

Resistance-Based Scaling of Noise Temperatures From 1 kHz to 1 MHz

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Abstract—A conventional technique for scaling temperatures in Johnson noise thermometry is via resistance ratios. We describe measurements from 1 kHz to 1 MHz using this approach via correlation methods in the frequency domain. We show that the effects of the mismatch in time constants of the input networks may be empirically treated to extract the noise power and temperature ratios in the low-frequency limit with uncertainties $< 60 \mu\text{K/K}$.

Index Terms—Noise, noise measurement, temperature, temperature measurement.

I. INTRODUCTION

NOISE thermometry involves the accurate measurement of noise-power ratios where one calculable noise source serves as a known reference to determine the power of a second thermal source at some unknown temperature T_1 [1], [2]. The usual measurement approach is to alternately connect the two signals to a noise voltage correlator (NVC) according to a scheme as shown in Fig. 1. Each two-port noise source (S_1 and S_2) is coupled to the inputs of two parallel amplifier chains with gains G_x and G_y via coupling networks N_{ix} and N_{iy} ($i = 1, 2$), and a switching network M . The coupling networks are essentially short transmission lines dominated by shunt capacitance and series inductance.

Conventional methods often rely on a known thermal source of a resistance R_2 at a temperature T_2 for the reference noise spectral density $S_2 = 4kT_2R_2$. If the unknown source of spectral density S_1 is likewise purely thermal, then the spectral noise-power density ratio may be approximated by

$$\frac{S_1}{S_2} = \frac{R_1T_1 [1 + \tau_{2,1}\omega + \tau_{2,2}^2\omega^2 + \tau_{2,3}^3\omega^3 + O(\omega^4)]}{R_2T_2 [1 + \tau_{1,1}\omega + \tau_{1,2}^2\omega^2 + \tau_{1,3}^3\omega^3 + O(\omega^4)]} \quad (1)$$

where $\tau_{i,j}$ denote the coefficients of the frequency terms for noise source i of order j in angular frequency ω . The coefficients are functions of the various time constants associated with the lumped parameters in the model. In general, $\tau_{i,j}\omega^j < 1$ for all j and $\omega/2\pi \leq 1$ MHz and terms proportional to ω^4 and higher powers $O(\omega^4)$ are very small and may be neglected.

Digital noise-voltage measurement systems operate at gains in excess of 10^5 and consequently have limited linearity. A standard practice is to equalize the powers of the two noise

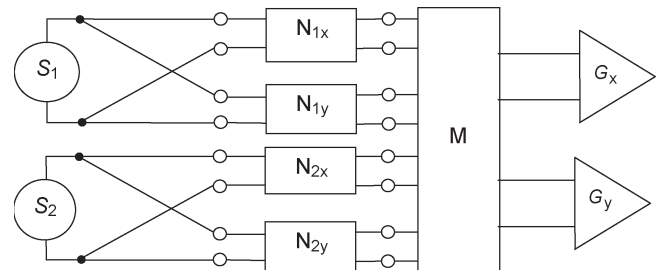


Fig. 1. Network representation of inputs to the noise voltage correlator with parallel channels x and y , sources S_1 and S_2 , switching network M , and coupling networks N_{ix} and N_{iy} .

sources to minimize errors from nonlinearity. For resistance-based systems, this imposes the constraint $R_1T_1 \approx R_2T_2$, or $S_1/S_2 \approx 1$. Therefore, a difference in the frequency response of the noise resistors and their coupling networks for any $T_1 \neq T_2$ will exist. This creates an imbalance in the quadratic time constants $\tau_{1,2}$ and $\tau_{2,2}$, which are predominately RC low-pass filters formed by the source resistances R_1 , R_2 and the cabling capacitances C_{1p} and C_{2p} in the coupling networks N_{ij} . Furthermore, the difference $\tau_{1,2}^2 - \tau_{2,2}^2$ will scale as $T_2^2 - T_1^2$ so long as $C_{1p} \approx C_{2p}$.

It is therefore important to properly account for the mismatched time constants in Johnson noise thermometry (JNT) measurement systems in order to avoid certain frequency-dependent errors. One traditional approach is to impose a narrow bandpass filter as part of the preamplifier circuits which has a low-pass cutoff frequency $f_c \ll 1/2\pi\tau_{1,2}$ [3]. In our case, however, there is no bandpass limit on the input other than what the coupling networks N_{ix} and N_{iy} naturally create such that $\tau_{1,2} \approx 17$ ns and $\tau_{2,2} \approx 79$ ns when $T_1/T_2 = 2.2$. Obtaining the largest possible bandwidth is required to achieve results of higher statistical precision in shorter measurement times.

In this paper, we report on measurements using only resistance-based scaling methods. This is in contrast to other ongoing work which employs a quantized voltage noise source (QVNS) for the reference [4]. While the QVNS system conveys important advantages by providing power scaling which is much less impedance dependent, the measurements shown here help illustrate the full extent of these advantages in comparison to thermal noise references using what are otherwise identical electronics. In either case, the optimization of the measurement bandwidth is important. There is a compromise between greater statistical precision for a given integration time versus possible systematic errors associated with source-dependent time constants and offsets. Using a large measurement bandwidth,

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we demonstrate that these time constants may be empirically modeled and also demonstrate simple compensation methods which approximately balance the effects out to 1 MHz. We also show that linear frequency dependence cannot be neglected in this process and propose an origin for such terms.

II. EXPERIMENTAL

We use a digital NVC to sample noise voltages from the two resistive sources. Time series data in the two parallel channels G_x and G_y are oversampled by $50 \text{ MSample} \cdot \text{s}^{-1}$ digitizers followed by digital signal processors which are down-sampled to $2^{21} \text{ Sample} \cdot \text{s}^{-1}$ to the computer. Correlated noise power spectra are calculated in real-time in the frequency domain using the FFTW [5] C-language libraries. This same system has been used for other noise thermometry work with a QVNS-based reference and is described elsewhere [6].

The reference noise source in this paper is made from two fixed-value metal-foil resistors as a matched differential pair of $R_2 = 125 \Omega$ each. The resistors are mounted inside a shielded probe which is thermally regulated in a stirred water bath at selected fixed temperatures in the range of 308 to 312 K, depending on the exact noise power matching requirements. The reference temperature is known via standard platinum resistance thermometers (SPRTs) calibrated on the International Temperature Scale of 1990 (ITS-90) [7].

The “unknown” source is produced by two Pt–W alloy resistors of nominal resistance 56.5Ω at 692 K arranged in a similar differential pair. The resistors are terminations to a pair of transmission lines made from silver wires mounted in quartz-insulated twin-bore rectangular tubes with a platinum shield connected to the commons of the two differential channel inputs. The assembly of resistors and quartz insulators is mounted in an overall quartz protection tube which is coated with a Pt film, forming a high-temperature noise probe. This probe is maintained at temperature in a thermal well of a vacuum furnace comparator along with ITS-90 calibrated SPRTs. The two-zone furnace is regulated at $T_1 \approx 692 \text{ K}$ with a stability of $\pm 3 \text{ mK}$ or better. Further details on the thermal aspects of the system are described elsewhere [8].

The probes are connected to the NVC inputs via conventional foam-polyethylene-insulated twisted-pair cables with low intrapair and pair-to-ground capacitance. The two probes and associated cabling form the noise sources S_i and their coupling networks N_{ij} which, together with the switching network M and preamplifier input impedances, determine the system time constants. As short transmission lines they have different 3 dB point frequencies due to the dissimilar source impedances and dissimilar admittances of the materials used in construction. This is particularly true of the loss tangents of the associated dielectrics.

III. MODELING

The coupling network N_{ij} is modeled in the lumped-parameter approximation as a short transmission line coupling a noise-source noise R_T (either S_1 or S_2) to the preamplifier as shown in Fig. 2. The equivalent noise-source voltage

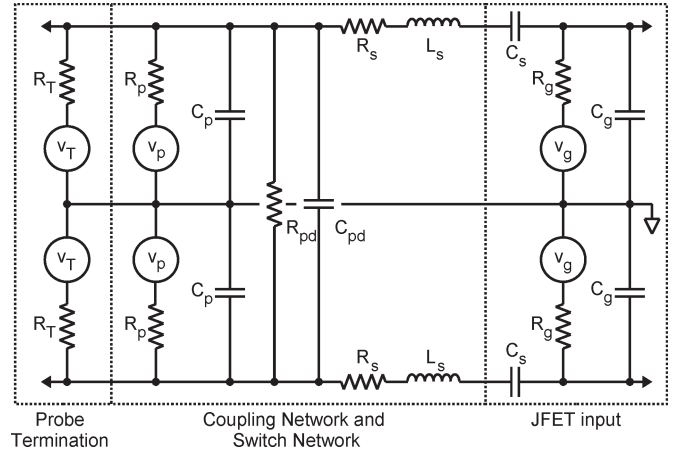


Fig. 2. Equivalent circuit model for the probe termination, coupling network, switch, and preamplifier input impedances. The left-hand side is connected to an identical network that connects with the second amplifier channel.

spectral density $\nu_T = \sqrt{4kTR}$ is from the resistor R_T at a fixed temperature. The noise sources ν_p and ν_g are extraneous sources in each channel; however, they are strongly attenuated and partly canceled in the cross-correlated ratio spectrum [8]. The remaining correlated component produces a $-3 \mu\text{K/K}$ shift in the ratio. The parallel capacitance of the cabling and probes is lumped together as C_p , which is modeled as a lossy capacitor, $C = C' - jC'' = C'(1 - j \tan \delta)$

$$Z_p = \frac{\tan \delta - j}{\omega C_p (1 + \tan^2 \delta)} \quad (2)$$

where $\tan \delta$ is the dielectric loss tangent. In addition, there is a parallel shunt resistance R_q due to the finite conductance of insulators at dc. However, this is only important for understanding small direct-current (dc) effects in the high-temperature probe where the quartz exhibits $R_q \approx 50 \text{ M}\Omega$ at $T_1 = 693 \text{ K}$.

The series impedances of the networks are as follows: inductance $Z_{L_s} = j\omega L_s$, resistance $Z_{R_s} = R_s \approx 1 \Omega$, and the dc blocking coupling capacitor $Z_{C_s} = 1/j\omega C_s$. The preamplifiers have n-type junction field-effect transistor (JFET) inputs with a $20 \text{ M}\Omega$ gate resistor R_g in parallel with each gate to pass the dc gate leakage current to ground. The JFET common-source input capacitance C_g and the gate resistor are treated as an equivalent input impedance Z_g of the preamplifier or

$$Z_g = \frac{R_g - j\omega C_g R_g^2}{1 + \omega^2 C_g^2 R_g^2} \quad (3)$$

The voltage transfer function $H(f)$ for the combined network attenuation from source to input is different for each noise-source S_1 and S_2 , but may be written generally as

$$H(f) \equiv \frac{V_{in}}{V_T} = \left[1 + \frac{R_T}{Z_g} + \frac{Z_{L_s} R_T}{Z_g Z_p} + \frac{R_s R_T}{Z_g Z_p} + \frac{Z_{C_s} R_T}{Z_g Z_p} + \frac{R_T}{Z_p} + \frac{Z_{L_s}}{Z_g} + \frac{R_s}{Z_g} + \frac{Z_{C_s}}{Z_g} \right]^{-1} \quad (4)$$

The cross-correlated spectral power density of input 1 referred to the output is then

$$S_1(f) = \nu_{T1}^2 |H_1(f)|^2 G_x(f) G_y(f) \quad (5)$$

and similarly for input 2.

The power transfer function $|H|^2$ expands to 45 terms in the denominator. While many of these terms are $< 1 \times 10^{-6}$, the most important frequency dependent terms are quadratic: $\omega^2 R_{T1}^2 (C_{p1} + C_g)^2$; $2\omega^2 L_{s1} (C_{p1} + C_g)$; and linear $2\omega R_{T1} C_{p1} \tan \delta_{p1}$, and similarly for input 2. In practice, sections of epoxy-fiber-circuit-board (FR4) can exhibit $\tan \delta_p \approx 0.025$ and quartz insulators exhibit $\tan \delta_q \approx 0.06$ at $T_q \approx 693$ K. In addition, a small dc term given by $2R_1/R_g$ creates an offset correction. The terms from the relative transfer function, $|H_1|^2/|H_2|^2$, in the spectral ratio S_1/S_2 expand to become differences, with a dominant quadratic term of $(\tau_{2,2}^2 - \tau_{1,2}^2)\omega^2$ where

$$\begin{aligned} \tau_{2,2}^2 - \tau_{1,2}^2 \approx & C_g^2 (R_{T2}^2 - R_{T1}^2) + 2C_g (R_{T2}^2 C_{p2} - R_{T1}^2 C_{p1}) \\ & + R_{T2}^2 C_{p2}^2 - R_{T1}^2 C_{p1}^2 - 2L_s (C_{p2} - C_{p1}) \quad (6) \end{aligned}$$

assuming $L_{s1} \approx L_{s2}$.

For a given input i , the networks N_{ix} and N_{iy} are themselves coupled through the “five-wire” (four-wire-plus-ground) connections of the differential resistance pair as shown in Fig. 2. This coupling produces an effective parallel capacitance $C_{p\text{eff}}$ for each gate due to parallel and series combinations of C_p and C_{pd} within the two coupled networks. The value of $C_{p\text{eff}}$ is 2 to 3 times that of the single-ended, single-network values of C_p depending on the exact amount of intrapair capacitance for the input. Similarly, the effective shunt resistance $R_{p\text{eff}}$ is lowered through parallel and series combinations of R_p and R_{pd} . In the case of the reference probe, the conductances R_{p2}^{-1} and R_{pd2}^{-1} are negligible so that $R_{p\text{eff}} \approx 0$ and $R_{g\text{eff}} = R_g/2 = 10$ M Ω . In the case of the high-temperature probe, the finite quartz insulation resistances $R_q \approx R_{qd}$ produce $R_{p\text{eff}} \approx R_q/3$. The effective capacitance determines the observed time constants in the spectral densities S_i , and the values of $R_{p\text{eff}}$ and $R_{g\text{eff}}$ determine the magnitude of the small offsets.

IV. RESULTS

The measured noise power ratio spectrum is fitted with polynomials of the general form

$$\frac{S_1}{S_2} \approx a_0 + a_1 f + a_2 f^2 + a_3 f^3 + O(f^4) \quad (7)$$

in order to extract the zero-frequency-limit value a_0 . We compare this to the value predicted by the ITS-90, $a_{90} = (T_{1,90} R_1)/(T_{2,90} R_2)$ based on measurements of the probes and SPRTs [8].

The time constant mismatch produces a quadratic term, $a_2 > 0$ if $T_1 > T_2$ [see Fig. 3(a)] and for our system $a_2 \approx 2.4 \times 10^{-7}$ kHz $^{-2}$.

The noise power from the high-temperature probe is nearly 25% higher than that of the reference at 1 MHz

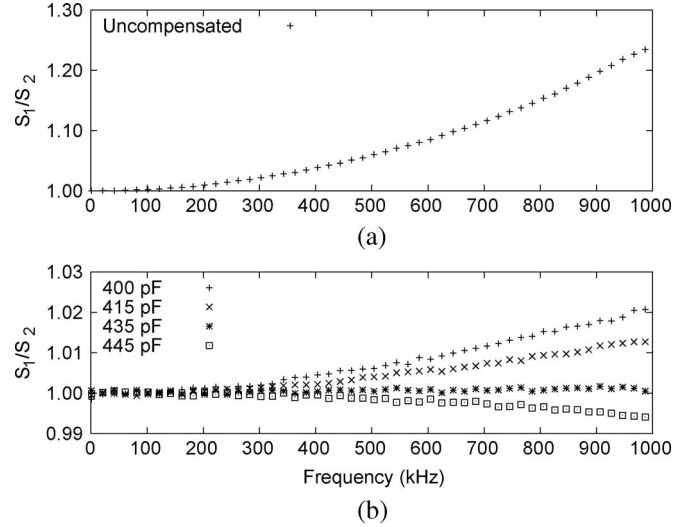


Fig. 3. Spectral noise power density ratio of an (a) uncompensated and (b) four capacitance-compensated runs to 1 MHz. The “unknown” probe is a $R_1 = 56.5$ Ω , $T_1 = 693$ K noise source and the reference probe is a $R_2 = 125$ Ω , $T_2 = 312$ K source.

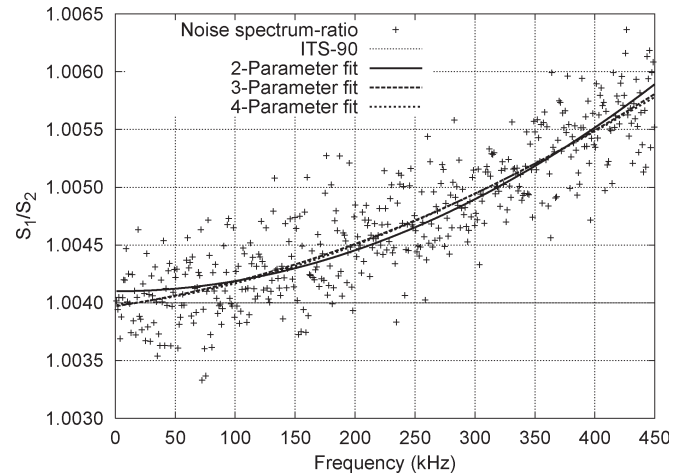


Fig. 4. Spectral noise power density ratio of a typical capacitance compensated run and the three polynomial fits to the data.

[Fig. 3(a)]. Fig. 3(b) shows the system running under the same conditions, except that the frequency responses of the inputs are capacitively compensated using a set of four ultrastable BaTiO $_3$ + TiO $_2$ composite capacitors (i.e., “NPO”-type) that are placed in parallel with each C_p . It can be seen that the ratio spectrum can be both under and over compensated with a modest change in the capacitance. The 435-pF compensation capacitors achieve a flat ratio spectrum to within 0.05% for the full 1-MHz bandwidth. The relationship between the changes in the capacitance to the change in S_1/S_2 is approximately 20 pF for a 1% change in the ratio.

We compare the results for a_0 in the special cases of a two-parameter fit (pure quadratic) $a_1 = a_3 = 0$ and a three-parameter fit (general quadratic) where $a_3 = 0$. Fig. 4 shows the three fit variations to the lowest 450-kHz band. The T-value $|a_1|/u(a_1)$ of the linear term a_1 is found to be statistically significant; however, the T-value of the cubic term, a_3 , is not, implying that the three-parameter fit is most appropriate. The residuals for the three fits are shown in Fig. 5 as well as a

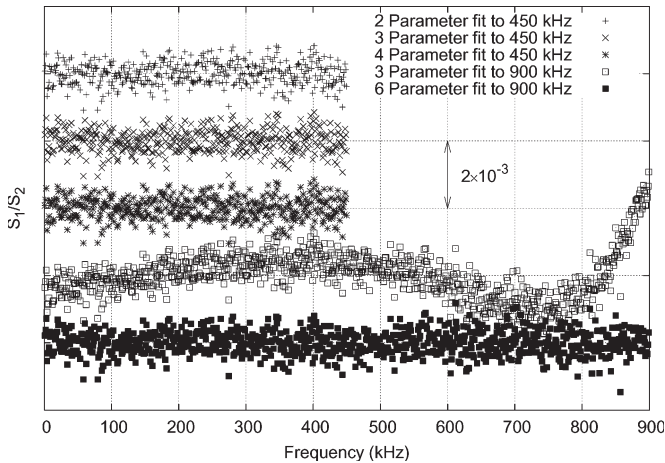


Fig. 5. Residuals of the polynomial fits to the spectral noise power density ratio of a typical capacitance compensated run. The origins of the residuals have been shifted for clarity.

three-parameter fit to 1 MHz, demonstrating that higher order terms are necessary to fit the frequency response of the system over the full bandwidth. We have chosen to limit the bandwidth to 450 kHz in order to reduce the number of parameters required for the fit and maximize the statistical precision of a_0 .

We have made 22 individual runs of 15 to 18 h in duration each at temperatures of 690 to 693 K for the “unknown” probe and 312 K for the reference probe to derive mean values for a_0 . Our current results for $(a_0 - a_{90})/a_{90}$ are: two-parameter fit $(62 \pm 37) \mu\text{K/K}$; three-parameter fit $(-29 \pm 43) \mu\text{K/K}$; and four-parameter fit $(-23 \pm 63) \mu\text{K/K}$. The two-parameter fit is consistently high which is expected from the positive sign of the linear term a_1 . The three-parameter and four-parameter fits yield similar results for $a_0 - a_{90}$, but since the statistical uncertainty in a_0 per fit scales nearly with the number of terms in the fit, the three-parameter fit is preferred. In contrast, a simple mean of a small bandwidth of the data will obviously lead to results that depend on the curvature of the ratio spectrum as can be seen in Fig. 4. If the frequency response of the inputs is sufficiently well matched and stable over the long integration times, the mean of a small low-frequency bandwidth may prove to be a suitable alternative to curve fitting.

V. CONCLUSION

Capacitive compensation on the high-temperature-probe input has been demonstrated as a useful technique to flatten the spectral noise ratio of a resistance-based JNT system. The advantages of capacitive compensation of the transmission line are that a well compensated ratio-spectrum can be parameterized by either a simple mean value or a low-order polynomial and the system is made insensitive to any nonlinearity in the preamplifiers. The linear term in the fit to the ratio-spectrum of $2\omega(R_2 - R_1)C_p \tan \delta$ is significant if $\tan \delta \geq 0.02$, which typically occurs in the quartz of the high-temperature probe and in fiber-epoxy-composite (e.g., “FR4”) printed circuit boards. The system time constants are easily extracted through fitting, but need not be accurately known. If proper impedance matching is applied, as is possible when using QVNS noise power

scaling, all such time constants will cancel in the relative ratio spectra except those which are temperature-dependent [2]. The total estimated standard uncertainty in $a_0 - a_{90}$ is $57 \mu\text{K/K}$ for our three-parameter data including type B effects [8]. These results are in good agreement with predictions based on ITS-90 temperatures and the sum of the known corrections using our method is less than $10 \mu\text{K/K}$.

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W. L. Tew, photograph and biography not available at the time of publication.

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