errata: final dimensions add to 7 and not 6 as shown

Evaluation of an Injection Molding Process Model Using the Calculus of Imprecision to Simultaneously Specify Tolerances and Process Parameters

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Abstract

The strong interrelationship between part geometry, material properties, tolerances, and process parameters for injection molded parts hinders tolerance allocation and process specification. The traditional design process of first optimizing tolerances and then setting process parameters to achieve these tolerances has the potential for sub-optimization. Rather simultaneous tolerance allocation and process specification is required. Unfortunately the injection molding modeling uncertainty hampers optimal tolerance and process specification. Consequently methods are needed for directly incorporating imprecision into these models. This paper advocates the use of imprecise quantities in existing analytical process models to simultaneously allocate tolerances and process specification for minimum manufacturing cost. A set mathematical approach called the Calculus of Imprecision (CoI) is developed to provide a general framework for including imprecision directly in existing process models. The CoI is a refinement of a worst-case interval approach but at various levels of plausibility with a reduced computational load.

***Keywords**: Design for manufacturing, manufacturing process modeling, fuzzy set theory, injection molding, model uncertainty, concurrent engineering, tolerance allocation, optimization.*

Introduction

The current market environment of rapid product development, short lead times, improved quality, and price competition has demanded more from product materials. Engineering plastics have emerged as a principal material choice due to advantageous properties that enable them to compete in applications where metals were traditionally utilized. Advantages of injection molding are the ability to combine several components into a single molded part to eliminate costly assembly operations [1], high production rates, high production volumes, and suitable mechanical properties at lower densities than competing metals. As a result, plastic utilization in the aerospace and automotive industries has increased dramatically as these industries strive to reduce overall product weight. Moreover, the broader market presence and increasing utilization of plastics have placed greater demands for increased quality and geometric complexity which has necessitated tighter part tolerances. Design for injection molding, whereby the product's functionality, tolerances, mold design, and process parameters are simultaneously considered, has emerged as a necessary design strategy [2]. Adherence to design for manufacturing heuristics such as found in Dixon and Poli [3] aid in reducing mold risk and product improvement. However, for more intricate parts the rules are insufficient and simulation models or analytical models are often used to determine process parameters and improve product characteristics with respect to manufacturability. Typically, designers optimize tolerance allocation based on functional requirements and then process engineers optimize the injection molding process to meet the design specification [4, 5]. Zhang [6] argues that both design and manufacturing must be simultaneously considered when allocating tolerances and demonstrates why in the machining domain. In this paper, we incorporate design requirements into the injection molding process model to simultaneously perform tolerance allocation and process specification for minimum manufacturing cost. The conceptualization of this approach is shown in Figure 1.

Injection molding process models are based on empirical studies and have complex relationships. Consequently, there exists a certain amount of model uncertainty. Furthermore, plastics have extremely complex material properties: non-Newtonian, non-isothermal rheology, and high correlation between parameters. Due to less than perfect process control, variation about the process parameter set points will occur and consequently the dimensions will also vary. The uncertainty and imprecision inherent in the process models cannot be ignored. Manufacturing process model uncertainty occurs as either stochastic random variation which can be modeled using statistics or as imprecision which can be modeled using fuzzy sets. Traditionally, only statistical methods have been used to deal with injection molding model uncertainty. There are, however, two major problems related to these methods: first, they are very computationally intensive: for every input parameter a sample must be taken from the distribution to determine a value in the output parameter. In a Monte-Carlo process simulation typically thousands of samples must be calculated. Second, even more importantly, these methods require that we know the probability distributions for each input parameter, and usually the exact values or shape of these probabilities are not known. Instead, only the intervals of possible error values are known. Injection molders generally do not have the data to support a statistical analysis. Practitioners can try to estimate the probabilities, but if the guess is wrong the output probabilities will be erroneous. More importantly, much of the process uncertainty is not of a stochastic nature but is due to the inherent model imprecision of ill-defined parameters and relationships.

An alternative approach is to directly model the imprecision as fuzzy sets or imprecise quantities. There has been an increased interest in modeling imprecision other than what can be described by stochastic uncertainty in engineering applications. The foundation of this approach is that many concepts cannot be accurately measured and modeled because imprecision is intrinsic to the parameters and relationships in these problems; such is the case for injection molding. In these situations, parameters can be modeled as imprecise quantities that restrict the value of a parameter to a partially ordered set. The injection molding process model relates machine control inputs such as mold temperature, melt temperature, and packing pressure with process outputs such as shrinkage and tolerances. The machine control input parameters can be represented by imprecise quantities that map through the process model to induce possibility distributions on the output. Inversely, preference distributions can be specified for the output and mapped through the process model to determine the plausibility range to restrict the input. Before set mappings through analytical injection molding process models can occur, two problems that arise due to the set operator's mathematical properties must be addressed. First there is, in general, no inverse for the extended algebraic operators addition and multiplication. Consequently fuzzy equations cannot be solved by inverting the operators. Second, when multiple occurrences of a parameter occur in a function the standard set mathematics overstates the imprecision of the result. Consequently, the result contains the actual set as a subset. These limitations hinder the application of traditional set mathematics. Direct incorporation of imprecision into existing process models is enabled here via the *Calculus of Imprecision* (CoI). The CoI overcomes the two limitations mentioned. The merits of modeling the imprecision are twofold: mapping sets through the model is computationally efficient compared to statistical approaches, and a form of robust design is accomplished such that if the process parameters remain within the specified range then the tolerances will remain within their specified range.

This paper examines the use of existing process models during the design process to determine optimal tolerance allocation and process parameter set points. The next section introduces the notation of imprecise quantities and presents the two problems encountered when using imprecise quantities in engineering models and related work to overcome these problems. Three operators; image, domain, and sufficient elements that comprise the CoI are presented to overcome these problems based on extensions from work conducted in interval analysis [7]. An injection molding process model is described and used as an example to demonstrate the benefits of the new operators for mapping imprecision in engineering models. The problem of simultaneously allocated tolerances and process parameters for minimum cost is solved for an example part.

Imprecise Quantities

An *imprecise quantity* Q is a partially ordered set of real numbers. Each element $x \in Q$ has an associated *membership value* $\mu_Q(x)$ representing the degree x belongs to Q. It is a mapping $\mu_Q : x \to [0,1]$. Common practice is to impose restrictions on the shape of $\mu_Q(x)$ to either a triangular or trapezoidal distribution [8, 9]. Here a triangular distribution will be used and is represented by a triple that defines the membership function's endpoints as,

$$x \to \langle \underline{x}, x, \overline{x} \rangle \tag{1}$$

An imprecise quantity defines a set of closed intervals called α-cut sets that are described by, $Q_\alpha = \left\{ x \middle| \mu_Q(x) \geq \alpha \right\}$, $\forall\ \alpha \in (0, 1]$. The α-cut set at α is represented by the interval,

$$Q_\alpha = [\underline{x}_\alpha, \overline{x}_\alpha] \tag{2}$$

Figure 2 shows the representation of expression (1) and an α-cut set at 0.5. The interval $[\underline{x}, \overline{x}]$ is the α-cut set at α=0 and is called the support set. The α-cut set at α=1 is a single value *x*. Throughout this paper the single bar notation will be used to represent the endpoints of the support set.

Fuzzy sets can model two facets of imprecision, either preference or possibility. *Preference* is when a designer specifies a range of acceptable values for a design parameter. For example, a functional tolerance can be represented by expression (1) to describe a preference for values that decreases the further from nominal. Thus, a fuzzy set tolerance is similar in concept to Taguchi's quality loss function [10], but is its inverse. *Possibility* is the plausibility that a parameter will assume a certain value. An illustration is the mold temperature in injection molding where a variation around the set point is observed as a range with gradation and can be represented by a possibility profile using expression (1). In injection molding the output in tolerances can be represented by preference distributions and the machine input controls by possibility distributions.

Issues and Limitations of Set Mathematics Applied to Engineering Systems

This section discusses the two anomalies associated with directly including imprecision in existing manufacturing process models.

Mapping Imprecise Quantities

Any crisp mapping function can be extended to fuzzy sets via the extension principle [11]. The membership function $\mu(x)$ is mapped by function f and induces $\mu(y)$ defined via the extension principle as,

$$\mu(y) = \sup\left\{\min\left\{\mu(x),\, \mu(m)\right\} \mid f(x) = y\right\} \tag{3}$$

$$\mu(y) = 0 \text{ if } f^{-1}(y) = \varnothing .$$

Alternatively, for an isotonic function f, the α-cut endpoints of the evaluated function are equal to that function evaluated on the α-cut endpoints of the individual parameters [12]. Formally, we state,

$$\left[f\left(Q_1, Q_2\right)\right]_\alpha = f\left(Q_{1\alpha}, Q_{2\alpha}\right), \quad \forall \alpha \in (0,1] \tag{4}$$

Equation (4) provides the justification used to evaluate functions based on the imprecise quantity's endpoints given by expression (1).

Figure 3 shows a monotonically increasing mapping (function) $f: R \to R$ and the inverse mapping $f^{-1}: R \to R$. Let $m \to \langle 1.5, 2, 2.5 \rangle$, $x \to \langle 5, 5.5, 6 \rangle$ and if the function f is $y = m \otimes x$, then the extended mapping follows the arrows from the x-axis to induce $y \to \langle 7.5, 10, 15 \rangle$. When $\mu(y)$ is mapped by the inverse, f^{-1}, then the extension principle follows the arrows from the y-axis to induce $\mu(x')$ on R and not $\mu(x)$. In Figure 3 the imprecise quantities x and x' are shown on the horizontal axis and y on the vertical axis. Note that $x \subseteq x'$, *i.e.* x is more *precise* than x'. In engineering applications it is commonly desirable to retrieve x but this is not possible when strictly using the extension principle. For example, if y was the desired tolerance output from injection molding then the correct constraint in the machine control input would be x and not x'. Use of x' as a machine control restriction would lead to tolerance deviations greater than the desired y and thus an unacceptably low yield rate.

The lack of an inverse is the first problem since algebraic equations of the form, $A \oplus X = B$ cannot be solved for unknown X as $X = B \ominus A$. The reason is the set operators for addition and multiplication, $\oplus$ and $\otimes$ are not group operators, but form a semi-group with identity 0 and 1 respectively [12]. The reason imprecise mappings lack an inverse is apparent when you consider that there is more than one forward mapping from the input to a single output value. (*e.g.* there are two combinations of x and m values, {10, 1.5} and {6, 2.5} that map through $y=mx$ to $y = 15$). The extension principle does not differentiate between which values are desired in the mapping but in physical systems there is an important distinction.

A second problem when mapping imprecise quantities through analytical models is that the multiple occurrence of a parameter in an expression causes incorrect results. Let $g(x) = \frac{x}{x-2}$ and an equivalent representation of the function is $f(x) = 1 + \frac{2}{x-2}$. If $x \to \langle 3, 4, 5 \rangle$ then $g(x) \to \langle 1, 2, 5 \rangle$ and $f(x) \to \langle 1.67, 2, 3 \rangle$. The function *g(x)* is called an *improper representation* of the function because it treats each occurrence x as a separate parameter with the same range, when the intent is that it is the same parameter. Therefore, *g(x)* obtains more imprecise results (a larger set of values) than *f(x),* the *proper representation* [12].

These mathematical properties pose significant hurdles to modeling imprecision directly in analytical models. Consequently, an alternative approach to including imprecise quantities in analytical process models is required.

Related Work in Set Mathematics

Buckley and Qu [13] examine the problem of solving linear and quadratic equations and present the conditions governing the existence of a solution when using the α-cut method. Dong and Wong [14] used a combinatorial interval analysis scheme to account for multiple occurrences of a parameter as part of an algorithm for computing fuzzy weighted averages (FWA). Inverses of functions can be determined by the FWA discretization algorithm using the internally stored discretized points only after the forward mapping is calculated. Thus, it is not possible to solve equations using this algorithm. Wood, *et al*., [15] and Otto, *et al.* [16] have extended this approach to encompass more functions and combination metrics. Klir [17] proposes constrained fuzzy arithmetic to overcome these problems, such that when a parameter appears multiple times in an algebraic expression then an equality constraint is included in the operation. This approach requires the modeler to add algebraic constraints to properly model the physical system.

The absence of an inverse and the multiple occurrence of parameters are well known problems in the domain of interval analysis [18, 19]. Ward, *et al.*, [20] have extensively examined the use of interval analysis in the mechanical engineering design domain. They developed three operators, the range operator and three inverses to range that are used to solve interval equations of three parameters. Finch and Ward [7] extended these results to arbitrary relationships over *n* parameters and show how to obtain useful information pertinent to the analysis of physical systems. They accomplish this by making an important distinction between physically dependent and independent parameters. It is this later work that is extended to imprecise quantities incorporated into existing analytical models that is presented in this paper.

Models of Physical Systems

The parameters in engineering application models have a domain specific connotation. The causality between the imprecise engineering parameters can be exploited to achieve better results in these domain specific models. Dubois, *et al*., [21] discuss the significance of controllable versus uncontrollable parameters in the context of job-shop scheduling. If the parameter is controllable then the fuzzy set represents preference for a value. Fuzzy sets of uncontrollable parameters represent a possibility distribution that constrain the values the parameter can assume. Likewise, the partitioning of parameters into design, tuning, and noise parameters has been advantageously applied by Otto and Antonsson [22] for mechanical engineering design. The distinction of the causality of parameters is significant to the interpretation of these engineering models. *Physically independent parameters* are those that temporally occur first and determine the physically dependent parameter values. The *physically dependent parameters* cannot be directly specified by the designer or process engineer. This notion of physical dependency does not correspond to the typical mathematical definition and will be demonstrated in the next section for analyzing the injection molding process.

Calculus of Imprecision

This section presents three operators adapted from [7] in the terminology relevant to imprecise quantities that will be used in analytical process models.

Definition 1: Decreasing Parameters Subset

The *decreasing parameters subset,* D_f is the subset of parameters for the function f such that the function $f(q_1, \ldots, q_n)$ with n parameters q is monotonically decreasing. $f(x, q_1, \ldots, q_n)$ is *monotonically decreasing* w.r.t. x if and only if for $x > x'$ and when $q_i,\ldots,q_n$ is constant, then $f(x, q_1, \ldots, q_n) < f(x', q_1, \ldots, q_n)$ [23].

Definition 2: Increasing Parameters Subset

The *increasing parameters subset,* I_f is the subset of parameters for when the function $f(q_1, \ldots, q_n)$ is monotonically increasing. A function $f(x, q_1, \ldots, q_n)$ is called *monotonically increasing* w.r.t. x if and only if for $x > x'$ and when $q_i,\ldots,q_n$ is constant, then $f(x, q_1, \ldots, q_n) \geq f(x', q_1, \ldots, q_n)$ [23].

Definition 3: Image

The image determines the possibility distribution of the physically dependent output from the input domain. This is equivalent to the extension principle and is considered “pessimistic” since it finds the largest possible set resulting from the physically independent parameters. It is included here to maintain a consistent notation with the inverses to image.

Image: $f(q_1,\ldots,q_n)=p$ then

$$p \to \left\langle f\left(\overline{D}_f,\underline{I}_f\right), f\left(D_f,I_f\right), f\left(\underline{D}_f,\overline{I}_f\right)\right\rangle$$

If $x \in D_f$ then the notation $\overline{D}_f$ denotes the parameters in D_f at their $\overline{x}$ values according to expression (1).

Definition 4: Domain

An inverse of the image is the domain operator. Domain determines the physically independent parameter such that the forward mapping will always be restricted by the physically dependent parameter *p*.

Domain: $f^{-1}(q_1,\ldots,q_n,p)=q_k$

for $p \in I_{f^{-1}}$ then $q_k \to$

$$\left\langle \begin{array}{l} f^{-1}\left(\left\{\underline{D}_{f^{-1}} \cup \underline{p}\right\},\left\{\overline{I}_{f^{-1}} - \overline{p}\right\}\right), \\ f^{-1}\left(\left\{D_{f^{-1}} \cup p\right\},\left\{I_{f^{-1}} - p\right\}\right), \\ f^{-1}\left(\left\{\overline{D}_{f^{-1}} \cup \overline{p}\right\},\left\{\underline{I}_{f^{-1}} - \underline{p}\right\}\right) \end{array} \right\rangle$$

for $p \in D_{f^{-1}}$ then $q_k \to$

$$\left\langle \begin{array}{l} f^{-1}\left(\left\{\underline{D}_{f^{-1}} - \underline{p}\right\},\left\{\overline{I}_{f^{-1}} \cup \overline{p}\right\}\right), \\ f^{-1}\left(\left\{D_{f^{-1}} - p\right\},\left\{I_{f^{-1}} \cup p\right\}\right), \\ f^{-1}\left(\left\{\overline{D}_{f^{-1}} - \overline{p}\right\},\left\{\underline{I}_{f^{-1}} \cup \underline{p}\right\}\right) \end{array} \right\rangle$$

Where $\left\{\underline{D}_{f^{-1}} - \underline{p}\right\}$ denotes the set of parameters when the expression $f^{-1}(q_1,\ldots,q_n,p)$ is decreasing less the parameter *p*. $\left\{\underline{I}_{f^{-1}} \cup \underline{p}\right\}$ denotes the set of monotonically increasing parameters and the parameter *p*.

Definition 5: Sufficient Elements

The independent parameters are partitioned into uncontrolled q' and controlled q'' subsets. Sufficient elements is an inverse of the image that determines the physically independent parameter sets in one partition such that adjusting the parameters in the second partition will map to every value in the physically dependent output set *p*.

SufElements: $f^{-1}(q', q'', p) = q_k$

$$for\ p \in D_{f^{-1}}\ then\ q_k \rightarrow$$

$$\left\langle \begin{array}{l} f^{-1}\left(\overline{I'}_{f^{-1}} \cup \overline{D''}_{f^{-1}}, \underline{p} \cup \underline{D'}_{f^{-1}} \cup \underline{I''}_{f^{-1}}\right), \\ f^{-1}\left(I'_{f^{-1}} \cup D''_{f^{-1}}, p \cup D'_{f^{-1}} \cup I''_{f^{-1}}\right), \\ f^{-1}\left(\underline{I'}_{f^{-1}} \cup \underline{D''}_{f^{-1}}, \overline{p} \cup \overline{D'}_{f^{-1}} \cup \overline{I''}_{f^{-1}}\right) \end{array} \right\rangle$$

$$for\ p \in I_{f^{-1}}\ then\ q_k \rightarrow$$

$$\left\langle \begin{array}{l} f^{-1}\left(\overline{D'}_{f^{-1}} \cup \overline{I''}_{f^{-1}}, \underline{p} \cup \underline{I'}_{f^{-1}} \cup \underline{D''}_{f^{-1}}\right), \\ f^{-1}\left(D'_{f^{-1}} \cup I''_{f^{-1}}, \underline{p} \cup I'_{f^{-1}} \cup D''_{f^{-1}}\right), \\ f^{-1}\left(\underline{D'}_{f^{-1}} \cup \underline{I''}_{f^{-1}}, \overline{p} \cup \overline{I'}_{f^{-1}} \cup \overline{D''}_{f^{-1}}\right) \end{array} \right\rangle$$

These three operators show how to obtain the parameters of expression (1). The entire membership function can be obtained via two methods, discretization [14, 15, 16] or the parametered fuzzy numbers approach [24]. Both methods are approximations but they reduce the computational complexity and obtain useful results. Giachetti and Young [24] analyzed fuzzy algebraic operators and set forth guidelines for determining the accuracy of the parametered fuzzy number approach. They defined a spread ratio, as $\lambda = \left(\underline{x} / x \right)$ for the left spread and $\rho = \left(x / \overline{x} \right)$ for the right spread. When $\lambda < 3.67$ and $\rho > 0.5$ then a linear approximation for α-cut endpoints between 0 and 1 yields results within 10% of the actual value. In the problems considered here this is always the case.

Injection Molding Process Model

Part shrinkage that occurs during the solidification stage is a significant factor determining tolerances. Shrinkage is a function of the material properties, part geometry, and the processing conditions temperature, pressure, and volume [25]. The relationship between shrinkage and tolerances is such that dimensions where shrinkage is more sensitive to process variation should be allocated looser tolerances than dimensions where shrinkage is not sensitive to process variation.

Volumetric shrinkage is,

$$S_v = \frac{v_o - v_e}{v_o} \tag{5}$$

where v_o is the specific volume when the gate freezes and v_e is the specific volume at room temperature (*i.e*. complete cool down). The linear shrinkage is approximately 1/3 of the volumetric shrinkage,

$$S_L = \frac{S_v}{3} \tag{6}$$

In injection molding, specific volume v is determined from PVT data and is estimated by an equation derived by Spencer and Gilmore [26] from empirical data,

$$v = \frac{R'(T - \tau)}{(P + \pi)} + w \tag{7}$$

where T is temperature (K), P is pressure (MPa), and the constants are given from PVT data for polypropylene, w = 0.62 g/cm^3, π = 162 MPa, τ = 0 K, and $R' = 0.202$ MPa-cm^3/g-K. Fuzzy values are an appropriate representation to model the imprecision of the process parameters of temperature, pressure, and volume since equation (7) defines a curve fitted to empirical data and consequently is intrinsically imprecise. Furthermore, machine control is less than perfect so small variations will occur about the process parameters set points.

Information pertinent to injection molding tolerance capabilities can be obtained by evaluating expressions (5), (6), and (7) with the image, domain and sufficient elements operators. Using the terminology of [7] the parameters in the injection molding example are classified based on their physical causality. The shrinkage is a *physically dependent parameter* since it is determined by the specific volume which is determined by the machine's packing pressure and melting temperature. Consequently, shrinkage is determined using the image operator. Both temperature and pressure are *physically independent* in expression (7) since they are determined first by the manufacturing expert and adjusted on the injection molding machine. If a desired shrinkage is specified first (*i.e*.

as a preference function) using expression (1) then the *physically independent* volume to achieve it can be found with the domain operator. In expression (7) pressure and temperature are classified as *controllable* since they are adjusted on the machine but for a given material such as polypropylene the parameters w, π, τ, and R' are classified *uncontrollable*. These terms are used in the sufficient elements operator to solve for the pressure such that for any temperature in the set T then every value in volumetric shrinkage could be achieved. Note that this notion of physical dependency is different than mathematical dependency since expression (7) could be rewritten to solve for T as a function of v and P but T would still be physically independent even though it becomes mathematically dependent in the rewritten equation.

Evaluation of Injection Modeling Process Model

The proceeding examination of the injection molding model and introduction of the three operators suggest a formal methodology for applying the CoI to analytical process models.

1. Identify physical dependency conditions of model parameters.
2. Identify parameters that are controllable and those that are uncontrollable.
3. Determine appropriate operator based on classification. If solving for y and $y = f(q_k)$ then: if y is dependent use image, if y is independent use domain, and if y is independent and controllable use sufficient elements.
4. Determine the increasing subset I_f and the decreasing subset D_f for each equation.
5. Solve model using the three operators.

The following sections demonstrate the application of the three operators image, domain, and sufficient elements to mapping imprecise quantities through analytical process models.

Image

The physically dependent volumetric shrinkage is determined using the image operator. Let $v_o \to \langle 0.86, 0.87, 0.88 \rangle$ g/cm^3 and $v_e \to \langle 0.83, 0.84, 0.85 \rangle$ g/cm^3 which are obtained from plastic PVT data for polypropylene. According to definitions 1 and 2 the set of decreasing variables D_f is $\{v_e\}$ and the set of increasing variables I_f is $\{v_o\}$. The image of expression (5) is,

$$S_v \rightarrow \left\langle \frac{\underline{v_o} - \overline{v_e}}{\underline{v_o}}, \frac{v_o - v_e}{v_o}, \frac{\overline{v_o} - \underline{v_e}}{\overline{v_o}} \right\rangle$$

$$S_v \rightarrow \langle 0.0116, 0.0345, 0.0568 \rangle \text{ cm/cm}$$

This is the induced possibility distribution of volumetric shrinkage that can be expected if the specific volumes are ill-defined and represented by possibility distributions. Even though v_o occurs multiple times in expression (5) it is treated here as a single parameter. If the extension principle is used then the result is,

$$S_v{}' \rightarrow \langle 0.0114, 0.0345, 0.0581 \rangle \text{ cm/cm}$$

The extension principle, as previously noted, does not distinguish that v_o is a single parameter and treats it as two separate values to obtain an incorrect range $S_v{}'$. The extension principle, without accounting for the physical realization of the model incorrectly overestimates the plausible range of shrinkage values, *i.e.* $S_v \subseteq S_v{}'$.

Domain

The physically independent specific volume is determined such that it is restricted to map forward into the desired volumetric shrinkage output. Expression (5) is rewritten as,

$$v_o = \frac{v_e}{1 - S_v} \tag{8}$$

In this expression the decreasing subset of parameters $D_{f^{-1}}$ is $\{S_v\}$ and the increasing subset of parameters $I_{f^{-1}}$ is $\{v_e\}$. The physically dependent parameter p from expression (5) is S_v, consequently $p \in I_{f^{-1}}$. The domain operator is used,

$$v_o \rightarrow \left\langle \frac{\overline{v_e}}{1 - \underline{S_v}}, \frac{v_e}{1 - S_v}, \frac{\underline{v_e}}{1 - \overline{S_v}} \right\rangle$$

and the packing volume is obtained as,

$$v_o \rightarrow \langle 0.86, 0.87, 0.88 \rangle \text{ g/cm}^3$$

This result is identical to the original specification and demonstrates that the domain operator is an inverse to the image operator. If the extension principle was used then the resulting packing volume would be,

$$v_o{}' \to \langle 0.84,\, 0.87,\, 0.90\rangle \text{ g/cm}^3$$

This represents a greater range than the original *i.e.*

$$v_o \subseteq v_o{}'$$

$v_o{}'$ is incorrect and occurs since the image of f^{-1} is not the inverse of the image of *f*.

Sufficient Elements

The sufficient elements is used to determine the packing pressure such that the temperature can be adjusted and still yield the desired volumetric shrinkage. Equation (7) is rewritten as a function of *T* and *v*,

$$P = \frac{R'(T-\tau)}{(v-w)} - \pi \tag{9}$$

Let $T \to \langle 420, 425, 430\rangle$ K. The relevant classification of subsets is: $I''_{f^{-1}}$ is $\{T\}$, $I'_{f^{-1}}$ is $\{w, R'\}$, $D''_{f^{-1}}$ is $\{\varnothing\}$, and $D'_{f^{-1}}$ is $\{\pi, \tau\}$. The physically dependent parameter *p* is *v* and $v \in D_{f^{-1}}$. The sufficient elements operator is applied,

$$P \to \left\langle \frac{\overline{R'}(\underline{T}-\underline{\tau})}{(\underline{v}-\overline{w})}, \frac{R'(T-\tau)}{(v-w)}, \frac{\underline{R'}(\overline{T}-\overline{\tau})}{(\overline{v}-\underline{w})} \right\rangle$$

$$P \to \langle 172, 181, 191\rangle \text{ MPa}$$

This is the range over which *P* can be adjusted with *T* to obtain every value in *v*. Otherwise using the extension principle,

$$P' \to \langle 164, 181, 199\rangle \text{ MPa}$$

$P \subseteq P'$ with the possible result of falling outside of the desired range of volumetric shrinkage if P' was used. These three examples demonstrated the necessity of the three operators image, domain, and sufficient elements to obtain more accurate results.

Simultaneous Tolerance Allocation and Process Specification

A non-linear optimization problem is formulated to simultaneously allocate tolerances and specify process parameters. The example problem, a polypropylene hinge to be injection molded, is shown in Figure 4.

The tolerance capabilities of dimension x_j are related to linear shrinkage by,

$$\left(\overline{x}_j - \underline{x}_j\right) \geq x_j S_L \tag{10}$$

The model constraints are given in Table 1. Not shown are constraints that provide lower and upper bounds on the values that a parameter can assume. The proposed CoI methodology is used and the model parameters classification is given in Table 2.

The objective is to minimize the cost given as,

$$Min \sum_{j=1}^{n} \frac{a_j}{\left(\overline{x_j} - \underline{x}_j\right)^2} + \frac{b}{\left(\overline{T}_o - \underline{T}_o\right)^2} + \frac{c}{\left(\overline{T}_e - \underline{T}_e\right)^2} + \frac{d}{\left(\overline{P} - \underline{P}\right)^2} \tag{11}$$

where a is a cost constant for dimension j and b, c, and d are cost constants for the injection molding process. The first term captures the concept that tighter tolerances require more expensive machining operations for fabricating the mold [27]. The last three terms model the inverse relationship that better process control requires more expensive and sophisticated injection molding equipment. In this example $a = 0.1$ for all dimensions and $b = c = d = 2000$. The explicit separation of mold machining costs and of injection molding equipment costs provide a means to achieve an optimal balance between the two.

A solution to the nonlinear optimization problem was found using the generalized reduced gradient algorithm [28] and is shown in Table 3. The tolerances determined agree with handbook suggestions as provided by [29]. In this example, if the process parameters are allowed to vary more than 10.7 C for temperature or 8.3 MPa for pressure than the geometric constraints cannot be assured. It is noted that without the new operators defined in the previous section the constraints would obtain incorrect results, *i.e.* the traditional application of set propagation would result in larger ranges. Exploitation of the physical causality of the injection molding process model enables a more aggressive approach of assigning tolerances and process parameters. Optimizing tolerances separately from process parameters would probably result in different tolerances and it is unlikely that a manufacturing engineer could determine the optimal process parameters by an iterative select and test method. Consequently, this problem demonstrates the importance of simultaneously considering process parameters and tolerances.

The results shown in Table 3 obtained ranges of values within which adjustments can be made to the process control parameters. The current injection molding technology can maintain control set points for temperature and pressure well within the given solution ranges for these parameters. Therefore, this solution can be considered a phase one optimization. A second optimization can be performed with this solution setting bounds on the results. A typical secondary objective would be to minimize cycle time and a technique such as recursive constraint bounding [30] could be used to determine control parameter set points based on quality measurements of the output.

Conclusion

In traditional design processes the tolerance allocation would be optimized first and then the manufacturing engineer would optimize process parameters based on those tolerances. This technique has the potential of sub-optimization. In this paper we advocate using nonlinear optimization techniques to minimize manufacturing cost while simultaneously allocating tolerances and process parameters. Consequently, process information is included in the design phase and sub-optimization is avoided. Impediments to accomplishing this is the inherent uncertainty of empirically developed process models. The primary contribution of this paper was the set mathematical approached called the Calculus of Imprecision for representing parameters as imprecise quantities in existing injection molding process models. The direct incorporation of imprecision into the existing process model was accomplished by extending the operators of image, domain, and sufficient elements developed by Finch and Ward [7] to imprecise quantities. The Calculus of Imprecision methodology was demonstrated to overcome two common problems encountered when analyzing uncertainty in analytical manufacturing process models; the lack of an inverse and the multiple occurrence of parameters in a relationship. The use of the three operators, image, domain, and sufficient elements obtained accurate results whereas traditional set mappings may lead to results that while mathematically correct are inconsistent with the physical process. The operators are particularly suited to process models where the physical causality of the model can be exploited to obtain improved results. While the process parameter variation may be better modeled by stochastic random variables, the set mathematical analysis refines a worst-case interval analysis but at various levels of plausibility that bound the actual solution. The worst-case interval analysis would require 2^n calculations for *n* parameters and the results are overly conservative. The approach advocated here is accomplished with a reduced computational load compared to both traditional interval analysis and statistical methods (*e.g.* Monte-Carlo simulation) since sets of information are being manipulated instead of single values [31]. Further work is required to classify engineering parameters and physical dependency to better evaluate models that contain imprecision.

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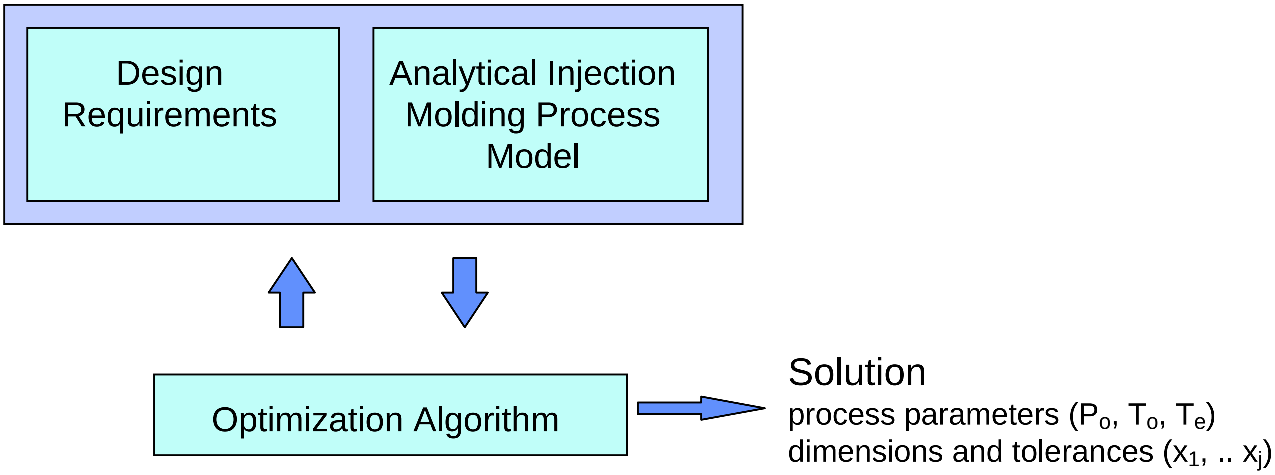

Figure 1. Conceptualization of simultaneous optimization of product and process

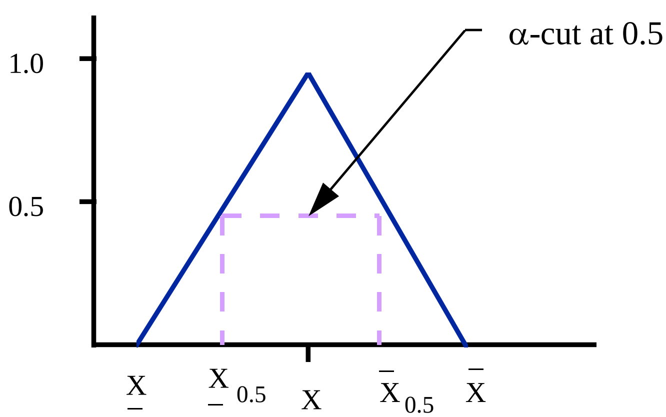

Figure 2. Triangular distribution for an imprecise quantity

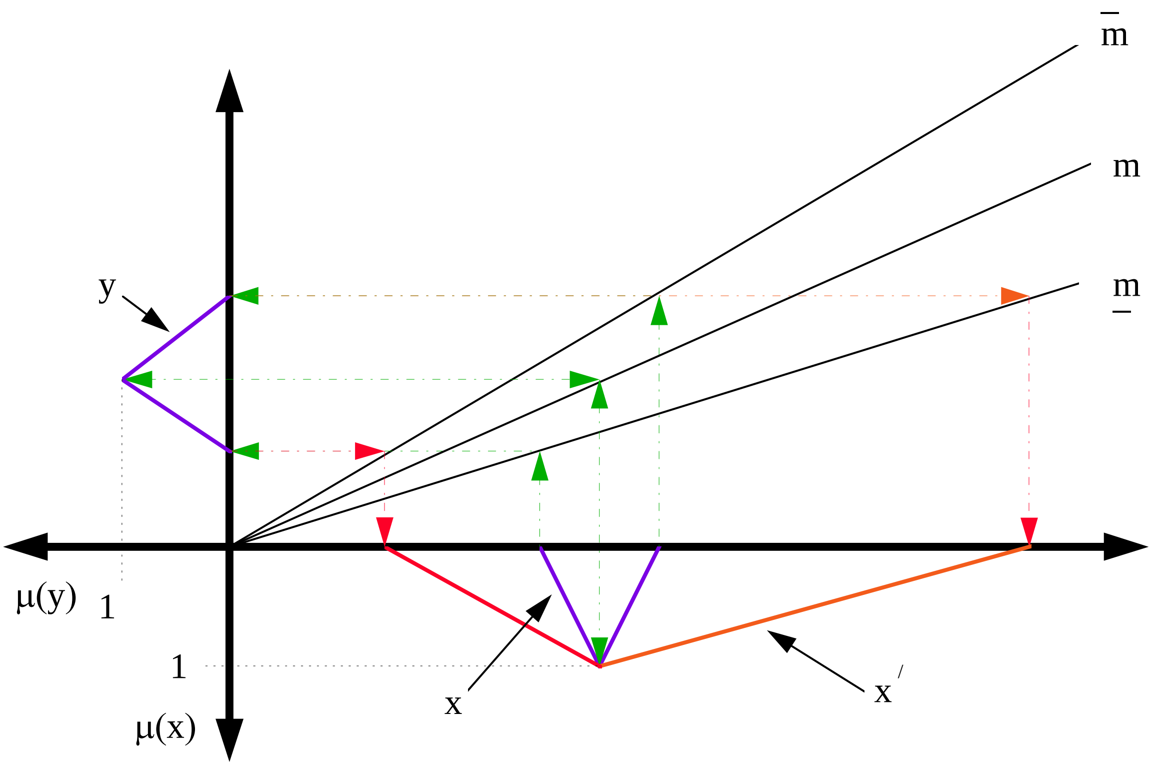

Figure 3. Mapping imprecise quantities

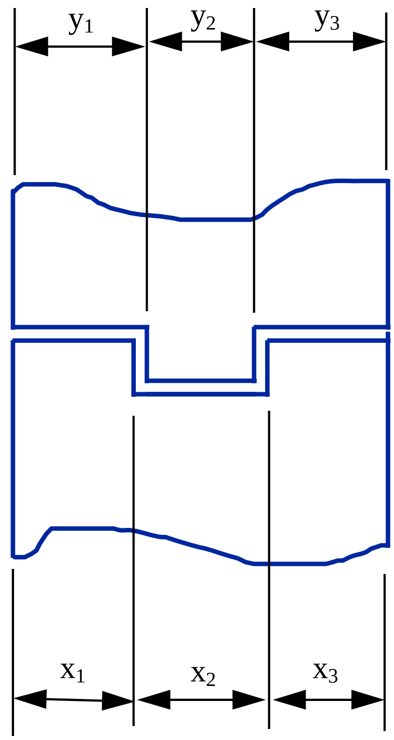

Figure 4. Injection molded hinge assembly

Table 1. Tolerance optimization constraints

constraint	description
$y_1 + y_2 + y_3 = 6$	design function requirement
$x_1 + x_2 + x_3 = 6$	design function requirement
$x_1 \le y_1$	mating requirement
$y_2 \le x_2$	mating requirement
$x_3 \le y_3$	mating requirement
$x_1 = x_3$	symmetry requirement
$y_1 = y_3$	symmetry requirement
$x_2 - y_2 \le 0.07$	fit requirement
$\left(\overline{x}_j - \underline{x}_j\right) \ge x_j S_L$	processing requirement for each dimension *j*
$v = \dfrac{R'(T-\tau)}{(P+\pi)} + w$	volume at given temperature and pressure
$S_v = \dfrac{v_o - v_e}{v_o}$	shrinkage given volumes

Table 2. Combined Part and Process Model Parameters

Parameter	Physical Dependency	Controllability
$x_1, x_2, x_3, y_1, y_2, y_3$	dependent	uncontrollable
$S_v \;\; S_L$	dependent	uncontrollable
T_o, T_e, P_o	independent	controllable

v_o, v_e	dependent	uncontrollable

Table 3. Solution to tolerance optimization problem

parameter	value
x_1	$\langle 2.843, 2.876, 2.909 \rangle$ cm
x_2	$\langle 1.215, 1.248, 1.281 \rangle$ cm
x_3	$\langle 2.843, 2.876, 2.909 \rangle$ cm
y_1	$\langle 2.888, 2.921, 2.954 \rangle$ cm
y_2	$\langle 1.125, 1.158, 1.191 \rangle$ cm
y_3	$\langle 2.888, 2.921, 2.954 \rangle$ cm
T_o	440 ± 10.7 K
T_e	340 ± 9.2 K
P_o	180 ± 8.3 MPa

Author's Biography

Ronald E. Giachetti is an Industrial Engineer Postdoctoral Research Associate in the Manufacturing Systems Integration Division at NIST. This position is funded by the National Research Council. His research interests are in the areas of modeling manufacturing systems, agile manufacturing, design-for-manufacturing, and fuzzy set applications in manufacturing. His current research is focused on modeling manufacturing process capabilities to support design for manufacturing. Dr. Giachetti received a BS in Mechanical Engineering from Rensselaer Polytechnic Institute, a MS in Manufacturing Engineering from Polytechnic University, and a Ph.D. in Industrial Engineering from North Carolina State University. He is a member of SME, IIE, and ASME.