

Temporal Range Registration for Unmanned Ground and Aerial Vehicles

R. Madhavan, T. Hong, and E. Messina

Intelligent Systems Division

National Institute of Standards and Technology

Gaithersburg, MD 20899-8230, U.S.A.

Tel: (301) 975-2865 Fax: (301) 990-9688

Email: raj.madhavan@ieee.org, {tsai.hong, elena.messina}@nist.gov

Abstract—An iterative temporal registration algorithm is presented in this paper¹ for registering 3D range images obtained from unmanned ground and aerial vehicles traversing unstructured environments. We are primarily motivated by the development of 3D registration algorithms to overcome both the unavailability and unreliability of Global Positioning System (GPS) within required accuracy bounds for Unmanned Ground Vehicle (UGV) navigation. After suitable modifications to the well-known Iterative Closest Point (ICP) algorithm, the modified algorithm is shown to be robust to outliers and false matches during the registration of successive range images obtained from a scanning LADAR rangefinder on the UGV.

Towards registering LADAR images from the UGV with those from an Unmanned Aerial Vehicle (UAV) that flies over the terrain being traversed, we then propose a hybrid registration approach. In this approach to air to ground registration to estimate and update the position of the UGV, we register range data from two LADARs by combining a feature-based method with the aforementioned modified ICP algorithm. Registration of range data guarantees an estimate of the vehicle's position even when only one of the vehicles has GPS information. Temporal range registration enables position information to be continually maintained even when both vehicles can no longer maintain GPS contact. We present results of the registration algorithm in rugged terrain and urban environments using real field data acquired from two different LADARs on the UGV.

I. INTRODUCTION

The National Institute of Standards and Technology (NIST) is developing architectures and algorithms for unmanned vehicles with funding from the Army Research Laboratory (ARL) and the Defense Advanced Research Projects Agency (DARPA). The NIST Highly Mobile Multipurpose Wheeled Vehicle (HMMWV) and an eXperimental Unmanned Vehicle (XUV) developed under the Army's Demo III program [1] serve as test beds for this research. These vehicles are commanded by the hierarchical, distributed, hybrid 4D/RCS architecture [2].

The 4D/RCS architecture developed for Demo III specifies the simultaneous representation of information about entities and events in a hierarchical distributed knowledge database

¹Commercial equipment and materials are identified in this paper in order to adequately specify certain procedures. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

wherein information is presented in a form that is ideally suited for path planning and task decomposition. Maps are populated both with knowledge from *a priori* sources such as digital terrain databases, and with knowledge from sensors. The range and resolution of maps at different levels are specified to correspond to the range and resolution of planning algorithms. This limits the amount of computational power required to maintain maps and symbolic data structures with a latency that is acceptable for planning and reactive processes at each level.

The position estimation for the above Unmanned Ground Vehicles (UGVs) relies on fusing Global Positioning System (GPS) reported estimates with other on-board navigation sensors. The required accuracy of the GPS estimates cannot be guaranteed for the entirety of a particular mission as the direct line of sight to the satellites cannot be maintained at all times. GPS can be lost due to multipathing effects and terrain conditions, especially for on-road driving tasks. Sufficiently accurate vehicle positions are necessary to derive correct locations of sensed data towards accurate representations of the world and for correctly executing planned trajectories and missions. In order to compensate for such unavailability and unreliability of GPS, another form of secondary position estimation becomes inevitable.

The following reasons also warrant the need to develop robust 3D data registration algorithms:

- Within RCS, the use of *a priori* maps would enhance the scope of the world model. These maps may take a variety of forms including survey and aerial maps and may provide significant information about existing topology and structures. In order to take advantage of this knowledge, research is needed to register these *a priori* maps with the sensor-centric maps. Additionally, for incorporating *a priori* knowledge into the world model, some form of weighting is required and this depends on how well the *a priori* data and the sensed information are registered.
- There is also the need to generate higher resolution *a priori* terrain maps as the current survey maps are too coarse for off-road autonomous driving and also for maintaining up-to-date representations of the world even if the maps are of higher resolution.

- Another potential application for registering LADAR data is the computation of *ground truth* as such registration is not dependent on time-based drift (unlike inertial navigation systems), vehicle maneuvers and terrain of travel.

Active range sensing has become an integral part of any unmanned vehicle navigation system due its ability to produce unambiguous, direct, robust, and precise images consisting of range pixels, for example, using LAser Detection And Ranging (LADAR) imagery. This is in direct contrast to passive sensing where the inference of range largely remains computationally intensive and not robust enough for use in natural outdoor environments. Depending on the speed of the vehicle, operating environment, and data rate, such range images acquired from a moving platform need to be registered to make efficient use of information contained in them for various navigation tasks within the 4D/RCS architecture.

One of the following two approaches is commonly employed for matching range images to *a priori* maps [3]:

- *Feature-based Matching*: In this approach, two sets of features, \mathcal{F}_i^1 and \mathcal{F}_j^2 , are extracted from two sets to be matched and then correspondences between features, \mathcal{F}_{ik}^1 and \mathcal{F}_{jk}^2 , $k \in i, j$, that are globally consistent, are found. The displacement between the two sets can then be computed to deduce the sensor pose.
- *Point Matching*: This approach directly works on two sets of data points, \mathcal{P}^1 and \mathcal{P}^2 , by minimizing a cost function of the form $\mathbf{F}(\mathbf{T}(\mathcal{P}^2), \mathcal{P}^1)$, where $\mathbf{T}(\mathcal{P}^2)$ is the second set of points subjected to a transformation \mathbf{T} . Any sensible cost is acceptable as long as its minimum corresponds to a *best* estimate of \mathbf{T} in some sense. Usually, the minimization leads to an iterative gradient-like algorithm.

For example, Kanade et al. [4] compared elevation maps obtained from 3D range images to determine vehicle location. A similar point matching approach has also been adopted by Shaffer [5]. Cox [6] proposes a point matching method for an indoor robot named Blanche where scan-points from an optical rangefinder are matched to an *a priori* map composed of straight line segments. Blanche’s position estimation system utilizes a robust matching algorithm which estimates the precision of the corresponding match/correction that is then optimally combined with odometric position to provide an improved estimate of robot position. Hoffman *et al.* employ a point matching algorithm for obtaining the inter-frame rotation and translation in a vision-based rover application [7]. Lu [8] finds corresponding points between two successive scans to compute the relative rotation and translation. An Iterative Dual Correspondence (IDC) algorithm is formulated based on closest point and matching range rules.

The major drawback of the above approaches is that their use is limited to structured office or factory environments rather than unstructured natural environments. Straightforward correlation-based schemes (for e.g., see [9]), in general, are unable to handle outliers. As cross-correlation calculates the

similarity, the two scans must be similar and thus this method cannot accommodate occlusions. This is easy to understand since if areas visible in one scan are not visible in another due to occlusion, then correlation of these scans may produce arbitrarily bad pose estimates. Also correlation usually places a high burden on computation especially when the scans are at different orientations.

In this paper, we present algorithms for registering 3D range images to overcome both the unavailability and unreliability of GPS within required accuracy bounds for UGV navigation. At the core of the registration process is a modified version of the well-known Iterative Closest Point (ICP) algorithm. These modifications render robustness to outliers, occlusions and false matches/spurious points. We then propose extensions to the ICP algorithm that make it possible to register range images obtained from a UGV to range images obtained from an Unmanned Aerial Vehicle (UAV). Registration of range data guarantees an estimate of the vehicle’s position even when only one of the vehicles has GPS information. Temporal range registration enables position information to be continually maintained even when both vehicles can no longer maintain GPS contact. We present results of the registration algorithm using real field data acquired from two different LADARs on the UGV.

The paper is organized as follows: Section II details the ICP algorithm with suitable modifications for robustness. Section III presents the results of the proposed algorithm when applied for registering consecutive range images obtained from two LADARs mounted on a moving UGV. Section IV extends the modified ICP algorithm for air to ground registration by a hybrid feature-based registration approach. Section IV-A elaborates the results of the hybrid approach when employed for registering aerial and ground range images. Finally, Section V provides conclusions and describes further research work.

II. ITERATIVE TEMPORAL RANGE REGISTRATION ALGORITHM

The ICP algorithm [10] can be summarized as follows: Given an initial motion transformation between two 3D point sets, a set of correspondences are developed between data points in one set and the next. For each point in the first data set, find the point in the second that is closest to it under the current transformation. It should be noted that correspondences between the two points sets is initially unknown and that point correspondences provided by sets of closest points is a reasonable approximation to the true point correspondence. From the set of correspondences, an incremental motion can be computed facilitating further alignment of the data points in one set to the other. This find correspondence/compute motion process is iterated until a predetermined threshold termination condition.

In its simplest form, the ICP algorithm can be described by the following steps:

1. For each point in data set \mathbf{D} , compute its closest point in data set \mathbf{M} . In this paper, this is accomplished via 3D

nearest point search from the set comprising N_D data and N_M model points.

2. Compute the incremental transformation (\mathbf{R}, \mathbf{T}) using Singular Value Decomposition (SVD) using correspondences obtained in step 1.
3. Apply the incremental transformation from step 2. to \mathbf{D} .
4. If relative changes in \mathbf{R} and \mathbf{T} are less than a threshold, terminate. Else go to step 1.

To deal with spurious points/false matches and to account for occlusions and outliers, the least-squares objective function that is to be minimized is weighted such that [11]:

$$\min_{(\mathbf{R}, \mathbf{T})} \sum_i w_i \|\mathbf{M}_i - (\mathbf{R}\mathbf{D}_i + \mathbf{T})\|^2 \quad (1)$$

where \mathbf{R} is a 3×3 rotation matrix, \mathbf{T} is a 3×1 translation vector and the subscript i refers to the corresponding points of the sets \mathbf{M} and \mathbf{D} .

If the Euclidean distance between a point x_i in one set and its closest point y_i in the other, denoted by $d_i \triangleq d(x_i, y_i)$, is bigger than the maximum tolerable distance threshold \mathcal{D}_{max} , then w_i is set to zero in Equation (1). This means that an x_i cannot be paired with a y_i since the distance between reasonable pairs cannot be very big. The value of \mathcal{D}_{max} is set adaptively in a robust manner by analyzing distance statistics.

Let $\{x_i, y_i, d_i\}$ be the set of original points, the set of closest points and their distances, respectively. The mean and standard deviation of the distances are computed as:

$$\mu = \frac{1}{N} \sum_{i=1}^N d_i; \quad \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (d_i - \mu)^2}$$

where N is the total number of pairs.

The pseudo-code for the adaptive thresholding of the distance \mathcal{D}_{max} is given below:

$$\begin{aligned} & \text{if } \mu < \mathcal{D} \\ \mathcal{D}_{max}^{itn} &= \mu + 3\sigma; \\ & \text{elseif } \mu < 3\mathcal{D} \\ \mathcal{D}_{max}^{itn} &= \mu + 2\sigma; \\ & \text{elseif } \mu < 6\mathcal{D} \\ \mathcal{D}_{max}^{itn} &= \mu + \sigma; \\ & \text{else } \mathcal{D}_{max}^{itn} &= \epsilon; \end{aligned}$$

where itn denotes the iteration number and \mathcal{D} is a function of the resolution of the range data.

During implementation, \mathcal{D} was selected based on the following two observations: (i) If \mathcal{D} is too small, then several iterations are required for the algorithm to converge and several good matches will be discarded, and (ii) if \mathcal{D} is too big, then the algorithm may not converge at all since many spurious matches will be included. The interested reader is referred to [11] for more details on the effect and selection of \mathcal{D} and ϵ on the convergence of the algorithm. At the end of this step, two corresponding point sets, $\mathbf{P}_M: \{\mathbf{p}_i\}$ and $\mathbf{P}_D: \{\mathbf{q}_i\}$ are available.

The incremental 3D transformation (rotation and translation) of step 2. is obtained as follows [12]:

- Calculate $\mathbf{H} = \sum_{i=1}^{N_D} (\mathbf{p}_i - \mathbf{p}_c)(\mathbf{q}_i - \mathbf{q}_c)^T$; $(\mathbf{p}_c, \mathbf{q}_c)$ are the centroids of the point sets $(\mathbf{P}_M, \mathbf{P}_D)$.

- Find the Singular Value Decomposition (SVD) of \mathbf{H} such that $\mathbf{H} = \mathbf{U}\mathbf{\Omega}\mathbf{V}^T$ where \mathbf{U} and \mathbf{V} are unitary matrices whose columns are the singular vectors and $\mathbf{\Omega}$ is a diagonal matrix containing the singular values.

- The rotation matrix relating the two point sets is given by $\mathbf{R} = \mathbf{V}\mathbf{U}^T$.

- The translation between the two point sets is given by $\mathbf{T} = \mathbf{q}_c - \mathbf{R}\mathbf{p}_c$.

This process is iterated as stated in step 4. until the mean Euclidean distance between the corresponding point sets \mathbf{P}_M and \mathbf{P}_D is less than or equal to a predetermined distance or until a given number of iterations is exceeded. For further details, see [13].

III. TEMPORAL REGISTRATION OF 3D RANGE IMAGES

In this section, we present the results of the modified iterative algorithm on two sets of LADAR data (henceforth referred to as UGVL1 and UGVL2). Utilizing knowledge about the LADAR mount position and calibration factors, the range information provided by the LADARs are transformed to cartesian coordinates.

UGVL1 data was obtained during field trials as the XUV traversed rugged terrain with vegetation. The LADAR was mounted on this UGV on a pan/tilt platform to increase its narrow 20° field of view. The range of the tilt motion is $\pm 30^\circ$ resulting in an effective field of view of about 90° . UGVL1 provides a range image of $32 \text{ lines} \times 180 \text{ pixels}$ where each data point contains the distance to a target in the operating environment. The angular resolution of this LADAR is $0.658^\circ \times 0.5^\circ$ in the horizontal and vertical directions, respectively.

UGVL2 data was collected from a sensor mounted on the HMMWV as the vehicle traversed urban environments. The effective field of view is $80^\circ \times 330^\circ$ thus providing an almost panoramic view of the environment with an angular resolution of 0.036° . The scan rate of UGVL2 is $1^\circ/\text{s} - 15^\circ/\text{s}$ providing 10000 pts/s with range up to 800 meters thus making it much lower than that of UGVL1 but the resulting 3D range image is of a much higher resolution. For more details on the LADARs, see [14].

Figures 1(a)–(b) show the results when the modified ICP algorithm is used to register 3D range images obtained from UGVL1 and Figures 1(c)–(d) show that for UGVL2. The number of model (\mathbf{M}) and data (\mathbf{D}) points for the two LADARs are $\{2857, 2878\}$ and $\{125396, 123826\}$, respectively.

In the case of UGVL2, the 3D point cloud was acquired from two different view points whereas for UGVL1, the 3D point cloud represents scan points that were acquired between two consecutive vehicle locations. Additionally, for UGVL1, range image \mathbf{D} was also translated a meter along each of the (x, y, z) axes in addition to the translation and rotation that the image underwent due to the motion of the vehicle. It is

important to note here that even though the range image points arrive in the same sequence for both the model and data sets, it is not guaranteed that both sets will have the same number of points as some facets of the LADAR data sets might return empty values.

As can be seen from Figure 1, the images are well registered. The mean distance (m) after registration for the above three cases are $\{0.11, 0.84, 0.43\}$, respectively.

IV. AIR TO GROUND FEATURE-BASED REGISTRATION

Another way to minimize the dependency on GPS for UGV navigation is to use aerial survey maps constructed using a downward-looking LADAR mounted on an Unmanned Aerial Vehicle (UAV). If the LADAR range images from the UGV can be registered to those from the UAV, then these results can serve as secondary position estimates in the event of absence or degradation of GPS.

In this section, we propose a hybrid approach by combining the modified ICP algorithm with a feature-based method for registering two sets of LADAR range images. The proposed approach is conceptually similar to Hebert *et al.* [15] who employ range imagery to compute vehicle displacement between two viewing positions by using a two-stage technique (feature matching followed by point matching). The advantage of our hybrid approach lies in the fact that the accuracy of the point matching technique is retained while keeping the computational burden under control as the feature-based method provides a good initial estimate for refinement.

The value of aerial imagery obtained via active range sensing for aiding ground vehicle navigation is being recognized within the UGV community. For example, in [16], aerial and ground views from unmanned vehicles are registered by extracting a geometrically consistent set of correspondences using surface signatures from which a registration transformation is estimated. It is not clear, given the computational burden associated with the extraction of surface signatures, whether this approach can be implemented in real-time. In [17], an aerial vehicle, a Flying Eye (FE), flies ahead of an UGV acting as a “scout” to detect difficult obstacles from an overhead perspective thus benefitting ground vehicle navigation. The above paper briefly mentions the need for registering the data from the FE to the ground vehicle but the details of the registration process are not presented. The hybrid approach proposed in this paper exploits the simplicity and speed of the iterative closest point algorithm thus lending itself to real-time implementation.

The underlying assumption in the iterative registration algorithm is that the rotation angle between the range images that need to be registered is not too large and also that these images are not too far apart. For the current case of UAV and UGV LADAR data, this assumption is overly restrictive and an aiding mechanism for the registration of the range images becomes necessary.

The correspondence determination step is the most difficult and computationally expensive step of the iterative algorithm.

Despite the apparent simplicity of this problem, establishing reliable correspondences is extremely difficult as the UGV is subjected to heavy pitching and rolling motion characteristic of travel over undulating terrain. This is further exacerbated by the uncertainty of the location of the sensor platform relative to the global frame of reference. In addition to these factors, noise inherently present in LADAR range images complicates the process of determining reliable correspondences. One solution to overcome the above deficiencies is to extract naturally occurring view-invariant features, for example, corners, from the LADAR scans. Such *control points* can then be used for establishing reliable registration with the ICP algorithm converging to the global minimum.

Towards guaranteeing robust and accurate registration, we first obtain the z translation value by estimating the ground z (elevation) values on the UGV and UAV LADAR data in the vicinity of the UGV’s current location. For the UGV, the ground values are obtained from the LADAR points that are within a given radius immediately in front of the vehicle and those for the UAV are obtained by finding the minimum of the LADAR values. Then we project the UAV and UGV LADAR data into the base ground planes as depicted in Figure 2(a) and construct the feature planes by using the Canny edge detector [18]. The corner features are detected based on the intersections of lines formed by edges. The corner features are independently extracted from both LADAR data sets by considering those points that are above a given height from the ground as shown in Figure 2(b).

The two sets of the projected corner points (UAV LADAR set: \mathbf{A} and UGV LADAR set: \mathbf{G}) are used to estimate a 2D translation. Given two sets of 2D corner points:

$$\mathbf{A} \triangleq \mathbf{a}_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{nj} \end{bmatrix}; j = 1, 2, \dots, n;$$

$$\mathbf{G} \triangleq \mathbf{g}_k = \begin{bmatrix} g_{1k} \\ g_{2k} \\ \vdots \\ g_{nk} \end{bmatrix}; k = 1, 2, \dots, n;$$

To find a translation along the x and y directions, we first calculate the means of sets \mathbf{A} and \mathbf{G} :

$$\bar{\mathbf{a}} = \frac{1}{n} \sum_{j=1}^n \mathbf{a}_j; \quad \bar{\mathbf{g}} = \frac{1}{n} \sum_{k=1}^n \mathbf{g}_k;$$

The difference between the means of x , y and that between the aerial and ground z values provide a rough estimate of the required 3D translation between the two sets of LADAR data. The 3D translational offset when applied to the UGV range image enables the ICP algorithm to provide reliable registration results.

A. Experimental Results for Air to Ground Registration

The UAV LADAR produces a 3D range image at up to 6000 terrain pts/s within a 100 meter scanning range. For additional

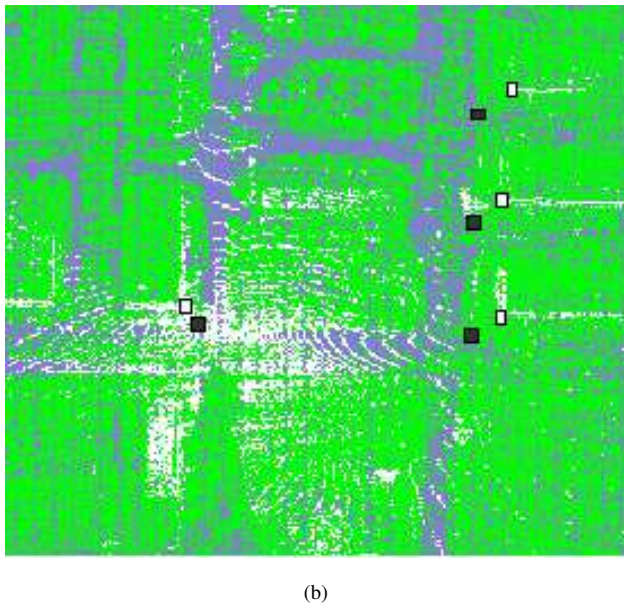
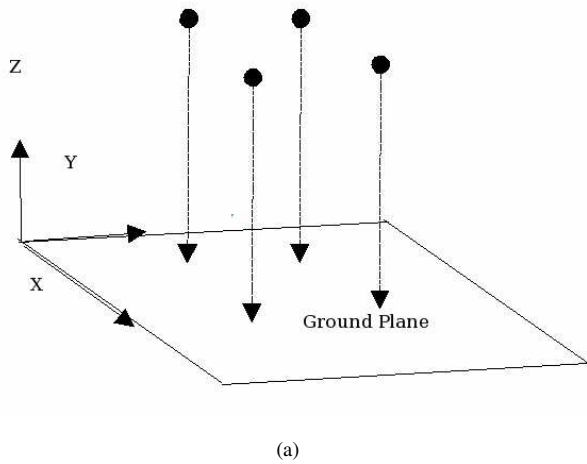


Fig. 2. Projection of LADAR data to base ground planes is shown in (a). The extracted features (corners) from the UGV (black) and UAV (white) LADARs are shown as white and black squares, respectively, in (b).

details, see [19], [20]. It provides an aerial survey map with significant information about existing topology and structures.

Figure 3 shows a top view of unregistered LADAR range images obtained from the UGV (in white) and UAV (in black) LADARs, respectively.

Figures 4(a)–(d) depict the results of the feature-based registration algorithm. Figure 4(a) shows a top view of the LADAR range images after applying the translation obtained using the corner features. Figure 4(c) shows the results of the iterative registration algorithm applied to the LADAR range images in (a). Figures 4(b) and (d) show a magnified view of stages depicted in Figures 4(a) and (c), respectively. From Figures 3 and 4, it is evident that the LADAR range images are registered. More results are available from [21].

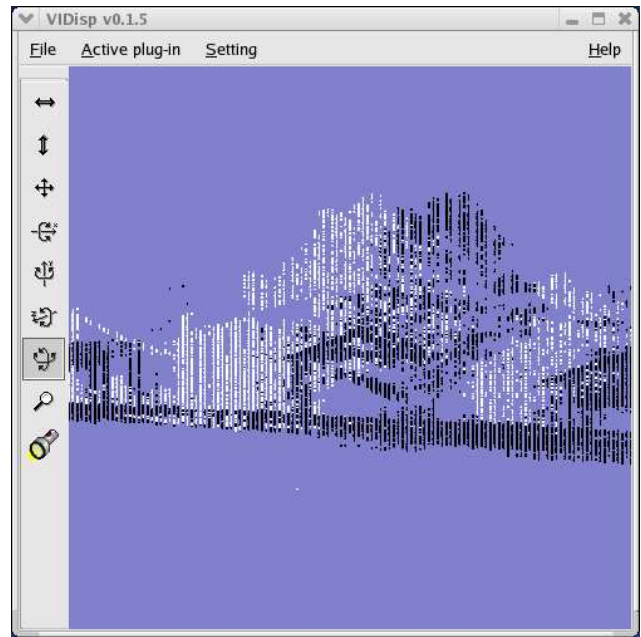
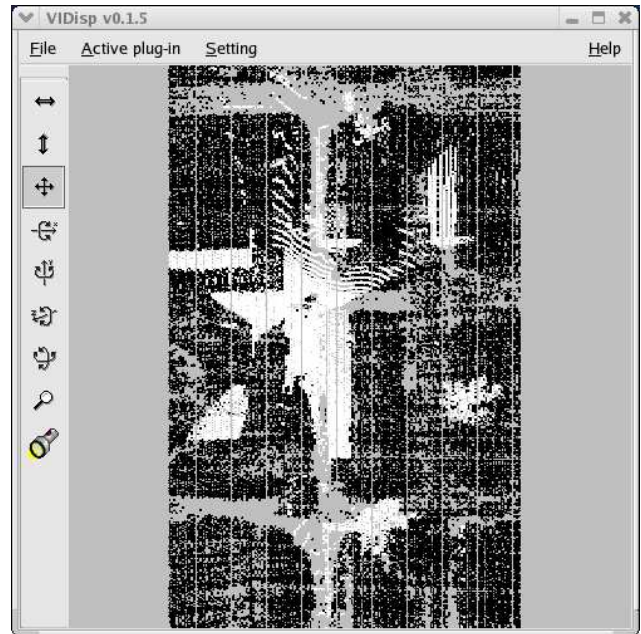
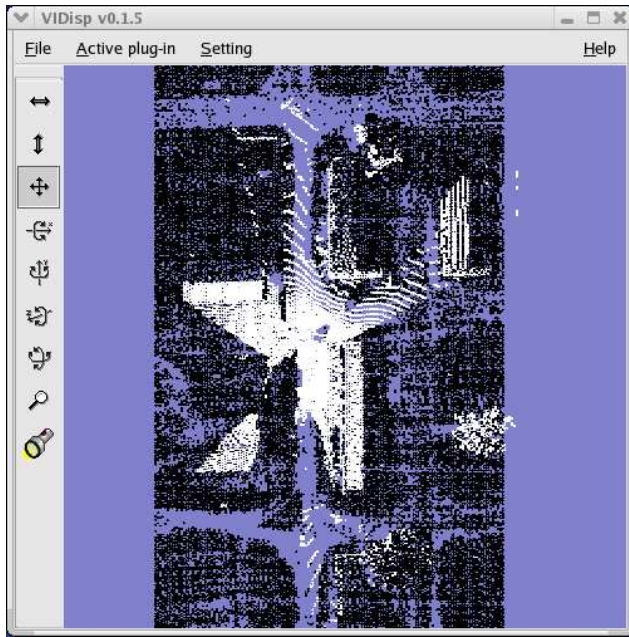
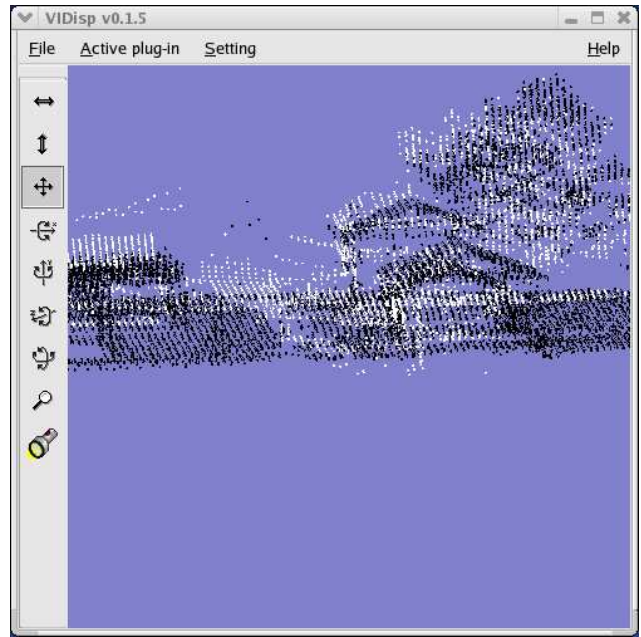


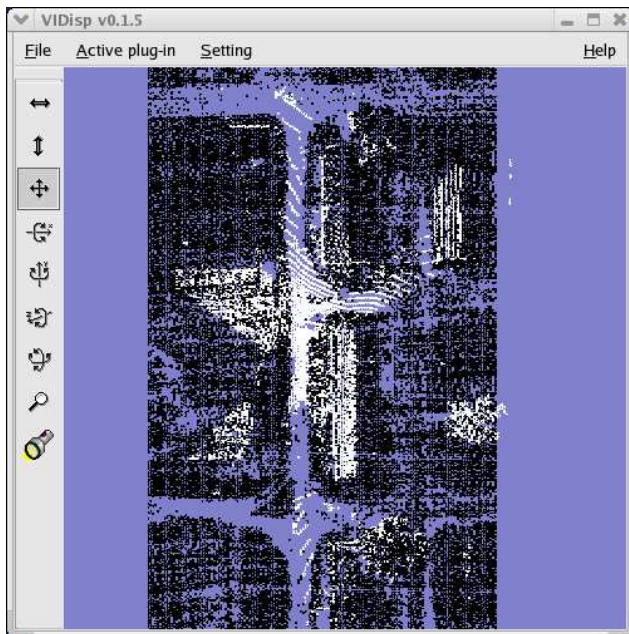
Fig. 3. A top view of unregistered UGV (black) and UAV (white) LADAR range images is shown in (a). A magnified side view of (a) is shown in (b).



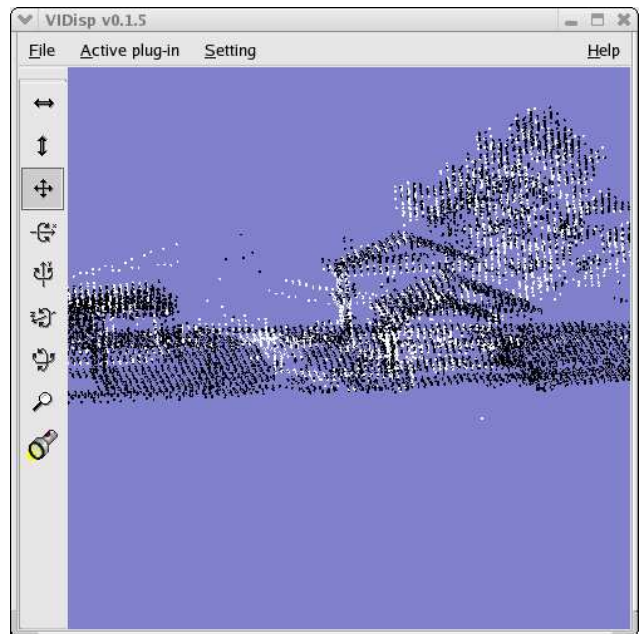
(a)



(b)



(c)



(d)

Fig. 4. A top view of the feature-based translation obtained using the extracted corners is shown in (a) and a magnified side view of the same is shown in (b). (c) shows a top view of the registered UAV (black) and UGV (white) LADAR range images obtained by utilizing the feature-based translation results and (d) is a magnified view of (c). See text for further details.

V. CONCLUSIONS AND FURTHER WORK

Registering 3D range images from unmanned ground and aerial vehicles was the main theme of this paper. The need for such registration is motivated by the requirement to continually estimate the position of the unmanned vehicle within accuracy bounds dictated by a particular mission even when the GPS position estimates are unreliable or unavailable. By making suitable modifications to the ICP algorithm it was shown that the modified algorithm provides reliable and robust registration in rugged terrain and urban environments for registering successive range images obtained from two different LADARs on a UGV.

The proposed algorithm was then extended to register aerial images obtained from a UAV with those from the UGV. A hybrid approach was proposed to this end by combining the modified ICP algorithm with a feature-based method. The feature-based hybrid approach was also shown to be effective in producing reliable registration for UGV navigation.

The results presented in the paper demonstrated the potential of this approach lending itself to real-time implementation. For practical purposes, the sets of LADAR data utilized in this paper can be assumed to be of the same resolution even though typically the aerial data tend to be of lower resolution than that of the UGV LADAR. To address this issue, we are currently developing schemes for use within the ICP algorithm that will inherently account for varying resolution in data sets that need to be registered. Towards this, we are also developing corner detection schemes using the Harris [22] and SUSAN corner detectors [23] on the 3D projected base ground planes.

Computing the correspondence is the most computationally expensive part of the algorithm. *kd-trees* have been proposed for faster correspondence where the complexity is reduced from $O(N_D N_M) \rightarrow O(N_D \log N_M)$. We have also employed Quaternions [24] (instead of SVD) to determine the 3D transformation but it results only in a slight improvement in the resultant registration for the tested field data.

The quality of the 3D registration will significantly improve if the uncertainty of the LADAR range images are taken into account and has been so verified for 2D laser scan registration [25], [26]. We are currently investigating the extension of these results to the 3D case. To quantify the accuracy of the registration results, we are investigating methods for estimating a covariance matrix of the error function that is minimized. We anticipate the covariance matrix to be useful when fusing the position estimates obtained via registration with other sensors.

REFERENCES

- [1] C. Shoemaker and J. Bornstein, "The Demo III UGV Program: A Testbed for Autonomous Navigation Research," in *Proceedings of the IEEE ISIC/CIRA/ISAS Joint Conference*, Sept. 1998, pp. 644–651.
- [2] J. Albus *et al.*, "4D/RCS Version 2.0: A Reference Model Architecture for Unmanned Vehicle Systems," National Institute of Standards and Technology, Tech. Rep. NISTIR 6910, 2002.
- [3] M. Hebert, T. Kanade, and I. Kweon, "3-D Vision Techniques for Autonomous Vehicles," in *Proceedings of the NSF Range Image Understanding Workshop*, 1988, pp. 273–337.
- [4] I. Kweon and T. Kanade, "High Resolution Terrain Map from Multiple Sensor Data," in *Proceedings of the IEEE International Workshop on Intelligent Robots and Systems*, vol. 1, 1990, pp. 127–134.
- [5] G. Shaffer, "Two-Dimensional Mapping of Expansive Unknown Areas," Ph.D. dissertation, Carnegie Mellon University, 1992.
- [6] I. Cox, "BLANCHE - An Experiment in Guidance and Navigation of an Autonomous Robot Vehicle," *IEEE Transactions on Robotics and Automation*, vol. 7, no. 2, pp. 193–204, 1991.
- [7] B. Hoffman, E. Baumgartner, T. Huntsberger, and P. Schenker, "Improved Rover State Estimation in Challenging Terrain," *Autonomous Robots*, vol. 6, no. 2, pp. 113–130, Apr. 1999.
- [8] F. Lu, "Shape Registration using Optimization for Mobile Robot Navigation," Ph.D. dissertation, Dept. of Computer Science, University of Toronto, 1995.
- [9] G. Weiβ, C. Wetzler, and E. von Puttkamer, "Keeping Track of Position and Orientation of Moving Indoor Systems by Correlation of Range-finder Scans," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 1, Sept. 1994, pp. 595–601.
- [10] P. Besl and N. McKay, "A Method for Registration of 3-D Shapes," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 14, no. 2, pp. 239–256, 1992.
- [11] Z. Zhang, "Iterative Point Matching for Registration of Free-Form Curves and Surfaces," *International Journal of Computer Vision*, vol. 13, no. 2, pp. 119–152, 1994.
- [12] K. Arun, T. Huang, and S. Bolstein, "Least-Squares Fitting of Two 3-D Point Sets," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 9, no. 5, pp. 698–700, 1987.
- [13] R. Madhavan and E. Messina, "Iterative Registration of 3D LADAR Data for Autonomous Navigation," in *Proceedings of the IEEE Intelligent Vehicles Symposium*, June 2003, pp. 186–191.
- [14] M. Shneier, T. Chang, T. Hong, G. Cheok, H. Scott, S. Legowik, and A. Lytle, "A Repository of Sensor Data for Autonomous Driving Research," in *Proceedings of the SPIE Unmanned Ground Vehicle Technology V*, Apr. 2003.
- [15] M. Hebert, C. Caillaes, E. Krotkov, I. Kweon, and T. Kanade, "Terrain Mapping for a Roving Planetary Explorer," in *Proceedings of the IEEE International Conference on Robotics and Automation*, vol. 2, 1989, pp. 997–1002.
- [16] N. Vandapel, R. Donamukkala, and M. Hebert, "Experimental Results in Using Aerial LADAR Data for Mobile Robot Navigation," in *Proceedings of the International Conference on Field and Service Robotics*, July 2003.
- [17] A. Stentz *et al.*, "Real-Time, Multi-Perspective Perception for Unmanned Ground Vehicles," in *Proceedings of the AUVSI Unmanned Systems Conference*, July 2003.
- [18] J. Canny, "A Computational Approach to Edge Detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 8, no. 6, pp. 679–698, Nov. 1986.
- [19] R. Miller and O. Amidi, "3D Site Mapping with the CMU Autonomous Helicopter," in *Proceedings of the 5th International Conference on Intelligent Autonomous Systems*, June 1998.
- [20] R. Miller, O. Amidi, and M. Delouis, "Arctic Test Flights of the CMU Autonomous Helicopter," in *Proceedings of the AUVSI 26th Annual Symposium*, 1999.
- [21] A. Downs, R. Madhavan, and T. Hong, "Registration of Range Data from Unmanned Aerial and Ground Vehicles," in *Proceedings of the Applied Imagery Pattern Recognition Workshop*, Oct. 2003.
- [22] C. Harris and M. Stephens, "A Combined Corner and Edge Detector," in *Proceedings of the Fourth Alvey Vision Conference*, Sept. 1988, pp. 147–151.
- [23] S. Smith and J. Brady, "SUSAN - A New Approach to Low Level Image Processing," *International Journal of Computer Vision*, pp. 45–78, May 1997.
- [24] B. Horn, "Closed Form Solution of Absolute Orientation using Unit Quaternions," *Journal of the Optical Society of America*, vol. 4, no. 4, pp. 629–642, Apr. 1987.
- [25] R. Madhavan and H. Durrant-Whyte, "Terrain Aided Localization of Autonomous Ground Vehicles," *Special Issue of the Journal of Automation in Construction (Invited)*, vol. 13, no. 1, pp. 69–86, Jan. 2004.
- [26] —, "Natural Landmark-based Autonomous Navigation using Curvature Scale Space," *Robotics and Autonomous Systems*, vol. 46, no. 2, pp. 79–95, Feb. 2004.