A Magnetic Levitation Technique for the Simultaneous Comparison of Mass Artifacts in Air and Vacuum

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***Abstract*—We describe a mass comparison system in which a test mass artifact in air can directly be compared to a standard mass artifact in a vacuum using the same high-precision comparator balance. The system uses a magnetic levitation technique to couple the weighing pan of the comparator balance in a vacuum to the test mass in air. We will discuss motivation, design considerations, and preliminary results.[1]**

***Index Terms*—Kilogram, magnetic levitation, mass, vacuum.**

I. Introduction

THE SI unit of mass is currently defined as "the mass of the international prototype of the kilogram" (IPK) [1]. Implicit in this definition is the storage and use of the IPK in air at the local atmospheric pressure. The IPK is used to calibrate national standards of platinum–iridium alloy, which are in turn used to disseminate the kilogram to the world. Periodic verifications of national standards against the IPK have shown a time-dependent drift [2], leading to the need for a redefinition of the kilogram in terms of an invariant constant of nature, such as the Planck's or Avogadro's constant. However, as part of its 2007 report to the International Committee of Weights and Measures (CIPM), the Consultative Committee for Mass and Related Quantities (CCM) recommended that no redefinition of the kilogram takes place until the results from the watt balance (measuring the Planck's constant) and Avogadro's experiments agree at the 95% level of confidence and until the relative standard uncertainty of the best realization of the definition of the kilogram does not exceed two parts in 10^8 at the level of 1 kg [3]. The major experiments being considered for the realization of a new definition of the kilogram all operate in a vacuum environment [4]–[6]. Assuming the continued dissemination of the kilogram based on calibrated artifacts, there will be a need to realize the new kilogram definition in air and in a vacuum. Hence, for the mass community, it is of critical importance to develop a suitable *mise en pratique* or set of instructions and recommendations for a practical and universally followed way of realizing the new definition (whatever it turns out to be) [7]. Transfer of the unit to a vacuum requires an unbroken traceability chain to the IPK, as well as characterization of the stability of the artifacts and their surfaces during transfers from air to a vacuum and *vice versa*. This requirement remains a major challenge for further advances in redefining the unit of mass. Therefore, there is a critical need for redefining how artifact mass metrology is maintained and disseminated. Furthermore, it requires the ability to measure artifacts in a vacuum and directly tie the vacuum measurements to both the IPK and the alternative definition with a measurement uncertainty that is equal to or less than the uncertainty of the alternative definition of the kilogram.

We are constructing a mass comparison system that allows for the simultaneous comparison of a standard artifact in vacuum with a test artifact in air using the same high-precision mass balance. It will be used to realize vacuum mass measurements with direct traceability to the IPK through the U.S. national prototypes. An "invisible coupling" between the mass balance, which is maintained in a vacuum environment, and the test mass artifact in air will be made using a magnetic levitation technique that maintains complete isolation between the atmospheric pressure and vacuum sections (see Fig. 1). The goal of the comparison system described here is to provide a link between the new kilogram definition in vacuum and mass artifacts in air with a relative uncertainty of about 1×10^{-8} on comparisons of 1-kg artifacts. To this end, a proof-of-concept apparatus was built that successfully magnetically coupled a 1-kg artifact to a mass balance in air. Although there were hardware and environmental issues that limited the performance of the comparison system, we were still able to achieve a repeatability of a few parts in 10^6 with this apparatus [8].

II. History and Motivation

A good review of magnetic suspension systems is given by Jayawant [9]. The first magnetic suspension balances were developed by Holmes [10] and Clark [11] prior to 1950. In 1955, Beams *et al.* [12] described a magnetic suspension balance that could detect changes in mass of 10^{-11}–10^{-12} kg when the total mass was on the order of 10^{-6}–10^{-7} kg and was used to study the adsorption of gases on surfaces. Beams *et al.* claimed that

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[1]Certain commercial equipment, instruments, or materials are identified in this paper. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the products identified are necessarily the best available for the purpose.

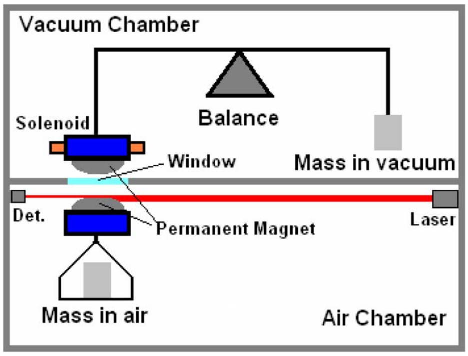


Fig. 1. Schematic of a magnetically coupled mass comparison system that functions in air and in a vacuum.

the precision of this balance was limited only by the "natural fluctuations" in the electronics and mechanical support system. Of particular interest is their note that the balance "is especially useful where it is necessary to weigh materials inside of sealed chambers which contain gases, vapors or liquids, or which are evacuated."

In the early 1960s, Gast developed a magnetic suspension balance, which was commercially produced by Sartorius [13]. Versions of this balance are now produced by Rubotherm [14] and are capable of weighing loads up to 80 g in a vacuum with a resolution of 1 μg. Abbess Instruments [15] also makes a magnetically levitated instrument for use in vacuum that can be operated as a mass balance, a force indicator, or an accelerometer. The maximum load is 240 g with a resolution of 0.1 μg.

The load and resolution required for the transfer to atmospheric air ($\sim 10^5$ Pa) of a new kilogram definition in vacuum precludes the use of any of the aforementioned commercially available instruments. For an alternative definition of the kilogram to rival the current artifact definition, a relative uncertainty of 2×10^{-8} or better is required; therefore, the necessary specifications for the balance are a load of 1 kg with a resolution of 10 μg. To achieve this precision, it was necessary to design and build our own magnetic suspension balance based on a commercially available high-precision mass balance that is specially modified for use in a vacuum environment.

It must be mentioned here that vacuum mass measurement is a new and fast growing area. Balances that can operate in a vacuum environment eliminate the need for an air buoyancy correction and its associated uncertainty. Currently, many National Measurement Institutes (NMIs) have efforts underway, including the National Physical Laboratory (NPL) [16], U.K.; the Bureau International des Poids et Mesures (BIPM) [17], Sevres, France; the National Metrology Institute of Japan (NMIJ) [18]; the Laboratoire National d'Essais (LNE) [19], Paris, France; and the Swiss National Laboratory (METAS) [20]. However, these approaches are limited by the necessity of the characterization of adsorption layers on the artifacts during vacuum-to-air exchanges and *vice versa* (for example, see [20]). In his review of surface contamination and the stability of mass standards, Davidson [21] identifies the following work to be done with regard to mass stability and surface effects:

1) the effect of transfer between air and vacuum on the value and stability of mass standards;

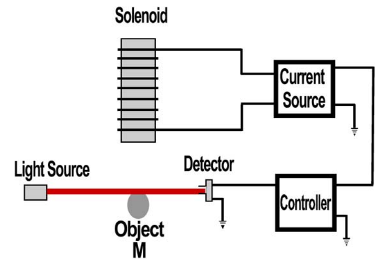


Fig. 2. Simple magnetic levitation system. A shadow detector and a feedback controller are used to suspend a ferromagnetic ball.

2) an investigation of the correlation between surface contamination (measured by surface analysis) and mass gain (measured gravimetrically);
3) an investigation into the long-term stability of mass standards stored in a vacuum or in an inert atmosphere.

There is clearly a need for the development of stable mass artifacts in order to provide the highest quality dissemination of mass metrology. Our laboratory is engaged in this effort, in conjunction with the development of the high-precision magnetic levitation balance. More information on this topic will be presented in a future publication.

III. Magnetic Suspension System

Magnetic suspension of a ferromagnetic object is conceptually straightforward; free suspension in air or in a vacuum is achieved when the sum of the forces acting on the object is zero. In the case of magnetic coupling, this amounts to the gravitational force on the object being perfectly balanced by the magnetic force, i.e.,

$$\vec{F} = m\,\vec{g} - \vec{f}(t). \tag{1}$$

In (1), F is the net force on the levitated object, m is the mass of the object, g is the acceleration due to gravity, and f is the magnetic force, which is time dependent. Fig. 2 illustrates a simple case of a magnetically suspended ferromagnetic ball. As a result of Earnshaw's theorem [22], stable levitation of a ferromagnetic object cannot be achieved through the exclusive use of a static magnetic field. Instead, it is necessary to regulate the magnetic field using an active feedback control system, e.g., a solenoid, whose magnetic field can be varied.

The controller and position sensor form the basis of a servo circuit that adjusts the magnetic field in order to maintain stable levitation. Paschall built a low-cost system similar to Fig. 2 and demonstrated the ability to measure the levitated ball's position with a resolution of 45 μm [23]. Referring to Fig. 3, a desired levitation position (vertical distance from the solenoid, i.e., Y_{desired}) is entered into the controller, which provides a current $i(t)$ to the solenoid; in turn, the magnetic field produced by the solenoid exerts an attractive force $F(t)$ on the ball and raises it to position Y. The sensor detects the position of the ball and feeds this back to the controller in the form of a voltage P that the controller uses to modify the solenoid current. In this iterative fashion, the position of the ball is stabilized, and the degree of stability depends on how sensitive the controller is

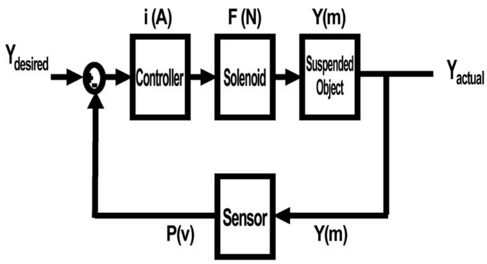


Fig. 3. Block diagram of magnetic suspension system showing the function of each element in the feedback loop.

to perturbations in the levitated position of the ball and how quickly it can respond to correct them. Note in Fig. 3 that the units associated with each variable appear in parentheses.

There is a large body of literature on controller design for magnetic levitation systems [24]–[31]. The force/current/air gap relationship of magnetic suspension systems is nonlinear, which places significant demands on the control techniques used [28]. Various methods are used to linearize the dynamic equations of motion and the electrical relationships in order to develop a controller that will provide the level of stability required for the application. For the present mass balance application, the desired noise floor is 0.02 mg on a 1-kg measurement, corresponding to a relative uncertainty of 2×10^{-8}.

In order to evaluate the magnetic fields produced by a given levitation design, we used commercially available finite-element analysis-based (FEA) simulation software to solve Maxwell's equations. The software is very versatile, allowing the use of both static and dynamic fields, as well as magnetic and nonmagnetic materials. Construction materials are chosen from a built-in library and configured into an array that can be solved both two and three dimensionally. From the FEA solution of the array, one can create flux plots and determine the forces acting on the levitated body. The force F on an object having a magnetic moment m in an external magnetic induction B may be calculated from the following vector equation [32]:

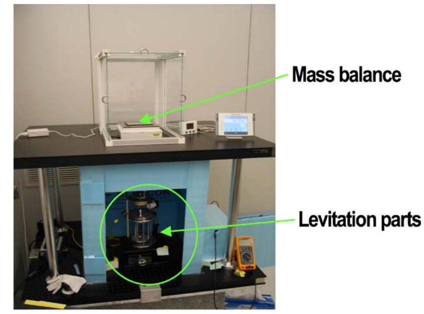
$$\vec{F} = \vec{\nabla}(\vec{m} \cdot \vec{B}). \tag{2}$$

Here, F is the force, m is the magnetic moment of the object, and B is the magnetic flux. Equation (2) states that the force varies as the gradient of the magnetic field; in other words, a *nonuniform* magnetic field is necessary for a force to be exerted on the object.

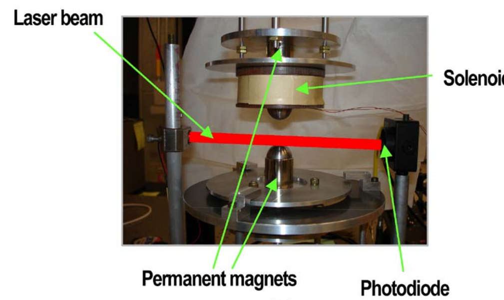
In order to evaluate the feasibility of this project, a proof-of-concept apparatus was built based on the results of many simulated configurations and will be described in the next section.

IV. Proof-of-Concept Apparatus

Magnetic levitation may be achieved with attractive or repulsive combinations of magnetic forces. After completing several simulations, we decided to build a proof-of-concept levitation system using an attractive magnetic force model. Although simulations indicated that the sensitivity of the vertical force versus gap distance may be a factor of 10 larger than for the models using a repulsive force, we used the attractive force design because of its relative simplicity and the presence of a horizontal restoring force. Fig. 4(a) and (b) shows photographs of the proof-of-concept apparatus. For simplicity, the proof-of-concept apparatus was constructed with both masses in air at atmospheric pressure. Just as in Fig. 1, the vertical position signal is provided by a "shadow sensor" photodiode that detects a laser beam that just skims the top of the magnetic pole that is attached to the levitated assembly. Referring to Fig. 4(b), the magnetic field is produced by a combination of ring-shaped permanent magnets and a solenoid. The permanent magnets are attached to both the stationary (upper) and levitated (lower) poles; the solenoid is fixed to the stationary pole and is wound with approximately 2000 turns of a 26-gauge insulated wire. The poles themselves are made from soft iron and have a nearly hemispherical shape. The permanent magnets have a coercivity of about 860 000 A/m and were chosen so that the magnetic induction is about 0.5 T when the poles are 10 mm apart, and the solenoid provides a coaxial "trimming" field in order to achieve stable levitation. The solenoid field is servo controlled about a position corresponding to a 1-V output of the photodiode detector. The output voltage of the detector swings from 0 V when completely dark to +1.5 V when the laser beam directly falls in the center of the sensor; the output voltage is converted to a proportional current in the range of ±200 mA by a homemade transistor-based voltage-to-current converter. During levitation, the laser beam is partially blocked by the levitated pole, reducing the photodiode output to approximately 1 V. The photodiode detector has a linear response of 0.8 V/mm to light levels corresponding to a change in vertical position of ±1 mm of the levitation parts. Any change in the detected light level will cause a corresponding change in the current fed to the solenoid, and the solenoid field will be appropriately increased or decreased to maintain stable levitation. Servo control is implemented with a simple proportional–integral–derivative



(a)



(b)

Fig. 4. (a) Proof-of-concept apparatus for a levitation mass comparison system. (b) Close-up of levitation parts in a proof-of-concept apparatus.

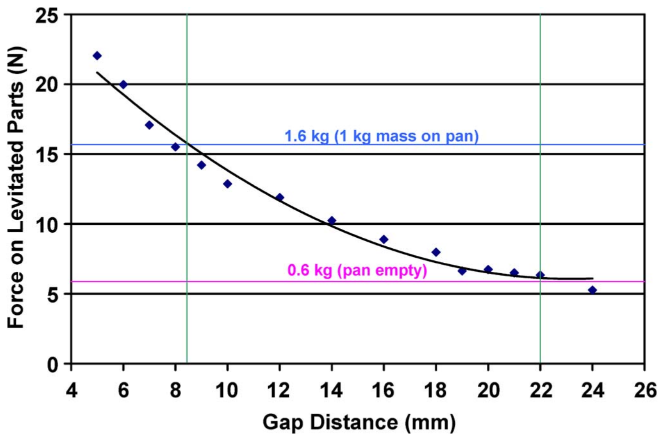


Fig. 5. Magnetic force as a function of distance between the fixed pole and the pole attached to levitated parts (gap distance).

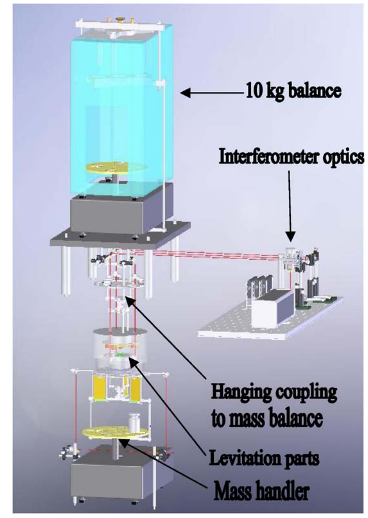


Fig. 6. Main components of the next-generation magnetic levitation mass comparison system.

position loop controller computer board using the photodiode input. Readings from the shadow detector are taken every 4 ms (250 Hz), establishing the response time of the control circuit.

The mass of the levitation parts, including the pan suspension, is 0.60 kg. Fig. 5 is a plot of magnetic force versus pole separation calculated from the FEA simulation software using the construction parameters of the proof-of-concept system. This plot shows that the stable separation position between the fixed and levitated parts (mass pan without a 1-kg mass) is 22 mm. When a 1-kg mass is placed on the pan, the total mass of the levitation parts is 1.60 kg, and it can be balanced at a separation distance of 8.5 mm.

A 10-kg balance was used in the proof-of-concept apparatus. The manufacturer's specified readability and repeatability of this balance are both 1 mg, but the precision of actual measurements will depend on environmental and experimental conditions. Initial measurements of a 1-kg mass under magnetic suspension gave a standard deviation of 11 mg, which is a factor of 10 higher than the balance's specification. Much of this noise was found to come from turbulent airflow and vibration within the laboratory, as well as instability of the position of levitation. The accuracy of mass measurements is also influenced by the choice of materials used in construction. For instance, a parasitic magnetic force was found between the levitation device and the test table; this was not unexpected, as the table had significant permanent magnetism. This extra force adds to the force provided by the magnetic poles so that when the test mass was placed on and off the levitated mass pan, the distance between the table and the levitated parts changed by 12 mm instead of the expected 13.5 mm (see Fig. 5). This effect causes the balance reading to be inaccurate, and we found that the extra magnetic force was the cause of a 1.2-g change in a 1-kg artifact's apparent mass. After replacing the ferromagnetic table with a nonmagnetic optical table, the difference between the 1-kg mass measurements on the balance pan and on the levitated pan was reduced to 33 mg. The proof-of-concept apparatus confirmed that stable levitation of a 1-kg mass was possible in air, and it identified magnetic interactions that would need to be corrected in order to eliminate spurious forces. Indeed, after enclosing the levitation apparatus with a stiff insulating foam board, replacing the original table with an all-aluminum model, and eliminating ferromagnetic materials in support apparatus (some inadvertently used nuts and screws in a laboratory jack support), the stability improved by a factor of 10 to the 1-mg level, which is the resolution limit of the balance.

In summary, we achieved stable levitation of a 1-kg mass at a pole separation of about 8.5 mm and a solenoid current of 0 ± 10 mA. It should be noted that under these conditions, we found the response of the solenoid current to changes in the levitated mass to be linear and approximately 2.7 mA/g.

V. Next-Generation Magnetically Coupled Balance

An improved magnetic levitation system that addresses the problems identified in the proof-of-concept apparatus has been designed and is under construction. The major improvements are given in the list that follows.

1) *Nonmagnetic environment.* All vacuum fittings, nuts, bolts, washers, and clamps are made of aluminum, brass, or phosphor bronze (the copper alloys were tested for residual magnetism prior to use). The apparatus now sits on an all-aluminum optical table that is anchored to concrete pillars via aluminum vibration dampening legs. No stainless steel components of any kind were used.
2) *Improved position detection.* The shadow detector is being replaced by a laser interferometer with a subnanometer resolution (see Fig. 6) with the goal of limiting the variation in magnetic force due to positional fluctuations to less than 100 μg. There are five interferometers altogether: three for vertical displacement measurements (in order to detect and correct tilt) and two for horizontal displacement measurements; we are also experimenting with Hall sensors and specially designed linear variable differential transformers as possible position indicators. Rotational control will be achieved using electrostatic plates [33].
3) *Vacuum-compatible high-resolution mass comparator.* There is a new vacuum-compatible balance with higher readability and stability; this new balance was specially

manufactured to be vacuum compatible, i.e., free of oil, paint, plastics, and other materials that would cause outgassing of contaminants. The balance has a maximum load of 10 kg and a resolution of 10 μg and has been specially modified to enable magnetic coupling to the levitation pan while maintaining the ability to weigh on its regular mass pan with the levitation parts decoupled.

4) *Magnetic shielding.* The magnet poles are encapsulated in magnetic shielding in order to prevent leakage fields that may influence the stability of the balance or cause spurious forces on magnetic objects in the room.

5) *Vacuum vessel.* The entire system is installed in a specially designed nonmagnetic vessel that contains two chambers separated by a glass interface in order to accommodate the simultaneous need for air and vacuum. The vessel was custom made of 6061 aluminum by the NIST Fabrication Technology Division. The upper and lower chambers are cylinders of about 84 cm in diameter and 56 cm in length. A separate aluminum cylinder closed at one end serves as a "bell jar cover" for the upper chamber. This removable bell jar is 84 cm in diameter and 58 cm in length and is sealed to the upper chamber with an elastomer gasket. Lifting the bell jar from the upper chamber allows access to the mass balance, interferometer optics, and environmental sensors. The vessel is designed so that either the upper or the lower chamber may be evacuated, although the most frequent configuration will be with the upper chamber (containing the mass balance) evacuated and the lower chamber at atmospheric pressure. The pumping system is completely "dry," consisting of a 240 L/s turbodrag pump backed by a piston pump that also serves to initially evacuate the chamber. Ultimate pressure is designed to be 10^{-3} Pa. Several glass viewports are included to allow direct observation of the system, as well as to allow the entrance and exit of laser beams for the interferometers.

6) *Automation.* The operation of the entire system will be automated. This includes the magnetic levitation, position detection, mass comparison, data acquisition, and analysis.

VI. Summary

A mass comparison system has been constructed for the purpose of comparing an artifact in air to a mass standard in a vacuum. The goal of this system is to establish a *mise en pratique* for the dissemination of a vacuum-defined kilogram, which may occur as early as 2011. The system uses a magnetic coupling arrangement to enable the weighing of an artifact in air by a mass balance in a vacuum. Outside of environmental noise (vibrations, thermal instability), the major limitation of the precision of the system is the ability to which the position of the levitated mass can be controlled. The next-generation system will use a laser interferometer-based position detector that has a resolution of less than 0.5 nm. Combined with other system improvements, including a more precise balance and a better control algorithm, we expect the precision to be on the order of 100 μg or better.

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