Traversability Metrics For Urban Search and Rescue Robots On Rough Terrain

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***Abstract*— Rough terrain, such as the rubble that we would expect to find in urban disaster areas, will likely impede robot mobility. The goal of this paper[1] is to find methods for quantifying the difficulty a robot should encounter traversing a region of rough terrain. We construct three metrics describing rough terrain robot mobility. In order to simplify the problem we assume that the rough terrain in question can be discretized in a certain manner and then we develop the metrics for this discretized version of the terrain. Two of these metrics reflect the difficulty a robot would have trying to move over the entire region of terrain, which is what we refer to as the coverability. The other metric describes the difficulty a robot would encounter attempting to move from some fixed point on the terrain to some other fixed point, which we call the crossability. We compute some coverability numbers for NIST step fields and briefly analyze the numerical data that are obtained.**

Keywords: *roughness, rough terrain, step field, traversability, coverability, crossability*

I. Introduction

When a robot is to be deployed in an urban disaster area we should expect it to encounter many different types of terrain that will pose varying degrees of difficulty to its mobility. For instance, rubble will often be present in such environments and the various properties of the rubble will greatly influence a robot's motion capabilities. Some of the aspects of the terrain that will affect the mobility of robots attempting to traverse it include, but are not limited to, the following:

- How rough is the terrain, i.e., how large are the small scale height variations of the terrain?
- What is the terrain's composition, i.e., what is it made of?
- Is the terrain stable or are there loose sections of the terrain?

In order to effectively use robotic tools for urban search and rescue, we must first come up with an accurate and robust system for classifying the traversability of the different types of terrain the robots will be encountering. It is too difficult to address all of the issues related to terrain traversability simultaneously so this paper will focus on classifying the traversability of terrain that is assumed to be uniform in composition and stable but has varying degrees of roughness.

We develop three different metrics for terrain traversability. Two of the metrics correspond to the difficulty a robot would have attempting to cover every part of the terrain. It is important to be able to measure such a quantity since a robot performing a search and rescue mission might have to cover all of the terrain in order to be sure that no victims are located in that region. The other metric corresponds to the difficulty a robot would have moving from a given point on the terrain to some other point. This is also a quantity we would like to be able to measure since we might have some information regarding where a victim is located so we may wish to send a robot directly to that location.

In Section II of this paper we briefly discuss some of the previous research that has been done in the field of rough terrain robot mobility. Section III defines some key terms and concepts that are needed in order to understand the work that is being described in this paper. In Section IV we develop two metrics for terrain coverability, which represents the difficulty a robot would have moving over every part of a region of rough terrain. Section V addresses terrain crossability, which is the difficulty encountered when a robot tries to move from some fixed point to another given point. Then, in Section VI we present some numerical results obtained by computing the coverability metrics for four different step fields. Finally, Section VII discusses some of the conclusions that can be drawn from the research that we have conducted.

II. Previous Work

A fairly large amount of research has been performed in the area of robot mobility, with much of the work on rough terrain mobility being done in the past ten to fifteen years. For a detailed survey of previous work, one should read chapter one of [1]. While some of the results of this research are useful in the construction of traversability metrics, most of it is not directly applicable for several key reasons.

Most past research on rough terrain robot mobility has focused on a detailed analysis of a specific type of robot traversing a region of rough terrain. In particular, researchers have come up with relatively complex mathematical models

[1]Commercial equipment and materials are identified in this paper in order to adequately specify certain procedures. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

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describing the static and dynamic stability of an individual robot moving on rough terrain, often for the purposes of path planning. For examples of such work see [2] and [3]. While such detailed models for specific robots may prove to be quite accurate, they fail to display a satisfactory degree of generality for our purposes since they cannot be applied to a wide enough range of robot geometries. Therefore, these robot-specific models should not be used to construct traversability metrics, which should mostly be classifying the inherent properties of the rough terrain. As a result, we must construct simpler, more general models that take into account only a minimal amount of information pertaining to the robot and focus more heavily on terrain characteristics.

Also, many traditional models of rough terrain mobility used for path planning represent obstacles and open space in a binary format. As a result, every point on the terrain is either considered an obstacle, meaning it cannot be traveled over, or completely open, meaning it poses no difficulty to robot mobility. This is undesirable for our purposes since there should be a continuous scale between terrain that can be traversed very easily and impassable obstacles. For a survey of traditional path planning methods, one should consult [4].

Although very little research has been done on objectively classifying rough terrain traversability for mobile robots, there are a few notable papers on this topic. For example, both [5] and [6] discuss the construction of a so-called traversability index, which is meant to classify the difficulty a robot would encounter attempting to traverse a region of terrain. In both of these papers, fuzzy logic is used to obtain the traversability index.

III. Definitions: Roughness, Traversability, and Step Fields

A. Measuring Roughness

Roughness is defined to be a measure of the small-scale variations in the height of a physical surface. Hence, for the purposes of this paper, we shall let rough terrain refer to terrain that is uniform in composition and stable but may display significant small-scale height variations. We expect terrain roughness to be directly related to robot traversability [7].

There is no single universally-accepted method for quantifying the roughness of a surface and the different methods of roughness classification may be suitable for different purposes. For example, many researchers use statistical roughness parameters such as average roughness (R_a), root mean square roughness (R_q), or the maximum peak height (R_p) to describe a surface's roughness [8]. Others feel that the fractal dimension of a surface is a good way to numerically characterize its roughness, although, it only makes sense to speak of the fractal dimension of a surface if that surface displays some sort of self-similarity at different magnification scales [9]. Still others construct their own roughness indices for their own specific purposes. For example, the roughness of natural water channels is often specified by a number called Manning's n-value [10].

B. From Roughness to Traversability

Even if we can decide on the appropriate method for quantifying the roughness of some patch of terrain, we will still need to find a way to go from terrain roughness to terrain traversability. We know that terrain roughness should be related to robot traversability, but we do not know the exact nature of this relationship.

This leads us to ask precisely that we mean by traversability. If we want the traversability of a patch of rough terrain to correspond to the difficulty a robot would have covering every part of the terrain, as a robot would likely have to do if it were performing a search of the region, then perhaps some modified roughness parameter would be a suitable estimate for traversability. However, if we want the traversability of a patch of rough terrain to correspond to the difficulty a robot would encounter getting from some fixed point to some other point, as would likely be the case if the robot had information regarding the location of a victim in need of assistance, then we would expect roughness to be a very bad proxy for traversability. Since both the ability to cover all of the terrain and the ability to cross it (from some fixed point to some other fixed point) are important for urban search and rescue robots, we must come up with different metrics for terrain traversability representing these different goals.

Hence, we define the coverability of some region of rough terrain to be a measure of the difficulty a robot would have moving over every section of that region. Similarly, we define the crossability of some region of rough terrain from point p to point q to be a measure of the difficulty a robot would have moving from point p to point q. We will make these definitions more precise later in this paper, when we express them mathematically as functions of the terrain topography and certain dimensions of the robot that is traversing the terrain.

C. Step Fields as an Approximation to Rough Terrain

In order to test the capabilities of urban search and rescue robots moving across rough terrain, it is necessary to have a describable, reconfigurable, repeatable test apparatus to challenge robot mobility. To this end, the National Institute of Standards and Technology (NIST) developed random step fields. A random step field consists of an array of square wooden blocks cut to assorted cubic unit lengths (a unit being the post width) and arranged in different geometric patterns. When several of these step fields are configured into a sequential series or side by side into a "field", they provide an abstract but easily fabricated surrogate for rubble, debris, or other complex ground environments. A picture of a robot traversing a group of step fields can be found in Figure 1.

The facts that step fields are a standard test apparatus used for challenging robot mobility and that they form good surrogates for rubble, debris, or other challenging ground conditions make them an excellent place to begin our analysis of rough terrain. We note that assuming that our rough terrain has the structure of a step field is not overly restrictive since

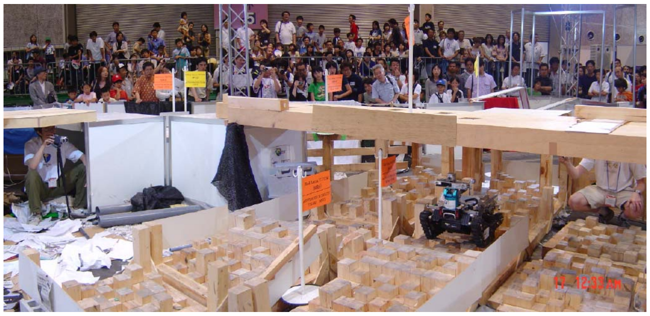

Fig. 1. A robot traversing step fields.

we can always discretize our terrain into a rectangular grid in order to obtain a step field structure.

Before we proceed, it will be useful to establish some standard notation that can be used for performing calculations relating to step fields. If we fix the post width to be one unit then it is clear that a step field with m rows of blocks and n columns of blocks can be completely described by an $m \times n$ matrix of real numbers with each entry representing the height of a certain post. For any such matrix A we will let SF_A denote the associated step field. In general, the NIST step fields are constructed to have the same number of rows and columns so a NIST step field will usually correspond to a square matrix A. However, for the sake of generality, we will establish metrics that can be applied to any rectangular matrix.

IV. Terrain Coverability

As mentioned in the previous section, the coverability of a certain region of terrain is defined to be some measure of the difficulty a robot would encounter if it were to move over the entire region. Since the robot must cover all the terrain, some sort of modified roughness parameter for the surface of the terrain should also serve as a relatively good estimate for coverability. In this section, we will introduce two different modified roughness parameters, each with its own strengths and weaknesses, that should serve as good metrics for coverability.

A. *Modified Average Roughness as a Metric for Coverability*

The most common parameter used to quantify the roughness of a surface is the average roughness, denoted by R_a. Originally, average roughness was used for two-dimensional, stylus-type profiling applications so average roughness is commonly defined by putting $R_a = \frac{1}{b-a}\int_a^b |\phi(x)|dx$ where the profile runs from $x = a$ to $x = b$ and $\phi(x)$ denotes the height of the profile relative to some best fitting line. For many applications this best fitting line is taken to be the horizontal mean line, i.e., the horizontal line with y-intercept $\bar{y} = \frac{1}{b-a}\int_a^b y(x)dx$ where $y(x)$ is the height of the profile at the point x. When this is the case, the formula for average roughness becomes $R_a = \frac{1}{b-a}\int_a^b |y(x) - \bar{y}|dx$.

It is not difficult to construct a three-dimensional definition of average roughness that is analogous to the two-dimensional definition that we have just described. Let S be a surface and let $\phi(x, y)$ denote the height of the surface S relative to a best fitting plane, cylinder, sphere, or other smooth surface Ω. We then define the average roughness, R_a, by writing $R_a = \frac{1}{Area(\Omega)}\int\int_\Omega |\phi(x, y)|dxdy$. As in the two-dimensional case, this best fitting surface is sometimes taken to be a horizontal plane, depending on the nature of the surface S and the application that is being considered.

When dealing with step fields, we shall always assume that the best fitting smooth surface Ω is a horizontal plane since step fields are meant to represent obstacles occurring on flat ground[2]. This assumption means that the formula for average roughness reduces to $R_a = \frac{1}{Area(\Re)}\int\int_\Re |z(x, y) - \bar{z}|dxdy$ where $\Re$ is the rectangular base of the step field, $z(x, y)$ is the height of the step field at the point (x, y), and $\bar{z}$ is the average height given by $\bar{z} = \frac{1}{Area(\Re)}\int\int_\Re z(x, y)dxdy$. In fact, since we are considering our rough terrain to be a step field, we can simplify this formula much further. Let SF_A be an $m \times n$ step field with associated matrix A and let $\Re$ denote the base of SF_A. If $A = (a_{ij})_{i=1,\ldots,m;j=1,\ldots,n}$ then we obtain

$$R_a = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|a_{ij} - \left(\frac{1}{mn}\sum_{k=1}^{m}\sum_{l=1}^{n} a_{kl}\right)\right| \tag{1}$$

Now, we must determine how we should alter the average roughness of a step field in order to have it more accurately reflect coverability. First of all, total roughness is going to be a much better estimate for coverability than average roughness since it will obviously be harder for a robot to cover a large patch of rough terrain than it would be to cover a smaller one of equal roughness. We will denote total roughness by TR so that we can write $TR = \sum_{i=1}^{m}\sum_{j=1}^{n}\left|a_{ij} - \left(\frac{1}{mn}\sum_{k=1}^{m}\sum_{l=1}^{n} a_{kl}\right)\right|$. Furthermore, we need to somehow take into account the dimensions of the robot covering the step field. For example, it should generally be easier for a larger robot to move over a post of height h than it would be for a smaller robot. Perhaps the most relevant dimension of the robot attempting to cover a step field is its wheel diameter if it is a wheeled vehicle or its track height if it is a tracked vehicle. Thus, letting d be the wheel diameter or track height of the robot in question, we consider the quantity $TR_d = \sum_{i=1}^{m}\sum_{j=1}^{n}\left|\frac{a_{ij} - \left(\frac{1}{mn}\sum_{k=1}^{m}\sum_{l=1}^{n} a_{kl}\right)}{d}\right|$. Finally, we do not expect the difficulty a robot would have moving over a step field to scale linearly in height. For example, it should be harder for a robot to move over one very tall post of height h than it would be to move over two smaller posts, each of size $\frac{h}{2}$. This leads us to define our first coverability parameter for a step field, which we shall denote by Cvr_1, to be

$$Cvr_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\left|\frac{a_{ij} - \left(\frac{1}{mn}\sum_{k=1}^{m}\sum_{l=1}^{n} a_{kl}\right)}{d}\right|^{p_1} \tag{2}$$

[2]The main reason that we require Ω to be horizontal is that if we allowed Ω to be any arbitrary plane then an inclined plane lying on flat ground, i.e., a ramp, would have $R_a = 0$. This is undesirable when dealing with robot mobility since ramps are clearly more difficult for robots to traverse than flat ground, especially if the ramp is very steep.

where $p_1 > 1$ can be chosen appropriately for different robots and different applications.

B. Another Modified Roughness Parameter as a Metric for Coverability

For reasons that will be discussed a little later in this paper, average roughness and the coverability measure Cvr_1 that we derived from it have some inherent shortcomings. Hence, it is worth coming up with another coverability metric that is obtained from a different roughness parameter. It is worth noting that while average roughness and, in turn, Cvr_1 can be defined for an arbitrary surface S, the roughness parameter that we construct here only really makes sense for a surface that has been discretized in some way, as is the case when dealing with a step field[3].

As mentioned previously, roughness is a measure of the small-scale height variations of a surface so it makes sense to consider a roughness parameter that is basically the sum of all of the height changes. In the case of a step field, there is a potential height change between any two neighboring posts. However, we must define precisely what we mean by two neighboring posts. There are two reasonable definitions that we could consider:

- Any given post has four neighbors, namely the posts directly above and below it and the posts directly to the left and the right of it. If a post is on the perimeter of the array then we consider its exterior neighbors to have a height of zero.
- Any given post has eight neighbors, namely the four neighbors listed above and the four posts that are located on its diagonals. Once again, posts on the perimeter of the array are considered to have exterior neighbors with height equal to zero.

Since a robot should effectively have a full 360 degree range of motion, it is better to use the second definition so we assume that each post has eight neighbors. Thus, for an $m \times n$ step field SF_A with associated matrix $A = (a_{ij})_{i=1,\ldots,m;j=1,\ldots,n}$ we can define a new total roughness parameter

$$
\begin{aligned}
\tilde{TR} \;=\; & 3\sum_{k=1}^{m}\left(|a_{k1}| + |a_{kn}|\right) \\
& +3\sum_{k=1}^{n}\left(|a_{1k}| + |a_{mk}|\right) \\
& -|a_{11}| - |a_{1n}| \\
& -|a_{1n}| - |a_{m1}| \\
& +\sum_{i=1}^{m}\sum_{j=1}^{n-1}\left|a_{ij} - a_{i(j+1)}\right| \\
& +\sum_{j=1}^{n}\sum_{i=1}^{m-1}\left|a_{ij} - a_{(i+1)j}\right| \\
& +\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}\left|a_{ij} - a_{(i+1)(j+1)}\right| \\
& +\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}\left|a_{i(j+1)} - a_{(i+1)j}\right|
\end{aligned}
\tag{3}
$$

that is obtained by adding up all the height changes.

As before, we take into account the wheel diameter or track height d of the robot and we take into account the fact that coverability should not scale linearly with height in order to define another coverability parameter

$$
\begin{aligned}
Cvr_2 \;=\; & 3\sum_{k=1}^{m}\left(\left|\frac{a_{k1}}{d}\right|^{p_2} + \left|\frac{a_{kn}}{d}\right|^{p_2}\right) \\
& +3\sum_{k=1}^{n}\left(\left|\frac{a_{1k}}{d}\right|^{p_2} + \left|\frac{a_{mk}}{d}\right|^{p_2}\right) \\
& -\left|\frac{a_{11}}{d}\right|^{p_2} - \left|\frac{a_{1n}}{d}\right|^{p_2} \\
& -\left|\frac{a_{m1}}{d}\right|^{p_2} - \left|\frac{a_{mn}}{d}\right|^{p_2} \\
& +\sum_{i=1}^{m}\sum_{j=1}^{n-1}\left|\frac{a_{ij} - a_{i(j+1)}}{d}\right|^{p_2} \\
& +\sum_{j=1}^{n}\sum_{i=1}^{m-1}\left|\frac{a_{ij} - a_{(i+1)j}}{d}\right|^{p_2} \\
& +\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}\left|\frac{a_{ij} - a_{(i+1)(j+1)}}{d}\right|^{p_2} \\
& +\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}\left|\frac{a_{i(j+1)} - a_{(i+1)j}}{d}\right|^{p_2}
\end{aligned}
\tag{4}
$$

where $p_2 > 1$ can be chosen appropriately for different robots and different applications.

C. Strengths and Weaknesses of Cvr_1 and Cvr_2

Neither Cvr_1 nor Cvr_2 serves as a perfect metric for the coverability of rough terrain and both have their own relative strengths and weaknesses. Here, we will discuss the advantages and shortcomings of both of these parameters.

The parameter Cvr_1 is closely related to R_a so it will have many of the same properties as average roughness. One nice quality of Cvr_1 is that it can be calculated for any surface S, even if the surface S has not been discretized. Furthermore, Cvr_1 will scale properly with partitions of the surface. For example, suppose that we take a step field SF_A and partition each post into four posts by cutting the dimensions of the base of each post in half in order to obtain a new step field $SF_{A'}$. It is not hard to see that the Cvr_1 values will be the same for both SF_A and $SF_{A'}$. This is good since SF_A and $SF_{A'}$ are effectively the same step field, at least as far as robot mobility is concerned. However, Cvr_1 has one large disadvantage in that it does not take into account the placement of the peaks

[3] Another case in which this parameter makes sense is when we have a triangulated surface.

and valleys relative to each other[4]. For example consider step fields SF_A and SF_B with associated matrices

$$A = \begin{pmatrix} 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 9 & 9 & 0 & 0 & 9 & 9 \\ 9 & 9 & 0 & 0 & 9 & 9 \\ 9 & 9 & 0 & 0 & 9 & 9 \\ 0 & 0 & 9 & 9 & 0 & 0 \\ 0 & 0 & 9 & 9 & 0 & 0 \\ 0 & 0 & 9 & 9 & 0 & 0 \end{pmatrix}.$$

For most practical purposes we would consider SF_B to be rougher and more difficult to cover than SF_A, even though both step fields have the same R_a value and, in turn, the same Cvr_1 value.

Now, consider Cvr_2. Unlike Cvr_1, the parameter Cvr_2 has the undesirable properties that it can only be expressed for surfaces that are discretized in some way and that it does not scale properly with partitions of the surface. However, the main advantage that it has over Cvr_1 is that it does take into account the relative placement of the peaks and the valleys of the surface. For instance, if SF_A and SF_B are the two step fields with associated matrices A and B defined above then the Cvr_2 value for SF_B will be higher than the Cvr_2 value for SF_A.

It is worth noting one more key difference between Cvr_1 and Cvr_2. Cvr_2 assumes that the ground surrounding the step field is flat and that the intersection of this flat ground with the rough step field may cause the robot some difficulty. In other words, Cvr_2 takes into account the problems that the robot may have when traveling along the perimeter of the step field. This is desirable, so long as the robot is expected to be affected by the outer perimeter of the field, as it clearly would be if it were entering or exiting the array of posts. On the other hand, Cvr_1 ignores the ground surrounding the step field so it does not consider the difficulty a robot may encounter on its outermost perimeter. As a result, Cvr_1 is more appropriate for a robot that starts off on the step field that it wishes to cover and can avoid any interaction with the outer edges of the posts along the perimeter. It would be quite easy to modify either Cvr_1 or Cvr_2 in order to ensure that they both do or both do not take into account the terrain surrounding the step field. Perhaps the easiest way to cause Cvr_1 to reflect the terrain surrounding the step field would be to augment the $m \times n$ matrix A by surrounding it by zeroes in order to obtain a new $(m+2) \times (n+2)$ matrix A' and then use Equation (2) on A'. For example, the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ would become

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Similarly, it is quite easy to make sure that Cvr_2 does not take the terrain surrounding the step field into account by simply removing the first two lines of equation 4, i.e., removing the terms $3\sum_{k=1}^{m}\left(\left|\frac{a_{k1}}{d}\right|^{p_2} + \left|\frac{a_{kn}}{d}\right|^{p_2}\right) + 3\sum_{k=1}^{n}\left(\left|\frac{a_{1k}}{d}\right|^{p_2} + \left|\frac{a_{mk}}{d}\right|^{p_2}\right)$ and $-\left|\frac{a_{11}}{d}\right|^{p_2} - \left|\frac{a_{1n}}{d}\right|^{p_2} - \left|\frac{a_{m1}}{d}\right|^{p_2} - \left|\frac{a_{mn}}{d}\right|^{p_2}$. However, we choose to leave Cvr_1 not representing the surrounding terrain and Cvr_2 representing the surrounding terrain in order to emphasize the fact that we may want to include or not include the terrain surrounding the step field in our model, depending on the application.

In summary, neither Cvr_1 nor Cvr_2 perfectly reflects rough terrain coverability. As a result, both metrics may prove to be useful in different circumstances so we shall use them both to represent the coverability of step fields.

V. Terrain Crossability

In addition to the ability to cover a patch of rough terrain, it is important for urban search and rescue robots to be able to move directly from some given point of that terrain to some other given point, i.e., to be able to cross the terrain in some sense. In this section we will work on developing a metric for terrain crossability that will take as inputs a topographical map of the terrain, the start and finish locations, and certain robot dimensions. It is worth noting that in the model that we develop for a robot crossing a patch of rough terrain, we focus more on keeping the model general enough to apply to different types of robots than we do on making it very accurate for some fixed robot. There is obviously going to be a large trade-off between accuracy for specific robot geometries and the generality and simplicity of the model and we choose to err on the side of generality.

A. Why Coverability and Crossability Require Different Metrics

While modified roughness parameters serve as reasonable measures for the coverability of rough terrain, it is quite easy to see that the relationship between roughness and crossability is not so simple. Consider a step field SF_A with associated matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This step field can have arbitrarily high roughness parameters and arbitrarily high values for Cvr_1 and Cvr_2 by taking a, the height of the post in the center of the array, large enough. However, for many robots it will be quite easy to cross this step field by traveling around the center post. Thus, we see

[4]If we did not require the best fitting plane Ω to be horizontal then Cvr_1 would probably reflect peak/valley placement a bit better. However, it would still not be perfect and this would introduce other problems so we maintain the requirement that Ω be horizontal.

that roughness and coverability do not necessarily correspond to crossability.

A better method for measuring the crossability of some region of rough terrain from point p to point q would be to measure the difficulty a robot would encounter trying to maneuver the least difficult path connecting p to q. However, in order to make all of this precise, we need to define what we mean by a path in a region of rough terrain and we need to define some cost function that can accurately reflect the difficulty of a path. Once we do this, the problem basically reduces to finding a least cost path through a graph, which is a well-studied problem and can be solved using path-planning algorithms such as A^*.

B. Defining a Path Through Rough Terrain

If we wish to find a least cost path through a region of rough terrain, we must first define what constitutes a path through the terrain. We begin by assuming that the state of the robot at any time can be described by giving a point in $\mathbb{R}^2$ representing the location of the center of mass of the robot in the xy-plane and a real number in the interval $[0, 2\pi)$ representing the direction that the robot is facing.

Next, we discretize the surface of the terrain in such a way that, without loss of generality, we can consider the region of rough terrain to be a step field[5]. This allows us to make the state space of the robot discrete as well by assuming that the xy-coordinates of the robot's center of mass always lie at the center of some post and by assuming that the robot is always facing in one of eight directions: north, northeast, east, southeast, south, southwest, west, or northwest. In accordance with standard mathematical convention, we will let east correspond to an angle of 0 radians, northeast correspond to an angle of $\frac{\pi}{4}$ radians, and so on. Hence, the state of the robot can always be expressed by a 3-tuple where the first two entries are the row and column of a post in the step field, respectively, and the third entry is a number in the set $\{\frac{t\pi}{4} : t = 0, 1, ..., 7\}$.

Now, for each robot state we need to define the set of states to which the robot can move. Suppose that the robot starts off in state $(i, j, \frac{t\pi}{4})$. For, our purposes, it makes sense to assume that the robot can move to one of three new states:

- $(i, j, \frac{t'\pi}{4})$ where $t' = t - 1\,(mod\,8)$. This corresponds to the robot turning 45 degrees in the clockwise direction. We will call this a move of type one.
- $(i, j, \frac{t''\pi}{4})$ where $t'' = t + 1\,(mod\,8)$. This corresponds to the robot turning 45 degrees in the counterclockwise direction. We will call this a move of type two.
- $(i + f_1(t), j + f_2(t), \frac{t\pi}{4})$ where $f_1(0) = 0$, $f_2(0) = 1$, $f_1(1) = -1$, $f_2(1) = 1$, $f_1(2) = -1$, $f_2(2) = 0$, $f_1(3) = -1$, $f_2(3) = -1$, $f_1(4) = 0$, $f_2(4) = -1$, $f_1(5) = 1$, $f_2(5) = -1$, $f_1(6) = 1$, $f_2(6) = 0$, $f_1(7) = 1$, and $f_2(7) = 1$. This corresponds to the robot moving forward to the next block. We will call this a move of type three.

With just these three options it is possible for a robot to get from any state to any other state in some finite number of moves[6].

Finally, we are ready to define a path through a step field. An ordered set of 3-tuples of the form $(i, j, \frac{t\pi}{4})$ described above such that the first 3-tuple represents the specified starting state p, the last 3-tuple represents the specified finishing state q, and each 3-tuple can be obtained from the previous one by one of the three valid robot moves described in the preceding paragraph is said to be a path from p to q.

C. Constructing the Cost Function

As mentioned before, the crossability from state p to state q of a region of rough terrain should be the cost of the least cost path connecting p to q, where the cost of a path is the difficulty that a robot would encounter trying to follow it. Thus, in order to calculate crossability, we need to construct this cost function. In other words, we need to define the cost of performing moves of type one, two, and three.

As done by Iagnemma and Dubowsky, we assume that the three main aspects of the path that affect robot mobility are the roughness of the terrain encountered along that path, the amount of turning required to follow that path, and the length of that path [1]. It is clear that a robot will have more difficulty traveling along a path that takes it over very rough terrain than it will have traveling along a similar path over perfectly flat terrain. Furthermore, we expect that it should be more difficult for a robot to follow a path that requires a lot of turning, especially if that turning occurs over rough terrain, than it would be for the robot to follow a straight path. Finally, it makes sense that, ceteris paribus, it is easier for a robot to travel a shorter path than a longer one.

Also, we assume that there are two major properties of the robot that will affect the difficulty it encounters along the path. First, we expect that the robot's wheel diameter or track height, which we again denote by d, will have a relatively large effect on its ability to maneuver a given path. This makes sense since robots with larger wheels or tracks should have less difficulty going over a bump of size h or traveling a distance of length l than robots with smaller wheels or tracks. Next, we expect that the dimensions of the base of the robot will be relevant, where the base of the robot is defined to be the convex hull of the wheels or tracks when the robot is placed on flat ground. These dimensions will determine the region of terrain about the center of mass that should be considered when accounting for the rough terrain encountered along the path.

In order to define the cost function, it is useful to first define a bit of simplified notation. Suppose that the current robot state is $(i, j, \frac{t\pi}{4})$. We let $F_1(i, j, \frac{t\pi}{4})$ be the set of posts in the step field that come in contact with the base of the robot as the robot performs a move of type one. Similarly,

[5]Once again, we could instead triangulate the terrain surface in order to make everything discrete. However, since we are focusing on step fields in this paper, we choose a discretization that allows us to reduce the problem to the case of a robot on a step field.

[6]Here, we assume that the robot can turn without moving its center of mass. This assumption is reasonable for the case of a skid steered robot but not particularly accurate for other steering designs.

let $F_2(i,j,\frac{t\pi}{4})$ and $F_3(i,j,\frac{t\pi}{4})$ be the sets of posts that the robot's base contacts as it performs a move of type two or three, respectively. Thus, $|F_m|$ denotes the number of posts that are in the set F_m for $m = 1, 2, 3$.

Now, we are ready to define the costs of performing moves of type one, two and three. Again, suppose that the robot is presently in state $(i,j,\frac{t\pi}{4})$. Let $cost_m(i,j,\frac{t\pi}{4})$, where $m \in \{1, 2, 3\}$, denote the cost of making a move of type m. We then say that

$$cost_m = \alpha_m \sum_{(k,l)\in F_m} \left| \frac{a_{kl} - \frac{1}{|F_m|}\sum_{(r,s)\in F_m} a_{rs}}{d} \right|^{\beta_m} + \frac{\gamma_m}{d} \tag{5}$$

where $\alpha_m > 0$, $\beta_m > 1$, and $\gamma_m > 0$ can all be chosen for different robots and different situations. Note that the symmetry between moves of type one and type two tells us that we should require $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, and $\gamma_1 = \gamma_2$. The cost of any given path can now be computed by just summing the costs of all of the individual moves that are required to follow that path.

By defining cost in this manner, we take into account all of the primary factors that we said should affect robot mobility including roughness, turning, path length, and robot dimensions. In particular, the $\alpha_m \sum_{(k,l)\in F_m} \left| \frac{a_{kl} - \frac{1}{|F_m|}\sum_{(r,s)\in F_m} a_{rs}}{d} \right|^{\beta_m}$ terms ensure that paths requiring the robot to move over large amounts of rough terrain and paths involving lots of turning on rough terrain will be considered more difficult than similar paths occurring on flat ground[7]. Additionally, the $\frac{\gamma_m}{d}$ terms imply that paths that are longer and paths that require a lot of turning will cost more than shorter and straighter paths over the same terrain.

D. Calculating Crossability by Finding the Least Cost Path

We have already determined that a reasonable estimate for the crossability of a step field from state p to state q is the cost of the least cost path connecting p to q. Hence, we formally define the crossability from state p to state q, denoted by $Crs_{p\to q}$, to be the cost of the least cost path going from state p to state q. Now that we have defined what constitutes a path over rough terrain and described how to calculate the cost of following such a path, the only thing that remains to be done is to explain the method in which we find the least cost path.

First, we construct the vertex set V of a digraph $G = (V, E)$ by letting each possible robot state be a vertex in a graph. Next, we construct the directed edge set E by saying that there is an edge directed from a vertex v_1 to another vertex v_2 if and only if the robot can get to the state corresponding to v_2 from the state corresponding to v_1 by performing a robot move of type one, two, or three. Finally, to each directed edge we associate the cost of performing the robot move that corresponds to that edge. In this way, we have reduced the problem to finding a least cost path through a digraph, which is a problem that has been studied in great detail. An existing algorithm, such as A^*, can be used to find the cost of the least cost path through such a digraph and this number can then be used to represent the crossability of the terrain.

VI. Numerical Results

In this section, we compute the coverability parameters Cvr_1 and Cvr_2 for several step fields produced by a NIST random step field generator[8]. Also, we will discuss the results that are obtained and see if the coverability values agree with our expectations and intuition. We have not yet performed any crossability calculations, but this is something that we would like to do in the near future.

The four step fields for which we compute coverability parameters can be seen in Figure 2. The digits 0, 1, 2, 3, and 4 represent posts of height $1\frac{3}{4}''$, $3\frac{1}{2}''$, $7''$, $10\frac{1}{2}''$, and $14''$, respectively. Furthermore, the step fields are surrounded by borders of height $3\frac{1}{2}''$, which we treat as two extra rows and columns for each step field so that all of the associated matrices for these step fields are 13×13. Note that the four step fields in Figure 2 are representative of the four different random step field layouts that NIST generates. The layout of SF_1 is known as the flat box layout, where the adjective flat describes the fact that there are no posts of size 4 and only four posts of size 3 and the terms box refers to the fact that those posts of size 3 make up a square box. Similarly, SF_2 is known as the flat cross layout since the four posts of size 3 form a cross. SF_3 is called the diagonal layout since there is a hill made up of posts of size 4 running across the diagonal of the field. Finally, the layout depicted in SF_4 is the hill layout and it is characterized by the column of posts of size 4 located in the middle of the step field.

The results of the calculations for the step fields shown in Figure 2 can be found in Table I (rounded to the nearest thousandth). For these calculations, we used $p_1 = p_2 = 2$ and assumed that $d = 7''$, i.e., the wheel diameter corresponds to about two post widths. The coverability values that were obtained agree roughly with our expectations in the sense that the diagonal step field (SF_3) and the hill step field (SF_4) yield significantly larger coverability numbers than the two flat step fields (SF_1 and SF_2) do. It is worth noting that the two coverability metrics Cvr_1 and Cvr_2 produce different relative orderings of the coverabilities of these four step fields with Cvr_1 implying that SF_4 is more difficult to cover than SF_3 and Cvr_2 implying the exact opposite. This reiterates the fact that Cvr_1 and Cvr_2 are very different parameters, each with its own advantages and disadvantages.

While it is useful to see what coverability values we obtain for a few actual step fields, these numerical results do not accurately test the coverability metrics and more detailed experiments should be conducted to suit this purpose. For example, one could run an experiment requiring subjects to

[7]We use a modified R_a parameter to represent roughness. We could have instead used a modified $\tilde{TR}$ parameter, where $\tilde{TR}$ is as described in section IV.

[8]A NIST random step field generator creates an eleven by eleven matrix where the entries of the matrix all lie in the set $\{0, 1, 2, 3, 4\}$ and the matrix is subject to certain rules. For example, the height difference between two horizontal or vertical neighbors can not exceed 2.

drive various robots over several step fields with the goal of covering each field and then have the subject rank the step fields in terms of difficulty to cover. Then, the relative difficulties of covering the step fields as ranked by the subjects can be compared to the relative coverability difficulties produced by the metrics Cvr_1 and Cvr_2.

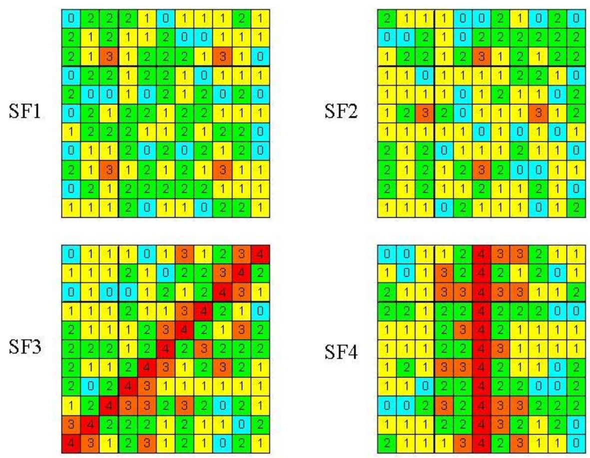

Fig. 2. Step Fields

TABLE I
COVERABILITY VALUES FOR SF_1, SF_2, SF_3, AND SF_4

	Cvr_1	Cvr_2
SF_1	14.013	143.875
SF_2	12.764	137.625
SF_3	37.246	227.750
SF_4	37.956	191.500

VII. CONCLUSIONS

Urban search and rescue robots are likely to encounter many difficult terrain conditions as they perform their required tasks. In particular, they will have to be able to move over and across terrain that displays a lot of small scale height variation, which we refer to as rough terrain. Thus, it is important to have metrics that describe the difficulty a robot would have attempting to traverse different regions of rough terrain. To this end, we have developed three different metrics for the traversability of rough terrain that take as inputs a topographical map of the terrain and some minimal information about the robot's dimensions. These metrics are derived for the case when the region of rough terrain under investigation is a step field, which is not an overly restrictive assumption since we can always discretize the terrain in order to give it a step field structure.

Two of these metrics, which we have denoted by Cvr_1 and Cvr_2, tell us how difficult it would be for a robot to cover, i.e., move over ever part of, a step field. They are effectively modified roughness parameters that are scaled by the wheel diameter or track height of the robot in question in order to make them dimensionless and in order to account for the effect of the size of the robot on traversability. Both Cvr_1 and Cvr_2 have their relative strengths and weaknesses so we use both of these quantities to describe coverability.

The third metric describes how difficult it would be for a robot to move from some point on a step field to some other point. In order to derive this metric, we discretized the state space of the robot and constructed a cost function that approximates how difficult it is for the robot to move from one state to another. Then, we defined the crossability from state p to state q, denoted by $Crs_{p\to q}$, to be the cost of the least cost path connecting p to q so that $Crs_{p\to q}$ describes the difficulty a robot would have moving from state p to state q.

Finally, we performed some computations and determined Cvr_1 and Cvr_2 values for four different step fields produced by a NIST random step field generator. The numerical results make intuitive sense but more detailed experiments should be performed in order to better test these coverability metrics. No crossability values have been calculated at this point, but this is an area in which we would like to devote more attention sometime in the near future.

ACKNOWLEDGMENTS

The first author would like to thank the United States Department of Homeland Security. His research towards this paper was performed on appointment as a U.S. Department of Homeland Security (DHS) Fellow from the Courant Institute of Mathematical Sciences at New York University under the DHS Scholarship and Fellowship Program, a program administered by the Oak Ridge Institute for Science and Education (ORISE) for DHS through an interagency agreement with the U.S. Department of Energy (DOE). ORISE is managed by Oak Ridge Associated Universities under DOE contract number DE-AC05-00OR22750. All opinions expressed in this paper are the authors' and do not necessarily reflect the policies and views of DHS, DOE, or ORISE.

In addition, he wishes to thank NIST for hosting his summer research and Elena Messina for the excellent guidance and mentoring that she provided.

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