# A Methodology for the Reduction of Imprecision in the Engineering Process

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**Abstract:** Engineering design is characterized by a high level of imprecision, vague parameters, and ill-defined relationships. In design, imprecision reduction must occur to arrive at a final product specification. Few design systems exist for adequately representing design imprecision, and formally reducing it to precise values. Fuzzy set theory has considerable potential for addressing the imprecision in design. However, it lacks a formal methodology for system development and operation. One repercussion of this is that imprecision reduction is, at present, implemented in a relatively ad-hoc manner. The main contribution of this paper is to introduce a methodology called *precision convergence* for making the transition from imprecise goals and requirements to the precise specifications needed to manufacture the product. A hierarchy of fuzzy constraint networks is presented along with a methodology for creating transitional links between different levels of the hierarchy. The solution methodology is illustrated with an example within which an imprecision reduction of 98% is achieved in only three stages of the design process. The imprecision reduction is measured using the *coefficient of imprecision*, a new metric introduced to quantify imprecision.

**Keywords:** Engineering design, design, decision making, constraint processing, fuzzy set theory, hierarchical modeling, concurrent engineering, precision convergence, coefficient of imprecision.

## 1 Introduction

The design process encompasses all of the activities which are performed to arrive at a final product specification. It is during the design process when an estimated 60 to 85% of the product's cost is determined [Zangwill, 1992; Eversheim, et al., 1995]. This has led to increased interest in the activities which comprise the design process. Rapid product development cycles require design methodologies to efficiently explore the design space to build products with reduced cost, improved functionality, and improved quality to compete in global markets.

One of the phases of design which is least supported is the transition from early design phases to the final design stages [Abeln, 1990]. This is the transition from vague and imprecise specifications to precise and exact values and is a major activity of the design process. Many systems only attempt to provide design support in the domain of well defined variables and specifications in which all values used during design must be known with certainty. This restriction to certainty limits the utility of these systems to the later stages of the design process. Although routinely performed, a lack of a formal mechanism for quantitatively representing the transition has hindered the development of tools to support the early phases of design.

A promising technology for representing the imprecision in design is fuzzy set theory. A few researchers have begun to explore this possibility. Zimmermann and Sebastian, (1994), believes fuzzy representation of imprecision will play an increasingly important role in design systems. Standard mathematical functions used in design can be applied to imprecise quantities through use of the extension principle [Zadeh, 1965]. The extension principle determines the possibility function of $f(\tilde{Q}_1, \tilde{Q}_2)$ to encompass all possible values that could result from combining the ranges of the arguments $\tilde{Q}_1$ and $\tilde{Q}_2$ [Dubois and Prade, 1993]. The spreads, which are representative of the imprecision, *expand* in fuzzy calculations. Operations on imprecise quantities will result in equally imprecise or more imprecise results. The imprecision of the results cannot decrease. The problem with supporting the entire design process using fuzzy set theory representation is that the operators used to compute fuzzy numbers *increase* the imprecision while a necessary condition to design an artifact which is manufacturable is to *decrease* the imprecision. Although some work has been done on an extension principle defined for "optimistic" functions, where the imprecision does not necessarily increase during calculations, it appears to have limited practical applicability [Dubois and Prade, 1988].

This problem of increased imprecision from operations on fuzzy sets can be illustrated using interval arithmetic. The relationship between fuzzy sets and intervals is well documented [Dubois and Prade, 1988; Klir and Yuan, 1995]. Operations on fuzzy sets are equivalent to performing interval arithmetic at each α-level. Fuzzy multiplication is related to interval multiplication where interval multiplication is defined as [Moore, 1966],

$$[a, b]*[c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)].$$

The product will always have greater imprecision (magnitude of interval [Kim, et al., 1995]) than the input values. This is referred to as a "pessimistic" calculation by Dubois and Prade (1988) (page 43). For a specific example of this problem, let us compute the volume of a cube with imprecisely defined edges. Each edge is defined by the triangular fuzzy number $\langle 2, 3, 4\rangle$ and the volume is the fuzzy product $\langle 2, 3, 4\rangle \otimes \langle 2, 3, 4\rangle \otimes \langle 2, 3, 4\rangle = \langle 8, 27, 64\rangle$. Intuitively, many engineers can accept the concept of a dimension with a tolerance $3\pm1$ (i.e. a triangular fuzzy number $\langle 2, 3, 4\rangle$) yet find it unacceptable that the tolerance for the volume is $27^{+37}_{-19}$ (i.e. a triangular fuzzy number $\langle 8, 27, 64\rangle$). We can see that the result is much more imprecise than the input values and that extended arithmetic can produce such a large spread of possible values that the results become meaningless. Consequently, incorporating a representation for imprecision into existing design systems and utilizing existing design methodologies is inadequate for operating with imprecise quantities, and it becomes imperative to develop methodologies for manipulating fuzzy numbers to obtain reasonable results.

In this paper we present a solution to this problem using a new approach, termed *precision convergence*, that takes advantage of inherent precision divergence resulting from fuzzy operations to produce *precision convergence* as we move from conceptual design to detailed design. To quantify the imprecision reduction, a new metric is also introduced -- the *coefficient of imprecision*. The new approach will be discussed and then described through an example - the design of a cellular telephone. However, prior to this discussion, brief discussions in current design theory, fuzzy set theory, and constraint networks will be presented.

## 2 Engineering Design

There are many definitions of design which can be found in the literature. Suh (1990) defines design as the "*continuous interplay between what we want to achieve and how we want to achieve it.*" Design is a mapping from the functional domain into the physical realm [Tomiyama, 1990], it is a decision-making process [Bradley and Agogino, 1993], it is a search process [Gero, 1990], and it is the formulation and satisfaction of constraints [Serrano and Goddard, 1987]. Design exhibits all of these traits and conforms to these definitions to varying degrees. The research in engineering design is focused on finding suitable theories of design, hopefully leading to methods to arrive at better designs [Dixon, 1988].

In studying design, researchers have found it convenient to classify the different types of design which are described as a method of design [N.N, 1993]. There are several definitions of design types [Baumann, 1982; Ehrlenspiel, 1985; Hintzen, et al., 1989; Eversheim, 1990; Pahl and Beitz, 1993]. In general, four types of design can be differentiated, original design, adaptive design, variant design, and design by fixed principles. An analysis by Hintzen, et al. (1989) shows that about 5% of design activities are original design, about 55% are adaptive design, about 20% are variant design, and the other 20% is design by fixed principles.

Regardless of the type of design problem encountered and how they are classified, it is generally acknowledged that the design process consists of stages of progressively finer detailed designs [Rodenacker, 1991; Koller, 1985; Pahl and Beitz, 1993; Roth, 1994; Seifert, 1989; Ehrlenspiel, 1983; Hubka, 1984]. Finger and Dixon (1989) provide a good review of the many and varied perspectives on representing the engineering design process. However, they note that their review is missing the large body of research published in German. We concur and for our purposes use the classification of Pahl and Beitz (1988), one of the few works available in translation. Pahl and Beitz (1988) list four design stages: clarification of task, conceptual design, embodiment design, and detail design. Let us describe the four design stages in more detail. *Clarification of task* is a problem formulation activity where the functional requirements are specified. *Conceptual design* is synthesis of an abstract structure that can be a solution to the design problem. *Embodiment design* is the development of an abstract concept into a preliminary scaled engineering drawing. *Detailed design* involves the specification of attribute values to the design parameters. Sufficient detail must be added to each stage for evaluation and analysis. Clearly, design is an iterative process that moves an imprecise concept to a precise definition of the product and how it is to be manufactured. Despite the different breakdown and terms used, there are similarities in all descriptions of the design stages and the design process in general.

### 2.1 Imprecision in Design

Imprecision is most notable in the early phases of the design process and has been defined as the choice between alternatives [Antonsson and Otto, 1995]. Attempts have been made to address the issue of imprecision and inconsistency in design employing intervals [Davis, 1987; Kim, et al., 1995; Navinchandra and Rinderle, 1990]. However, these were found unsatisfactory for the general simultaneous engineering problem. Other approaches to representing imprecision in design include using utility theory, implicit representations using optimization methods, matrix methods such as Quality Function Deployment, probability methods, and necessity methods. These methods have all had limited success in solving design problems with imprecision. Antonsson and Otto (1995) provide an extensive review of these approaches and the reader is directed to their paper for a more detailed discussion.

Reusch (1993) examined the general problem of imprecision and inconsistency in design, and concluded that the problems are well suited to be solved using fuzzy technology. There are two aspects of imprecision when modeled with fuzzy sets - a preference view and a plausibility view. Imprecision can be defined as the preference a designer has for a particular value but will accept other values to a lesser degree. This interpretation of using fuzzy sets to model preference was put forward by Dubois [1987] and demonstrated in the domain of mechanical engineering by Wood and Antonsson [1989]. Imprecision can also be the plausibility of a value under a given possibility distribution. There exists important conceptual differences between the two based on whether the parameter is controllable or not [Dubois and Prade, 1995].

Young, et al. (1995) classify the different sources of imprecision found in engineering design as: relationship imprecision, data imprecision, linguistic imprecision, and inconsistency imprecision. *Relationship imprecision* is the ambiguity that exists between the design parameters. *Data imprecision* is when a parameter's value is not explicitly known. *Linguistic imprecision* arises from the qualitative descriptions of goals, constraints, and preferences made by humans. *Inconsistency imprecision* arises from the inherent conflicting objectives among various areas in a product's life-cycle. Regardless of the different sources of imprecision we believe it can be modeled using fuzzy set theory.

## 3 Fuzzy Set Theory

Fuzzy set theory is a generalization of classical set theory. In normal set theory an object is either a member of a set or not a member of the set. There are only two states. This is referred to as a *crisp set*. Fuzzy sets contain elements to a certain degree. Thus, it is possible to represent an object which has partial membership in a set. The membership value of element $x$ in a fuzzy set is represented by $\mu(x)$ and is normalized such that the membership degree of element $x$ is in [0, 1]. All elements $x$ such that $\mu(x) \geq \alpha$ define an "$\alpha$-level" which is a bounded interval.

Operations on fuzzy sets are performed across these α-levels, either by discretizing μ into a set of α-levels with $\alpha \in [0,1]$ or by treating μ as a continuum, and then applying the extension principle. This forms the basis of fuzzy set theory [Zadeh 1965]. Since the boundaries of inclusion in a set are *fuzzy* and not definite, we are able to directly represent ambiguity or imprecision in our models.

Fuzzy sets can represent linguistic terms and imprecise quantities. Linguistic terms are used to model the imprecision in natural language statements such as "tall" or "inexpensive". A fuzzy quantity is a set defined on $\Re$ of real numbers. It represents information such as, "about 5 inches". Thus, 4.9 inches would be a member of this set. 4.5 inches may also be a member of the set but to a lesser degree. Fuzzy numbers have a membership function that is normal, piecewise continuous, and convex. The value with membership of 1 is called the modal value. If all three of these properties hold for the membership function then this is a LR fuzzy number [Dubois and Prade, 1988; Zimmermann, 1985]. Standard mathematical operators can be used with fuzzy numbers through application of the extension principle, and specific implementations of these operations are fuzzy operators [Dubois and Prade, 1988; Zadeh 1965].

## 4 Constraint Satisfaction

The representation of a constraint satisfaction problem, defined as a constraint network problem, can be defined as follows (this is adapted from Dechter and Pearl, 1988): **Fuzzy Constraint Network Problem**: A fuzzy constraint network problem consists of a set of *n* fuzzy variables, $\tilde{X} = \left\{\tilde{X}_1, \tilde{X}_2, \cdots, \tilde{X}_n\right\}$, and a set of *m* constraints, $C = \{C_1, C_2, \cdots, C_m\}$. A fuzzy variable $\tilde{X}_i$ has its domain, $\Omega_i$, which defines the set of values that the variable can have. A constraint $C_i$ is a *k-ary relation* on $\tilde{X}_i' = \left\{\tilde{X}_{i1}, \tilde{X}_{i2}, \cdots, \tilde{X}_{ik}\right\} \subseteq \tilde{X}$, i.e., $C_i\left(\tilde{X}_{i1}, \tilde{X}_{i2}, \cdots, \tilde{X}_{ik}\right)$, and is a subset of the Cartesian product $\Omega_{i1} \times \Omega_{i2} \times \cdots \times \Omega_{ik}$.

In this formulation each constraint is satisfied to a degree $\mu_{C_i} \in [0,1]$, depending on the instantiation of the variables. This is the membership value of the constraint. A solution of the network is defined as an assignment of values to all the variables such that the constraints are satisfied. The constraints are satisfied when $\mu_{C_i} \geq \alpha_S$ where $\alpha_S$ is the *system truth threshold*. It is a level of satisfaction a solution must fulfill within the entire network to be accepted by the designer. This value is set *a priori* by the user [Young, et al., 1996].

The fuzzy constraint processing system used to implement the precision convergence method described in this paper is FuzCon. In Young, et al. (1996) the system and its operator set is described along with a brief review of prior fuzzy constraint processing work. FuzCon is the latest generation in a series of constraint processing systems which include; SPARK [Young, et al., 1992], SATURN [Fohn, et al., 1993], and, JUPITER [Liau, et al., 1995]. The approach taken in these systems is to view each constraint as a logic sentence with an associated truth value. (See Greef, et al., (1995) for an elaboration of the equivalency of a logic-based system and the constraint satisfaction network problem defined above.) These are interactive constraint processing systems which aid the user by propagating user assignments through the network in an attempt to satisfy all the constraints using direct calculations. This approach supports a feature-rich representation schema and supports omni-directional constraint propagation. Finding a solution that satisfies the constraints is supported by a sub-system that identifies the source of a constraint violation and assists in correcting the problem. This type of system is crucial to successfully modeling design because:

1. Representing design information requires a wide variety of data types that constrain the design through complex relationships beyond the equality relation.

2. Design is an iterative process that draws upon the knowledge of the user requiring that propagation not be restricted to a single direction nor a prescribed starting point.

## 5 Related Work

Other researchers have also begun to explore the use of fuzzy set theory in design. Many of these systems are for conducting design evaluation [Knosala and Pedrycz, 1987; Müller and Thärigen, 1994]. Fuzzy set theory has also been applied to specific design problems such as design for assembly [Jackson, et al., 1993]. However, none of these systems profess to support the entire design process. They are all targeted to a specific type of design problem or to a known and specific phase of the design process.

Fuzzy constraint processing has been studied by Kim and Cho (1994) who extended the definitions for numeric and interval based constraint networks to fuzzy constraint networks. Ruttkay (1994) discusses different methods for determining the joint satisfaction of constraints. Fargier (1994) and Dubois, et al., (1995) have used fuzzy constraint networks to model flexible temporal constraints in job-shop scheduling. Bowen, et al. (1992) discuss the link between fuzzy constraint satisfaction and semantics in classical logic. Of these methods the constraint model of Dubois, et al. (1995) has the greatest similarities to the model used here.

Recent work by Chen and Otto (1995) used fuzzy set theory to solve a variational system of constraints for CAD. However, the approach is limited to numeric variables and equality constraints and can only be solved when the number of variables equals the number of constraints. Wood and Antonsson (1989) developed the *Method of Imprecision* (MoI) which uses fuzzy set theory to model the preferences of a designer. Instead of direct calculations the MoI technique allows a designer to rank the coupling between design parameters and performance parameters. Those with the strongest coupling become candidates for modification in an attempt to reduce the imprecision of the output. In this sense, it is related to Taguchi's method [Otto and Antonsson (1993a)] and to utility theory [Otto and Antonsson (1993b)].

The computations in these two approaches are performed using a discretized solution algorithm developed by Dong and Wong (1987) and modified by Wood, et al., (1992). The algorithm has computational complexity of the order $O( M2^{N-1} k)$, where $N$ is the number of imprecise parameters, $M$ is the number of $\alpha$-levels into which the membership function is divided, and $k$ is the number of multiplications and divisions in the function, $f(\tilde{d})$, containing all the calculations. The algorithm was initially limited to performing calculations on a single expression, but has been extended to perform calculations for a system of equations [Chen and Otto, 1995]. Clearly, in addition to representational restrictions, these approaches are computationally bounded when applied to even small-sized problems and indicate a new approach is needed to support modeling imprecision in design using fuzzy set theory.

Giachetti and Young (1996) and Giachetti (1996) have developed a parametric representation which facilitates efficient computation of fuzzy arithmetic on triangular and trapezoidal fuzzy numbers. It has computational complexity of order $O(k)$ where $k$ is the number of arithmetic operations in the equations. Additionally, the fuzzy constraint system developed by Young, et al. (1996) is rich in features with the ability to represent linguistic variables and terms, fuzzy variables and numbers, and crisp variables and numbers in complex relations that include not only crisp equality, but also fuzzy equality relations, fuzzy and crisp inequality relations, and fuzzy and crisp logical relations. These developments set the stage for the development of the *precision convergence* methodology presented in this paper.

## 6 Hierarchical Model of Design

To be successful, a design must have an overall monotonically decreasing trend with regard to imprecision. This is shown graphically in Figure 1. Different stages of the design process require different knowledge representation methods. These are identified along the abscissa of the figure. The different stages through which a design passes can be modeled by decomposing the design process into a hierarchy of constraint networks. This hierarchy matches the movement of decisions from product conception to product manufacture. The hierarchy proceeds from higher level, abstract models of the product to lower level, detailed models of the product. A constraint network is a node in the hierarchy. The top node of the hierarchy is the root node. A constraint network can be linked from above and from below to other networks in the hierarchy. The approach of decomposing the design problem into a hierarchy of constraint networks conforms to the formal hierarchical model presented in O'Grady, et al. (1994).

Figure 2 shows a hierarchy of fuzzy constraint networks. The constraints are represented as circles and the variables are represented as solid-line links. The dashed-line links connect different levels of the hierarchy. They represent a variable's value in a higher level of the hierarchy becoming a constraint in a lower level of the hierarchy. These dashed-line links are *transition links* through which precision convergence is realized. In the example presented in the next section, the *clarification of task* constraint network is the root node of the hierarchy. The next lower level network is the *conceptual design* network. The lowest level network in Figure 2 is the *embodiment design* network.

A *transition link* connects a variable's value in one level of the hierarchy to a constraint in a lower level of the hierarchy. As an example of linking between constraint networks in different levels of the design hierarchy, let us examine transition link T connecting the *conceptual design* network to the *embodiment design* network. In the *conceptual design* network, the fuzzy variable $\tilde{t}$ represents the phone thickness. Through constraint propagation a solution is found to the *conceptual design* network that satisfies its constraints. As part of this solution, $\tilde{t}$ is assigned a value. We now move down the design hierarchy to the next stage in the design process – *Embodiment Design*.

The *embodiment design* stage for the cellular phone is modeled by the *embodiment design* constraint network. Via transition link T, the value assigned to $\tilde{t}$ as part of the solution to the *conceptual design* network defines constraint C7 in the *embodiment design* network. A solution to the *embodiment design* constraint network must satisfy the relationship defined by constraint C7. Since operations on fuzzy numbers only produce values with increased imprecision, the fuzzy sets selected for use in this lower-level constraint network must have sufficiently reduced imprecision such that their combined effect will not exceed the imprecision of T within constraint C7. This bounds the solution in the *embodiment design* phase and has the affect of forcing the designer to work backwards from imprecise quantities to more precise quantities. Successive applications of this principle – higher levels set requirements for lower levels - reduces the overall imprecision as the design progresses through the design stages. We call the result of this principle *precision convergence*.

## 7 Precision Convergence Using Fuzzy Constraint Satisfaction -- An Example

In this example taken from Young, et al. (1995), we design a cellular telephone in which we balance phone weight, battery life, battery technology, phone selling price, manufacturing cost, and time-to-market. The design parameters are:

- Phone weight - ranges between 0.2 and 1.0 kg.
- Battery life - ranges between 2 and 10 hours.
- Time-to-Market - ranges between 9 and 24 months.
- Selling Price - ranges between $50 and $200.
- Manufacturing Cost - ranges between $20 and $90.
- Battery technology is one of the following: new technology, nickel-cadmium type 1, nickel-cadmium type 2, alkaline.

Although these variables are stated as quantitative ranges, values for them at the *clarification of task* stage are discussed in terms such as *undesirable, less desirable, more or less satisfactory, more desirable, very desirable*. Chen and Hwang (1992) examined this type of representation and concluded that typically descriptive terms are rarely less than three, rarely more than ten, and typically five. Figure 3 restates the problem variables using descriptive terms and shows how each variable's term is mapped into a common set of terms. Figure 4 shows an example mapping for phone weight from its quantitative range onto five triangular fuzzy numbers whose combined range is normalized between zero and one. The mapping onto a normalized range allows us to transform the different data types into a common *fuzzy linguistic* representation so we can more easily compute with them. They are now each a fuzzy linguistic variable whose possible values are one of the descriptive terms in Figure 3. As an example, the variable **Phone weight** can now only be assigned one of the five descriptive terms {high weight, above average weight, average weight, below average weight, low weight}. The solution to the problem will be a descriptive term for each variable selected from the possible descriptive terms listed in Figure 3. It is then mapped back into its quantitative range to identify a more restrictive range (See Figure 4). In this way, a solution reduces the imprecision for each variable.

Continuing the problem statement, the variables are related to each other through the following types of relationships.

**Marketing Requirements**
The relationship of *phone weight* and *battery life* to *selling price*.

**Production Requirements**
The relationship of *time-to-market* to *manufacturing cost*.

**Cost Requirements**
The relationship of *selling price* to *manufacturing cost*.

**1st Technology Restrictions**
The relationship of *battery technology* to *phone weight* and to *battery life*.

**2nd Technology Restrictions**
The relationship of *battery technology* to *time-to-market* and *manufacturing cost*.

These requirements constitute design restrictions and can be represented as a fuzzy constraint network in which the requirements are constraints interconnected through shared variables. The fuzzy constraint network for this problem is the root node shown in Figure 2. The marketing constraint is shown in Figure 5. In the marketing constraint we see that one of four conditions must be met to satisfy the constraint. Examining the first condition in the constraint we have the following,

> **Phone weight** *is* low weight and
> **Battery life** *is* long life and
> **Selling price** *is* high

This condition expresses marketing's view that customers will pay more for a light weight phone that lasts a long time. In the constraint, "*is*" is a fuzzy linguistic operator that compares a linguistic variable to a descriptive term and is an example of modeling *relationship imprecision*. **Phone weight**, **Battery life**, and **Selling price** are linguistic variables, and "low weight", "long life", and "high" are descriptive terms from Figure 3. These are examples of modeling *linguistic imprecision*. The remaining constraints have similar relationships based on the concerns of the production, marketing, management, and technological departments within the company. Using fuzzy linguistic variables, descriptive terms and linguistic operators allows us to build a model using imprecise descriptions typically employed in requirement specifications while supporting those descriptions with a well defined mathematical basis and processing approach. In this way, the highest level constraint network of Figure 2 is a *fuzzy constraint network* and addresses the "clarification of task phase" in Pahl and Beitz's (1988) model. The fuzzy constraint network is described in Young, et al. (1995) and solved using fuzzy constraint satisfaction techniques. It was shown to have a solution only when a compromise could be found between two conflicting constraints -- the 2nd technology restrictions and the production requirements. Conflicting constraints are common in design and are an example of *inconsistency imprecision*. The ability to find compromise solutions through a sound mathematical basis is a recently identified strength of fuzzy constraint satisfaction [Martin-Clouaire, 1993; Lang and Schiex, 1993; Young, et al., 1995].

The solution to the clarification-of-task is as follows [Young, et al., 1995]:

| | |
|---|---|
| **Phone weight**: | below average (0.28 to 0.6 kg) |
| **Battery life**: | above average (6 to 9.2 hours) |
| **Battery technology**: | nickel-cadmium type 2 |
| **Time-to-market**: | average (13.5 to 19.5 months) |
| **Selling price**: | lower than average ($65 to $125) |

We can see in the solution all the variables have a reduced range and are more precise than when we began. This is the objective of strategic planning and is not unexpected. However, by using fuzzy technology we now have a formal basis for what has previously been an adhoc procedure. This approach, in and of itself, is useful. However, the issue is how to vertically integrate this solution with the next stage to continue the progression towards more precise results. To show how this might be accomplished, let us focus upon one of the parameters from the solution of the strategic planning problem -- the phone weight. In the solution, the phone's weight is determined to be "below average" which maps onto a reduced range of acceptable weights defined by the interval 0.28 to 0.6 kg which is represented by the fuzzy number $\langle .28, .44, .60\rangle$. $\langle .28, .44, .60\rangle$ is a triple describing the triangular fuzzy number shown in Figure 6 and is an example of *data imprecision*. In the next lower level of the design hierarchy the phone's weight becomes the upper bound for the phone's mass in constraint C7 of the *conceptual design* constraint network. A solution to the *conceptual design* phase must satisfy constraint C7.

### 7.1 Conceptual design network

The second network in the design hierarchy represents the *conceptual design* phase. The solution to the *clarification of task* constraint network becomes a requirement for conceptual design. To specify the requirement we state it as a fuzzy constraint in which the mass of the cellular phone must be less than or equal to the triangular fuzzy number $\tilde{M}$ (represented by the triple $\langle .28, .44, .60\rangle$ ). Thus, $\tilde{M}$ is an upper bound on the mass of the phone and receives its value via transition link M shown connecting these two constraint networks in Figure 2. Table 1 lists the constraints for this level of the example problem. The *conceptual design* constraint network is the second level in Figure 2. The objective at this second level is to determine the phone's dimensions (length, width, and thickness) subject to the constraints.

The designer has a concept of a thin, slender phone and selects values appropriately. The values selected for width and thickness are: $\tilde{w} = \langle 5, 7, 9\rangle$ cm. and $\tilde{t} = \langle 1.5, 2, 2.5\rangle$ cm. The mass is calculated in constraint C1 automatically by constraint propagation, and is

$\tilde{m} = \langle 254, 882, 2295 \rangle$ g. This result is automatically checked by constraint propagation against constraint C2 using possibility theory [Dubois and Prade, 1988],

$$\mu_{C2} = Poss\left(\tilde{m} \mid \tilde{\lesssim} M\right) = 0.57 \quad .$$

Since $\mu_{C2} \geq \alpha_S$ , where $\alpha_S$ is the system threshold for determining acceptability, the relationship defined in constraint C2 is satisfied. The problem solution is an approximate size for the cellular phone that satisfies the requirements from the *clarification of task* phase. In finding a solution to the *conceptual design* constraint network, the constraint propagation process infers fuzzy values to assign to the fuzzy variables such that the application of fuzzy multiplication in constraint C1 produces a value that does not violate the upper bound for the phone's mass in constraint C2. The solution is produced by satisfying constraint C2, through successive applications of fuzzy arithmetic operators. Because the application of a fuzzy arithmetic operator always results in an increase in imprecision, a side effect of using constraint processing technology is to effectively solve the problem backwards so that the fuzzy values assigned to variables $\tilde{l}$ , $\tilde{w}$, and $\tilde{t}$ (representing length, weight and thickness) are more precise than the fuzzy value for the phone's mass. In the next lower level of the design hierarchy the phone's width and thickness become upper bounds on thickness and width which the embodiment design must satisfy.

### 7.2 Embodiment Design of the Cellular Phone Mouthpiece

The solution to the *conceptual design* constraint network sets the fuzzy values for the dimensions of the phone. Its solution was determined by satisfying constraints which modeled the intended use of the phone. In the *embodiment design* phase, the designer selects components which will become part of the phone as manufactured. The selection of these components as well as the relationship among them and other variables must further reduce the imprecision involved in the design problem to correspond as close as possible to the specifications of standard components. The standard component's specifications are the lower limit on the level of imprecision and represent the production process variation shown in Figure 1. When this level of imprecision is achieved, the precision convergence process can terminate.

In our example, we concentrate on a subcomponent of the cellular phone, the mouthpiece design and the selection of a particular mouthpiece from those available in inventory. Tables 2 shows the constraints for the *embodiment design* fuzzy constraint network. Table 3 shows the inventory data on which the relation constraint, C3, operates. This constraint network is the bottom node in the design hierarchy of Figure 2. The value's assigned to variables in solving the *conceptual design* constraint network become constraints in the *embodiment design* constraint network via transition links W and T. These transition links become constraints C2 and C7 in the *embodiment design* network.

The designer selects the fuzzy set $\langle 0.20, 0.25, 0.30 \rangle$ as the fuzzy value for the shell thickness, $\tilde{S}_T$. The designer also selects the fuzzy set $\langle 0.15, 0.2, 0.25 \rangle$ as the fuzzy value for the clearance, variable $\tilde{c}_T$, between the mouthpiece and the phone's outer shell in the thickness direction. The designer selects part numbers from the mouthpiece inventory data table that are automatically propagated to constraints C1 and C6, inferring values for $\tilde{w}$ and $\tilde{t}$. Constraint propagation automatically checks requirements constraints, C2 and C7. A partial ordering can be produced based on $\mu_{C2}$ and $\mu_{C7}$, which shows how well the mouthpieces satisfy these constraints. The designer is free to choose any mouthpiece which satisfies the constraints where, $\mu \geq \alpha_S$. The partially ordered set of mouthpieces provides the designer with knowledge as to which mouthpiece best satisfies the preference set in prior design stages. As an example, mouthpiece MP-01 satisfies constraints C2 and C7 at fuzzy set membership values of $\mu_{C2} = 1.0$ and $\mu_{C7} = 0.75$ with an overall width of $\tilde{w} = \langle 4.7, 4.9, 5.1 \rangle$ and $\tilde{t} = \langle 1.95, 2.15, 2.35 \rangle$. Alternatively, the designer can search for a system truth threshold level, $\alpha_S$, at which the constraint network is satisfied. The $\alpha_S$ value is indicative of the solution's quality. As $\alpha_S$ approaches one the solution quality is better and as $\alpha_S$ approaches zero the solution quality is worse.

### 7.3 Detailed Design

The detailed design stage could be accomplished using a parametric CAD system. The physical layout of the product has been determined, the parameters have converged to within the capabilities of the production process so the nominal dimensions and tolerances are defined by the fuzzy numbers in the solution set. The procedure followed in the previous stages can be continued where the solution values from embodiment design become constraints in detailed design. Using a parametric CAD system, these constraints can be represented, thus ensuring the continuation of the original design objectives into the detailed design stage.

## 8 A Metric for the Reduction of Design Imprecision

Common measures of fuzziness are defined as the lack of distinction between a fuzzy set and its complement. The less a set differs from its complement the fuzzier it is [Klir and Yuan, 1995]. Wood and Antonsson (1989) showed that these measures are inadequate for quantifying design imprecision and developed a new metric to measure the design imprecision of a fuzzy set. This new metric, called the gamma function, quantifies the spread of the membership function about the mode value and is given by the expression,

$$D(\tilde{C}) = \sum_{i=1}^{|X|} \left( e^{\alpha_{\tilde{C}}(x_i)} - 1 \right) \qquad (1)$$

However, this metric is not adequate to compare imprecise quantities of different units and scale because it only accounts for the absolute spread of the fuzzy set. It does not account for the relative difference in magnitude different imprecise quantities may have. A similar problem exists in statistics in comparing the variability of samples drawn from populations that differ in scale or units. In such cases the standard deviations are scaled by dividing each by its own mean to produce the coefficient of variation as a relative measure of variability. In an analogous manner, we can compare the relative imprecision of disparate fuzzy sets by modifying the gamma function to account for the relative scale of the fuzzy set. We do this by scaling expression (1) by the mode of the fuzzy set, denoted by *b*. We call this the *coefficient of imprecision* and define it as,

$$c(\tilde{A}) = \frac{\int_a^c (e^{\alpha_{\tilde{A}}(x)} - 1)\, dx}{b} \qquad (2)$$

A crisp number has a measure of fuzziness equal to 0. The higher the coefficient of imprecision, the more imprecise the underlying membership function. When applied to crisp intervals, the *mode* parameter is determined by, $b = (c-a)/2$. This measure of fuzziness appears to correctly rank a mixture of triangular fuzzy numbers and crisp intervals regardless of the mode value *b*.

The coefficients of variation for the design parameters in the solutions to the example constraint networks are computed and compiled in Table 4. In the *clarification of task* constraint network the original range for the mass was reduced to $\tilde{m}$ = $\langle .28, .44, .60\rangle$ with $c(\tilde{m})$ = 0.522. The *conceptual design* constraint network reduced this mass into three separate variables with lower coefficients of imprecision as listed in Table 4. All three values show a reduction in imprecision from the imprecision in the mass. The hierarchy of constraint networks reduced the imprecision from 2.29 to a low value of 0.287 for a 87% total reduction in imprecision from the *clarification of task* stage to the *conceptual design* stage. Further precision convergence is realized by making each of the three product dimensions a constraint in a lower level network. An example of *embodiment design* is presented using variables for thickness and width. This network demonstrates a method for reducing the number of feasible alternatives, and leads to the selection of a discrete part from a component database. The new thickness and width are $\langle 1.95, 2.15, 2.35\rangle$ and $\langle 4.7, 4.9, 5.1\rangle$. This reduces the imprecision further, and when measured as a percentage of the initial mass, a 98% reduction in imprecision is realized as the design progressed from the *clarification of task* stage to the *embodiment design* stage. The solution to the *embodiment design* constraint network reduces the imprecision to a magnitude approaching the tolerances of the manufacturing processes. This signals the crossover from a fuzzy analysis of imprecision to the uncertainty from the stochastic variation that is an inherent property of the manufacturing processes. This termination is shown in Figure 1 where the imprecision line intersects the uncertainty line.

## 9 Conclusions

Imprecision is an inherent characteristic of engineering design. All designs begin with goals and requirements that are descriptive statements of what functions a product should perform. These imprecise product specifications are transformed by the design process to the precise specifications necessary for manufacturing. A methodology was outlined, whereby a concurrent engineering design problem, could be structured so that the effect of constraint propagation would be a reduction in design imprecision. A total reduction of 98% was achieved in the example problem. Using a hierarchy of fuzzy constraint networks we decomposed the design process according to levels of decision making appropriate to the chronological design stages. The hierarchy models information at different levels of abstraction and corresponds to actual design processes. It facilitates team based design since conceptual design decisions are usually made by different people than embodiment design decisions. This precision convergence approach is quantifiable. We can determine when the imprecision in a design has been sufficiently reduced to the point that it matches the imprecision inherent in the production process. This gives us a termination metric for the design iteration cycle, and also a means to map inherent production process variation to its impact on the design.

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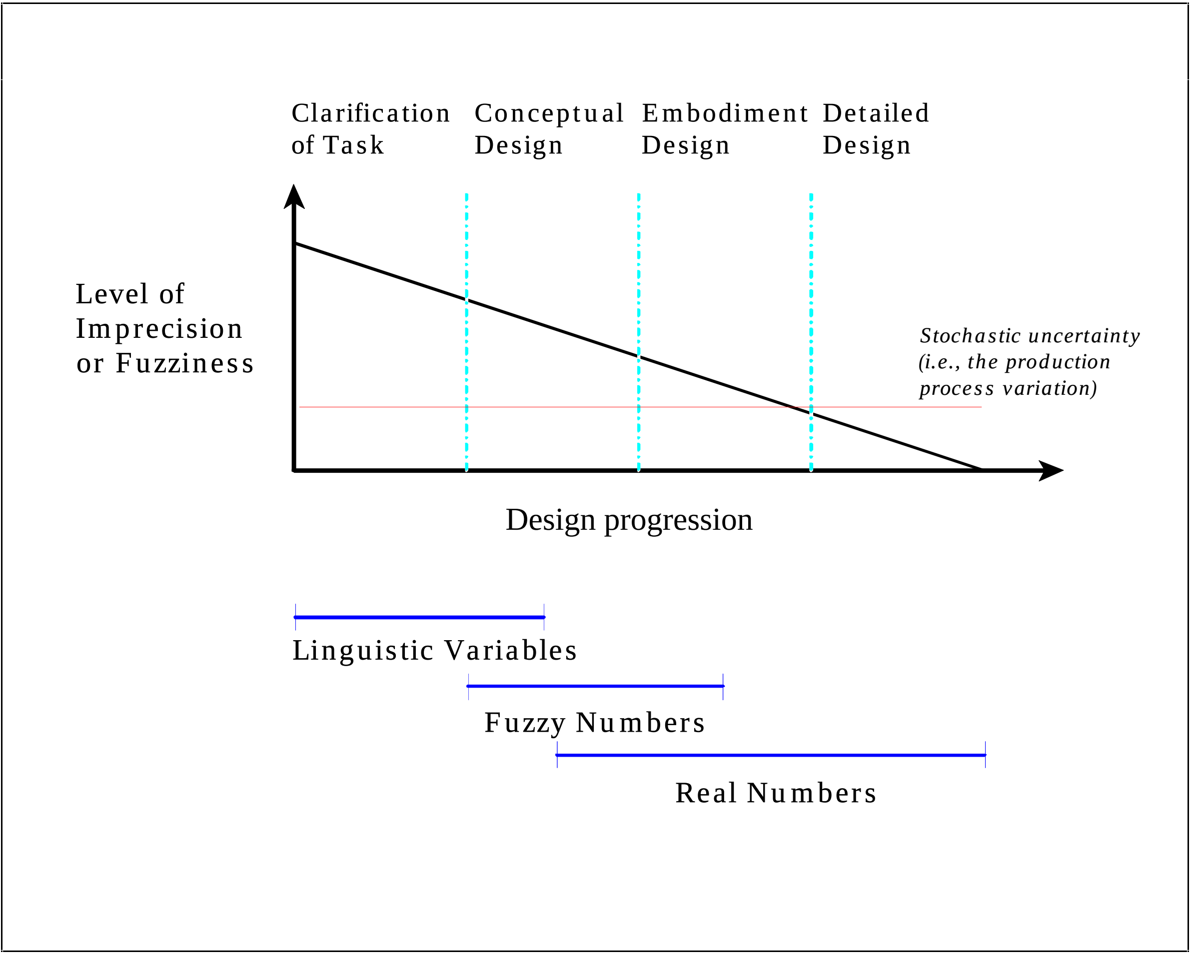


**Figure 1: Design stages versus imprecision level and the type of variables to use.**

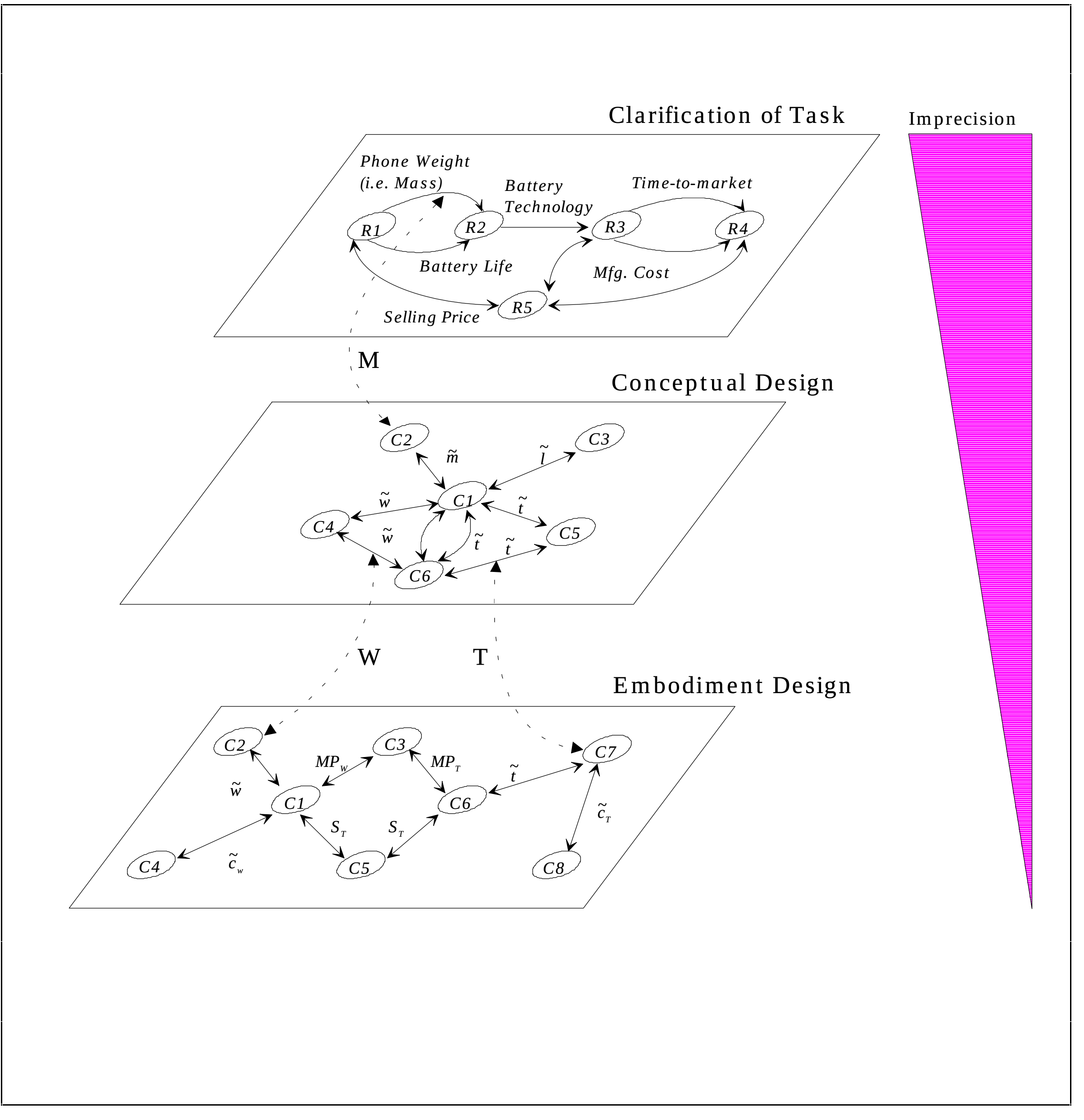


**Figure 2: Design hierarchy for the cellular phone example.**

| | |
|---|---|
| **Battery Technology** **(**not a fuzzy number) | one of the following: {new technology, nickel-cadmium type 1, nickel-cadmium type 2, alkaline} |

| **Variable** | **Satisfaction Range** | | | | |
|---|---|---|---|---|---|
| | **undesirable** *but not unacceptable* | **less desirable** | **more or less satisfactory** | **more desirable** | **very desirable** |
| **Phone weight** | high weight | above average weight | average weight | below average weight | low weight |
| **Battery life** | short life | below average life | average life | above average life | long life |
| **Time-to-market** | long | longer than average | average | shorter than average | short |
| **Selling price** | high | higher than average | average | lower than average | low |
| **Manufacturing cost** | high | higher than average | average | lower than average | low |

**Figure 3:** **Typical terms used to describe the range for each variable for *the clarification of task* constraint network and their mapping onto a common term set.**

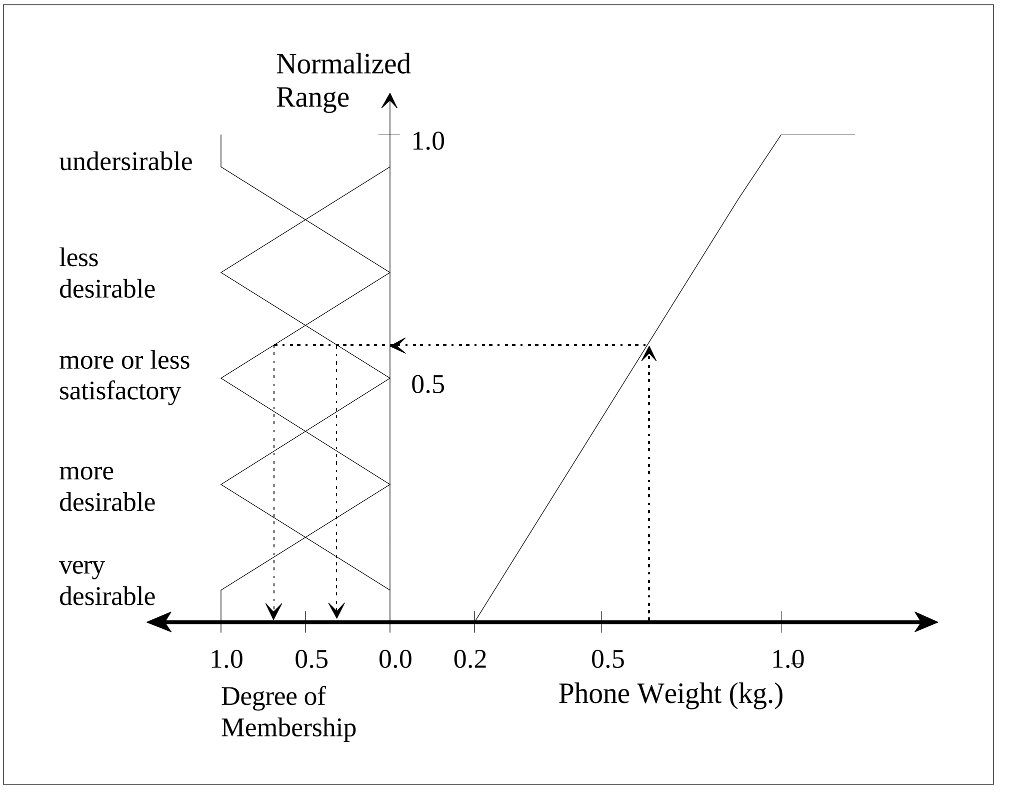


**Figure 4: An example mapping for the phone weight to a normalized range of linguistic terms.**

> **Phone weight** *is* low weight and **Battery life** *is* long life and **Selling price** *is* high
>
> or
>
> **Phone weight** *is* low weight and **Battery life** *is* above average life and
> **Selling price** *is* average
>
> or
>
> **Phone weight** *is* below average weight and **Battery life** *is* long life and
> **Selling price** *is* average
>
> or
>
> **Phone weight** *is* below average weight and **Battery life** *is* above average life and
> **Selling price** *is* lower than average

**Figure 5: The marketing constraint expressed using fuzzy linguistic variables, descriptive terms and linguistic operators.**

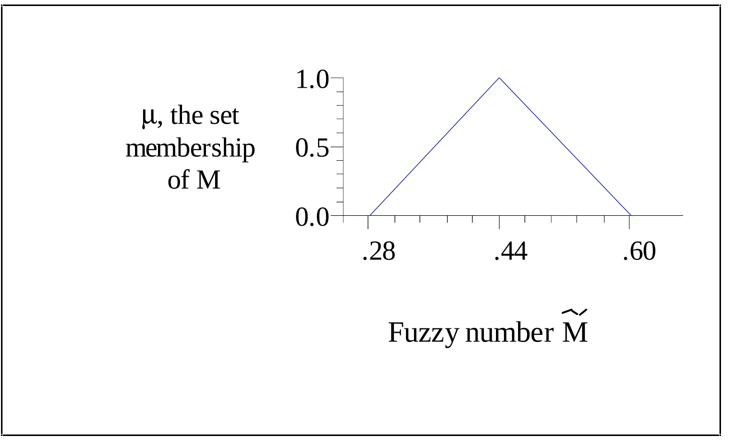


**Figure 6.** **Triangular Fuzzy Set for the phone's weight (i.e., mass), $\tilde{M} = \langle .28, .44, .60 \rangle$.**

**Table 1:** **Fuzzy constraints for the *Conceptual Design* constraint network.**

| | Fuzzy Constraint | Description |
|---|---|---|
| C1 | $\tilde{m} = \tilde{\rho} \otimes \tilde{l} \otimes \tilde{w} \otimes \tilde{t}$ | Mass calculation |
| C2 | $\tilde{m} \ \tilde{\lesssim} \ M$ | Upper bound of mass (from the solution of the *clarification of task* network.) |
| C3 | $\tilde{l} = 1.\tilde{5} \otimes \tilde{P}$ | Length calculation, where length is about 1.5 times a typical pocket depth. |
| C4 | $4 \leq \tilde{w} \leq 12$ | Typical pocket width. |
| C5 | $\tilde{t} \leq 3$ | Typical thickness. |
| C6 | $2\tilde{t} \oplus \tilde{w} \ \tilde{\lesssim} \ W$ | A cellular phone will fit into a pocket if the sum of its thickness and width are approximately less than a typical pocket width. |

**Fuzzy Variables:**

$\tilde{m}$ - calculated mass (g.)

$\tilde{\rho}$ - approximate density (g/cm3) for a cellular phone with below average weight and above average battery life using nickel-cadmium type 2 technology

$\tilde{l}$ - length (cm.)

$\tilde{w}$ - width (cm.)
[its value becomes a requirement for the *conceptual design* network via transition link W ]

$\tilde{t}$ - thickness (cm.)
[its value becomes a requirement for the c*onceptual design* network via transition link T ]

$\tilde{<} M$ - upper boundary on mass (g.)
[M receives its value from the *clarification of task* network via transition link M ]

$\tilde{P}$ - typical pocket length (cm.)

$\tilde{<} W$ - typical pocket width (cm.)

Where: $\tilde{1.5} = \langle 1.3, 1.5, 1.7\rangle$, $\tilde{P} = \langle 13, 14, 15\rangle$, $\tilde{l} = \langle 16.9, 21, 25.5\rangle$, and $\tilde{\rho} = \langle 2, 3, 4\rangle$ g/cm$^3$,
and $\otimes$ and $\oplus$ are fuzzy multiplication and fuzzy addition.

**Table 2: Fuzzy constraints for the *Embodiment Design* constraint network.**

| Constraint No. | Fuzzy Constraint | Description |
|---|---|---|
| C1 | $\tilde{w} = 2\tilde{S}_T \oplus MP_w \oplus 2\tilde{c}_w$ | Width calculation. |
| C2 | $\tilde{w} \;\tilde{<}\, W$ | Upper bound of width. [from the solution of the *conceptual design* network via transition link W] |
| C3 | MP(PN, $MP_l$, $MP_T$, $MP_W$, $MP_{WT}$) | Relation that defines the structure of the mouthpiece inventory data. |
| C4 | $\tilde{c}_w \geq 0.25$ | Clearance should be larger than 0.25 cm. for wires. |
| C5 | $0.2 \leq \tilde{S}_T \leq 0.5\ cm.$ | Shell thickness should be in this range to meet strength and weight constraints. |
| C6 | $\tilde{t} = MP_T \oplus 2\tilde{S}_T \oplus 2\tilde{c}_T$ | Thickness calculation. |
| C7 | $\tilde{t} \;\tilde{<}\, T$ | Upper bound of thickness. [from the solution of the *conceptual design* network via transition link T] |
| C8 | $0.2 \leq \tilde{c}_T \leq 0.4$ | Clearance must be large enough not to interfere with lead protrusion and small enough not to muffle sound. |

**Embodiment Design Variables:**

$\tilde{w}$ - width (cm.)

$\tilde{t}$ - thickness (cm.)

PN - Part number of mouth piece.

$MP_{WT}$ - Mouth piece weight (g.)

$MP_W$ - Mouth piece width (cm.)

$MP_L$ - Mouth piece length (cm.)

$MP_T$ - Mouth piece thickness (cm.)

*T* - Upper bound on thickness = ⟨1.5, 2, 2.5⟩ cm.
[receives its value via transition link T from the *conceptual design* network]

*W* - Upper bound on width = ⟨5, 7, 9⟩ cm.
[receives its value via transition link W from the *conceptual design* network]

$\tilde{S}_T$ - Shell thickness of plastic case (cm.)

$\tilde{c}_w$ - Clearance between mouth piece and shell. (cm.)

$\tilde{c}_T$ - Clearance between mouth piece and shell in thickness direction. (cm.)

**Table 3: Mouthpiece inventory data used by constraint C3.**

| **Part Number (PN)** | **$MP_L$ (cm)** | **$MP_W$ (cm)** | **$MP_T$ (cm)** | **$MP_{WT}$ (g.)** |
|---|---|---|---|---|
| **MP-01** | 4 | 4 | 1.25 | 100 |
| **MP-02** | 5 | 5 | 1.25 | 125 |
| **MP-03** | 6 | 6 | 1.5 | 150 |
| **MP-04** | 6.5 | 6.5 | 1.75 | 150 |
| **MP-05** | 7 | 7 | 1.75 | 200 |

**Table 4:** ***Coefficient of Imprecision* for design parameters in the example problem.**

| **Design Parameter and its corresponding fuzzy set** | | **Design Stage in which the parameter appears** | ***Coefficient of Imprecision,*** $c(\tilde{A})$ |
|---|---|---|---|
| Phone weight$_{Clarification\ of\ task} \in [0.2, 1.0]$ | | *Clarification of task* | 2.29 |
| $\tilde{m}_{Clarification\ of\ task}$ | $= \langle .28, .44, .60 \rangle$ | *Clarification of task* | 0.522 |
| $\tilde{l}_{Clarification\ of\ task}$ | $= \langle 16.9, 21, 25.5 \rangle$ | *Clarification of task* | 0.287 |
| $\tilde{w}_{Conceptual\ Design}$ | $= \langle 5, 7, 9 \rangle$ | *Conceptual Design* | 0.410 |
| $\tilde{t}_{Conceptual\ Design}$ | $= \langle 1.5, 2, 2.5 \rangle$ | *Conceptual Design* | 0.359 |
| $\tilde{w}_{Embodiment\ Design}$ | $= \langle 4.7, 4.9, 5.1 \rangle$ | *Embodiment Design* | 0.059 |
| $\tilde{t}_{Embodiment\ Design}$ | $= \langle 1.95, 2.15, 2.35 \rangle$ | *Embodiment Design* | 0.032 |