	$-0.700584546433113\times10^{48}$
27	22	36	$-0.705772623326374\times10^{65}$
28	24	24	$0.166861176200148\times10^{53}$
29	28	28	$-0.300475129680486\times10^{61}$
30	32	22	$-0.668481295196808\times10^{51}$
31	32	32	$0.428432338620678\times10^{69}$
32	32	36	$-0.444227367758304\times10^{72}$
33	36	36	$-0.281396013562745\times10^{77}$

Table 34 Coefficient and exponents of the auxiliary equation $v_{3u}(p,T)$ for subregion 3u

i	I_i	J_i	n_i
1	−12	14	$0.122088349258355\times10^{18}$
2	−10	10	$0.104216468608488\times10^{10}$
3	−10	12	$-0.882666931564652\times10^{16}$
4	−10	14	$0.259929510849499\times10^{20}$
5	−8	10	$0.222612779142211\times10^{15}$
6	−8	12	$-0.878473585050085\times10^{18}$
7	−8	14	$-0.314432577551552\times10^{22}$
8	−6	8	$-0.216934916996285\times10^{13}$
9	−6	12	$0.159079648196849\times10^{21}$
10	−5	4	$-0.339567617303423\times10^{3}$
11	−5	8	$0.884387651337836\times10^{13}$
12	−5	12	$-0.843405926846418\times10^{21}$
13	−3	2	$0.114178193518022\times10^{2}$
14	−1	−1	$-0.122708229235641\times10^{-3}$
15	−1	1	$-0.106201671767107\times10^{3}$
16	−1	12	$0.903443213959313\times10^{25}$
17	−1	14	$-0.693996270370852\times10^{28}$
18	0	−3	$0.648916718965575\times10^{-8}$
19	0	1	$0.718957567127851\times10^{4}$
20	1	−2	$0.105581745346187\times10^{-2}$
21	2	5	$-0.651903203602581\times10^{15}$
22	2	10	$-0.160116813274676\times10^{25}$
23	3	−5	$-0.510254294237837\times10^{-8}$
24	5	−4	-0.152355388953402
25	5	2	$0.677143292290144\times10^{12}$
26	5	3	$0.276378438378930\times10^{15}$
27	6	−5	$0.116862983141686\times10^{-1}$
28	6	2	$-0.301426947980171\times10^{14}$
29	8	−8	$0.169719813884840\times10^{-7}$
30	8	8	$0.104674840020929\times10^{27}$
31	10	−4	$-0.108016904560140\times10^{5}$
32	12	−12	$-0.990623601934295\times10^{-12}$
33	12	−4	$0.536116483602738\times10^{7}$
34	12	4	$0.226145963747881\times10^{22}$
35	14	−12	$-0.488731565776210\times10^{-9}$
36	14	−10	$0.151001548880670\times10^{-4}$
37	14	−6	$-0.227700464643920\times10^{5}$
38	14	6	$-0.781754507698846\times10^{28}$

Table 35 Coefficient and exponents of the auxiliary equation $v_{3v}(p,T)$ for subregion 3v

i	I_i	J_i	n_i
1	−10	−8	$-0.415652812061591\times10^{-54}$
2	−8	−12	$0.177441742924043\times10^{-60}$
3	−6	−12	$-0.357078668203377\times10^{-54}$
4	−6	−3	$0.359252213604114\times10^{-25}$
5	−6	5	$-0.259123736380269\times10^{2}$
6	−6	6	$0.594619766193460\times10^{5}$
7	−6	8	$-0.624184007103158\times10^{11}$
8	−6	10	$0.313080299915944\times10^{17}$
9	−5	1	$0.105006446192036\times10^{-8}$
10	−5	2	$-0.192824336984852\times10^{-5}$
11	−5	6	$0.654144373749937\times10^{6}$
12	−5	8	$0.513117462865044\times10^{13}$
13	−5	10	$-0.697595750347391\times10^{19}$
14	−5	14	$-0.103977184454767\times10^{29}$
15	−4	−12	$0.119563135540666\times10^{-47}$
16	−4	−10	$-0.436677034051655\times10^{-41}$
17	−4	−6	$0.926990036530639\times10^{-29}$
18	−4	10	$0.587793105620748\times10^{21}$
19	−3	−3	$0.280375725094731\times10^{-17}$
20	−3	10	$-0.192359972440634\times10^{23}$
21	−3	12	$0.742705723302738\times10^{27}$
22	−2	2	$-0.517429682450605\times10^{2}$
23	−2	4	$0.820612048645469\times10^{7}$
24	−1	−2	$-0.188214882341448\times10^{-8}$
25	−1	0	$0.184587261114837\times10^{-1}$
26	0	−2	$-0.135830407782663\times10^{-5}$
27	0	6	$-0.723681885626348\times10^{17}$
28	0	10	$-0.223449194054124\times10^{27}$
29	1	−12	$-0.111526741826431\times10^{-34}$
30	1	−10	$0.276032601145151\times10^{-28}$
31	3	3	$0.134856491567853\times10^{15}$
32	4	−6	$0.652440293345860\times10^{-9}$
33	4	3	$0.510655119774360\times10^{17}$
34	4	10	$-0.468138358908732\times10^{32}$
35	5	2	$-0.760667491183279\times10^{16}$
36	8	−12	$-0.417247986986821\times10^{-18}$
37	10	−2	$0.312545677756104\times10^{14}$
38	12	−3	$-0.100375333864186\times10^{15}$
39	14	1	$0.247761392329058\times10^{27}$

Table 36 Coefficients and exponents of the auxiliary equation $v_{3w}(p,T)$ for subregion 3w

i	I_i	J_i	n_i
1	−12	8	$-0.586219133817016\times10^{-7}$
2	−12	14	$-0.894460355005526\times10^{11}$
3	−10	−1	$0.531168037519774\times10^{-30}$
4	−10	8	0.109892402329239
5	−8	6	$-0.575368389425212\times10^{-1}$
6	−8	8	$0.228276853990249\times10^{5}$
7	−8	14	$-0.158548609655002\times10^{19}$
8	−6	−4	$0.329865748576503\times10^{-27}$
9	−6	−3	$-0.634987981190669\times10^{-24}$
10	−6	2	$0.615762068640611\times10^{-8}$
11	−6	8	$-0.961109240985747\times10^{8}$
12	−5	−10	$-0.406274286652625\times10^{-44}$
13	−4	−1	$-0.471103725498077\times10^{-12}$
14	−4	3	0.725937724828145
15	−3	−10	$0.187768525763682\times10^{-38}$
16	−3	3	$-0.103308436323771\times10^{4}$
17	−2	1	$-0.662552816342168\times10^{-1}$
18	−2	2	$0.579514041765710\times10^{3}$
19	−1	−8	$0.237416732616644\times10^{-26}$
20	−1	−4	$0.271700235739893\times10^{-14}$
21	−1	1	$-0.907886213483600\times10^{2}$
22	0	−12	$-0.171242509570207\times10^{-36}$
23	0	1	$0.156792067854621\times10^{3}$
24	1	−1	0.923261357901470
25	2	−1	$-0.597865988422577\times10^{1}$
26	2	2	$0.321988767636389\times10^{7}$
27	3	−12	$-0.399441390042203\times10^{-29}$
28	3	−5	$0.493429086046981\times10^{-7}$
29	5	−10	$0.812036983370565\times10^{-19}$
30	5	−8	$-0.207610284654137\times10^{-11}$
31	5	−6	$-0.340821291419719\times10^{-6}$
32	8	−12	$0.542000573372233\times10^{-17}$
33	8	−10	$-0.856711586510214\times10^{-12}$
34	10	−12	$0.266170454405981\times10^{-13}$
35	10	−8	$0.858133791857099\times10^{-5}$

Table 37 Coefficients and exponents of the auxiliary equation $v_{3x}(p,T)$ for subregion 3x

i	I_i	J_i	n_i
1	−8	14	$0.377373741298151\times10^{19}$
2	−6	10	$-0.507100883722913\times10^{13}$
3	−5	10	$-0.103363225598860\times10^{16}$
4	−4	1	$0.184790814320773\times10^{-5}$
5	−4	2	$-0.924729378390945\times10^{-3}$
6	−4	14	$-0.425999562292738\times10^{24}$
7	−3	−2	$-0.462307771873973\times10^{-12}$
8	−3	12	$0.107319065855767\times10^{22}$
9	−1	5	$0.648662492280682\times10^{11}$
10	0	0	$0.244200600688281\times10^{1}$
11	0	4	$-0.851535733484258\times10^{10}$
12	0	10	$0.169894481433592\times10^{22}$
13	1	−10	$0.215780222509020\times10^{-26}$
14	1	−1	-0.320850551367334
15	2	6	$-0.382642448458610\times10^{17}$
16	3	−12	$-0.275386077674421\times10^{-28}$
17	3	0	$-0.563199253391666\times10^{6}$
18	3	8	$-0.326068646279314\times10^{21}$
19	4	3	$0.397949001553184\times10^{14}$
20	5	−6	$0.100824008584757\times10^{-6}$
21	5	−2	$0.162234569738433\times10^{5}$
22	5	1	$-0.432355225319745\times10^{11}$
23	6	1	$-0.592874245598610\times10^{12}$
24	8	−6	$0.133061647281106\times10^{1}$
25	8	−3	$0.157338197797544\times10^{7}$
26	8	1	$0.258189614270853\times10^{14}$
27	8	8	$0.262413209706358\times10^{25}$
28	10	−8	$-0.920011937431142\times10^{-1}$
29	12	−10	$0.220213765905426\times10^{-2}$
30	12	−8	$-0.110433759109547\times10^{2}$
31	12	−5	$0.847004870612087\times10^{7}$
32	12	−4	$-0.592910695762536\times10^{9}$
33	14	−12	$-0.183027173269660\times10^{-4}$
34	14	−10	0.181339603516302
35	14	−8	$-0.119228759669889\times10^{4}$
36	14	−6	$0.430867658061468\times10^{7}$

Table 38 Coefficients and exponents of the auxiliary equation $v_{3y}(p,T)$ for subregion 3y

i	I_i	J_i	n_i
1	0	−3	$-0.525597995024633 \times 10^{-9}$
2	0	1	$0.583441305228407 \times 10^{4}$
3	0	5	$-0.134778968457925 \times 10^{17}$
4	0	8	$0.118973500934212 \times 10^{26}$
5	1	8	$-0.159096490904708 \times 10^{27}$
6	2	−4	$-0.315839902302021 \times 10^{-6}$
7	2	−1	$0.496212197158239 \times 10^{3}$
8	2	4	$0.327777227273171 \times 10^{19}$
9	2	5	$-0.527114657850696 \times 10^{22}$
10	3	−8	$0.210017506281863 \times 10^{-16}$
11	3	4	$0.705106224399834 \times 10^{21}$
12	3	8	$-0.266713136106469 \times 10^{31}$
13	4	−6	$-0.145370512554562 \times 10^{-7}$
14	4	6	$0.149333917053130 \times 10^{28}$
15	5	−2	$-0.149795620287641 \times 10^{8}$
16	5	1	$-0.381881906271100 \times 10^{16}$
17	8	−8	$0.724660165585797 \times 10^{-4}$
18	8	−2	$-0.937808169550193 \times 10^{14}$
19	10	−5	$0.514411468376383 \times 10^{10}$
20	12	−8	$-0.828198594040141 \times 10^{5}$

Table 39 Coefficients and exponents of the auxiliary equation $v_{3z}(p,T)$ for subregion 3z

i	I_i	J_i	n_i
1	−8	3	$0.244007892290650 \times 10^{-10}$
2	−6	6	$-0.463057430331242 \times 10^{7}$
3	−5	6	$0.728803274777712 \times 10^{10}$
4	−5	8	$0.327776302858856 \times 10^{16}$
5	−4	5	$-0.110598170118409 \times 10^{10}$
6	−4	6	$-0.323899915729957 \times 10^{13}$
7	−4	8	$0.923814007023245 \times 10^{16}$
8	−3	−2	$0.842250080413712 \times 10^{-12}$
9	−3	5	$0.663221436245506 \times 10^{12}$
10	−3	6	$-0.167170186672139 \times 10^{15}$
11	−2	2	$0.253749358701391 \times 10^{4}$
12	−1	−6	$-0.819731559610523 \times 10^{-20}$
13	0	3	$0.328380587890663 \times 10^{12}$
14	1	1	$-0.625004791171543 \times 10^{8}$
15	2	6	$0.803197957462023 \times 10^{21}$
16	3	−6	$-0.204397011338353 \times 10^{-10}$
17	3	−2	$-0.378391047055938 \times 10^{4}$
18	6	−6	$0.972876545938620 \times 10^{-2}$
19	6	−5	$0.154355721681459 \times 10^{2}$
20	6	−4	$-0.373962862928643 \times 10^{4}$
21	6	−1	$-0.682859011374572 \times 10^{11}$
22	8	−8	$-0.248488015614543 \times 10^{-3}$
23	8	−4	$0.394536049497068 \times 10^{7}$

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