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KINEMATIC FIXTURING WITH RESPECT TO A PLANE USING CONTACT SENSING

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ABSTRACT

This paper presents a method for determining the location of geometric elements that compose the external features of referencing fixtures. Since in most applications parts that are handled in robotic work-cells are on a worktable or a floor, this paper focuses on fixture geometries that reside on a plane of known location. The location of the unknown geometric elements are found using contacts to the geometric elements and spatial constraints between the geometric elements. Geometric equations for contacts between lines, planes, points, spheres, and cylinders are derived. Spatial constraint equations are also derived. An algorithm is given for locating the geometric elements that form the fixture. The algorithm uses the contact equations and spatial constraint equations to locate the geometric elements. To illustrate the use of this algorithm, two examples are described in detail.

INTRODUCTION

Part referencing is the process of determining the relative location of a part with respect to a tool (such as a machine tool, a robot, or a material handling system) or with respect to a world coordinate system. Part referencing is used in robot calibration (see, for example, Roth, Mooring and Ravani (1987) or Hollerbach (1988)) and computer controlled manufacturing (see, for example, Duffie et. al. (1984) or Slocum (1988)).

Since both part referencing and calibration require measurement of relative locations between two objects, referencing fixtures are usually used to simplify the sensing function and to improve repeatability. Recently Nederbragt and Ravani (1997a) have developed a design theory for design of tactile sensing mechanical fixtures. Their design methods uses group theory to exploit the symmetry of different measuring arrangements.

In this paper, we extend this work by developing a method for determining the location of geometric elements based on the contacts that are made to these elements. We consider only a common and practical situation were the geometric elements are positioned on a reference plane of known location. This commonly occurs in part referencing because the parts (and fixture) being located frequently reside on a table of known height and orientation. In other words, the location of the table surface is known in the sensor frame – also referred to as the end effector frame (see Figure 1). If a proper set of geometric elements are used and the necessary number and type of contacts are made to the geometric elements, then a reference frame can be created using the methods shown here. Much of the background necessary for understanding this paper can be found in Nederbragt and Ravani (1997a).

The organization of the paper is as follows. First, we discuss referencing based on contact sensing. This includes a discussion of the type of contacts that can be used and the number and type needed to create a reference frame. We then derive geometry-based equations for each type of contact. We also derive spatial-constraint equations that relate the geometric elements of the reference fixture to each other. Finally, we describe a method for solving the equations. Two examples are given to illustrate the use of this new method.

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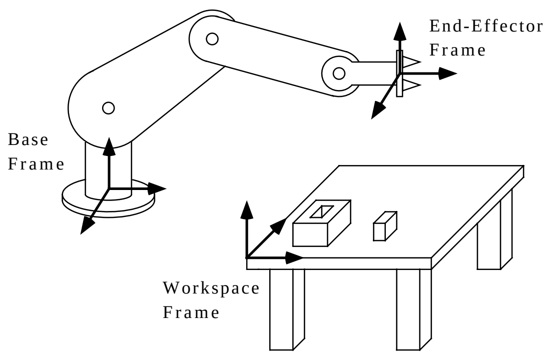

Figure 1. REFERENCING GIVEN A KNOWN PLANE

REFERENCING BASED ON CONTACT SENSING

Referencing using contact sensing involves bringing a sensing element and a geometric element (that belongs to the reference fixture) into contact with one another, activating the sensor, and measuring the location of the contact in the sensor coordinate system. This was the case, for example, in the system described by Duffie et al. (1984) where a touch sensor was attached to the end of a robot and it was moved until it contacted the spherical surface of a fixture. In such a system the location of the touch is only known in the robot manipulator frame; however, the shape of the touch surface on the fixture is completely known.

If several touches are made to the surface, then enough information may be obtained to determine the relative location of the two frames. Duffie et al. (1984) used a fixture consisting of three separate spheres of known radii. They found that four separate touches to each of the spheres made it possible to determine the location of the fixture with respect to the robot. McCallion and Pham (1984) used three non-collinear touches to a plane to determine its location in space, and, using three perpendicular planes of a cube, the relative location of the robot to the fixture was found.

When designing a contact-sensing fixture, one has to geometrically and physically combine appropriate geometric elements such as planes, spheres, and cylinders (to create the reference fixture) with a proper number, type, and arrangement of contacts to determine the relative location of the reference fixture coordinate frame to the contact sensor coordinate frame.

In this paper, the geometric elements that can form the reference fixture include lines, planes, points, cylinders, and spheres. Moreover, the shape of the contact probe can also be any of these shapes. Currently, the most common shape for a contact probe is a sphere; however, all five shapes have application in reference fixture design.

Nederbragt (1997) gave a complete list of all of the contact combinations that could be used to create a complete reference frame (using the five geometric shapes described earlier). The enumeration of contact sets used a group theory notation to rep-

Table 1. ENUMERATION OF CONTACT COMBINATIONS THAT INCLUDE A "PLANE-TO-PLANE" CONTACT AND RESULT IN A DIMENSION OF ZERO (Nederbragt and Ravani, 1997c)

Geometric element	Number of types
$\{G_P\}\{G_P\}\{G_P\}$	1
$\{G_P\}\{G_P\}\{S\}$	1
$\{G_P\}\{G_P\}\{S\cdot T_1\}$	1
$\{G_P\}\{G_P\}\{G_P\cdot R\}$	2
$\{G_P\}\{G_P\}\{S\cdot S\}$	2
$\{G_P\}\{G_P\}\{S\cdot C\}$	3
$\{G_P\}\{G_P\}\{S\cdot T_2\}$	2
$\{G_P\}\{G_P\}\{C\cdot R\cdot C\}$	3
$\{G_P\}\{S\}\{S\}$	1
$\{G_P\}\{S\}\{S\cdot T_1\}$	1
$\{G_P\}\{S\}\{G_P\cdot R\}$	2
$\{G_P\}\{S\}\{S\cdot S\}$	2
$\{G_P\}\{S\}\{S\cdot C\}$	3
$\{G_P\}\{S\}\{S\cdot T_2\}$	2
$\{G_P\}\{S\}\{C\cdot R\cdot C\}$	3
$\{G_P\}\{S\cdot T_1\}\{S\cdot T_1\}$	1
$\{G_P\}\{S\cdot T_1\}\{G_P\cdot R\}$	2
$\{G_P\}\{S\cdot T_1\}\{S\cdot S\}$	2
$\{G_P\}\{S\cdot T_1\}\{S\cdot C\}$	3
$\{G_P\}\{S\cdot T_1\}\{S\cdot T_2\}$	2
$\{G_P\}\{S\cdot T_1\}\{C\cdot R\cdot C\}$	3
$\{G_P\}\{G_P\cdot R\}\{G_P\cdot R\}$	3
$\{G_P\}\{G_P\cdot R\}\{S\cdot S\}$	4
$\{G_P\}\{G_P\cdot R\}\{S\cdot C\}$	6
$\{G_P\}\{G_P\cdot R\}\{S\cdot T_2\}$	4
$\{G_P\}\{G_P\cdot R\}\{C\cdot R\cdot C\}$	6
$\{G_P\}\{S\cdot S\}\{S\cdot S\}\{S\cdot S\}$	4
$\{G_P\}\{S\cdot S\}\{S\cdot S\}\{S\cdot C\}$	9
$\{G_P\}\{S\cdot S\}\{S\cdot S\}\{S\cdot T_2\}$	6
$\{G_P\}\{S\cdot S\}\{S\cdot S\}\{C\cdot R\cdot C\}$	9
$\{G_P\}\{S\cdot S\}\{S\cdot C\}\{S\cdot C\}$	12
$\{G_P\}\{S\cdot S\}\{S\cdot C\}\{S\cdot T_2\}$	12
$\{G_P\}\{S\cdot S\}\{S\cdot C\}\{C\cdot R\cdot C\}$	18
$\{G_P\}\{S\cdot S\}\{S\cdot T_2\}\{S\cdot T_2\}$	6
$\{G_P\}\{S\cdot S\}\{S\cdot T_2\}\{C\cdot R\cdot C\}$	12
$\{G_P\}\{S\cdot S\}\{C\cdot R\cdot C\}\{C\cdot R\cdot C\}$	12
$\{G_P\}\{S\cdot C\}\{S\cdot C\}\{S\cdot C\}$	10
$\{G_P\}\{S\cdot C\}\{S\cdot C\}\{S\cdot T_2\}$	12
$\{G_P\}\{S\cdot C\}\{S\cdot C\}\{C\cdot R\cdot C\}$	18
$\{G_P\}\{S\cdot C\}\{S\cdot T_2\}\{S\cdot T_2\}$	9
$\{G_P\}\{S\cdot C\}\{S\cdot T_2\}\{C\cdot R\cdot C\}$	18
$\{G_P\}\{S\cdot C\}\{C\cdot R\cdot C\}\{C\cdot R\cdot C\}$	18
$\{G_P\}\{S\cdot T_2\}\{S\cdot T_2\}\{S\cdot T_2\}$	4
$\{G_P\}\{S\cdot T_2\}\{S\cdot T_2\}\{C\cdot R\cdot C\}$	9
$\{G_P\}\{S\cdot T_2\}\{C\cdot R\cdot C\}\{C\cdot R\cdot C\}$	12
$\{G_P\}\{C\cdot R\cdot C\}\{C\cdot R\cdot C\}\{C\cdot R\cdot C\}$	10

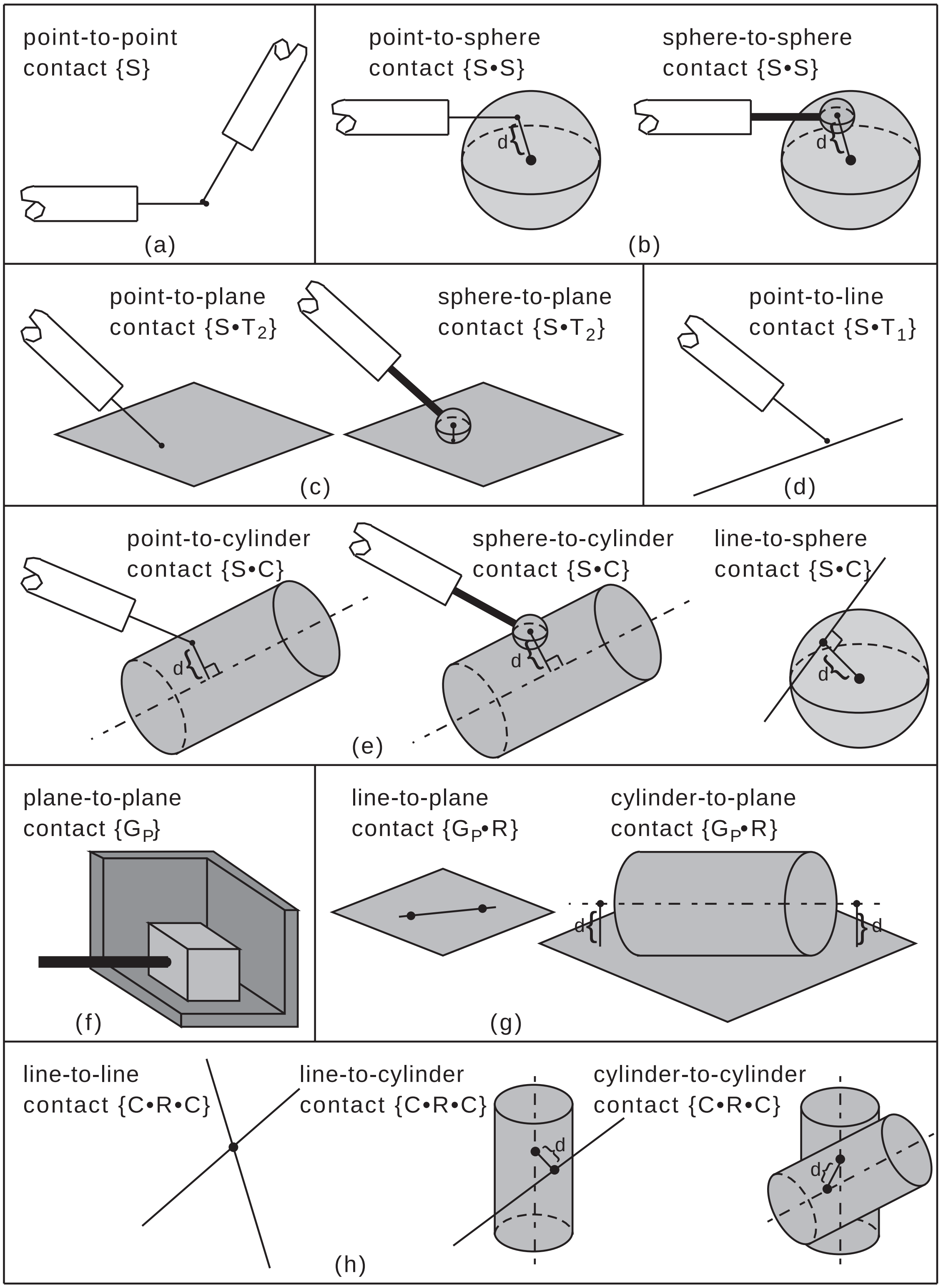

Figure 2. CONTACT TYPES

resent each type of contact. The different types of contacts are shown in Figure 2 with their group representation. In this paper, we are studying the contact sets that include a plane of known location. Table 1 lists all of the contact sets that include a known plane (the known plane is represented by the first $\{G_P\}$ element in each row).

CONTACT EQUATIONS

In order to create a reference frame using a set of geometric elements, we need to locate each geometric element using the contacts made to it. Therefore, we need an equation for each of the contacts illustrated in Figure 2. It is advantageous to represent the geometric elements in a form that makes the resulting equations easy to solve. Figure 3 shows the representations for each geometric element that we will use – namely, lines, cylinders, points, spheres, and planes. The point and sphere are simply represented by a point coordinate and radius. The radius is set equal to zero for a point. A plane is represented by the standard equation:

$$n_x x + n_y y + n_z z + c = 0 \tag{1}$$

where n_x, n_y, and n_z are the components of the normal $\vec{n}$ to the plane. Representing a cylinder or a line is more difficult because a simple equation for a line does not exist. The representation used for this paper assumes that the line or center line (in the case of a cylinder) intersects the x-y plane once. Hence, the line is not parallel to the x-y plane. In most cases, this should not cause a problem. Since the line intersects the x-y plane, there must be a point $(x_o, y_o, 0)$ that belongs to the line. The direction that the line follows can be represented by the vector $< x, y, 1 >$. Therefore, the line can be represented using four variables: x_o, y_o, x, and y. Plücker coordinates (McCarthy, 1990) can also be used to represent a line using only four independent variables (actually six variables and two quadratic equations); however, Plücker coordinates will require additional quadratic equations to be solved. Hence, we do not want to use them here.

In the following subsections, the equations representing each type of contact are derived for later use. The equations are based on the geometric representations described in Figure 3.

Contact type $\{S\}$: point-to-point contact

A point-to-point contact requires that two points come into contact. Mechanical contacts occur when three-dimensional objects make contact; hence, a point-to-point contact is an approximation to the real world case. Moreover, it is difficult to position two point-like geometries next to each other. Hence, a point-to-point contact is not useful for describing mechanical contacts. However, this is a useful contact representation for a non-mechanical contact. For example, if a scanning system can

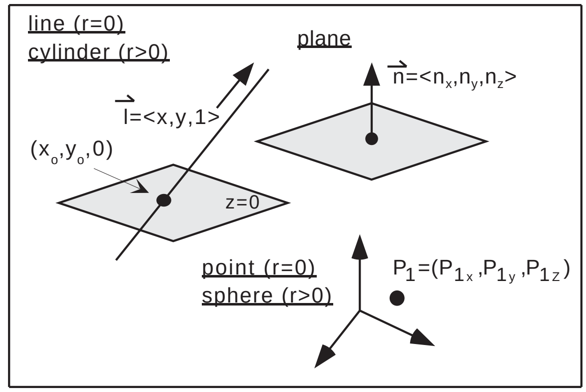

Figure 3. GEOMETRIC ELEMENT REPRESENTATIONS

locate a particular reference point in space, then this type of contact is applicable.

The equation for dealing with a point-to-point contact is simple. Since the two points occupy the same location, the two points must be the same. This can be represented simply as

$$P_1 = P_2 \tag{2}$$

where P_1 belongs to the probing side and P_2 belongs to the referenced side.

Contact type $\{S \cdot S\}$: point-to-sphere and sphere-to-sphere contacts

Point-to-sphere and sphere-to-sphere contacts occur when a touch probe comes into contact with a sphere. If the touch probe is very small it can be modeled as a point (a point-to-sphere contact). If accuracy is critical, then, a spherical touch probe should be treated as a sphere (a sphere-to-sphere contact).

Let P_3 be the center of the major sphere, and let R be its radius. Let P_4 be the center of the probe, and let r be its radius ($r = 0$ for a point contact). The distance between the probe center and the center of the referencing sphere must be $R + r$ when contact occurs. Using the equation for the distance between two points (Anton, 1984), the equation for this type of contact can be represented as

$$R + r = \sqrt{(P_{4x} - P_{3x})^2 + (P_{4y} - P_{3y})^2 + (P_{4z} - P_{3z})^2}. \tag{3}$$

Squaring eq. 3, we obtain

$$(R + r)^2 = (P_{4x} - P_{3x})^2 + (P_{4y} - P_{3y})^2 + (P_{4z} - P_{3z})^2. \tag{4}$$

Contact type $\{S \cdot T_2\}$: point-to-plane and sphere-to-plane contacts

Point-to-plane and sphere-to-plane contacts occur when a touch probe comes into contact with a plane. As with contact

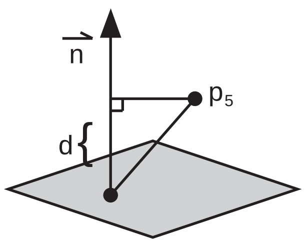

Figure 4. DISTANCE BETWEEN A POINT AND A PLANE

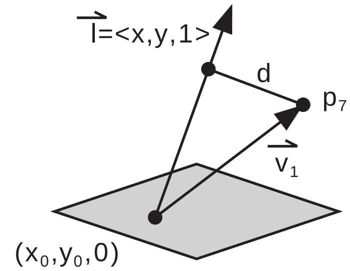

Figure 5. SHORTEST DISTANCE BETWEEN A POINT AND A LINE

type $\{S \cdot S\}$, if the touch probe is very small it can be modeled as a point (a point-to-plane contact). If accuracy is critical, then a spherical touch probe should be treated as a sphere (a sphere-to-plane contact).

Let P_5 be the center of the probe, and let r be its radius ($r = 0$ for a point contact). The plane is represented by eq. 1 with normal $\vec{n} =< n_x, n_y, n_z >$. The distance between the probe center and the plane during contact is r. The equation for the shortest distance between a point and a plane is

$$d = \frac{|n_x P_{5x} + n_y P_{5y} + n_z P_{5z} + c|}{\sqrt{n_x{}^2 + n_y{}^2 + n_z{}^2}} \tag{5}$$

where d is the shortest distance (Anton, 1984) (see Figure 4). Since the shortest distance between a plane and a sphere during contact is r, $d = r$. Squaring eq. 5, we get

$$r^2(n_x{}^2 + n_y{}^2 + n_z{}^2) = (n_x P_{5x} + n_y P_{5y} + n_z P_{5z} + c)^2. \tag{6}$$

Equation 6 is a quadratic equation with four unknowns – namely, n_x, n_y, n_z, and c. However, $\vec{n}$ can be normalized to reduce the number of unknowns to three. If $r = 0$, then eq. 6 simply becomes

$$n_x P_{5x} + n_y P_{5y} + n_z P_{5z} + c = 0. \tag{7}$$

Contact type $\{S \cdot T_1\}$: point-to-line contact

A point-to-line contact requires that a line and a point come into contact. Again, mechanical contacts occur when three-dimensional objects make contact; hence, a point-to-line contact is an approximation to a real-world case. Moreover, it is difficult to position point-like and line-like geometries next to each other. Hence, a point-to-line contact is not useful for describing mechanical contacts. However, this can be a useful representation for a non-mechanical contact.

All points on a line, using the notation described earlier, can be represented by $(x_o, y_o, 0) + a < x, y, 1 >$ where $a \in \mathbb{R}$. Let P_6 be the point in contact with the line. The following equation must be satisfied:

$$(x_o, y_o, 0) + a < x, y, 1 >= P_6. \tag{8}$$

Equation 8 can be treated as three equations – one for each component direction. If the point is being located using the line, then there are four unknowns – namely, P_{6x}, P_{6y}, P_{6z}, and a. If the line is being located using the point, then there are five unknowns – namely, x_o, y_o, x, y, a.

contact type $\{S \cdot C\}$: point-to-cylinder, sphere-to-cylinder, and line-to-sphere contacts

A point-to-cylinder contact and a sphere-to-cylinder contact occurs when a touch probe comes into contact with a cylinder. Again, if the touch probe is very small it can be modeled as a point (a point-to-cylinder contact). If accuracy is critical, then a spherical touch probe should be treated as a sphere (a sphere-to-cylinder contact). The line-to-sphere contact is an entirely different contact; however, the equation governing its behavior is the same. This type of contact occurs if a probe wire or beam comes into contact with a sphere.

Let R be the radius of the cylinder for the point-to-cylinder contact and the sphere-to-cylinder contact. Let R also be the radius of the sphere for the line-to-sphere contact. Let r be the radius of the probe sphere for the sphere-to-cylinder contact (r will be zero for the other two contacts). Let P_7 be the point of contact for the point-to-cylinder case and the center of the sphere for the sphere-to-cylinder case and line-to-sphere case.

Based on the geometry of the contact, the shortest distance between P_7 and the line/center line must be $R + r$. An equation using this distance can be derived using vector algebra (Nederbragt and Ravani, 1997b). Figure 5 illustrates the variables and geometry used. The shortest distance equation is

$$d = \frac{|\vec{l} \times \vec{v}_1|}{|\vec{l}|} \tag{9}$$

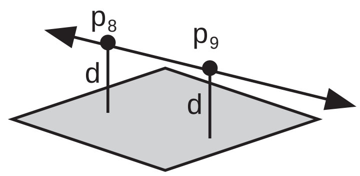

Figure 6. CYLINDER-TO-PLANE CONTACT

where $\vec{l} = < x, y, 1 >$, $d = R + r$, and $\vec{v}_1 = < P_{7x} - x_o, P_{7y} - y_o, P_{7z} - 0 >$. Taking the square of eq. 9, we get

$$(R+r)^2(x^2+y^2+1) = \\ | < x,y,1 > \times < P_{7x} - x_o, P_{7y} - y_o, P_{7z} > |^2. \quad (10)$$

Contact type $\{G_P\}$: plane-to-plane contact

A plane-to-plane contact simply means that the probing plane is resting on the referencing plane. In other words, the two planes are coincident. This occurs, for example, if a cubical probe comes into contact with three mutually perpendicular planes. In this case, the three planar surfaces would have plane-to-plane contact with the cube (see Figure 2f). It should be noted that it is difficult to get a cubical probe to mate properly with three mutually perpendicular planar surfaces without some human intervention. In Table 1 a $\{G_P\}$ element is included at the beginning of every set combination. This particular $\{G_P\}$ represents the plane of known location; hence, it is a ”virtual” plane-to-plane contact since a plane-to-plane contact will determine the location of a plane.

Contact type $\{G_P \cdot R\}$: line-to-plane and cylinder-to-plane contacts

A line-to-plane contact or a cylinder-to-plane contact occurs when a line or cylinder rests on a plane. If a line rests on a plane then all points on the line also reside in the plane. If a cylinder contacts the plane, then all points on the center line have a shortest distance R to the plane where R is the radius of the cylinder. The easiest way to handle these contacts is to use two arbitrary points on the line or center line, eq. 6, and the Radius R. Using two points is sufficient for representing the geometric constraint imposed by a line-to-plane or cylinder-to-plane contact (see Figure 6).

Contact type $\{C \cdot R \cdot C\}$: line-to-line, line-to-cylinder, and cylinder-to-cylinder contacts

The line-to-line contact occurs if a probe wire or beam comes into contact with another wire or beam. A line-to-cylinder contact or a cylinder-to-cylinder contact occurs when a probe wire or beam comes into contact with a referencing cylinder. The

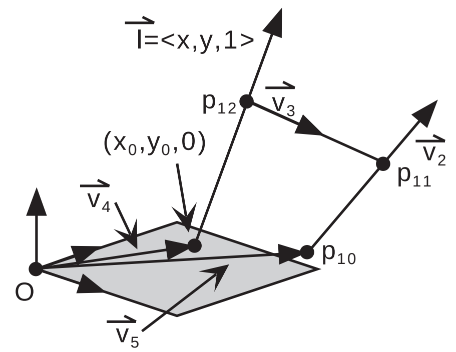

Figure 7. DISTANCE BETWEEN TWO LINES

wire could have a significant radius; Therefore, it could be modeled as a thin cylinder.

In all three cases, the contact equation can be derived in the same way. The shortest distance between the two lines/center-lines must be equal to the combination of the radii of the two cylinders/lines. An equation representing the shortest distance between two lines is described in Faux and Pratt (1979). Using a similar method, we develop an equation for our use.

Let $\vec{l}$ and $(x_o, y_o, 0)$ describe the unknown line (or center line) l. Let $\vec{v}_2$ and P_{10} describe the known line (or center line) v_2. Let $P_{12}P_{11}$ represent the shortest connector between the two lines. Let $\vec{v}_4$ be the position vector to $(x_o, y_o, 0)$. Let $\vec{v}_5$ be the position vector to P_{10}. Let $\vec{v}_3$ be a unit vector along $P_{12}P_{11}$ from l to v_2.

Using the variables described earlier, a vector loop equation can be written:

$$\vec{v}_4 + a\vec{l} + d\vec{v}_3 = \vec{v}_5 + b\vec{v}_2 \quad (11)$$

where $a\vec{l} = P_{12} - (x_o, y_o, 0)$, $b\vec{v}_2 = P_{11} - P_{10}$, and d is the shortest distance between the two lines. The shortest line between l and v_2 must be perpendicular to l and v_2. Hence, $\vec{v}_3$ must be perpendicular to $\vec{l}$ and $\vec{v}_2$. This development leads to the equation:

$$\vec{v}_3 = (k)\frac{\vec{l} \times \vec{v}_2}{|\vec{l} \times \vec{v}_2|} \quad (12)$$

where k equals either minus one or plus one. The variable k is necessary to change the direction of the cross product to insure that it points in the same direction as $\vec{v}_3$. Taking the dot product of each component in eq. 11 with $\vec{v}_3$, we get

$$\vec{v}_4 \cdot \vec{v}_3 + d = \vec{v}_5 \cdot \vec{v}_3. \quad (13)$$

Two terms drop out of the equation because $\vec{l}$ and $\vec{v}_2$ are perpendicular to $\vec{v}_3$. Rewriting eq. 13, we get

$$R+r=(\vec{v}_5-\vec{v}_4)\cdot\vec{v}_3 \tag{14}$$

where $R+r$ is the distance between our two lines/center lines. Substituting eq. 12 into eq. 14, we get

$$R+r=(\vec{v}_5-\vec{v}_4)\cdot(k)\frac{\vec{l}\times\vec{v}_2}{|\vec{l}\times\vec{v}_2|}. \tag{15}$$

Equation 15 can be rewritten as

$$k(R+r)|\vec{l}\times\vec{v}_2|=(\vec{v}_5-\vec{v}_4)\cdot(\vec{l}\times\vec{v}_2) \tag{16}$$

Squaring eq. 16 and including the unknown components of $\vec{l}$ and $\vec{v}_4$, we get

$$(R+r)^2|<x,y,1>\times\vec{v}_2|^2 = [(\vec{v}_5-<x_o,y_o,0>)\cdot(<x,y,1>\times\vec{v}_2)]^2. \tag{17}$$

FIXTURE GEOMETRY EQUATIONS

We now can derive an equation for each type of contact shown in Figure 2. However, we still need equations for representing the spatial constraints between the geometric elements that compose the external features of a referencing fixture. Use of these spatial-constraint equations can significantly reduce the number of contacts needed to determine the location of the geometric elements. For example, given a sphere-point fixture with a distance of d between the center of the sphere and the point, an equation relating the distance d between the point and sphere can be created, see Figure 8. Given the point-to-point contact, only two point-to-sphere contacts are necessary (three point-to-sphere contacts are normally necessary) to locate the center of the sphere. This is due to the extra constraint imposed by the spatial relationship between the point and the sphere. Spatial-constraint equations between any two geometric elements – namely, a sphere, a cylinder, a plane, a line, and a point – need to be determined.

The spatial constraints between points and any of the other geometric elements are already known from Section 3. The spatial constraint between two points in space is represented by eq. 4. Equation 4 can also be used with a point and a sphere. The spatial constraint relating a point and plane in space is represented by eq. 6 where r should be replaced with the distance between the plane and point. The spatial constraint relating a point and a cylinder or line is represented by eq. 10 where $r+R$ should be replaced by the shortest distance between the point and the line (or center line).

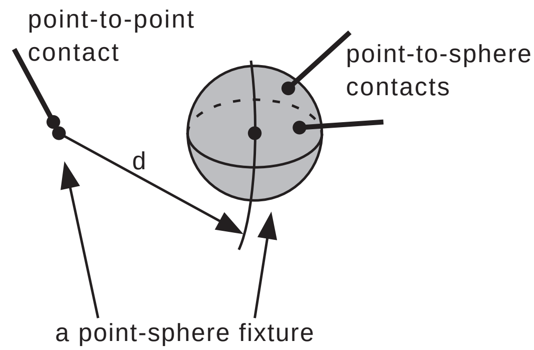

Figure 8. CONTACTS TO A POINT-SPHERE FIXTURE

The spatial constraints between spheres and any of the other geometric elements are the same as the point cases except that the center of the sphere is used in replacement of the point.

The spatial constraint between two planes in space can be determined using the angle between the two normals of the planes. This can be represented as

$$\cos\theta=\frac{\vec{n}_1\cdot\vec{n}_2}{||\vec{n}_1||||\vec{n}_2||} \tag{18}$$

where $\vec{n}_1$ and $\vec{n}_2$ are the normals for the planes, and θ is the angle between the normals. This is commonly referred to as the definition of the scalar product (Anton, 1984).

The spatial constraint between a plane and line (or cylinder) can also be represented by eq. 18. The only difference is that one of the normals should be replaced by a vector pointing in the direction of the line (or center line of the cylinder).

The spatial constraint between a line (or cylinder) and another line (or cylinder) can be represented by two equations. One equation constrains the distance between the two lines/center lines (eq. 17). The other equation constrains the angle between the two lines/center lines (eq. 18).

With the equations developed in this section and the previous section and a set of contacts to a group of geometric elements (that meets the general requirements described in Nederbragt and Ravani (1997a)), the location of the geometric elements can be determined. An algorithm for solving these equations is given in the next section.

SOLVING FIXTURE GEOMETRIES THAT INCLUDE A KNOWN PLANE

We now have the equations necessary to solve any of the cases listed in Table 1. Solving these equations to determine the locations of the geometric elements may result in multiple numeric solutions. For example, three point contacts to one sphere will result in two possible locations for the center of the sphere.

In many cases, several of the extra solutions will be complex; hence, they can be neglected.

Algorithm

A method for solving the equations (that represent the cases described in Table 1) are given in the following seven steps.

Step 1 Develop numeric equations for all given contacts. In other words, substitute all numerical data into the needed contact equations described in Section 3.

Step 2 Develop the equations that relate the geometric elements in the referencing fixture to each other. Since there is always a plane of known location in each case, there will always be one equation that relates each geometric element of unknown location to the known plane. For example, given a fixture containing a point, a line, and a plane (of known location), there are spatial constraints between the line and point, the line and plane, and the point and plane. Numeric equations that represent the spatial constraints can be created using the equations described in Section 4.

Step 3 Solve for geometric elements where the contact and spatial constraint equations can be solved without involving unknown variables from the other geometric elements. For example, given a plane (of known location) and two sphere fixture where there are two point-to-sphere contacts to one sphere and one point-to-sphere contact to the other sphere, the sphere with two point-to-sphere contacts can be located without using variables from the second sphere. However, the second sphere cannot be located without the use of variables from the first sphere.

Step 4 Solve for geometric elements where the contact and spatial constraint equations can be solved without involving unknown variables from the other geometric elements but using the previous results. Using the two sphere/one plane example from Step 3, the second sphere with one contact can be located using the equation that relates the first sphere to the second sphere, the equation that relates the plane to the second sphere, the equation for the point-to-sphere contact, and the numeric values obtained for the first sphere.

Step 5 Solve for the simplest set of geometric elements using results obtained from Steps 3 and 4 (if there are results from these steps).

Step 6 Using the results from Step 5 return to Step 4 until all geometric element locations are found.

Step 7 With all of the geometric element locations known, a reference frame can be created using the geometric element properties. In many cases, several possible solutions will exist. The extra – incorrect – solutions need to be eliminated. Methods for eliminating these extra solutions are currently being investigated. Examples are now given to demonstrate the algorithm.

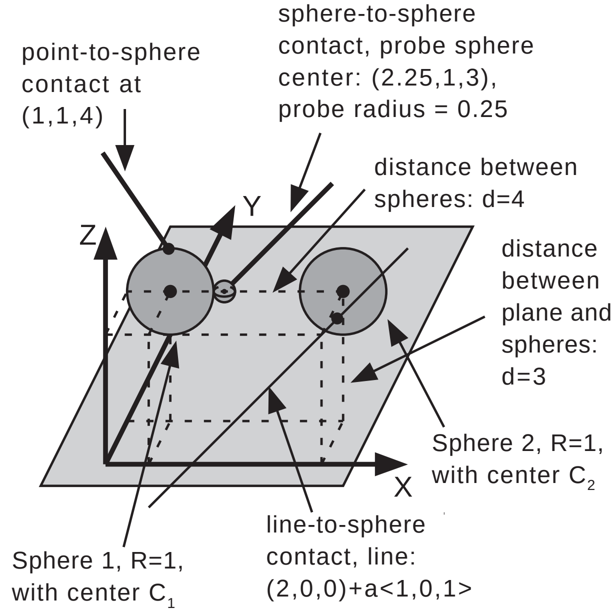

Figure 9. EXAMPLE 1

Example 1

To demonstrate the algorithm described in Section 5.1, an example is given. The referencing fixture for this example consists of two spheres and a known plane. Figure 9 shows the geometric elements that compose the external features of this referencing fixture and the spatial constraints between the geometric elements. It also shows three contacts to the spheres. There are two contacts to Sphere 1: a point-to-sphere contact and a sphere-to-sphere contact. There is one contact to Sphere 2: a line-to-sphere contact. From Table 1, this is the $\{G_P\}\{S \cdot S\}\{S \cdot S\}\{S \cdot C\}$ case.

Using our algorithm, the first step is to determine the equations for each contact. The point-to-sphere contact is represented using eq. 4 where $R = 1$, $r = 0$, P_4=C_1, and P_5 is the location of the point contact, $(1,1,4)$. After entering this information into eq. 4, we get

$$(1+0)^2 = (1-C_{1x})^2 + (1-C_{1y})^2 + (4-C_{1z})^2. \tag{19}$$

The sphere-to-sphere contact to Sphere 1 is also represented using eq. 4 where $R = 1$, $r = 0.25$, P_4=C_1, and P_5 is the location of the center point of the spherical probe, $(2.25,1,3)$. After entering these data into eq. 4, we get

$$(1+.25)^2 = (2.25-C_{1x})^2 + (1-C_{1y})^2 + (3-C_{1z})^2. \tag{20}$$

The line-to-sphere contact to Sphere 2 is represented using eq. 10 where $R = 1$, $r = 0$, P_4=C_2, $x_o = 2$, $y_o = 0$, $x = 1$, and $y = 0$. After entering these data into eq. 10, we get

$$2 = 2{C_{2y}}^2 + (C_{2x} - 2 - C_{2z})^2. \qquad (21)$$

We have developed all of the contact equations. Now, we need to find the spatial constraint equations that relate the geometric elements. Since we have a plane, and two spheres, we need an equation to relate the spheres to each other, and we need equations that relate the plane to each sphere.

The equation for relating the plane to each sphere is eq. 6. The distance between the sphere center and the plane is 3. The equation of the plane is simply $z = 0$ (this is known ahead of time). Hence, the normal to the plane is $< 0, 0, 1 >$. The center of Sphere 1 is C_1 and the center of Sphere 2 is C_2. Using eq. 6 we get

$${C_{1z}}^2 = 9 \qquad (22)$$

for Sphere 1, and

$${C_{2z}}^2 = 9 \qquad (23)$$

for Sphere 2.

The equation that relates the two spheres is eq. 4 where $R + r = 4$, $P_4 = C_1$, and $P_5 = C_2$. Substituting this into eq. 4, we get

$$(4)^2 = (C_{2x} - C_{1x})^2 + (C_{2y} - C_{1y})^2 + (C_{2z} - C_{1z})^2. \qquad (24)$$

We have all of the contact equations and the geometric element spatial constraint equations. We now need to solve for the location of any geometric elements that do not involve unknown variables from the other geometric elements. Equation 19, eq. 20, and eq. 22 can be used to find the center of Sphere 1. Using a mathematics package and these equations, four solutions are found. There is only one real result:

$$C_1 = (1, 1, 3), \qquad (25)$$

which, from analysis of the example, is the correct result.

Using the result from Step 3, we now proceed with Step 4. The second sphere can now be located using eq. 21, eq. 23, and eq. 24, and eq. 25. Using a mathematics package to solve the equations, we get eight solutions. Six of these solutions are complex; hence, they can be neglected. The other two solutions are: $C_2 = (1, 5, 3)$ and $C_2 = (-0.9355, 4.5005, 3.0000)$. From the example, we can tell that $C_2 = (1, 5, 3)$ is the correct solution.

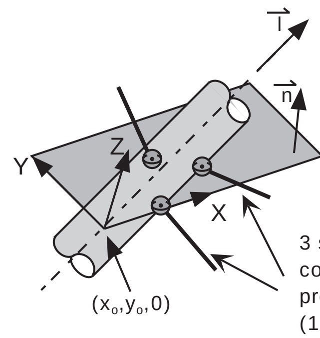

Figure 10. EXAMPLE 2

Example 2

To further demonstrate the algorithm described in Section 5.1, another example is given. This time a fixture consisting of a cylinder and a known plane is used. Figure 10 shows the geometric elements and their relative positions. It also shows the three sphere-to-cylinder contacts. From Table 1, this is the $\{G_P\}\{S{\cdot}C\}\{S{\cdot}C\}\{S{\cdot}C\}$ case where $\{G_P\}$ represents the known plane, and the three $\{S{\cdot}C\}$'s are the three sphere-to-cylinder contacts.

Using our algorithm, the first step is to determine the equations for each contact. The sphere-to-cylinder contacts are represented using eq. 10. For all three cases, the radius of the probe sphere is 0.25, and the radius of the cylinder is 1. The center of the probe sphere for the three contacts are $(1, -1.25, 1)$, $(2, -1.25, 2)$, and $(0.1161, 0, 1.8839)$. The normal to the plane is $< 0, 0, 1 >$. The equation for the plane is $z = 0$. The angle between the plane normal and the center line of the cylinder is 45 degrees. These values correspond to a cylinder with $x_o = 0$, $y_o = 0$, $x = 1$, and $y = 0$. Since we already know the solution, we can verify the solution obtained from solving the contact equations.

The contact equations for the three sphere-to-cylinder contacts are

$$\begin{aligned} 0 =& 2x - x^2 + 2x_o - 2xx_o - {x_o}^2 - 2.5y \\ &- 2.5xy + 2.5xx_oy - 0.4375y^2 + 2x_oy^2 \\ &- {x_o}^2y^2 - 2.5y_o - 2.5x^2y_o - 2yy_o \\ &- 2xyy_o + 2xx_oyy_o - {y_o}^2 - x^2{y_o}^2 - 1, \end{aligned} \qquad (26)$$

$$
\begin{aligned}
0 =& 8x - 4x^2 + 4x_o - 4xx_o - {x_o}^2 - 5y \\
& - 5xy + 2.5xx_oy - 6.4375y^2 + 4x_oy^2 \\
& - {x_o}^2y^2 - 2.5y_o - 2.5x^2y_o - 4yy_o \\
& - 4xyy_o + 2xx_oyy_o - {y_o}^2 - x^2{y_o}^2 - 4,
\end{aligned} \tag{27}
$$

and

$$
\begin{aligned}
0 =& 0.4375x - 1.9865x^2 + 0.2322x_o \\
& - 3.7678xx_o - {x_o}^2 - 2y^2 + 0.2322x_oy^2 \\
& - {x_o}^2y^2 - 3.7678yy_o - 0.2322xyy_o \\
& + 2xx_oyy_o - {y_o}^2 - x^2{y_o}^2 + 1.549.
\end{aligned} \tag{28}
$$

We have all of the contact equations; however, the equations are large. Hence, anything that we can do to simplify the equations should be done. Each of the three equations share several equation components that have the same coefficients. If we subtract one equation from the other two, then we can significantly simplify two of the three equations. Subtracting eq. 26 from eq. 27 gives us

$$
\begin{aligned}
0 =& 6x - 3x^2 + 2x_o - 2xx_o - 2.5y - 2.5xy \\
& - 6y^2 + 2x_oy^2 - 2yy_o - 2xyy_o - 3.
\end{aligned} \tag{29}
$$

Subtracting eq. 26 from eq. 28 gives us

$$
\begin{aligned}
0 =& -1.5625x - 0.9865x^2 - 1.7678x_o \\
& - 1.7678xx_o + 2.5y + 2.5xy - 2.5xx_oy \\
& - 1.5625y^2 - 1.7678x_oy^2 + 2.5x^2y_o \\
& + 2.5y_o - 1.7678yy_o + 1.7678xyy_o + 2.5490.
\end{aligned} \tag{30}
$$

We need to find the equation relating the plane to the cylinder. Equation 18 constrains the angle between the center line of the cylinder and the normal of the plane. In our case, the angle is 45 degrees. Substituting our known values into eq. 18, we get

$$
x^2 + y^2 - 1 = 0. \tag{31}
$$

We have all of the contact equations and the geometric element spatial constraint equations. We now need to solve for the location of any geometric elements that do not involve unknown variables from the other geometric elements. In this example, we only need to find the cylinder. Equations 26, 29, 30, and 31 can be used to find the cylinder. Solving these equations with a mathematics package gives us 11 solutions. Of these 11 solutions, only three are real. These solutions are given in Table . As can be seen from the table, we did get the correct solution. Again, methods are needed to eliminate the unwanted solutions.

Table 2. SOLUTIONS TO EXAMPLE 2

Sol'n	x_o	y_o	x	y
1	-1.7678	-1.25	1	0
2	2.5872	-1.2440	-0.6193	0.7851
3	0	0	1	0

Dealing with Multiple Solutions

As can be seen in the examples, several solutions can result from the analysis. There are things that can be done to eliminate many of these cases. If the solutions are complex, then, obviously, they can be neglected. If the results are real, then, using the contact access direction for the probe, it may be possible to eliminate incorrect solutions (Nederbragt and Ravani, 1997b). Moreover, it may be possible to reduce the order of the equations using algebraic methods prior to analysis. This can significantly reduce the complexity involved in solving the problem and reduce the number of results obtained. These methods are currently being studied by the authors.

MEASUREMENT ERROR ANALYSIS

Another important issue is analysis of measurement error. Because the geometric contact locations vary from contact case to contact case, a measurement error analysis must be performed for every individual case in order to determine the error in the final result. The authors are investigating analysis methods for determining the error in the geometric element locations based on contact measurement errors.

CONCLUSION

This paper presents a systematic method for determining the location of fixturing geometries based on contact analysis. In the near future, methods for dealing with multiple solutions and measurement errors will be presented.

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