COMPARISONS BY PTB, NIST, AND LNE-LADG IN AIR AND NATURAL GAS WITH CRITICAL VENTURI NOZZLES AGREE WITHIN 0.05 %

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Abstract: The PTB, NIST, and LNE-LADG compared gas flow measurement standards in air and in natural gas in the pressure range from 0.9 to 42 bar using four critical nozzles with ISO standard toroidal shape [1]. The four facilities generated calibrated flows through the four nozzles to determine curves of the discharge coefficient (C_d) versus Reynolds number (Re) spanning the range from 2×10^5 to 2×10^7 in Re. Three of the facilities used air; the fourth facility used natural gas. The data show the laminar-to-turbulent transition at throat Reynolds numbers in the interval 1×10^6 to 2×10^6. All of the laboratories' discharge coefficients agreed within 0.15 % or less, well within the uncertainty of the comparison. Comparing the average C_d's for the four nozzles, all of the laboratories agree within 0.05 %. The geometry of the nozzles was measured with coordinate measuring machines at NIST and PTB. The throat diameter measurements from the two institutes agreed within 4 μm or better. The dimensional measurements were used to generate theoretical C_d curves, for both the laminar and turbulent regimes, for air and natural gas. The theoretical C_d curves agree with the experimental measurements within 0.15 % or less. The results show that critical nozzles calibrated in air may be used as working standards in natural gas with an uncertainty less than 0.2 %, provided that gas' properties are known with sufficient accuracy.

1. INTRODUCTION

Flow comparisons between national laboratories are used to demonstrate equivalence and proficiency, validate uncertainty analyses, and to support fair trade. It is difficult to span the wide range of pressures, flows, and gas species that exist in large gas flow labs. The critical flow venturi (CFV or critical nozzle) is a widely accepted transfer standard for gas flow comparisons. In this work, we used them over a wide range of conditions, together with theoretical predictions of the discharge coefficient, to bridge gaps between the facilities compared.

We calibrated four critical nozzles of nominally 25 mm throat diameter in four flow facilities: 1) the NIST 26 m^3 *PVTt* standard with air, 2) a PTB working standard turbine meter in air, previously calibrated with the PTB Nozzle Rig, 3) the PTB / pigsar standards in high pressure natural gas, and 4) a set of working standard nozzles in air, previously calibrated with the LNE-LADG Piscine standard. Each laboratory used its own pressure and temperature instrumentation as well as upstream and downstream piping.

2. CALIBRATION FACILITIES

2.1 NIST 26 m³ *PVTt* Standard

The four nozzles (see Fig. 1) were calibrated with the NIST 26 m^3 *PVTt* standard [2] in March, 2005 (see Fig. 2). The NIST *P*ressure-*V*olume-*T*emperature-*t*ime or *PVTt* standard measures mass flow by diverting flow from the meter under test into an initially evacuated collection tank of known volume for a measured time interval. Mass flow is calculated by multiplying the tank volume by the density change of the gas attributed to filling process divided by the collection time. Here, the density change is determined using an equation of state in conjunction with initial and final pressure and temperature measurements.

Each of the four nozzles was tested at a minimum of three flows. A more extensive data set was collected for two of the nozzles (*i.e.*, 1691-3 and 1691-5) to capture the laminar to turbulent transition. At each flow, at least three measurements were collected on two different occasions and used to produce averages at each of these flows. Thus, the plotted data for each of the four nozzles are averages of six or more individual calibration measurements.

Fig. 1 The four critical venturis ("nozzles") used as the transfer standard during the comparison.

For CFV 1691-3 the meter was calibrated, taken out of line, reinstalled, and recalibrated to assess possible uncertainties from "*take out put back*". The standard uncertainty associated with the "*take out put back*" test was determined to be less than 0.02 % and this value was used for the other three nozzles as well. The nozzles were tested with dry air (dew point ≤ -40 °C) at pressures between 200 kPa and 700 kPa.

Mass flow measurements from the 26 m^3 *PVTt* standard have a standard uncertainty (k=1) of 0.045 %. Standard uncertainties in the pressure (0.01 %) and temperature measurements (0.015 %) associated with the nozzles, along with the standard uncertainty of the molecular weight and critical flow function (0.02 %) and the take out put back uncertainty leads to an expanded uncertainty (k = 2) for the C_d values measured at NIST of 0.11 %.

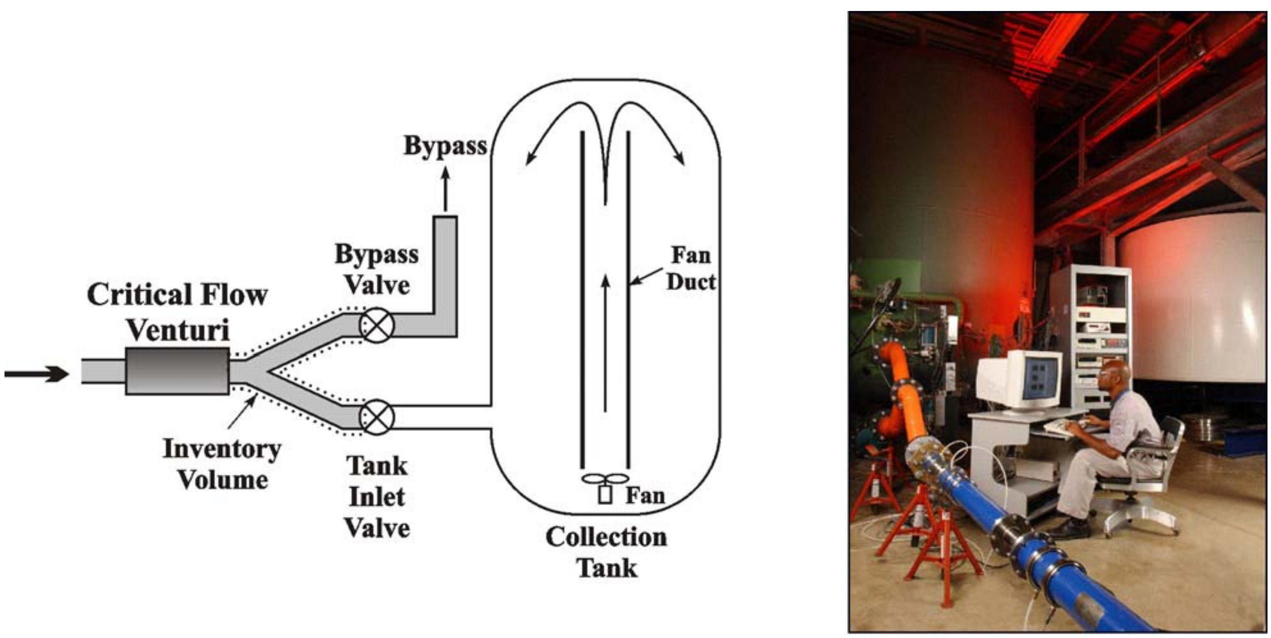


Fig. 2 The NIST 26 m^3 *PVTt* standard.

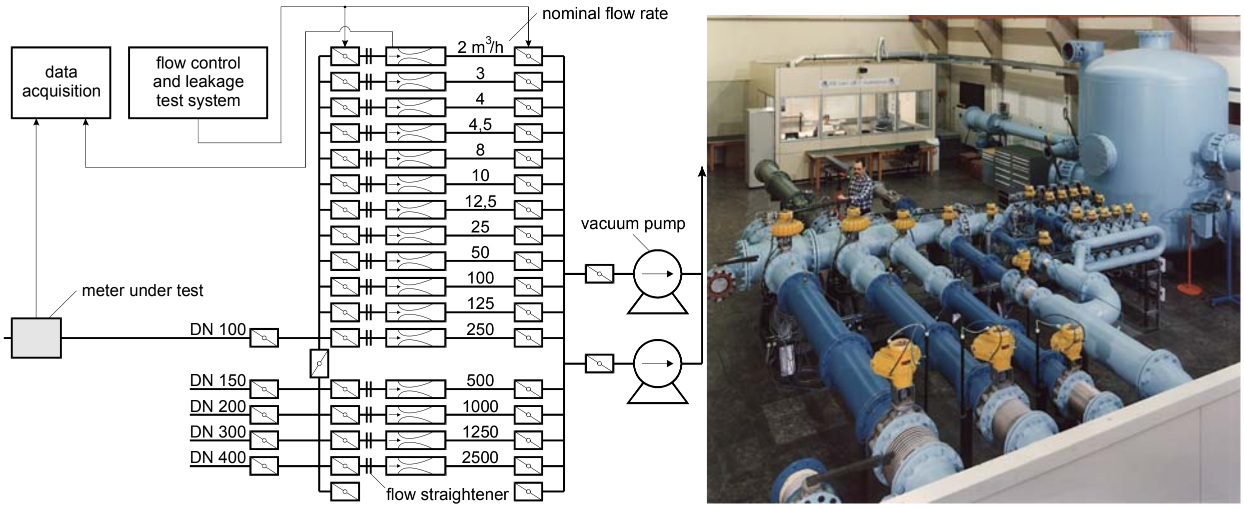


Fig. 3 The PTB Air Flow Standard.

2.2 PTB Air Flow Standard

Critical nozzles with volume flows > 100 m^3/h can be calibrated at the PTB with atmospheric air using a transfer meter. The traceability chain starts with the PTB bell prover [3] which is used to calibrate sonic nozzles at flows less than < 100 m^3/h with an uncertainty of 0.06 % (k=2). Such nozzles are used in a secondary test rig (the PTB Nozzle Rig, see Fig. 3) to calibrate meters under test [4]. The uncertainty of the PTB Nozzle Rig is about 0.08 %.

The calibration of sonic nozzles needs a further step, because the capacity of the vacuum pumps does not allow a calibration of two nozzles in series. Therefore we calibrate first a transfer meter (a turbine meter or positive displacement device) with very good short term reproducibility and then we calibrate the nozzle with the transfer meter. The pressure losses inside the test rigs allow us to vary the inlet pressure of the nozzles under test over a range of about 90 kPa to 100 kPa.

As the equipment for measurement of pressure and temperature at the transfer meter is identical for both steps of calibration, we have an additional contribution of uncertainty due to the reproducibility of less than 0.03 %.

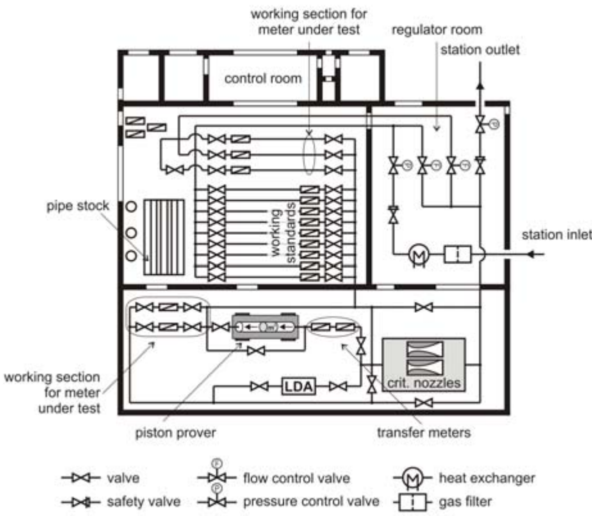
The composition of the air is the standard composition of dry air with compensation for humidity (measured with dew point meters). The influence of the humidity is calculated as given in Aschenbrenner [5]. Density and compressibility are calculated in accordance to Giacomo [6].

Finally, we claim an uncertainty of $U(C_{\mathrm{d,PTB,air}})$ = 0.12 % (k=2) for the C_{d} value of a nozzle of such a size calibrated with air at atmospheric conditions. Uncertainty contributions related to the nozzle throat diameters are omitted since all laboratories used the same values.

2.3 PTB / pigsar Flow Standard

The pigsar test facility uses natural gas at pressures between 16 and 50 bar (see Fig. 4). It is described in detail in the references [7, 8]. The traceability is based on the geometrically measured volume of a high pressure piston prover and gas density measurements made with a buoyancy balance.

The natural gas used at pigsar is a North Sea gas from the Groningen region with a typical composition given in Table 1. The critical flow factor, C^* is calculated using the AGA8-DC92 state equations according to Schley [9]. To estimate the uncertainty of this value, C^* was also calculated with the state equations of GERG2004 [GERG] [10, 11] for a wide variety of gas compositions. Also, a variance analysis was performed considering the uncertainty of gas composition. All results are documented in a reference [12].



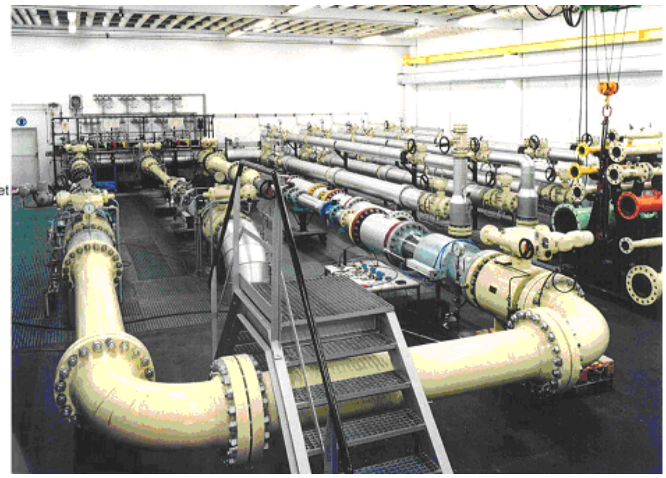


Fig. 4 The PTB / pigsar Flow Standard

Table 1 Typical composition of the natural gas used at PTB / pigsar.

Component	Mole Fraction (%)	*U* (%, *k* = 2)
CH_4	83	0.3
N_2	9.5	0.2
CO_2	1.5	0.2
C_2H_6	4.5	0.2
C_3H_8	1	0.1
C_4H_{10} and others	< 0.5	0.06

The uncertainty of the volumetric flow for a meter under test is claimed as 0.16 % (k=2). Together with additional uncertainties of the critical flow factor C^* ($U(C^*)$ = 0.065%, k=2) and molar mass ($U(M)$ = 0.1 %, k=2) we get finally an uncertainty for the C_d of the nozzles of $U(C_{d,PTB,NG})$ = 0.18 % (k = 2). The PTB / pigsar flow measurements used to calculate C_d are harmonized based on the results of the recent International Committee for Weights and Measures (CIPM) Key-Comparison KC5a [13] in natural gas.

Fig. 5 The LNE-LADG Piscine Standard.

2.4 LNE-LADG Standards

The four nozzles were calibrated at the LNE-LADG facilities against a set of working standard nozzles, previously calibrated with the Piscine *PVTt* standard (see Fig. 5). Dry air near ambient temperature with molecular weight of 28.966 g/gmole was used and the uncertainty of C^* is estimated to be 0.03 %.The uncertainty of the C_d values from LNE-LADG is 0.22 % (k=2). More details about the calibration facilities used can be found in the references [14]. The LNE-LADG flow measurements used to calculate C_d are harmonized based on the results of the recent CIPM-Key-Comparison KC5b [15] in high pressure air and nitrogen.

2.5 Correlation between LNE-LADG and PTB / pigsar Harmonized Values

The harmonized values reported by LNE-LADG and PTB / pigsar, although based on different test (*i.e.*, the KC5a and the KC5b, respectively), have common uncertainty sources and are therefore correlated. The correlation coefficient can be calculated by applying the method of propagation of uncertainty to the KC5a and KC5b results. If this is done, the correlation coefficient between LNE-LADG and PTB / pigsar can be shown to be less than 0.05. Based on this low value, the harmonized values reported by the two laboratories are nearly independent.

2.6 Nozzle Geometry

The dimensions of the four nozzles were measured at NIST by the Precision Engineering Division using a coordinate measuring machine with uncertainty better than 1 µm at a 95 % confidence level [16]. The throat diameter, d, and the radius of curvature at the throat, r_c, were determined indirectly from measurements made around the circumference at cross sections up and downstream from the throat. At each of 17 cross sections 0.5 mm apart, 12 radial measurements spaced 30° apart were made. At each cross section, an effective radius was calculated by dividing the area of a best fit ellipse by π and taking the square root. Derivatives of a polynomial fit of these radii as a function of streamwise position were used to find the throat diameter and the radius of curvature at the throat. Figure 6 shows the shape of the nozzle wall in the region 4 mm up and downstream from the throat and Fig. 7 shows the departure from circularity for 5 cross sections near the throat. All four nozzles are circular within 4 µm.

PTB also performed dimensional measurements on the four nozzles to determine the throat diameter and radius of curvature. The PTB length lab measured eight surface lines at the wall of the nozzle in the axial or z-direction. Each surface line is measured in the range of ±1 mm around the throat with 400 points (every 5 µm). Hence, we obtain eight points in the x-y-plane for every z-value. Then we applied a regression of a circle to each of this set of 8 points to obtain an average radius for each z-position. The radius as a function of axial position can be evaluated to obtain the minimum or radius of curvature (as well as other quantities such as deviation from circularity, shape deviations, and axial errors).

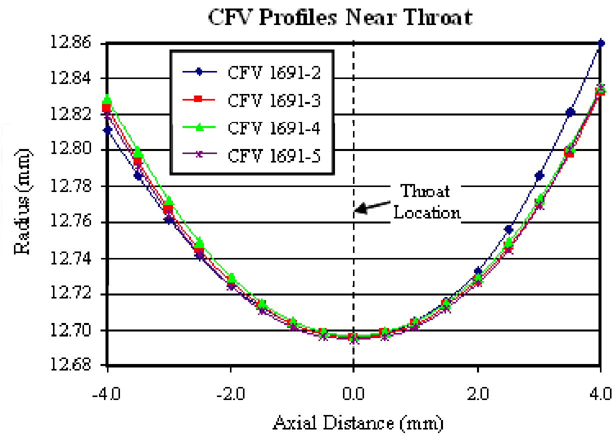


Fig. 6 Profiles of the nozzle walls near the throat of the four nozzles.

The NIST and PTB results for the throat diameter and radius of curvature are listed in Table 2 along with another quantity of interest for theoretical C_d predictions, the throat curvature ratio: $\Omega = d/(2\ r_c)$. The throat diameters of the four nozzles are the same size within 5 µm (0.02 %) according to the NIST dimensional data and within 10 µm (0.04 %) according to the PTB data.

When we compared the NIST and PTB results for r_c and Ω we found differences of 7 % or less for three of the nozzles, but of 34 % for nozzle 1691-2. The curvature difference can be explained by differences in the measurement grid used by the two labs or the streamwise length analyzed for the radius of curvature calculation. The dimensional measurements of PTB detected very small deviations from the ideal form near the throat, but these deviations are too small to influence the character of the flow significantly. The values of Ω for these nozzles fall below the minimum specification given in the ISO standard which calls for values from 0.227 to 0.278. Although the nozzle was machined to fall within the ISO specifications, it is likely that polishing after machining decreased the value of Ω. Therefore the radius of curvature is not constant in the throat region and the calculated value depends on the arc length considered. Fortunately, the theoretical C_d values are not strongly dependent on Ω and a 10 % change in the curvature ratio leads to less than 0.01 % change in the C_d values over the range of Re in this comparison.

To maintain consistency for the flow comparison, the NIST values of throat diameter were used to calculate C_d for all of the laboratories, and the average of the NIST values for the curvature ratio were used in the theoretical C_d calculations

Table 2 Throat diameters, radius of curvature at the throat, and curvature ratios for the four nozzles, based on dimensional measurements made at NIST and PTB.

SN	*d* NIST (mm)	*d* PTB (mm)	r_c NIST (mm)	r_c PTB (mm)	Ω NIST (-)	Ω PTB (-)
1691-2	25.3932	25.3968	68	103	0.186	0.123
1691-3	25.3910	25.3919	66	71	0.192	0.178
1691-4	25.3935	25.3917	62	62	0.204	0.205
1691-5	25.3883	25.3869	71	67	0.180	0.188

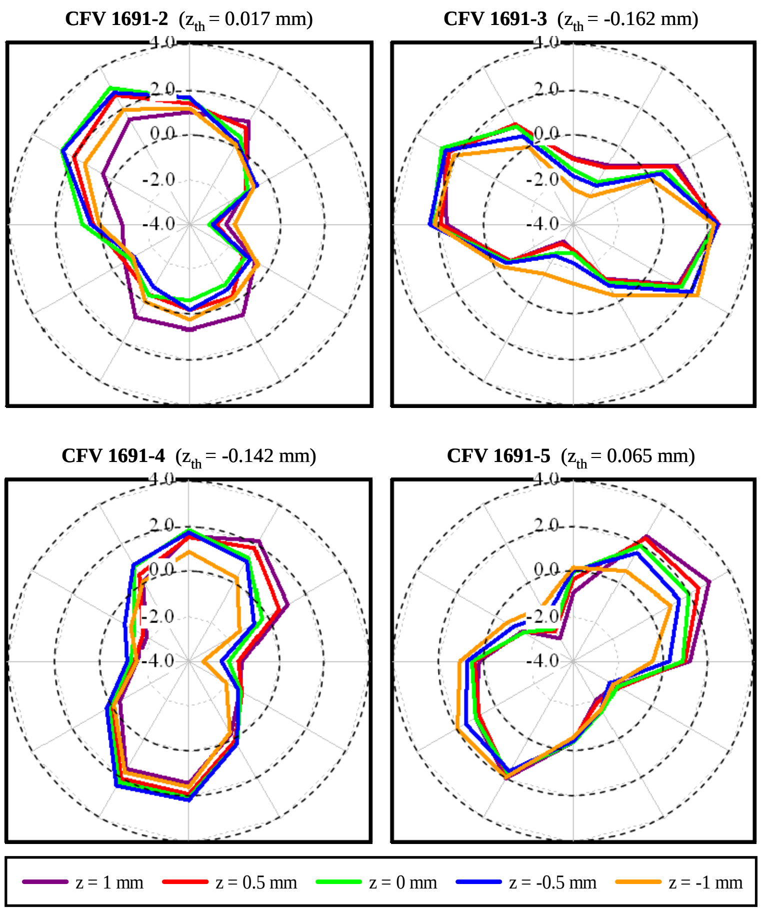


Fig. 7 The departure from circularity for the four transfer standard nozzles, i.e. difference between each of the 12 measured radii and the average radius in μm.

2.7 Theoretical Calculations of C_d

The details of the calculation of the discharge coefficient are given in other publications [17] and therefore are only briefly described here. The discharge coefficient for a critical nozzle can be calculated by combining solutions for the inviscid, core flow with solutions for the boundary layer along the nozzle wall. For the core flow, the solutions of Hall [18] were used. The boundary layer solution is tailored for whether laminar or turbulent conditions are present, and transition is expected around Re = 10^6. Mickan [17] gives a detailed description of how integral methods can be used to predict the boundary layer flow. He shows that there is no significant difference from the model developed by Geropp [19] for the laminar boundary layer (there is only a small difference in the models due to the difference in the nozzle shape used by Geropp and the toroidal shape used here). For the turbulent boundary layer, integral methods were applied to the energy equation [17].

The theoretical curve plotted in Fig. 8 is based on an average curvature ratio for the four nozzles. The curvature ratios for the four nozzles are close enough to each other that using the average values introduces no more than 0.01 % uncertainty to the theoretical C_d curves over the range of Re plotted. We considered Ω_{PTB} for CFV 1619-2 an outlier and did not include this result in the averaging process. Additional geometric measurements will be performed at NIST to validate this assumption.

Figure 8 also shows confidence bounds for the calculated C_d values. The uncertainty of the theoretical C_d's includes the uncertainty of the radius of curvature (10 %) and the uncertainty introduced by the difference between the true wall temperature and the adiabatic wall temperature assumed in theoretical calculations (10 K). The C_d uncertainty resulting from these component uncertainties was determined by varying the parameters of the mathematical model. The uncertainty of the calculated C_d is approximately 0.05 % for the range of Re discussed here.

3. RESULTS

The results of the comparison are shown in Fig. 8, a plot of C_d versus Re for the experimental measurements made in each laboratory (symbols) along with lines representing the theoretical predictions for C_d. Data for all four nozzles are presented on the same plot: different colors are used to differentiate data for each nozzle. The PTB measurements made at the two highest Re values are for natural gas and the two lowest Re sets of data from PTB are for air. Theoretical C_d curves are presented for both air and natural gas at high values of Re.

LNE-LADG and NIST calibrated the nozzles in overlapping Re range from 9×10^5 to 2×10^6 and the average C_d values of the four nozzles from the two laboratories agree within 0.05 %, even through the laminar to turbulent transition. At the highest flows (Re = 8×10^6 to 2×10^7), LNE-LADG and PTB / pigsar have overlapping calibration data that agree within 0.05 % (comparing the average of the four nozzles), despite being measured with two different gases. Moreover, the air based LNE-LADG results fall slightly below the natural gas based PTB / pigsar results, in agreement with the theoretical predictions.

NIST and PTB have no overlapping flows, but using the LNE-LADG data and the theoretical C_d curves as a bridge, we estimate that the three flow standards from these two laboratories also agree with each other within 0.05 %.

4. DISCUSSION AND CONCLUSIONS

The uncertainty of experimentally measured C_d values from each flow standard is given in a prior section of this paper and the values are shown graphically in Fig. 8. The uncertainty of theoretical C_d values is about 0.05 % in this region of Reynolds number. The largest differences between C_d's from different laboratories or the theoretical predictions is 0.15 %, and the differences are generally less than 0.1%, much less than the claimed uncertainties of the measurements and the theoretical calculations. The measured differences are well within our expectations based on uncertainty estimates and indicate that all participants are equivalent to each other. The difference in average C_d values for measurements made in air and natural gas was less than 0.05 %. This result is very encouraging for the application of nozzles calibrated in air for use as working standards in natural gas.

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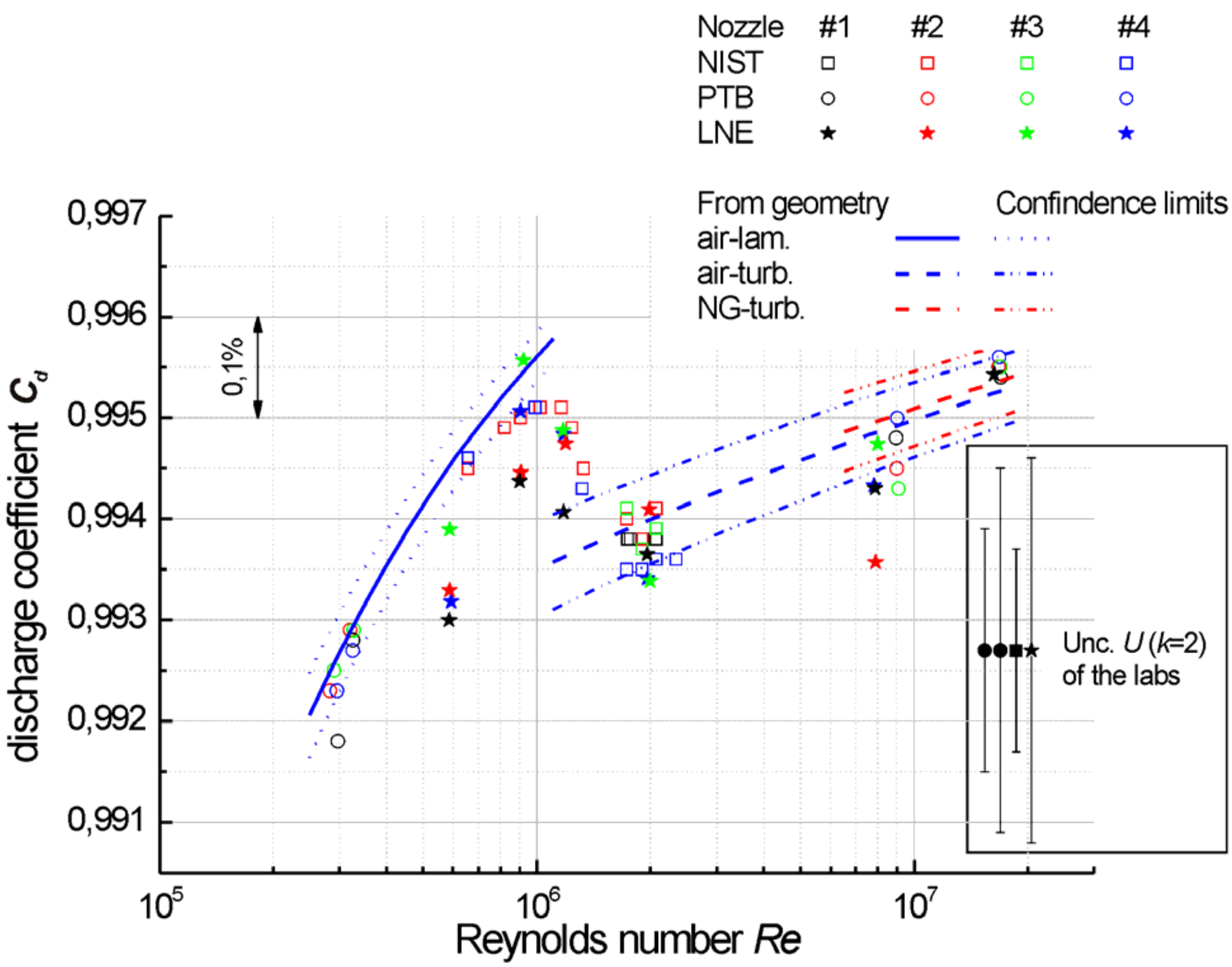

Fig. 8 Discharge coefficients of four critical venturi nozzles determined from experimental measurements at NIST, PTB, and LNE / LADG and by theoretical calculations based on dimensional measurements of the nozzle geometry. The extra lines are confidence limits for the theoretical calculations, see the legend.

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