Dynamic Modeling and Robust Controller Design of a Two-Stage Parallel Cable Robot

So-Ryeok Oh, Kalyan K. Mankala and Sunil K. Agrawal *
University of Delaware, Newark, DE - 19716. Email: mankala@me.udel.edu, agrawal@me.udel.edu

James S. Albus
NIST, Gaithersburg, MD 20899-8230. Email: james.albus@nist.gov

Abstract. Cable robots have been extensively used for the loading and unloading of cargo in shipping industries. In this paper, we look at a two stage cable robot, i.e., a cable robot with two moving platforms connected in series. The sea condition introduces disturbance into the system. This disturbance is considered while modeling the dynamics of the two stage cable robot. A robust controller is designed which can assure robust tracking of the desired end-effector trajectory in the presence of the disturbance. Simulation results presented show the effectiveness of the controller.

Keywords: Dynamics, Modeling, Two Stage Cable Robot, Robust Control, Crane ship, Cargo Transfer

1. Introduction

One of the motivational applications behind this study is a crane ship which is used to transfer cargo from a container carrier to a smaller, lighter ship at sea when a port is not available for the container ship. This transfer operation can be unstable unless the sea condition is properly accounted for. During the last few decades, many researchers have studied wire suspended mechanisms, such as a crane, to deal with this problem. Patel*et al.* (1978) derived the model of a ship-mounted crane incorporating the coupled motions of the crane and the ship. Schellin *et al.*(1989) extended the model to three dimensions allowing for all ship motions and load pendulations, and damping in the cable. Nayfeh *et al.* (2002) designed and implemented a controller that suppressed cargo pendulation on most common commercial cranes. Shiang *et al.* (1999) investigated four cable robotic crane to provide improved cargo handling. Based on the concept of parallel platforms, NIST (1992) has developed the ROBOCRANE which can control the position, velocity, and force of tools and heavy machinery in all six degrees-of-freedom (x, y, z, roll, pitch, yaw).

This paper deals with a new type of a crane robot with two moving platforms in series, a conceptual design developed by NIST for skin-

* corresponding author

to-skin open-ocean transfer of cargo [Schulz *et al.* (1999)]. The upper stage is motivated from keeping the suspension cables from contacting containers adjacent to the target container, and also to increase the system redundancy. The lower stage is designed to engage the container. This two-stage design promises the desired goal of transfer of cargo from one ship to another safe. This two stage cable robot can also be used with helicopters for different rescue and transport missions.

In this paper, we present a dynamic model for the two stage cable robot, incorporating the disturbance from the sea condition in section 2. Section 3 deals with the design of a robust controller for robust tracking of the desired end effector trajectory in the presence of disturbance. Section 4 presents some simulation results.

2. System Dynamics

Fig. 1 shows the kinematic chain of the two-stage cable robot. It is composed of an upper spreader ($\mathcal{B}$) and a lower spreader-end-effector ($\mathcal{C}$), which are connected by 12 cables to a rotator ($\mathcal{A}$). The rotator is rigidly attached to a body of large mass, generally a ship or a helicopter. 12 actuators are mounted on the rotator to control the 12 cables. Out of the twelve cables, six cables are directly connected to the upper spreader and the other 6 cables pass through 6 pulleys on the lower spreader and are then attached to the upper spreader.

The motion of the rotator is coupled to the motion of the upper spreader and the end effector through the cables. Due to the sea condition, a ship is subjected to a disturbance motion. In the helicopter case, it cannot hover perfectly. There is always a disturbance present from the wind. This disturbance motion coming on to the rotator has to be modeled while considering the dynamics of the cable structure. So the rotator cannot be thought of as being inertially fixed, but may be considered to have a *"prescribed"* motion coming from the environment.

The motion of end effector can be analysed relative to the inertial frame or relative to a frame attached to the rotator. From the application point of view, if the end effector is supposed to transport things from one part of the ship to another, then it will be easy to specify its desired motion in the frame attached to the ship, i.e., the rotator frame. But if the target is in the inertial frame, then the desired motion of the end effector, relative to rotator frame cannot be precisely specified. In that case, it is good to specify the motion of the end effector in the inertial frame. Again, if the target of the end effector is in another ship, then we cannot express the desired motion of the end effector in either of the frames precisely. In such a case, generally a visual feedback is necessary to keep track of the target. So it looks like either formulations are equally good. But if we select to express the motion of the end-effector relative to the frame attached to the rotator, we expect to separate out the disturbance from the dynamic equations, which in turn will make the design of robust controllers easier.

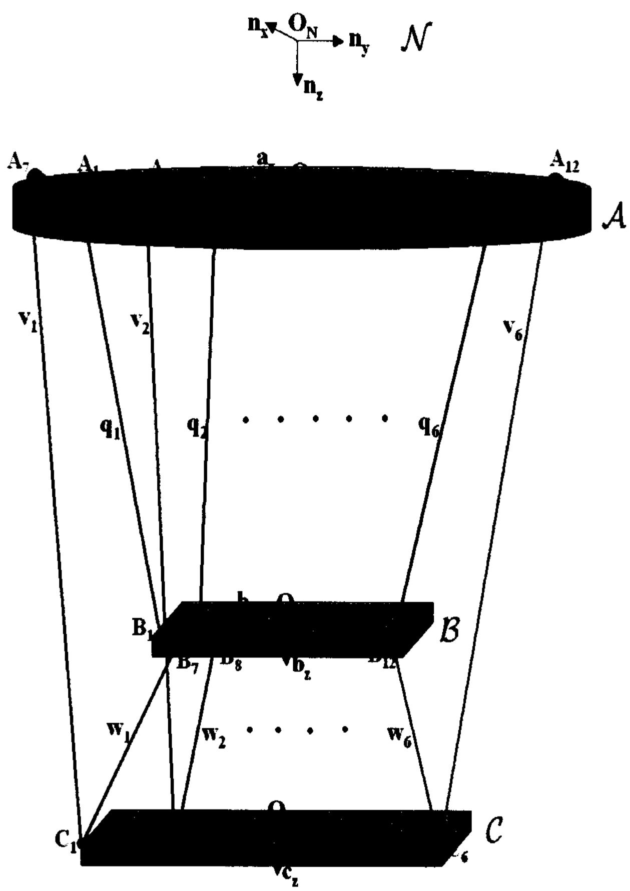

Figure 1. A sketch of a two stage cable system along with geometric parameters.

2.1. KINEMATICS

Consider an inertial coordinate frame $\mathcal{N}$ with origin O_N and basis vectors $\mathbf{n_x}, \mathbf{n_y}, \mathbf{n_z}$. The rotator of the robot has a coordinate frame $\mathcal{A}$ fixed to it with origin O_A and basis vectors $\mathbf{a_x}, \mathbf{a_y}, \mathbf{a_z}$. The upper spreader (end-effector) has a coordinate frame $\mathcal{B}$ ($\mathcal{C}$) fixed to it with origin O_B (O_C) and basis vectors $\mathbf{b_x}, \mathbf{b_y}, \mathbf{b_z}$ ($\mathbf{c_x}, \mathbf{c_y}, \mathbf{c_z}$).

The position, O_A of $\mathcal{A}$, is described by $\mathbf{x}_A = (x_A, y_A, z_A)^T$ along three coordinate directions $\mathbf{n_x}$, $\mathbf{n_y}$, $\mathbf{n_z}$. We choose the orientation of frame $\mathcal{A}$ to be given by a space-three rotation sequence of ψ_A about $\mathbf{n_x}$, θ_A about $\mathbf{n_y}$, and ϕ_A about $\mathbf{n_z}$. Let $\mathbf{\Psi}_A = (\psi_A, \theta_A, \phi_A)^T$. So the configuration of frame $\mathcal{A}$, at any instant is described by $\mathbf{X_A} = (\mathbf{x}_A, \mathbf{\Psi}_A) = (x_A, y_A, z_A, \psi_A, \theta_A, \phi_A)^T$. For the rotator, a vector written in terms of coordinate frame $\mathcal{A}$ can be written in terms of inertial frame using the rotation matrix, ${}^{\mathcal{N}}R_A$ which is given as

$$ {}^{\mathcal{N}}R_A = \begin{bmatrix} C\theta_A C\phi_A & S\psi_A S\theta_A C\phi_A - S\phi_A C\psi_A & C\psi_A S\theta_A C\phi_A + S\phi_A S\psi_A \\ C\theta_A S\phi_A & S\psi_A S\theta_A S\phi_A + C\phi_A C\psi_A & C\psi_A S\theta_A S\phi_A - C\phi_A S\psi_A \\ -S\theta_A & S\psi_A C\theta_A & C\psi_A C\theta_A \end{bmatrix}. \tag{1} $$

If the angular velocity of $\mathcal{A}$ w.r.t. $\mathcal{N}$ is, $\omega_{a1}\mathbf{a_x} + \omega_{a2}\mathbf{a_y} + \omega_{a3}\mathbf{a_z}$, from rigid body kinematics,

$$ {}^{\mathcal{N}}\omega_{\mathcal{AN}} = \begin{bmatrix} \omega_{a1} \\ \omega_{a2} \\ \omega_{a3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S\theta_A \\ 0 & C\psi_A & S\psi_A C\theta_A \\ 0 & -S\psi_A & C\psi_A C\theta_A \end{bmatrix} \begin{bmatrix} \dot{\psi}_A \\ \dot{\theta}_A \\ \dot{\phi}_A \end{bmatrix} = P_A \dot{\Psi}_A, \tag{2} $$

Here, we have introduced a notation ${}^{A}\omega_{\mathcal{BC}}$, which means the angular velocity vector of a frame $\mathcal{B}$ w.r.t. frame $\mathcal{C}$, expressed in a frame A. The notation holds for all the vectors used in this paper.

As already mentioned, we would like to analyze the motion of the end-effector relative to the frame attached to the rotator $\mathcal{A}$. So we define the position and orientation of $\mathcal{B}$ and $\mathcal{C}$ in $\mathcal{A}$, by $\mathbf{X_i} = [\mathbf{x}_i, \mathbf{\Psi}_i] = [(x_i, y_i, z_i), (\psi_i, \theta_i, \phi_i)]^T$, $i = b, c$. Small letters are used to indicate that the position and orientation quantities are described with respect to frame $\mathcal{A}$.

If the angular velocity of $\mathcal{B}$ w.r.t. $\mathcal{A}$ is, $\omega_{b1}\mathbf{b_x} + \omega_{b2}\mathbf{b_y} + \omega_{b3}\mathbf{b_z}$, from rigid body kinematics,

$$ {}^{\mathcal{B}}\omega_{\mathcal{BA}} = \begin{bmatrix} \omega_{b1} \\ \omega_{b2} \\ \omega_{b3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S\theta_b \\ 0 & C\psi_b & S\psi_b C\theta_b \\ 0 & -S\psi_b & C\psi_b C\theta_b \end{bmatrix} \begin{bmatrix} \dot{\psi}_b \\ \dot{\theta}_b \\ \dot{\phi}_b \end{bmatrix} = P_b \dot{\Psi}_b, \tag{3} $$

where $\dot{\Psi}_b = (\dot{\psi}_b, \dot{\theta}_b, \dot{\phi}_b)^T$.

Similarly if the angular velocity of $\mathcal{C}$ w.r.t. $\mathcal{A}$ is $\omega_{c1}\mathbf{c_x} + \omega_{c2}\mathbf{c_y} + \omega_{c3}\mathbf{c_z}$, then from rigid body kinematics, we get,

$$ {}^{\mathcal{C}}\omega_{\mathcal{CA}} = \begin{bmatrix} \omega_{c1} \\ \omega_{c2} \\ \omega_{c3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -S\theta_c \\ 0 & C\psi_c & S\psi_c C\theta_c \\ 0 & -S\psi_c & C\psi_c C\theta_c \end{bmatrix} \begin{bmatrix} \dot{\psi}_c \\ \dot{\theta}_c \\ \dot{\phi}_c \end{bmatrix} = P_c \dot{\Psi}_c, \tag{4} $$

where $\dot{\Psi}_c = (\dot{\psi}_c, \dot{\theta}_c, \dot{\phi}_c)^T$.

Angular acceleration of $\mathcal{B}$, ${}^{\mathcal{N}}\alpha_{\mathcal{BN}}$ in inertial frame, $\mathcal{N}$, can be derived as follows,

$$
\begin{aligned}
{}^{\mathcal{N}}\omega_{\mathcal{BN}} &= {}^{\mathcal{N}}\omega_{\mathcal{AN}} + {}^{\mathcal{N}}\omega_{\mathcal{BA}} \\
&= {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}} \\
\Rightarrow {}^{\mathcal{N}}\alpha_{\mathcal{BN}} &= \frac{d}{dt}\,{}^{\mathcal{N}}\omega_{\mathcal{BN}} \\
&= \frac{d}{dt}\left[{}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}}\right] \\
&= {}^{\mathcal{N}}\dot{R}_{\mathcal{A}}\,{}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}\dot{\omega}_{\mathcal{AN}} + {}^{\mathcal{N}}\dot{R}_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}} \\
&\quad + {}^{\mathcal{N}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\dot{\omega}_{\mathcal{BA}} \\
&\quad (\text{where } {}^{\mathcal{N}}R_{\mathcal{B}} = {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}R_{\mathcal{B}})
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\text{but, } {}^{\mathcal{N}}\dot{R}_{\mathcal{A}}\,{}^{\mathcal{A}}\omega_{\mathcal{AN}} &= {}^{\mathcal{N}}\omega_{\mathcal{AN}} \times {}^{\mathcal{N}}R_{\mathcal{A}}\,{}^{\mathcal{A}}\omega_{\mathcal{AN}} \\
&= {}^{\mathcal{N}}\omega_{\mathcal{AN}} \times {}^{\mathcal{N}}\omega_{\mathcal{AN}} = 0 \\
\text{similarly, } {}^{\mathcal{N}}\dot{R}_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}} &= {}^{\mathcal{N}}\omega_{\mathcal{BN}} \times {}^{\mathcal{N}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}} = {}^{\mathcal{N}}\omega_{\mathcal{BN}} \times {}^{\mathcal{N}}\omega_{\mathcal{BA}} \\
&= {}^{\mathcal{N}}\omega_{\mathcal{AN}} \times {}^{\mathcal{N}}\omega_{\mathcal{BA}} \\
&= {}^{\mathcal{N}}R_{\mathcal{A}}\left({}^{\mathcal{A}}\omega_{\mathcal{AN}} \times {}^{\mathcal{A}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}}\right) \\
\text{therefore, } {}^{\mathcal{N}}\alpha_{\mathcal{BN}} &= {}^{\mathcal{N}}R_{\mathcal{A}}\,[{}^{\mathcal{A}}\alpha_{\mathcal{AN}} + {}^{\mathcal{A}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\alpha_{\mathcal{BA}} \\
&\quad + ({}^{\mathcal{A}}\omega_{\mathcal{AN}} \times {}^{\mathcal{A}}R_{\mathcal{B}}\,{}^{\mathcal{B}}\omega_{\mathcal{BA}})]
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
\text{Where, } {}^{\mathcal{A}}\alpha_{\mathcal{AN}} &= {}^{\mathcal{A}}\dot{\omega}_{\mathcal{AN}} = [\dot{\omega}_{a1} \quad \dot{\omega}_{a2} \quad \dot{\omega}_{a3}] \\
&= \dot{P}_a\dot{\Psi}_a + P_a\ddot{\Psi}_a
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
{}^{\mathcal{B}}\alpha_{\mathcal{BA}} &= {}^{\mathcal{B}}\dot{\omega}_{\mathcal{BA}} = [\dot{\omega}_{b1} \quad \dot{\omega}_{b2} \quad \dot{\omega}_{b3}] \\
&= \dot{P}_b\dot{\Psi}_b + P_b\ddot{\Psi}_b
\end{aligned}
\tag{8}
$$

Similarly, we can derive angular acceleration of $\mathcal{C}$, α_C in inertial frame $\mathcal{N}$ as

$$
{}^{\mathcal{N}}\alpha_{\mathcal{CN}} = {}^{\mathcal{N}}R_{\mathcal{A}}\left[{}^{\mathcal{A}}\alpha_{\mathcal{AN}} + {}^{\mathcal{A}}R_{\mathcal{C}}\,{}^{\mathcal{C}}\alpha_{\mathcal{CA}} + \left({}^{\mathcal{A}}\omega_{\mathcal{AN}} \times {}^{\mathcal{A}}R_{\mathcal{C}}\,{}^{\mathcal{C}}\omega_{\mathcal{CA}}\right)\right] \tag{9}
$$

Fig. 1 also shows cable attachment points $C_1, \cdots, C_6$ on the end-effector, $B_1, \cdots, B_{12}$ in the frame $\mathcal{B}$, and $A_1, \cdots, A_7$ on the rotator. Let $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$ denote the vectors from O_A, O_B, O_C to the cable attachement points, A_i, B_i, C_i repectively, expressed in their local frames, $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$. Let us also define 6 vectors $\mathbf{q}_i$ connecting $\mathcal{B}$ to $\mathcal{A}$, 6 vectors $\mathbf{v}_i$ connecting $\mathcal{C}$ to $\mathcal{A}$, and 6 vectors $\mathbf{w}_i$ connecting $\mathcal{C}$ to $\mathcal{B}$ (see Fig. 1), in inertial frame, $\mathcal{N}$.

$$\begin{bmatrix} \mathbf{q}_1 = \overrightarrow{B_1A_1} & \mathbf{q}_2 = \overrightarrow{B_2A_2} & \mathbf{q}_3 = \overrightarrow{B_3A_3} \\ \mathbf{q}_4 = \overrightarrow{B_4A_4} & \mathbf{q}_5 = \overrightarrow{B_5A_5} & \mathbf{q}_6 = \overrightarrow{B_6A_6} \\ \mathbf{v}_1 = \overrightarrow{C_1A_7} & \mathbf{v}_2 = \overrightarrow{C_2A_8} & \mathbf{v}_3 = \overrightarrow{C_3A_9} \\ \mathbf{v}_4 = \overrightarrow{C_4A_{10}} & \mathbf{v}_5 = \overrightarrow{C_5A_{11}} & \mathbf{v}_6 = \overrightarrow{C_6A_{12}} \\ \mathbf{w}_1 = \overrightarrow{C_1B_7} & \mathbf{w}_2 = \overrightarrow{C_2B_8} & \mathbf{w}_3 = \overrightarrow{C_3B_9} \\ \mathbf{w}_4 = \overrightarrow{C_4B_{10}} & \mathbf{w}_5 = \overrightarrow{C_5B_{11}} & \mathbf{w}_6 = \overrightarrow{C_6B_{12}} \end{bmatrix} \tag{10}$$

We can express the vectors, $\mathbf{q}_i, \mathbf{v}_i, \mathbf{w}_i$ in terms of cable attachment points and origins of local frames. For example, we can express $\mathbf{q}_1$ connecting B_1 to A_1, in the inertial frame $\mathcal{N}$ as

$$\begin{aligned} \mathbf{q}_1 &= -\overrightarrow{O_BB_1} - \overrightarrow{O_AO_B} + \overrightarrow{O_AA_1} \\ \text{But, } \overrightarrow{O_BB_1} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 \\ \overrightarrow{O_AO_B} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{x}_b \\ \overrightarrow{O_AA_1} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{a}_1 \\ \Rightarrow \mathbf{q}_1 &= -\, {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 - {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{x}_b + {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{a}_1 \\ &= {}^{\mathcal{N}}R_{\mathcal{A}} \left(-\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 - \mathbf{x}_b + \mathbf{a}_1 \right) \\ &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{q}}_1 \end{aligned} \tag{11}$$

where $\bar{\mathbf{q}}_1 = \left(-\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 - \mathbf{x}_b + \mathbf{a}_1 \right)$. We can express $\mathbf{v}_1$ connecting C_1 to A_7 in the inertial frame $\mathcal{N}$ as

$$\begin{aligned} \mathbf{v}_1 &= -\overrightarrow{O_CC_1} - \overrightarrow{O_AO_C} + \overrightarrow{O_AA_7} \\ \text{But, } \overrightarrow{O_CC_1} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 \\ \overrightarrow{O_AO_C} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{x}_c \\ \overrightarrow{O_AA_7} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{a}_7 \\ \Rightarrow \mathbf{v}_1 &= -\, {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{x}_c + {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{a}_7 \\ &= {}^{\mathcal{N}}R_{\mathcal{A}} \left(-\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - \mathbf{x}_c + \mathbf{a}_7 \right) \\ &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{v}}_1 \end{aligned} \tag{12}$$

where $\bar{\mathbf{v}}_1 = \left(-\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - \mathbf{x}_c + \mathbf{a}_7 \right)$. Simlarly, we can express $\mathbf{w}_1$ as,

$$\begin{aligned} \mathbf{w}_1 &= -\overrightarrow{O_CC_1} - \overrightarrow{O_BO_C} + \overrightarrow{O_BB_1} \\ \text{But, } \overrightarrow{O_CC_1} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 \\ \overrightarrow{O_BO_C} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, (\mathbf{x}_c - \mathbf{x}_b) \\ \overrightarrow{O_BB_1} &= {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 \\ \Rightarrow \mathbf{w}_1 &= -\, {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - {}^{\mathcal{N}}R_{\mathcal{A}}\, (\mathbf{x}_c - \mathbf{x}_b) + {}^{\mathcal{N}}R_{\mathcal{A}}\, {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 \\ &= {}^{\mathcal{N}}R_{\mathcal{A}} \left(-\, {}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - \mathbf{x}_c + \mathbf{x}_b + {}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1 \right) \\ &= {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{w}}_1 \end{aligned} \tag{13}$$

where $\bar{\mathbf{w}}_1 = \left(- \,{}^{\mathcal{A}}R_{\mathcal{C}}\, \mathbf{c}_1 - \mathbf{x}_c + \mathbf{x}_b + \,{}^{\mathcal{A}}R_{\mathcal{B}}\, \mathbf{b}_1\right)$.

we can also write, $\mathbf{q}_i = q_i\hat{\mathbf{q}}_i$, $\mathbf{v}_i = v_i\hat{\mathbf{v}}_i$, and $\mathbf{w}_i = w_i\hat{\mathbf{w}}_i$, $i = 1, \cdots, 6$. So the length l_i of cable i is given by

$$l_i = \begin{cases} q_i(\mathbf{X}), & i = 1, \cdots, 6. \\ v_{i-6}(\mathbf{X}) + w_{i-6}(\mathbf{X}), & i = 7, \cdots, 12. \end{cases} \tag{14}$$

where $\mathbf{X} = [\mathbf{X}_A\ \mathbf{X}_b\ \mathbf{X}_c]^T$. On defining $\mathbf{l} = [l_1, \cdots, l_{12}]^T$, the position kinematics of the robot is captured in the following nonlinear map,

$$\mathbf{l} = \mathbf{l}(\mathbf{X}) \tag{15}$$

2.2. Rigid Body Dynamics

2.2.1. *Force Equation*

Rotator, upper spreader, end-effector are three different bodies present in the system. Since the rotator has no motion (except disturbance motion) because of its relatively large mass, we are not interested in writing the equations of motion for this body. So, we write equations of motion only for the upper spreader and the end-effector. By doing force balance on the upper speader we get the following equations of motion,

$$\begin{aligned} m_b \frac{d^2\mathbf{x}_B}{dt^2} &= \sum \mathbf{F}_{ext} \\ &= m_b\mathbf{g} + [\,\hat{\mathbf{q}}_1 \quad \cdots \quad \hat{\mathbf{q}}_6 \quad -\hat{\mathbf{w}}_1 \quad \cdots \quad -\hat{\mathbf{w}}_6\,]\,\mathbf{T} \end{aligned} \tag{16}$$

where $\mathbf{T}$ is the vector defining tension in different cables whose directions are given by the coefficient matrix. We know, $\mathbf{x}_B = (\mathbf{x}_A + {}^{\mathcal{N}}R_{\mathcal{A}}\, \mathbf{x}_b)$. Also we can write, $\hat{\mathbf{q}}_i = \mathbf{q}_i/q_i = {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{q}}_i/q_i$. Similarly, we have $\hat{\mathbf{v}}_i = {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{v}}_i/v_i$, $\hat{\mathbf{w}}_i = {}^{\mathcal{N}}R_{\mathcal{A}}\, \bar{\mathbf{w}}_i/w_i$. Substituting these in the above equation we get,

$$\begin{aligned} m_b\ddot{\mathbf{x}}_b + \overbrace{m_b\, {}^{\mathcal{N}}R_{\mathcal{A}}^T \left(\ddot{\mathbf{x}}_A + {}^{\mathcal{N}}\ddot{R}_{\mathcal{A}}\mathbf{x}_b + 2\, {}^{\mathcal{N}}\dot{R}_{\mathcal{A}}\dot{\mathbf{x}}_b\right) - {}^{\mathcal{N}}R_{\mathcal{A}}^T m_b\mathbf{g}}^{D_{\mathbf{x},b}} \\ = [\, \tfrac{\bar{\mathbf{q}}_1}{q_1} \quad \cdots \quad \tfrac{\bar{\mathbf{q}}_6}{q_6} \quad -\tfrac{\bar{\mathbf{w}}_1}{w_1} \quad \cdots \quad \tfrac{-\bar{\mathbf{w}}_6}{w_6}\,]\,\mathbf{T} \end{aligned} \tag{17}$$

As already mentioned, frame $\mathcal{A}$ is subjected to disturbance from the environment it is located in. This disturbance manifests as perturbation in $\mathbf{x}_A$ and ${}^{\mathcal{N}}R_{\mathcal{A}}$. Assuming that we know cable lengths at any time, $D_{\mathbf{x},b}$ is the only term in the above equation that gets affected because of this disturbance. So this term can be treated as disturbance for the design of robust controllers for $\mathbf{T}$.

2.2.2. *Moment Equation*

Doing Moment balance on the upper speader, we get the following equations of motion:

$$I_B^{\mathcal{N}} \alpha_{B\mathcal{N}} + {}^{\mathcal{N}}\omega_{B\mathcal{N}} \times I_B \, {}^{\mathcal{N}}\omega_{B\mathcal{N}} = [\mathbf{r_{B1}} \times \hat{\mathbf{q}}_1 \ \cdots \ \mathbf{r_{B6}} \times \hat{\mathbf{q}}_6 \quad \mathbf{r_{B7}} \times \hat{\mathbf{w}}_1 \ \cdots \ \mathbf{r_{B12}} \times \hat{\mathbf{w}}_6]\mathbf{T}, \quad (18)$$

where I_B is the inertia matrix of upper spreader w.r.t. inertial frame $\mathcal{N}$. Also $I_B = {}^{\mathcal{N}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{N}}R_{\mathcal{B}}^T = {}^{\mathcal{N}}R_{\mathcal{A}} \, {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b ({}^{\mathcal{N}}R_{\mathcal{A}} \, {}^{\mathcal{A}}R_{\mathcal{B}})^T$, where I_b is the inertia matrix of upper spreader w.r.t. to frame $\mathcal{B}$. $\mathbf{r_{B1}}$ is the position vector from O_B to the attachment point B_1 expressed in inertial frame. So we can write $\mathbf{r_{B1}} = {}^{\mathcal{N}}R_{\mathcal{B}} \, {}^{\mathbf{B}}\mathbf{r_{b1}} = {}^{\mathcal{N}}R_{\mathcal{A}} \, {}^{\mathcal{A}}R_{\mathcal{B}} \, {}^{\mathbf{B}}\mathbf{r_{b1}}$, where ${}^{\mathbf{B}}\mathbf{r_{b1}}$ is the position vector expressed in frame $\mathcal{B}$. We can also write $\hat{\mathbf{q}}_1 = {}^{\mathcal{N}}R_{\mathcal{A}} \, \bar{\mathbf{q}}_1 / q_1$. So, $\mathbf{r_{B1}} \times \hat{\mathbf{q}}_1 = {}^{\mathcal{N}}R_{\mathcal{A}} \left({}^{\mathcal{A}}R_{\mathcal{B}} \, {}^{\mathbf{B}}\mathbf{r_{b1}} \times \bar{\mathbf{q}}_1 / q_1 \right)$. Using this, R.H.S. of Eq. (18) can be written as

$$\text{R.H.S.} \ = {}^{\mathcal{N}}R_{\mathcal{A}} \ [\ {}^{\mathcal{A}}R_{\mathcal{B}} \, {}^{\mathbf{B}}\mathbf{r_{b1}} \times \tfrac{\bar{\mathbf{q}}_1}{q_1} \cdots - \ {}^{\mathcal{A}}R_{\mathcal{B}} \, {}^{\mathbf{B}}\mathbf{r_{b12}} \times \tfrac{\bar{\mathbf{w}}_6}{w_6} \,] \, \mathbf{T} \quad (19)$$

From Kinematics, we have ${}^{\mathcal{N}}\omega_{\mathcal{BN}} = {}^{\mathcal{N}}R_{\mathcal{A}} \ {}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{N}}R_{\mathcal{A}} \ {}^{\mathcal{A}}R_{\mathcal{B}} \ {}^{\mathcal{B}}\omega_{\mathcal{BA}}$ and ${}^{\mathcal{N}}\alpha_{\mathcal{BN}} = {}^{\mathcal{N}}R_{\mathcal{A}} \left[{}^{\mathcal{A}}\alpha_{\mathcal{AN}} + {}^{\mathcal{A}}R_{\mathcal{B}} \, {}^{\mathcal{B}}\alpha_{\mathcal{BA}} + \left({}^{\mathcal{A}}\omega_{\mathcal{AN}} \times {}^{\mathcal{A}}R_{\mathcal{B}} \ {}^{\mathcal{B}}\omega_{\mathcal{BA}} \right) \right]$. Substituting these equations in L.H.S. of Eq. (18) we get,

$$\begin{aligned} \text{L.H.S.} = {}^{\mathcal{N}}R_{\mathcal{A}} \Big\{ & {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{B}}\alpha_{\mathcal{BA}} + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T \, {}^{\mathcal{A}}\alpha_{\mathcal{AN}} + \\ & {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T ({}^{\mathcal{A}}\omega_{\mathcal{AN}} \times {}^{\mathcal{A}}R_{\mathcal{B}} \ {}^{\mathcal{B}}\omega_{\mathcal{BA}}) + ({}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{A}}R_{\mathcal{B}} \ {}^{\mathcal{B}}\omega_{\mathcal{BA}}) \\ & \times ({}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T \, {}^{\mathcal{A}}\omega_{\mathcal{AN}} + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{B}}\omega_{\mathcal{BA}}) \Big\} \end{aligned} \quad (20)$$

Using Eqs. (2), (3), (7), (8), the above equation can be modified as,

$$\begin{aligned} \text{L.H.S.} = {}^{\mathcal{N}}R_{\mathcal{A}} \Big\{ & {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b (\dot{P}_b \dot{\Psi}_b + P_b \ddot{\Psi}_b) \\ & + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T (\dot{P}_A \dot{\Psi}_A + P_A \ddot{\Psi}_A) + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T (P_A \dot{\Psi}_A \times {}^{\mathcal{A}}R_{\mathcal{B}} \, P_b \dot{\Psi}_b) \\ & + \Big[\Big(P_A \dot{\Psi}_A + {}^{\mathcal{A}}R_{\mathcal{B}} \, P_b \dot{\Psi}_b \Big) \times \Big({}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T P_A \dot{\Psi}_A + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b P_b \dot{\Psi}_b \Big) \Big] \Big\}. \end{aligned}$$

We can group terms in the above equation as,

$$\begin{aligned} \text{L.H.S.} = {}^{\mathcal{N}}R_{\mathcal{A}} \Big\{ & {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b (\dot{P}_b \dot{\Psi}_b + P_b \ddot{\Psi}_b) + {}^{\mathcal{A}}R_{\mathcal{B}} \, P_b \dot{\Psi}_b \times {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b P_b \dot{\Psi}_b \\ & + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T (\dot{P}_A \dot{\Psi}_A + P_A \ddot{\Psi}_A) + {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T (P_A \dot{\Psi}_A \times {}^{\mathcal{A}}R_{\mathcal{B}} \, P_b \dot{\Psi}_b) \\ & + \Big(P_A \dot{\Psi}_A + {}^{\mathcal{A}}R_{\mathcal{B}} \, P_b \dot{\Psi}_b \Big) \times {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b \, {}^{\mathcal{A}}R_{\mathcal{B}}^T P_A \dot{\Psi}_A \\ & + P_A \dot{\Psi}_A \times {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b P_b \dot{\Psi}_b \Big\}. \end{aligned}$$

The above equation can be compactly written as

$$\text{L.H.S.} = {}^{\mathcal{N}}R_{\mathcal{A}} \left\{ {}^{\mathcal{A}}R_{\mathcal{B}} \, I_b P_b \ddot{\Psi}_b + F_{\mathbf{O},b} + D_{\mathbf{O},b} \right\} \quad (21)$$

where

$$\begin{aligned} F_{O,b} &= {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b \dot{P}_b \dot{\Psi}_b + {}^{\mathcal{A}}R_{\mathcal{B}}\, P_b \dot{\Psi}_b \times {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b P_b \dot{\Psi}_b \\ D_{O,b} &= {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b\, {}^{\mathcal{A}}R_{\mathcal{B}}^T (\dot{P}_A \dot{\Psi}_A + P_A \ddot{\Psi}_A) \\ &\quad + {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b\, {}^{\mathcal{A}}R_{\mathcal{B}}^T (P_A \dot{\Psi}_A \times {}^{\mathcal{A}}R_{\mathcal{B}}\, P_b \dot{\Psi}_b) \\ &\quad + \left(P_A \dot{\Psi}_A + {}^{\mathcal{A}}R_{\mathcal{B}}\, P_b \dot{\Psi}_b\right) \times {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b\, {}^{\mathcal{A}}R_{\mathcal{B}}^T P_A \dot{\Psi}_A \\ &\quad + P_A \dot{\Psi}_A \times {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b P_b \dot{\Psi}_b\,. \end{aligned}$$

Substituting for the modified forms of LHS and RHS in Eq. (18), we get

$$\begin{aligned} &{}^{\mathcal{A}}R_{\mathcal{B}}\, I_b P_b \ddot{\Psi}_b + F_{O,b} + D_{O,b} \\ &\quad = [\ {}^{\mathcal{A}}R_{\mathcal{B}}\, {}^{\mathbf{B}}\mathbf{r_{b1}} \times \tfrac{\bar{\mathbf{q}}_1}{q_1} \cdots - {}^{\mathcal{A}}R_{\mathcal{B}}\, {}^{\mathbf{B}}\mathbf{r_{b12}} \times \tfrac{\bar{\mathbf{w}}_6}{w_6}\ ]\, \mathbf{T}. \end{aligned} \tag{22}$$

As in the force equation, even here $D_{O,b}$ is the only term that depends on the perturbation in frame $\mathcal{A}$. So we can treat this term as disturbance term to design Robust Controllers. We can combine both force and moment equations as

$$\overbrace{\begin{bmatrix} m_b I_3 & 0_3 \\ 0_3 & {}^{\mathcal{A}}R_{\mathcal{B}}\, I_b P_b \end{bmatrix}}^{M_b} \begin{bmatrix} \ddot{\mathbf{x}}_b \\ \ddot{\Psi}_b \end{bmatrix} + \overbrace{\begin{bmatrix} 0_{3\times 1} \\ F_{O,b} \end{bmatrix}}^{F_b} + \overbrace{\begin{bmatrix} D_{\mathbf{x},b} \\ D_{O,b} \end{bmatrix}}^{D_b} = \overbrace{\begin{bmatrix} \bar{\mathbf{q}}_1/q_1 & \cdots & -\bar{\mathbf{w}}_6/w_6 \\ {}^{\mathcal{A}}R_{\mathcal{B}}\, {}^{\mathbf{B}}\mathbf{r_{b1}} \times \bar{\mathbf{q}}_1/q_1 & \cdots & -\, {}^{\mathcal{A}}R_{\mathcal{B}}\, {}^{\mathbf{B}}\mathbf{r_{b12}} \times \bar{\mathbf{w}}_6/w_6 \end{bmatrix}}^{J_b} \mathbf{T}. \tag{23}$$

where, I_3, 0_3, and $0_{3\times 1}$ are 3×3 identity matrix, 3×3 zero matrix, and 3×1 zero vetor, respectively. Similarly, we can write the equations of motion for end-effector as

$$\overbrace{\begin{bmatrix} m_c I_3 & 0_3 \\ 0_3 & {}^{\mathcal{A}}R_{\mathcal{C}}\, I_c P_c \end{bmatrix}}^{M_c} \begin{bmatrix} \ddot{\mathbf{x}}_c \\ \ddot{\Psi}_c \end{bmatrix} + \overbrace{\begin{bmatrix} 0_{3\times 1} \\ F_{O,c} \end{bmatrix}}^{F_c} + \overbrace{\begin{bmatrix} D_{\mathbf{x},c} \\ D_{O,c} \end{bmatrix}}^{D_c} = \overbrace{\begin{bmatrix} \bar{\mathbf{v}}_1/v_1 & \cdots & -\bar{\mathbf{w}}_6/w_6 \\ {}^{\mathcal{A}}R_{\mathcal{C}}\, {}^{\mathbf{C}}\mathbf{r_{c1}} \times \bar{\mathbf{v}}_1/v_1 & \cdots & -\, {}^{\mathcal{A}}R_{\mathcal{C}}\, {}^{\mathbf{C}}\mathbf{r_{c6}} \times \bar{\mathbf{w}}_6/w_6 \end{bmatrix}}^{J_c} \mathbf{T}. \tag{24}$$

Note that in the above equation, in the term, ${}^{\mathbf{C}}\mathbf{r_{ci}}$, i goes from 1 to 6, and then repeats again from 1 to 6 instead of from 7 to 12. This is because v_i and w_i cable pass through the same attachement point, i.e., there are only 6 attachment points on the end-effector. Combining Eq. (23) and Eq. (24) leads to

$$M(\underline{\mathbf{x}})\ddot{\underline{\mathbf{x}}} + F(\underline{\mathbf{x}}) + D(\underline{\mathbf{x}}, \mathbf{x}_A) = J(\underline{\mathbf{x}})\mathbf{T}, \tag{25}$$

$$M(\underline{\mathbf{x}}) = \begin{bmatrix} M_b & O \\ O & M_c \end{bmatrix}, \quad F(\underline{\mathbf{x}}) = \begin{bmatrix} F_b \\ F_c \end{bmatrix},$$
$$D(\mathbf{x}) = \begin{bmatrix} D_b \\ D_c \end{bmatrix}, \quad J(\underline{\mathbf{x}}) = \begin{bmatrix} J_b \\ J_c \end{bmatrix}. \tag{26}$$

Note that $\underline{\mathbf{x}} = [\mathbf{x}_b, \Psi_b, \mathbf{x}_c, \Psi_c]$.

3. Robust Controller

3.1. Sliding Mode Control

The theory of variable structure systems (VSS) with sliding mode has been studied in detail during the last thirty years. It rests on the concept of changing the structure of the controller in order to obtain desired response. There are several advantages in VSS; e.g. high speed response, good transient performance, insensitive to certain parameter variations and external disturbances [Utiks(1976), Utkin(1979)], while standard control schemes such as the computed torque or inverse method is very sensitive to parametric uncertainty, i.e., to imprecision on manipulator inertias, geometry, loads, or friction terms. Hence, the VSS approach has been widely applied to the design of many practical control system, such as servo system, robot manipulators, and flight control systems, etc.

The sliding mode control (SMC), which belongs to a class of VSS, is considered for the control of the dual-stage cable robot in the presence of uncertainties, since SMC not only provides a robust and accurate response, but also makes the system response insensitive to changes in the system parameters and load disturbances. In the following derivation, it is assumed that all the cables keep positive tensions during the motion. Also, the actuators are ideal and cable stiffness is longitudinally large to instantaneously carry the wrench torque to the end-effector. In addition, the control states $\underline{\mathbf{x}}$ and the motion of taget $\underline{\mathbf{x}}_d$ are detected by a camera attached to the rotator.

First of all, we define a sliding surface and Lyapunov function in the following equations.

$$\mathbf{s}_{6\times 1} = \dot{\underline{\mathbf{x}}} + \mathbf{\Lambda}(\underline{\mathbf{x}} - \underline{\mathbf{x}}_d), \tag{27}$$

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s}, \tag{28}$$

where $\Lambda = \begin{bmatrix} \lambda_1 & & O \\ & \ddots & \\ O & & \lambda_{12} \end{bmatrix}$. Differentiating Eqn. (28) w.r.t time leads to

$$\begin{aligned} \dot{V} &= \mathbf{s}^T\dot{\mathbf{s}}, \\ &= \mathbf{s}^T(\ \ddot{\underline{\mathbf{x}}} + \Lambda\dot{\underline{\mathbf{x}}}\ ), \\ &= \mathbf{s}^T[\ M^{-1}(\underline{\mathbf{x}})(J(\underline{\mathbf{x}})\mathbf{T} - F(\underline{\mathbf{x}}) - D(\underline{\mathbf{x}}, \mathbf{x}_A)\ ) + \Lambda\dot{\underline{\mathbf{x}}}\ ]. \end{aligned} \tag{29}$$

To make $\dot{V}$ negative, we select the control law as

$$\mathbf{u} = J^{-1}(\underline{\mathbf{x}}) \ [\ F(\underline{\mathbf{x}}) + M(\underline{\mathbf{x}})(-\Lambda\dot{\underline{\mathbf{x}}} - Ksgn(\mathbf{s})\ )\ ] \ , \tag{30}$$

where $K = \begin{bmatrix} k_1 & & O \\ & \ddots & \\ O & & k_{12} \end{bmatrix}$, $sgn(\mathbf{s}) = \begin{bmatrix} sgn(s_1) \\ \vdots \\ sgn(s_{12}) \end{bmatrix}$, which leads to

$$\begin{aligned} \dot{V} &= \mathbf{s}^T[-M^{-1}(\underline{\mathbf{x}})D(\underline{\mathbf{x}},\mathbf{x}_A) - Ksgn(\mathbf{s})] \ , \\ &\leq \sum_{i=1}^{12} |\mathbf{s}_i| (\sum_{j=1}^{12} |(M^{-1})_{ij}|\overline{D}_j - k_i) \ , \\ &= -\sum_{i=1}^{12} \eta_i |s_i| \ . \end{aligned} \tag{31}$$

We select $k_i = \sum_{j=1}^{6} |(M^{-1})_{ij}|\overline{D}_j + \eta_i$ and $|D(\underline{\mathbf{x}},\mathbf{x}_A)|_i \leq \overline{D}_i, \ i = 1, \cdots, 12$. The total energy decreases since $\dot{V} \leq 0$ and the invariant set to satisfy $\dot{V} = 0$ has only $s_i = 0$ as its candidates. Hence, there does not exist any other points where system may get stuck. Hence, the equilibrium at $\mathbf{x}_d$ is globally asymptotically stable as long as cables are in tension.

The idealized form of the methodology results in perfect tracking of the required signal. However, certain non-idealities associated with its implementation cause the trajectory to chatter along the sliding surface, resulting in the generation of an undesirable high-frequency component in the trajectory. Not only is the high-frequency component undesirable in itself, but also it may excite high-frequency dynamics associated with the control system which have been neglected in the course of modeling. To avoid chattering, we replace the $sgn()$ function by the $sat()$ function given as

$$sat(\frac{s_i}{\Phi_o}) = \begin{cases} sgn(\frac{s_i}{\Phi_o}), \ |s_i| > \Phi_o \\ \frac{s_i}{\Phi_o}, \ otherwise \end{cases} \ , \tag{32}$$

where we used a constant boundary layer thickness as $\Phi = \Phi_o \ [1, \cdots, 1]^T$. Hence the control law now looks like

$$\mathbf{u} = J^{-1}(\underline{\mathbf{x}}) \ [\ F(\underline{\mathbf{x}}) + M(\underline{\mathbf{x}})(-\Lambda\dot{\underline{\mathbf{x}}} - Ksat(\frac{\mathbf{s}}{\Phi_o})\ )\ ] \ , \tag{33}$$

3.2. Bound on disturbance

To design successfully the proposed controller for the uncertain system given in Eq. (26), we have to estimate the bound of the disturbance term $D(\underline{\mathbf{x}},\mathbf{x}_A)$. Because of the complexity of $D(\underline{\mathbf{x}},\mathbf{x}_A)$, we illustrate the method by only a translational motion of the rotator $(\mathbf{x}_A)$. Hence, we get $\mathbf{x}_A = (x_a, y_a, z_a, 0, 0, 0)$, which simplifies ${}^{\mathcal{N}}R_{\mathcal{A}} = I, \ {}^{\mathcal{N}}\dot{R}_{\mathcal{A}} = O, \ \ddot{R}_{\mathcal{A}} = O, \alpha_{\mathbf{a}} = \omega_{\mathbf{a}} = 0$. Given the above conditions, $D(\underline{\mathbf{x}}, \mathbf{x}_A)$ reduce to

$$D(\underline{\mathbf{x}}, \mathbf{x}_A) = \begin{bmatrix} -m_b\ddot{x}_a \\ -m_b\ddot{y}_a \\ -m_b\ddot{z}_a - m_b g \\ O_{3\times 1} \\ -m_c\ddot{x}_a \\ -m_c\ddot{y}_a \\ -m_c\ddot{z}_a - m_c g \\ O_{3\times 1} \end{bmatrix}. \tag{34}$$

If we prescribe the motion of the sea as a sinusoidal function of the form $A_i sin(\omega_i t)$, $i = x, y, z$, $D(\underline{\mathbf{x}}, \mathbf{x}_A)$ is bounded by a state independent vector $\overline{D}$ as

$$|D(\underline{\mathbf{x}}, \mathbf{x}_A)| \leq \overline{D} = \begin{bmatrix} m_b\omega_x^2 A_x \\ m_b\omega_y^2 A_y \\ m_b\omega_z^2 A_z + m_b g \\ O_{3\times 1} \\ m_c\omega_x^2 A_x \\ m_c\omega_y^2 A_y \\ m_c\omega_z^2 A_z + m_c g \\ O_{3\times 1} \end{bmatrix}. \tag{35}$$

4. Simulations

A simulation for the two stage cable robot implementing the sliding mode controller was developed in Matlab Simulink. We list all the parameter values in Table 1, which are actual values shown in NIST's design of the 2 stage cable robot.

m_b and m_c are the masses of the upper spreader and the end-effector respectively, I_{ij} is the j^{th} diagonal entry of a frame i's inertia matrix in the local frame, and r_{ij} stands for the position vector between j^{th} cable attachment point and the origin on a frame i. m_c includes the load and the end-effector mass. In this simulation, we prescribed a sea motion as a simple periodic function $\mathbf{x}_A = [0, 0, 1 * sin(0.5\pi t), 0, 0, 0]$, only allowing a vertical motion. From the result of Section 3, $\overline{D} = [0, 0, (2\pi)^2 m_b, O_{1\times 5}, (2\pi)^2 m_c, O_{1\times 3}]^T$, where we treated the gravitational terms $(-m_b g, -m_c g)$ as known terms. In addition, the set point $\underline{\mathbf{x}}_d = [0, 0, 6, O_{1\times 5}, 10, O_{1\times 3}]^T$ and initial states $\underline{\mathbf{x}}_0 = [0, 0, 5, O_{1\times 5}, 8, O_{1\times 3}]^T$. In this case, there doesn't exist any orientational motion since the stabilizing controller keeps all the angles from deviating from their initial values, which make the inertia matrix M be diagonalized. Hence,

Table I. Simulation parameters

Param.	Value	Param.	Value
m_b	100	r_{c1}	$(1.03, 3.04, 0)$
m_c	500	r_{c2}	$(-1.03, 3.04, 0)$
I_{b1}	33.67	r_{c3}	$(-1.03, -1.01, 0)$
I_{b2}	8.66	r_{c4}	$(-1.03, -1.01, 0)$
I_{b3}	41.66	r_{c5}	$(1.03, -1.01, 0)$
I_{c1}	1550	r_{c6}	$(1.03, -1.01, 0)$
I_{c2}	180	λ_o	5
I_{c3}	1727	η_o	5
r_{b1}	$(1, 2, 0)$	r_{a1}	$(0, 3.6, 0)$
r_{b2}	$(-1, 2, 0)$	r_{a2}	$(0, 3.6, 0)$
r_{b3}	$(-1, 2, 0)$	r_{a3}	$(-3.1, -1.8, 0)$
r_{b4}	$(0, -2, 0)$	r_{a4}	$(-3.1, -1.8, 0)$
r_{b5}	$(0, -2, 0)$	r_{a5}	$(3.1, -1.8, 0)$
r_{b6}	$(1, 2, 0)$	r_{a6}	$(3.1, -1.8, 0)$
r_{b7}	$(-1, 2, 0)$	r_{a7}	$(1, 3, 0)$
r_{b8}	$(1, 2, 0)$	r_{a8}	$(-1, 3, 0)$
r_{b9}	$(-1, 2, 0)$	r_{a9}	$(-1, -1, 0)$
r_{b10}	$(1, -2, 0)$	r_{a10}	$(-1, -1, 0)$
r_{b11}	$(1, 2, 0)$	r_{a11}	$(1, -1, 0)$
r_{b12}	$(-1, -2, 0)$	r_{a12}	$(1, -1, 0)$

from the Section 3, the control gain $\mathbf{k}$ is given by

$$\mathbf{k} = \begin{bmatrix} 0 \\ 0 \\ (0.5\pi)^2 \\ O_{5\times1} \\ (0.5\pi)^2 \\ O_{3\times1} \end{bmatrix} + \eta_o \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \tag{36}$$

As shown in Fig. 2, the control performance looks good and the cable tensions oscillate between 400N and 1.2kN. In this simulation, it appears that the cables do not become slack.

5. Conclusions

This paper dealt with a new type of a cable mechanism, a two stage cable robot with two moving platforms connected in series. Such cable robots can be used to transfer cargo for crane ships to a smaller ship at sea. The disturbance is

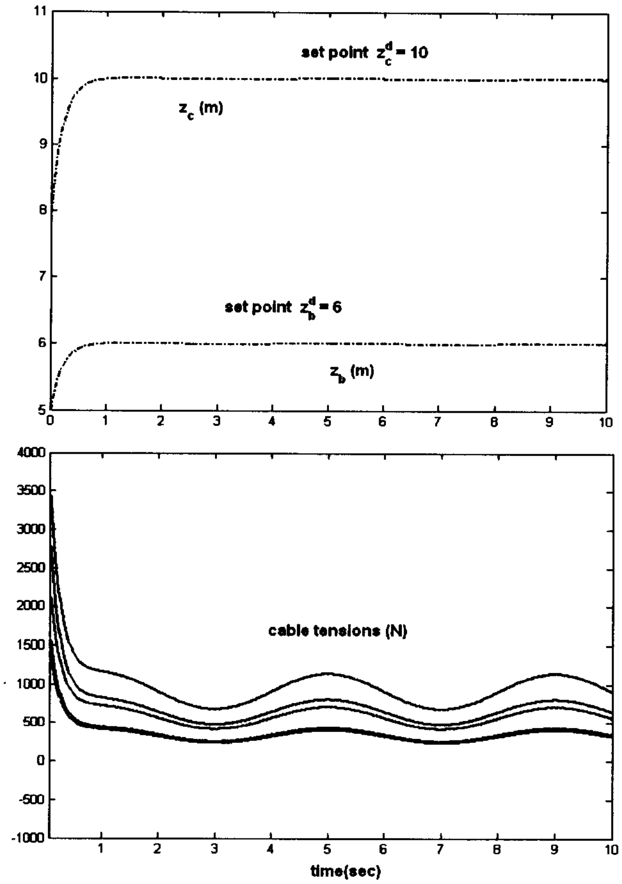

Figure 2. State and input trajectories of the sliding mode controller under a perturbation $z_c = sin(0.5\pi t)$

considered while modeling the dynamics of the two stage cable robot. A robust controller was designed which can assure robust tracking of the desired end-effector trajectory in the presence of the disturbance. Simulation results was presented to show the effectiveness of the controller.

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