

# Accurate determination of equilibrium state between two pressure balances using a pressure transducer

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## Abstract

To determine an equilibrium state between two pressure balances accurately, the measurement method using a precise pressure transducer and two air-operated constant volume valves (CVV) is proposed in this paper. The advantages of the proposed method are as follows: (1) by the usage of two air-operated CVV, the pressure generated by the pressure balance can be connected and disconnected quickly to the transducer without volume change in the hydraulic circuit or heat transfer from the operator, (2) by managing the time intervals between measurements equally, the method proposed can compensate for the effect of the drift component in the successive values measured by the transducer used and (3) the short time stability of the pressure generated by each pressure balance used can be evaluated quantitatively at each pressure. From the measurement results, it was revealed that the equilibrium state could be determined accurately using the method proposed, and the differences between this method and the conventional fall-rate method were sufficiently small.

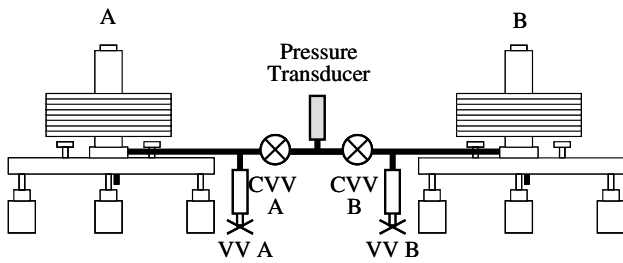
## 1. Introduction

A pressure balance is one of the most important devices used to generate accurate pressures by loading a known mass on a known effective area of the piston–cylinder assembly. For the calibration and characterization of a pressure balance, a cross-float measurement of two pressure balances is widely used in scientific research and in industry [1, 2]. In a cross-float measurement, the pressure balance to be calibrated is cross-floated against another previously calibrated pressure balance with known effective area, which provides the value for the calibration. Usually, small fractional masses on either pressure balance are adjusted until the equilibrium pressure for the two pressure balances is obtained. To determine whether the pressures generated by two pressure balances are equal or not, two methods are mainly used at present. One is the method of observing the fall-rate, which is obtained from observing the floating position of the piston of the pressure balance with time, and the other is the method of observing the differential pressure using a sensitive differential pressure cell. In both

methods, a relative resolution of the order of  $10^{-6}$  can be currently obtained to determine the equilibrium state if the performances of the pressure balances and measuring devices used are sufficiently good. In this study, a new method was used in which a precise transducer was connected first to the standard and then to the test device. The method for estimating the fractional mass to be placed on the pressure balance in order to obtain the equivalent pressure of two pressure balances is also discussed.

## 2. Apparatus

Figure 1 shows the schematic drawing of the apparatus used in this study. There are two pressure balances, two air-operated constant volume valves (CVV), two variable volumes and one transducer, which are connected using the high-pressure tubing in a hydraulic circuit. The CVV can be operated remotely by changing the air pressure supplied to the valve [3]. The CVV can connect and disconnect the pressure generated by



**Figure 1.** Schematic drawing of the apparatus, CVV: air-operated CVV, VV: variable volume.

the pressure balance to the transducer quickly with no volume change in the hydraulic circuit and no heat transfer from the operator. Two variable volumes are used to pressurize the system pressure in the circuit and to adjust the piston position of each pressure balance. To measure the pressures generated, an absolute pressure transducer, which uses two quartz crystal resonators, is used [4]. The measuring range of the transducer used is about 276 MPa and the resolution of the transducer can be selected by changing the integration time of data recording. In this experiment, the integration time was set to 3.425 s for one pressure measurement. The resolution in pressure measurement with this integration time was investigated from the data measured by the transducer and was found to be about  $5 \times 10^{-8}$  of the full scale (15 Pa). The details for the resolution are described in section 4. A customized computer program written for the measurement was used to sample all the data, which include the temperatures, piston positions of both piston–cylinder assemblies and the measured pressure compensated by the temperature.

### 3. Measurement principle

To achieve a target pressure, the approximate large mass, which is calculated using the nominal area of each piston–cylinder assembly, was applied on each pressure balance. Then, the entire system including the two pressure balances was pressurized using two variable volume injections. Normally, there is a difference between the pressures generated by each pressure balance when the fractional mass is not used. Therefore, after floating the piston of either pressure balance, CVV B (see figure 1) was closed to keep the position. In this state, the pressure generated by pressure balance A was applied to the transducer. After that, the piston positions of the pressure balances A and B were adjusted using the variable volumes A and B, respectively, to the target range, which was usually near the centre of the stroke. After pressurizing the system and waiting until the adiabatic heat had dissipated, the data collection program was started. Table 1 shows the sequence of procedures for the operation and data sampling with time. First, the program sampled the pressure generated by pressure balance A using the transducer six times every 10 s. The measurements were completed in 1 min. From the six values obtained from this measurement, the average value was calculated and is indicated as  $I_{A,1}$ . One minute after starting, CVV A was closed and CVV B was opened. The time needed for the switch was less than a few seconds at maximum. In this state, the pressure generated by pressure balance B was applied to the transducer and the program continued to mark

**Table 1.** Procedures for operation and sampling with time. O: open, C: close, M: measuring, W: waiting,  $I_{A-j}$  and  $I_{B-j}$ : average of the values measured from pressure balance A and B, respectively,  $DI_j$ : difference between  $I_{B-j}$  and the mean value of  $I_{A-j}$  and  $I_{A-j+1}$ .

	Interval/s	Valve A	Valve B	Operation	$I$	$DI$
Start	60	O	C	M	$I_{A,1}$	
	30	C	O	W		
	60			M	$I_{B,1}$	$DI_1$
	30	O	C	W		
	60			M	$I_{A,2}$	
	30	C	O	W		
	60			M	$I_{B,2}$	$DI_2$
	30	O	C	W		
	60			M	$I_{A,3}$	
	30	C	O	W		
	60			M	$I_{B,3}$	$DI_3$
	30	O	C	W		
	60			M	$I_{A,4}$	
	30	C	O	W		
	60			M	$I_{B,4}$	$DI_4$
	30	O	C	W		
	60			M	$I_{A,5}$	
	30	C	O	W		
	60			M	$I_{B,5}$	$DI_5$
	30	O	C	W		
	60			M	$I_{A,6}$	
	30	C	O	W		
	60			M	$I_{B,6}$	
	30	O	C	W		
End	60			M	$I_{A,6}$	

time. The program automatically began to sample the pressure generated by pressure balance B using the same procedure described above, 90 s after starting, and the average value,  $I_{B,1}$ , was obtained. Thereafter, the pressure generated by each pressure balance was alternately measured by the transducer by switching the valves in the circuit as indicated in table 1 and figure 1. In this method,  $I_{A-j}$  and  $I_{B-j}$  show the  $j$ th average values measured from pressure balance A and B, respectively. These measurements were iterated until  $j = 5$ . The difference,  $DI_j$ , is the difference between  $I_{B-j}$  and the mean value of  $I_{A-j}$  and  $I_{A-j+1}$ , and is calculated as follows:

$$DI_j = I_{B-j} - (I_{A-j} + I_{A-j+1})/2. \tag{1}$$

The difference between the pressures generated by two pressure balances,  $DP$ , is calculated as follows:

$$DP = f \cdot DI, \tag{2}$$

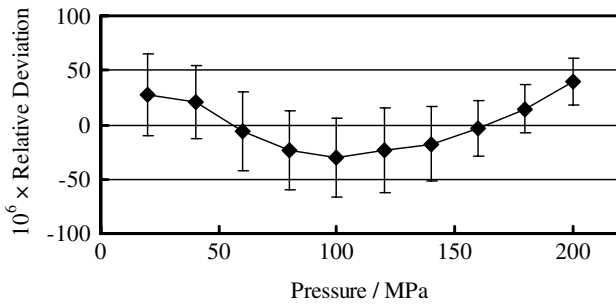
where  $f$  is the scaling factor of the transducer used and is defined by

$$P = f \cdot I + P_0, \tag{3}$$

where  $P$  is the actual pressure applied,  $I$  is the value indicated by the pressure transducer and  $P_0$  is the pressure expected at  $I = 0$ . In this paper, the unit of  $P$  and  $P_0$  is megapascal, the unit of  $I$  is the unit of transducer and the unit of  $f$  is megapascal per unit of transducer. The uncertainty of  $DP$  ( $k = 1$ ) is calculated from

$$[u(DP)]^2 = [u(f) \cdot DI]^2 + [f \cdot u(DI)]^2. \tag{4}$$

If the pressure generated by pressure balance A at its reference level,  $P_A$ , is known, the pressure generated by



**Figure 2.** Example of the characteristic of the transducer expressed as the deviations from the best fitting straight line.

pressure balance B at the reference level,  $P_B$ , is obtained as follows:

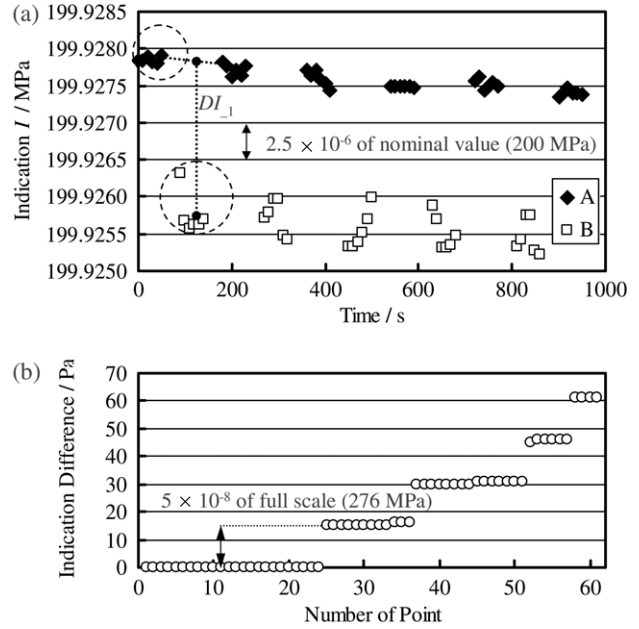
$$P_B = P_A + DP + (\rho_f - \rho_a) \cdot g \cdot \Delta h, \quad (5)$$

where  $(\rho_f - \rho_a) \cdot g \cdot \Delta h$  is the head correction, with  $\rho_f$  the density of the working fluid,  $\rho_a$  the air density,  $g$  the local acceleration due to gravity and  $\Delta h = h_A - h_B$  the vertical distance between the reference levels of both pressure balances.

#### 4. Results

The scaling factor,  $f$ , and  $P_0$  can be calculated using a regression statistics program by fitting equation (3) to the data of the relationship between the applied pressure and the indication of the transducer. These parameters can be obtained for each calibration since the known pressure generated by the reference pressure balance is applied to the transducer in ‘A’ measurement in the A(BA)<sup>n</sup> method at all the pressure points in each calibration. The deviation of the data from the fitted function is also considered in the uncertainty of  $f$ ,  $u(f)$ . With this procedure, the effects of the long-term stability and zero shift of the transducer on  $f$  are expected to be negligible. In this experiment, the pressure generated by pressure balance A was applied to the transducer in the pressure range from 20 MPa to 200 MPa in steps of 20 MPa. Figure 2 shows an example of the characteristic of the transducer used. The deviations of the data from the best fitting straight line are calculated, and the average value of the deviations to the full scale of the transducer is plotted at each pressure on the figure. The error bars show the standard deviations of the data. The coefficient,  $f$ , for a measurement was determined to be  $f = 0.99996$  MPa/(unit of transducer). As shown in figure 2, the relative deviations are normally within  $5 \times 10^{-5}$  of the full scale. Considering the environmental effects on  $f$ , the standard uncertainty ( $k = 1$ ),  $u(f)$ , was safely estimated as  $5 \times 10^{-4}$  MPa/(unit of transducer) in the pressure range measured. This value of  $u(f)$  gives a band on how large  $u(DP)$  should be for a given and/or derived uncertainty. Generally, the smaller  $DI$  makes the smaller  $u(DP)$ . For example, if the relative difference  $DI/I$  is less than  $10^{-4}$  with the relative uncertainty  $u(f)/f = 5 \times 10^{-4}$ , the uncertainty component  $u(f) \cdot DI/P$  would be less than  $5 \times 10^{-8}$ .

Figure 3(a) shows an example of the indications measured by the transducer during the measurement performed according to the method shown in table 1. In the figure, the vertical space between horizontal lines corresponds to



**Figure 3.** (a) Example of indication of the transducer during measurement and (b) indication difference between rearranged readings, evaluated from the data plotted in (a).

$2.5 \times 10^{-6}$  of the nominal value. From the measurement, six values of  $I_A$  and five values of  $I_B$  were obtained. The scatter of the data of  $I_B$  is larger than that of  $I_A$ . To express the scatter of each  $I_A$  or  $I_B$ , the standard deviation of the mean was calculated as  $s_A/\sqrt{6}$  or  $s_B/\sqrt{6}$ , where  $s_A$  or  $s_B$  is the standard deviation from six points. The measurement shown in figure 3(a) was repeated four times with ascending and descending pressures. The uncertainty of  $I_A$  and  $I_B$ ,  $u(I_A)$  and  $u(I_B)$  was calculated as the average of 24 values of  $s_A/\sqrt{6}$  and 20 values of  $s_B/\sqrt{6}$ , respectively. Compared with the stability of the pressure balance used, the transducer has a relatively large drift component with time. However, the rate of the drift is almost constant during short periods; therefore, the effect of the drift component on evaluating the difference value,  $DI$ , can be greatly mitigated by managing the time intervals between the measurements equally using the A(BA)<sup>n</sup> method described above. Concerning the difference,  $DI$ , 20 values from the complete measurements at each pressure were obtained. The uncertainty of  $DI$  by Type A evaluation,  $u_a$ , is calculated as the standard deviation of the mean,  $s_{DI}/\sqrt{20}$ , where  $s_{DI}$  is the standard deviation from 20 values of  $DI$ . The combined uncertainty ( $k = 1$ ) of  $DI$ ,  $u(DI)$ , is evaluated from  $u(I_A)$ ,  $u(I_B)$  and  $u_a$  by the root sum of squares method [5]:

$$[u(DI)]^2 = [u(I_A)]^2 + [u(I_B)]^2 + u_a^2. \quad (6)$$

From the data plotted in figure 3(a), the resolution of the pressure transducer was evaluated quantitatively. First, all the plotted pressure values of 66 points were rearranged in order of magnitude and the differences between two sequential pressure values were calculated. Figure 3(b) shows the differences arranged in order of magnitude. For convenience, the differences of the first 61 points are plotted. As shown in the figure, the resolution of the transducer in the pressure measurement is about 15 Pa, which corresponds to about

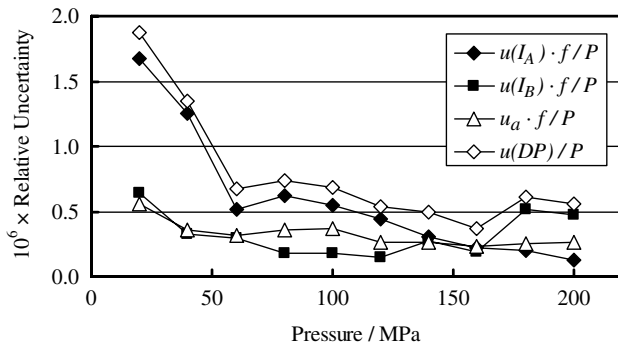


Figure 4. Uncertainty evaluation of  $u(I_A)$ ,  $u(I_B)$ ,  $u_a$  and  $u(DP)$ , ( $k = 1$ ).

$5 \times 10^{-8}$  of the full scale of the transducer. The resolution was also confirmed at other pressure points using the same method.

Figure 4 shows the evaluation results of the relative uncertainties as absolute values,  $u(I_A) \cdot f / P$ ,  $u(I_B) \cdot f / P$ ,  $u_a \cdot f / P$  and  $u(DP) / P$ , in units of  $10^{-6}$  ( $k = 1$ ), calculated at each pressure from 20 MPa to 200 MPa.  $u(DP) / P$  was calculated using equations (4) and (6) and was the same as  $u(DI) \cdot f / P$  within  $10^{-9}$  in this example. Examining the sources in the uncertainty of  $u(DP)$ , the uncertainty arising from  $u(I_A)$  is relatively large in the lower pressure range, and  $u(I_B)$  is the dominant component in the higher pressure range above 180 MPa. In this method,  $u(I_A)$  and  $u(I_B)$  are regarded as short period stabilities of the pressure generated by the pressure balances A and B and can be evaluated, respectively, as shown above. In the conventional fall-rate method or the differential pressure cell method, it was difficult to obtain the quantitative value of the stability for each pressure balance separately since the two pressure balances were connected in the measurement. For the present results, the maximum uncertainties in  $u(DP) / P$  at 20 MPa and 40 MPa exceed  $1 \times 10^{-6}$ . However, as understood from figure 4, the major uncertainty source comes from  $u(I_A)$ , which shows the stability of pressure balance A. Use of a more stable pressure balance should reduce the overall uncertainty.

From the difference pressure  $DP$  obtained, the fractional mass,  $m_A$  and  $m_B$ , that should be placed on the pressure balance A or B to obtain the equilibrium pressure can be approximately estimated, if the head correction between two pressure balances is sufficiently small, as follows:

$$m_A = M_A \cdot DP / P_A \quad \text{or} \quad m_B = -M_B \cdot DP / P_B, \quad (7)$$

where  $M$  is the total mass of the loaded weights including the piston. For the example shown in figure 3(a), the fractional mass  $m_B$ , which should be placed on the pressure gauge B to obtain the equilibrium pressure, was calculated. After adjusting the fractional mass, the relative difference,  $DP / P$ , was less than  $1 \times 10^{-6}$ .

Figure 5 shows the relative differences of the effective areas of test pressure balance B obtained from the conventional fall-rate method and the proposed method under the same condition in all cases of  $3 \times 10^{-6}$  or less. The relative combined uncertainties of the effective areas, including Type B effects independent of the equilibrium evaluation, are in the range  $2 \times 10^{-5}$  to  $3 \times 10^{-5}$  approximately as a function of

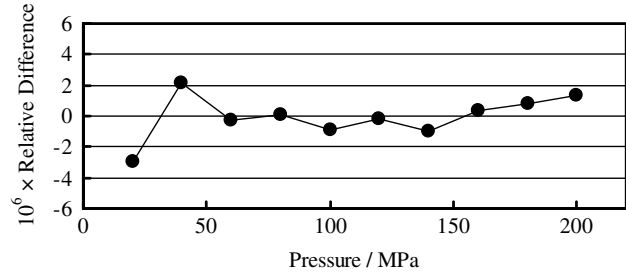


Figure 5. Relative differences of the effective areas obtained from the proposed method and the fall-rate method.

pressure. Therefore, the relative differences from the two methods are sufficiently small compared with the relative uncertainties of the effective areas as shown in figure 5. As mentioned previously, the uncertainty in the proposed method is dependent on the relative difference  $DI / I$ ; however, this component is sufficiently small if  $DI / I$  is less than  $10^{-4}$ . One national metrology institute (NMIJ/AIST) participated in key comparison CCM.P-K7 in the range 10 MPa to 100 MPa of hydraulic pressure using the proposed method. In their result, no systematic error caused by this method was found and the good repeatability in the uncertainty evaluation was demonstrated. The details are described in the comparison report [6].

### 5. Conclusions

To determine the differential pressure accurately, the measurement method using a precise pressure transducer and two air-operated CVV was proposed in this paper. From the measurement results, it was revealed that the differential pressure could be determined accurately using the convenient method proposed, and the differences between the present and the conventional fall-rate methods were sufficiently small. The advantages of the proposed methods are described. (1) By the usage of two air-operated CVV, the pressure generated by the pressure balance can be connected and disconnected quickly to the transducer without any volume change in the hydraulic circuit and any heat transfer from operator. (2) By managing the time intervals between measurements equally using the program developed, the present method can generally remove the effect of the drift component in the successive values measured by the transducer used. (3) The short time stability of the pressure generated by each pressure balance used can be evaluated quantitatively at each pressure.

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