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Design and Workspace Analysis of a 6-6 Cable-Suspended Parallel Robot

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Abstract – In this paper, we study the design and workspace of a 6-6 cable-suspended parallel robot. The workspace volume is characterized as the set of points where the centroid of the MP (MP) can reach with tensions in all suspension cables at a constant orientation. This paper attempts to tackle some aspects of optimal design of a 6DOF cable robot by addressing the variations of the workspace volume and the accuracy of the robot using different geometric configurations, different sizes and orientations of the MP. The global condition index is used as a performance index of a robot with respect to the force and velocity transmission over the whole workspace. The results are used for design analysis of the cable-robot for a specific motion of the MP.

1. INTRODUCTION

Parallel robots generally have larger load capacities, faster and more accurate motions and larger stiffness throughout their workspace as compared to the serial ones [6]. These attributes associated with the parallel robots make them quite attractive in real world applications. The increased popularity of parallel robots has opened many questions.

One particular question that this paper attempts to address is "what is an optimal design"? An optimal design of a robot generally refers to one which has the largest workspace, fewest singularities, large force output, high stiffness throughout its workspace, high accuracy, etc. Maximizing the workspace volume is one of the goals of an optimal design. Performance indices are defined to measure the different aspects that encompass an optimal design.

This paper focuses on a particular type of parallel robot, namely the cable suspended parallel robot. Traditional parallel robots are driven by prismatic actuators. Some common prismatic actuators include hydraulic and pneumatic pistons and motor driven lead screws. These systems tend to be large, heavy, and cumbersome. Cable suspended parallel robots are slightly different from the traditional parallel robots. In a cable suspended parallel robot, the moving platform (MP) is suspended and manipulated by the attached cables that are connected to the base.

Performance indices are defined to measure the different aspects that encompass an optimal design. The performance index is a term used when referring to methods that compare the quality of one parallel robot design to another. One common performance index used is the condition number of the Jacobian matrix of the robot. The condition number, further explained in section IV, is used to determine how close the robot is, in its particular position and orientation, to a singularity configuration. Another commonly used performance index is the global condition index (GCI), which is a measure of kinematic dexterity of the robot over the whole workspace [5].

Some reported research on cable-suspended robots are NIST Robocrane [1], ultra-high speed robot, tendon-driven Stewart platforms, parallel wire mechanism for measuring a robot pose, controller designs for cable-suspended robot [2], and design and workspace analysis of planar cable-suspended robots ([3], [4]). A three dimensional cable suspended robot, the ROBOCRANE, was created by the National Institute of Standard Technology (NIST). A systematic analysis for the particular geometry selected in the design of the cable robots by NIST, is still lacking.

This paper attempts to tackle some aspects of optimal design of a 6DOF cable robot by addressing the variations of the workspace volume and the accuracy of the robot using different geometric configurations, different sizes and orientations of MP. The workspace volume for the cable robots is characterized by the set of points where the center of mass of moving platform can be positioned while all cables are in tensions.

The organization of the paper is as follows: Section II explains the detailed kinematic and geometric modeling of the 6-6 cable robot. Section III uses the modeling to obtain the forces in the cables and defines the workspace for the cable robot. Section IV deals with global condition index (GCI), which are used to measure the quality of performance of the robot over the whole workspace. Based on the underlying modeling and workspace analysis, workspace volume and GCI are simulated in Section V for different geometry configurations, sizes of moving platform and orientations. Section VI uses the results of Section V to attain the best design of the cable-robot for a desired motion of the moving platform.

II. KINEMATIC AND GEOMETRIC MODELING

The model used is that of a 6-6 cable suspended parallel robot. The base points of the manipulator $b_1, \ldots, b_6$ are all contained within the same plane ($\mathbf{z}_O = 0$) as shown in Fig. 1. The points are placed at some radial distance r_{base} from the base coordinate system $\boldsymbol{F_O}$ that is located at O the center of the base platform (BP). The MP similarly has a set of connection points $a_1, \ldots, a_6$ located at a distance r_{end} from the moving coordinate frame $\boldsymbol{F_E}$ attached to O_E the center of mass of the MP. These points are located on the ($\mathbf{z}_E = 0$) plane relative to frame $\boldsymbol{F_E}$.

The position vector of point b_i on the base is defined by

$$ {}^{O}\mathbf{b}_i = \begin{bmatrix} r_{base}\cos(\beta_i) \\ r_{base}\sin(\beta_i) \\ 0 \end{bmatrix} \tag{1} $$

where r_{base} is the radial distance from the base coordinate frame $\mathcal{F}_O$. The variable β_i denotes the angular location of point b_i on BP with respect to axis $\mathbf{x}_0$.
The position vectors of the connection points on the MP are as follows:

$$ {}^{E}\mathbf{a}_i = \begin{bmatrix} r_{end}\cos(\alpha_i) \\ r_{end}\sin(\alpha_i) \\ 0 \end{bmatrix} \tag{2} $$

The variable α_i denotes the angular location of each connection point on the MP with respect to $\mathbf{x}_E$. The position vector from point b_i to point a_i, i.e., the cable vector shown in Fig. 2, can be expressed with respect to frame $\mathcal{F}_O$ as

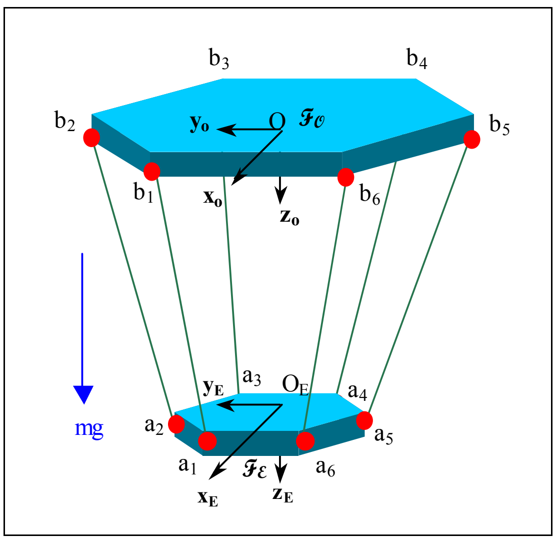

Fig. 1 A schematic of a 6-6 cable robot under study

$$ {}^{O}\mathbf{l}_i = {}^{O}\mathbf{p}_E + {}^{O}\mathbf{R}_E\,{}^{E}\mathbf{a}_i - {}^{O}\mathbf{b}_i \tag{3} $$

where ${}^{O}\mathbf{p}_E$ represents the position vector of point O_E with respect to O. ${}^{O}\mathbf{R}_E$ is the rotation matrix of MP with respect to BP using a fixed axis rotation sequence of ψ, θ, and ϕ about $\mathbf{x}_0$, $\mathbf{y}_0$, and $\mathbf{z}_0$ axes, respectively. The magnitude of each ${}^{O}\mathbf{l}_i$ vector is

$$ l_i^2 = \left\| {}^{O}\mathbf{l}_i \right\| = l_{ix}^{\;2} + l_{iy}^{\;2} + l_{iz}^{\;2} \tag{4} $$

Using the time derivative of the kinematic constraint equations, the relation between the cable velocity vector $\dot{\mathbf{q}}$ and the twist of MP, i.e., $\mathbf{t}$ can be related by the Jacobian matrix as follows:

$$ \dot{\mathbf{q}} = \mathbf{J}\mathbf{t} \tag{5} $$

where

$$ \dot{\mathbf{q}} = \begin{bmatrix} \dot{l}_1 & \dot{l}_2 & \dot{l}_3 & \dot{l}_4 & \dot{l}_5 & \dot{l}_6 \end{bmatrix}^T \tag{6} $$

$$ \mathbf{t} = \begin{bmatrix} {}^{O}\dot{\mathbf{p}}_E^{\;T} & {}^{O}\boldsymbol{\omega}_E^{\;T} \end{bmatrix}^T \tag{7} $$

Here, $\boldsymbol{\omega}_E^T$ is the angular velocity vector of the MP with respect to frame $\mathcal{F}_O$. The Jacobian matrix of the cable robot is written such that its i[th] row is

$$ \mathbf{j}_i = \begin{bmatrix} \left(\dfrac{{}^{O}\mathbf{l}_i}{l_i} \right)^T & \left({}^{O}\mathbf{a}_i \times \dfrac{{}^{O}\mathbf{l}_i}{l_i} \right)^T \end{bmatrix} \tag{8} $$

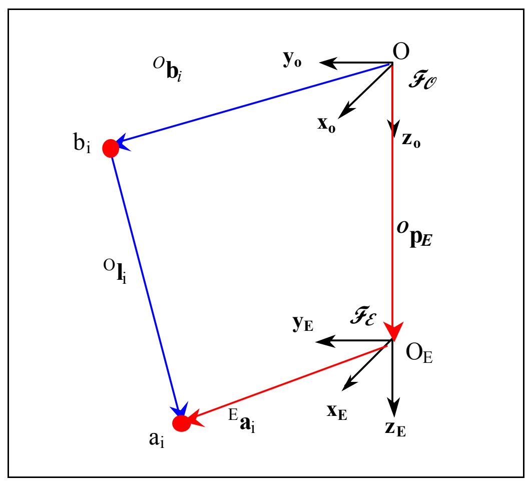

Fig. 2 A sketch of cable $\boldsymbol{i}$ and position vectors of BP and MP

III. WORKSPACE ANALYSIS

The static equilibrium of the cable suspended parallel robot is used to find the force of each cable. Force and moment balance on the MP is

$$ \begin{aligned} \sum \mathbf{F} &= m\mathbf{g} - \sum_{i=1}^{6} \mathbf{T}_i = \mathbf{0} \\ \sum \mathbf{M}_{O_E} &= -\sum_{i=1}^{6} \left({}^{O}\mathbf{a}_i \times \mathbf{T}_i \right) = \mathbf{0} \end{aligned} \tag{9} $$

where it can be seen that there are no moments applied to MP and the only external force is the gravitational force. All other external forces and moments are ignored. To relate the external forces represented by (9) to the forces in the cables, the dual relationship between kinematics and statics can be used as follows:

$$\mathbf{f}_{ext} = \mathbf{J}^T \mathbf{s} \tag{10}$$

where $\mathbf{s}$ is the vector of tensions present in each cable, J is the Jacobian matrix of the robot defined in (8) and $\mathbf{f}_{ext}$ is a 6 dimensional vector containing the external forces and moments given by

$$\mathbf{f}_{ext} = \begin{bmatrix} 0 & 0 & m\mathbf{g} & 0 & 0 & 0 \end{bmatrix}^T \tag{11}$$

The first three rows in (11) represent the external forces and similarly the last three rows represent the external moments in the x, y, and z direction, respectively. Now to obtain the equations for the force of each cable, Equation (11) is rearranged in the following manner

$$\mathbf{s} = \tilde{\mathbf{J}} \mathbf{f}_{ext} \tag{12}$$

where

$$\tilde{\mathbf{J}} = \left(\mathbf{J}^T\right)^{-1} \tag{13}$$

The workspace volume for the cable robot is later on characterized by the set of points where the center of mass of the MP can be positioned while all cables are in tensions. To this end, at each point within the possible workspace, the equation describing the force in each cable is used to see if tension is obtainable.

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IV. PERFORMANCE INDICES

The quality of performance of a robot with respect to the force and velocity transmission can be addressed by using the conditioning of the Jacobian matrix of the robot.
The global conditioning index (GCI) supplies the designer with a performance index tool that can be used. As the GCI approaches zero the workspace has a bad GCI and as the GCI approached one the workspace has a good GCI. Due to the difficulty of obtaining the exact solution to the integral a discrete version of GCI can be expressed as

$$GCI = \frac{\sum \left(\frac{1}{\kappa} \right)}{w} \tag{14}$$

where w is number of points in the workspace and the numerator is the summation of the inverse of each condition number κ in the workspace grid. Equation (14) is much easier to compute the GCI [5].

V. SIMULATION STUDIES AND ANALYSIS

A program was created in Matlab for workspace analysis of cable suspended robots. The program inputs include the connection points of the base and MP, the radius of these connection points on both platforms, the given orientation of the MP, the desired search volume and incremental step size for the mesh search. To keep consistency the step size was fixed along all three axes to 0.4, the search for possible workspace volume was fixed to the region of ($-8 \le x_0 \le 8$, $-8 \le y_0 \le 8$, $0 \le z_0 \le 10$), and finally $r_{base} = 6$ was fixed constant. Once a particular geometry, MP size, and MP orientation is selected, the program checks every point in the volume to see if the cables yield tensions when the MP is positioned at each point of the possible workspace volume. If the point in question does yield tensions, the condition number of the Jacobian at the point is computed. After all the points in the search volume are checked, a global condition index is computed over the workspace of good points, where “good points” refers to points that result in tension in all cables.

A. Different Geometry Configurations

Four different geometric designs were studied and compared. These designs included a geometric configuration for $\gamma = 45^O$, 30^O, 15^O and 0^O, where γ is the angle between BP connection points (b_1, b_2), (b_3, b_4), (b_5, b_6) and MP connection points (a_1, a_6), (a_2, a_3), (a_4, a_5). The design for $\gamma = 45^O$, 30^O, and 15^O is symmetric semi-hexagons and the design for $\gamma = 0^O$ is where the MP and BP are equilateral triangles similar to the 3-3 Stewart platform. The orientation of the MP to the BP is in such a way that the long side of MP would line up with the short side of the BP when $\boldsymbol{F_E}$ is aligned with $\boldsymbol{F_O}$. Dissimilar BP to MP geometries is not considered in the current study that is, mismatching the γ angles for the BP to MP.

B. Different Ratios of MP to BP

The size of the MP relative to the BP was also varied from the same size to smaller sizes for each γ value. The results are depicted in Fig. 3 for the constant orientation ($\psi = 0^O$, $\theta = 0^O$, $\phi = 0^O$). As it is shown for all geometries, as the ratio of MP/BP increases the workspace volume also increases.

C. Global Condition Index

The global condition index was used as a performance index in order to determine from a global perspective view how well a cable robot performed in each specific geometric configuration and different sizes of MP. The results are shown in Fig. 4. It can readily be inferred from the results that 0 degree geometric configuration has the best performance among others. Moreover, higher ratio of MP to BP results higher GCI, namely better performance.

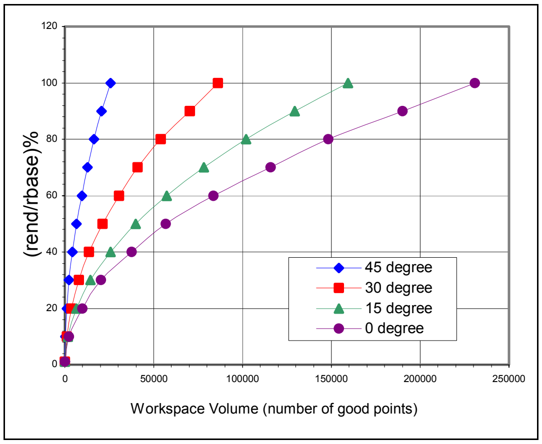

Fig. 3 Workspace volume versus ratio of MP to BP for orientation (ψ = 0^O, θ = 0^O, ϕ = 0^O)

D. Different Orientations

In attempting a broader range of studies besides those concerning geometric shapes and configurations, different orientations were studied. Due to the lengthy computational time, only two geometries were considered and three ratios of the MP to the BP. The geometries include γ = 0^O and γ = 45^O and the ratios are r_{end}/r_{base} = 100%, 50%, 1%. This amounts to six different cable suspended parallel robots.
All six of these robots were analyzed at all discrete combinations of

Table 1: Set of Orientations	
	Degrees
ψ	0, 10, 20, 30, 330, 340, 350
θ	0, 10, 20, 30, 330, 340, 350
ϕ	0, 10, 20, 30, 40, 50, 60, 300, 310, 320, 330, 340, 350

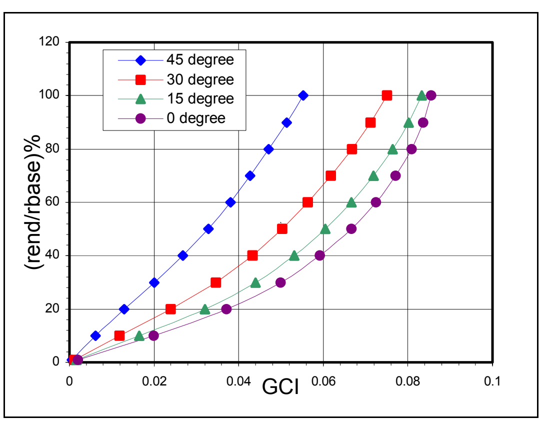

Fig. 4 GCI versus ratio of MP to BP for orientation **(ψ = 0^O, θ = 0^O, ϕ = 0^O)**

Due to the large amount of data points, only a select number of plots will be shown in this paper. The results displayed in Fig. 5 are the changes in the workspace volume for γ = 0^O for r_{end}/r_{base} = 100%, 50%, 1%. The three surfaces in each plot correspond to a constant value of ϕ. The top most surface in every plot is for r_{end}/r_{base} = 100%, the next for 50%, and the bottom 1%. The plot was made from a discrete analysis where the step size is 10^O for the angle. As the MP moves from the (ψ = 0, θ = 0, ϕ = 0) orientation the workspace volume decreases. The workspace volume was the largest when the ratio of the MP to the BP is 100% or the same size for any γ value. As expected, the workspace volume for a positive rotation of the MP is exactly matched by a mirrored negative rotation of the MP. This is due to the symmetric geometry chosen for the MP and the BP. For the γ = 0^O geometry the workspace volume decreases and eventually ceases to exist as the ϕ rotation about the $\mathbf{z_O}$ axis approaches 60^O. It is noted that at this orientation the MP aligns itself with the BP. Any additional rotation about the $\mathbf{z_O}$ axis is physically not possible for the cables would have to appose equilibrium and the cables are only capable of pulling not pushing.

To reduce the extensive amount of data pertaining to the multiple orientations observed, averages of the workspace volume are used. All the workspace volume values for different roll and pitch angles pertaining to a fixed value of ϕ (rotation about the $\mathbf{z_O}$-axis) were averaged for each geometry and size.

This data is then computed over the set of ϕ orientations and is presented in Fig. 6 for γ = 0^O. From the graph it is observed that the workspace volume value decreases as the MP rotates itself from the orientations pertaining to ϕ = 0^O, and then gradually decreases as it approaches ϕ = $\pm 60^O$ from ϕ = 0^O.

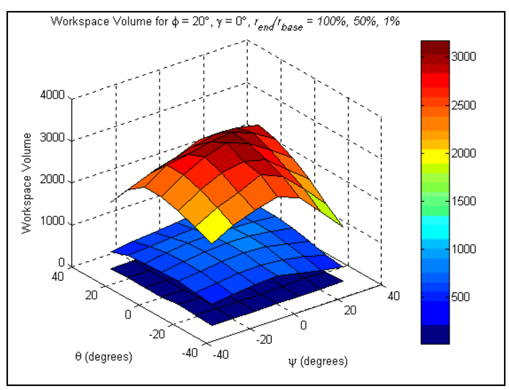

Fig. 5 Workspace volume for ϕ = 20^O, γ = 0^O, r_{end}/r_{base} = 100%, 50%, 1%

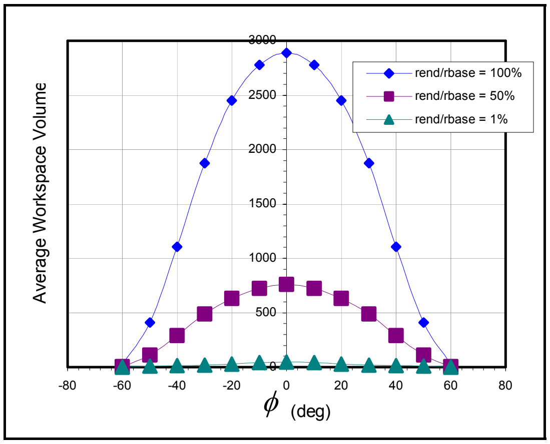

Fig. 6 Average workspace volume vs. ϕ for geometry $\gamma = 0^O$

Similar results can be given for the GCI values. To encompass a more general perspective of the behavior of the condition number over the entire space into one value, the GCI is created. After the MP moved through each point in the region under study and all the "good" points were obtained then the GCI was calculated to get a general understanding of the condition number for the entire workspace. The GCI values are displayed Fig. 7 in a manner similar to that for the workspace volume results.

To reduce the extensive amount of data pertaining to the multiple orientations observed, averages of the GCI value are used. All GCI values for different ψ and θ orientations pertaining to a fixed value of ϕ were averaged for each geometry and size. This data is then computed over the set of ϕ orientations and is presented in Fig. 8 for $\gamma = 0^O$. From the graph it is observed that the GCI value increases slightly as the MP rotates itself from the orientations pertaining to $\phi = 0^O$, and then gradually decreases as it approaches $\phi = \pm 60^O$ from $\phi = 0^O$.

VI. DESIGN PROBLEM

In this section, a design problem is presented to see if the workspace trends presented in the previous sections are applicable to the specific problem considered in this section. Different robot geometries and sizes are compared to observe how well one design is able to orient itself perpendicular to a known surface. A possible application where this particular design analysis may be valuable includes tasks such as welding, painting, washing, surface inspection, pick and place, etc. For such applications, it is desirable to choose a parallel robot that is able to manipulate itself over the largest surface area with the highest GCI value. Choosing a robot with the largest workspace volume is quite straightforward, for it yields increased versatility. A design with a high GCI is one with a globally well-conditioned jacobian. This directly correlates to the degree in which the robot is able to accurately position and orientate itself in the workspace and transmit forces and torques.

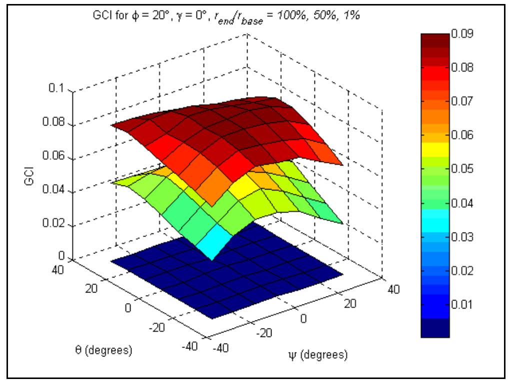

Fig. 7 GCI for $\phi = 20^O$, $\gamma = 0^O$, r_{end}/r_{base} = 100%, 50%, 1%

A. Design Problem Statement

The objective of this design problem is to determine which cable suspended parallel robot is able to orient itself perpendicular to the surface in question with the largest workspace volume and the maximum GCI. The surface considered is a simple plane defined as ($-8 \le x_0 \le 8$, $-8 \le y_0 \le 8$, $z_0 = 5$).

Four different cable suspended parallel robots are considered. In Section V, the prevailing ratio r_{end}/r_{base} = 100% persistently yielded the maximum workspace volume and the largest GCI values. Based on this information, we have chosen to only consider this ratio for our current design problem. The four robots studied will be those identified by $\gamma = 45^O, 30^O, 15^O, 0^O$. To extend the design problem, the MP for each robot is rotated through a discrete set of orientations about the $\mathbf{z_O}$ axis defined by ($-60^O \le \phi \le 60^O$) with a step size of 10^O. It is understood that ($\psi = 0$, $\theta = 0^O$) must be true for the MP to be perpendicular to the plane. Each robot is evaluated at every mesh point defining the surface for a constant orientation and the cables are checked for positive tensions. If tensions exist then the condition number is calculated at that point and after all the points are checked, the GCI is calculated.

B. Design Results

The results for the proposed design problem revealed similar trends to that obtained in the general workspace analysis portrayed in Section 5. As observed from the graph of the workspace volume vs. ϕ shown in Fig. 9 the geometry $\gamma = 0^O$ yields the largest workspace for comparative orientations.

The behavior of the GCI values is similarly displayed in Fig. 10 where it is noted that the abrupt end of the curve is where no points exist at that orientation. It can be seen that the $\gamma = 0^O$ geometry has the best GCI for comparative orientations.

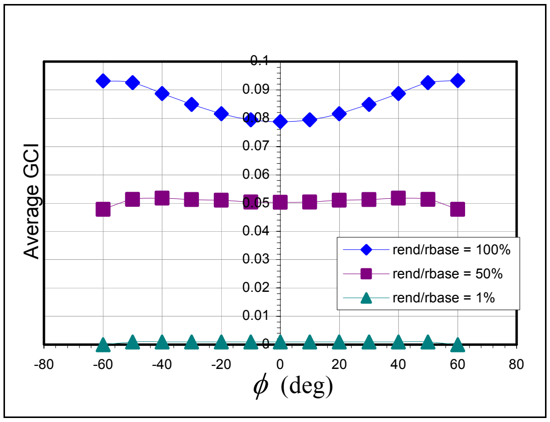

Fig. 8 Average GCI versus ϕ for $\gamma = 0^O$

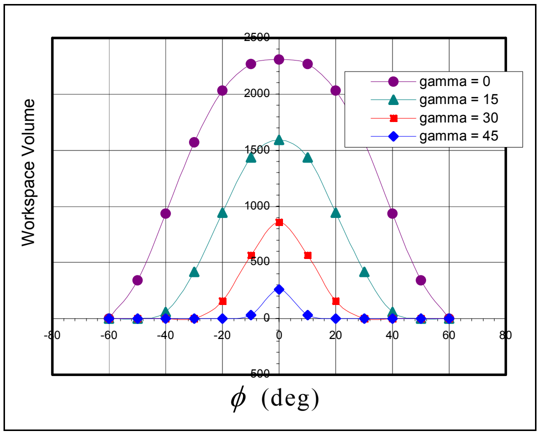

Fig. 9 Workspace volume vs. ϕ for r_{end}/r_{base} = 100%, plane at $z_O = 5$

VII. CONCLUSIONS

Design and workspace of 6-6 cable-suspended robots were studied in this paper. The variations of workspace volume and global condition index of the robot versus geometric configurations, size of the MP and different orientations were determined. The result was used for design analysis of a robot for a specific motion of the MP. In this study, it appears that for any geometry the largest workspace volume occurs when the MP is the same size as the BP. Furthermore, the $\gamma = 0^O$ geometry has the largest workspace as well as the highest GCI of any other γ value for the same orientations and MP to BP ratios. The results portrayed in the preceding sections can be used as a rule of thumb when designing similar cable suspended parallel robot.

VIII. ACKNOWLEDGMENTS

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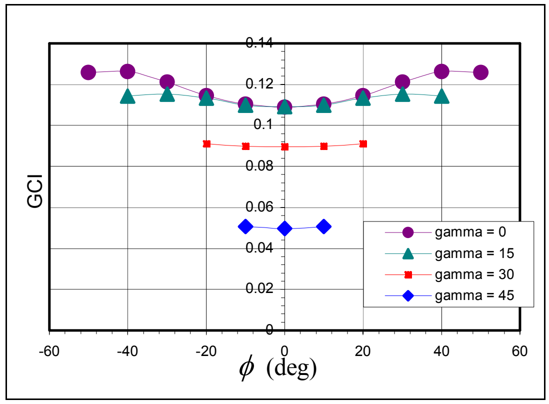

Fig. 10 GCI vs. ϕ for r_{end}/r_{base} = 100%, plane at $z_O = 5$

IX. REFERENCES

[1] J. Albus, R. Bostelman, and N. Dagalakis,,``The NIST Robocrane``, Journal of Robotic Systems, vol.10, no.5, 1992, pp.709-724.

[2] A. B. Alp, and S. K. Agrawal, "Cable Suspended Robots: Design, Planning, and Control", in Proceedings of 2002 IEEE International Conference on Robotics and Automation, Washington, pp.4275-4280.

[3] G. Barrett, and C. M. Gosselin, "Kinematic Analysis and Design of Planar Parallel Mechanisms Actuated with Cables", in Proceedings of 2000 ASME Design Engineering Technical Conference, DETC/MECH-14091.

[4] A. Fattah, and S. K. Agrawal, "Workspace and Design Analysis of Cable-Suspended Planar Parallel Robots", in Proceedings of 2002 ASME Design Engineering Technical Conference, 27th Biennial Mechanisms and Robotics Conference. DETC2002/MECH-34330.

[5] C. Gosselin and J. Angles, "A Global Performance Index for the Kinematic Optimization of Robotic Manipulators", Journal of Mechanical Design, vol. 113, September 1991, pp.220-226.

[6] Jean-Pierre Merlet, "Parallel Robots", Kluwer Academic Publishers, 2000.