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DESIGN OF A NOVEL FIXTURE FOR PART REFERENCING

Walter W. Nederbragt*

National Institute of Standards and Technology
Building 304, Room 06
Gaithersburg, MD 20879
Email: wwneder@cme.nist.gov

Bahram Ravani

Department of Mechanical and Aeronautical Engineering
University of California, Davis
Davis, California 95616
Email: bravani@ucdavis.edu

ABSTRACT

This paper presents a novel design for a practical tactile-sensing mechanical fixture. The fixture consists of two tactile-sensing surfaces, a cylindrical surface and a planar surface. A mathematical procedure is developed for constructing a reference frame using point-to-surface contact locations on the fixture and the geometry of the fixture. A numerical example is given to further illustrate the use of this approach. Finally, the design of the fixture is described and a picture of the laboratory demonstration unit is shown.

INTRODUCTION

Part referencing is the process of determining the relative location of a part with respect to a tool (such as a machine tool, a robot, or a material handling system) or with respect to a world coordinate system. Part location data is necessary for automated machine tool programming and part processing. In manufacturing, mechanical fixtures have been designed (see, for example, Duffie et. al. (1984) or Slocum (1988)) that would allow repeatable positioning of a pallet with respect to a machine tool at a predetermined location. In robot calibration, the position of the end-effector is usually measured at a set of predetermined locations using some form of a sensing system. This data is then combined with joint encoder readings from the same set of locations to update the kinematic parameters of the robot in its programming system (see, for example, Roth, Mooring and Ravani (1987) or Hollerback (1988)) to improve its positioning accuracy. Since both part referencing and calibration require mea-

surement of relative locations between two objects, mechanical fixtures are usually used to simplify the sensing function and to improve repeatability.

Mechanical fixtures are used (most of the time) in conjunction with touch or tactile sensing. The tactile-sensing mechanical fixtures use combinations of geometric elements to create a reference frame. The two most common mechanical reference fixtures are a three-sphere fixture (see, for example, Duffie et al. (1984)) and a cubical fixture (see, for example, McCallion and Pham (1984)). The three-sphere fixture requires three separate tactile-sensing spheres to operate. The cubical fixture requires three separate tactile-sensing planes to operate. In both cases a specific number of point-to-surface contacts to each of the tactile surfaces of the fixture are necessary to locate the fixture. In this paper, using a result from the theoretical fixture design methods given in Nederbragt and Ravani (1997a), we present a simple to manufacture, tactile-sensing fixture that only requires two tactile-sensing surfaces, one planar surface and one cylindrical surface, to make a complete reference measurement. The mathematics necessary to create a unique reference frame for this fixture are given next.

MATHEMATICAL DETERMINATION OF THE FIXTURE FRAME LOCATION

The mechanical fixture described in this paper uses point contacts to the surfaces of the fixture to locate the fixture and create a reference frame. From Nederbragt and Ravani (1997b), it is known that three point contacts to the planar surface and four point contacts to the cylindrical surface will uniquely locate the

* Address all correspondence to this author.

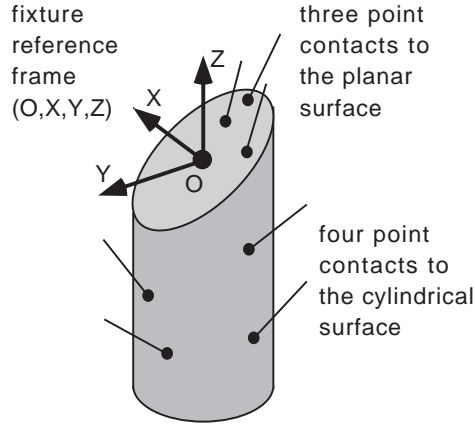


Figure 1. CYLINDER-PLANE FIXTURE

fixture (see Figure 1). Using these points, we need to create a reference frame consisting of three unique mutually perpendicular vectors and a unique origin. Four steps are necessary to create the frame. Step one, the normal to the planar surface needs to be found using the three point locations on this surface. Step two, a vector along the axis of the cylindrical surface needs to be found using the normal from step one and the four points on the cylindrical surface. Step three, using the vector along the axis of the cylindrical surface and the points on the cylindrical surface, a point on the axis needs to be found. Step four, using the results from the previous steps the reference frame needs to be created.

Let $x_1, x_2, x_3,$ and x_4 be four point contact locations on the tactile-sensing cylindrical surface. Let $x_5, x_6,$ and x_7 be three points on the tactile-sensing planar surface. Using points $x_5, x_6,$ and $x_7,$ we need to find the normal to the planar surface. This can easily be done by creating two vectors, $\vec{v}_1 = x_6 - x_5$ and $\vec{v}_2 = x_7 - x_5,$ and taking the cross product between the two, $\vec{n} = \vec{v}_1 \times \vec{v}_2$ (see Figure 2). The equation for the plane can be calculated using \vec{n} and a point on the planar surface (e.g., x_5).

With the normal \vec{n} to the planar surface known, we can now find a vector along the axis of the cylindrical surface. Let $\vec{s} = \langle s_x, s_y, 1 \rangle$ be this vector. The value of s_z is set to one to reduce the necessary computations. This can cause a problem if the axis of the cylinder is parallel with the x - y plane. However, this is very unlikely.

Using the four points on the cylindrical surface, we can create an equation with two unknowns, s_x and $s_y.$ The derivation of the equation can be found in Nederbragt and Ravani (1997c). The equation is:

$$\begin{bmatrix} s_x & s_y & 1 \end{bmatrix} \begin{bmatrix} n_{1x} & n_{2x} & n_{3x} \\ n_{1y} & n_{2y} & n_{3y} \\ n_{1z} & n_{2z} & n_{3z} \end{bmatrix} \begin{bmatrix} \|\vec{p}_1 \times \vec{s}\|^2 \\ \|\vec{p}_2 \times \vec{s}\|^2 \\ \|\vec{p}_3 \times \vec{s}\|^2 \end{bmatrix} = 0. \quad (1)$$

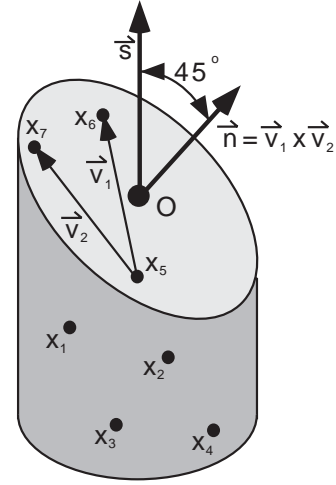


Figure 2. ANALYSIS OF THE COORDINATE FRAME ATTACHED TO THE PLANE-CYLINDER FIXTURE

where $\vec{p}_i = x_i - x_4,$ $\vec{n}_1 = \vec{p}_2 \times \vec{p}_3,$ $\vec{n}_2 = \vec{p}_3 \times \vec{p}_1,$ and $\vec{n}_3 = \vec{p}_1 \times \vec{p}_2.$ This cubic equation described the cylindrical surface in terms of the four points on its surface and the two unknowns. We have one equation and two unknowns, hence, we need another equation to find s_x and $s_y.$

If the inclination of the plane is set to a specific angle (other than zero or ninety degrees) relative to the axis of the cylindrical surface, then, using vector algebra (Kreyszig, 1988), another equation can be found. For simplicity we will let the angle between the axis of the cylindrical surface and the plane be 45 degrees. With the angle between the plane and axis equal to 45 degrees, the angle between \vec{n} and \vec{s} must also be 45 degrees (see Figure 2).

From vector algebra (Kreyszig, 1988), the following equation (commonly referred to as the scalar product) is known:

$$\frac{\vec{k} \cdot \vec{l}}{|\vec{k}| |\vec{l}|} = \cos(\theta) \quad (2)$$

where θ is the angle between \vec{k} and $\vec{l}.$ Using equation 2, \vec{s}, \vec{n} (which has been normalized), and the angle between them, the following equation is obtained,

$$\frac{\vec{n} \cdot \vec{s}}{|\vec{s}|} = \cos(45) = \frac{1}{\sqrt{2}}. \quad (3)$$

Taking the square of equation 3, we obtain

$$2(\vec{n} \cdot \vec{s})^2 = \vec{s} \cdot \vec{s}. \quad (4)$$

Expanding equation 4 using the components of \vec{n} and \vec{s} , we get

$$2(n_x s_x + n_y s_y + n_z)^2 = s_x^2 + s_y^2 + s_z^2. \quad (5)$$

Equations 1 and 5 can be solved to find \vec{s} using a standard mathematics package. Since equation 1 is cubic and equation 5 is quadratic, we will get up to six possible solutions for \vec{s} . These solutions represent different cylindrical surfaces with different radii that satisfy the equations. Since the radius of the fixture is known to the user, it can be used to eliminate the incorrect solutions (Nederbragt and Ravani, 1997c). With \vec{s} known, we can now find a point, $q = (q_x, q_y, q_z)$, on the axis of the cylindrical surface using the method described in Nederbragt and Ravani (1997c). With the position of both surfaces known it is now possible to create a reference frame.

Let $\vec{Z} = \vec{s}$, $\vec{Y} = \vec{n} \times \vec{s}$, and $\vec{X} = \vec{Y} \times \vec{Z}$ be our three unique mutually perpendicular reference frame vectors. Now we need an origin for the frame. The point where the axis of the cylindrical surface intersects the planar surface is unique and easily determinable, hence it would make an ideal origin. We denote this point as o . The axis of the cylindrical surface can be written as

$$(o_x, o_y, o_z) = (q_x, q_y, q_z) + l \langle s_x, s_y, s_z \rangle \quad (6)$$

where $l \in \mathbb{R}$. If equation 6 is substituted into the equation for the planar surface, then a value for l can be found that, when substituted back into equation 6, will determine the location of the point of intersection. In the next section, an example is given to demonstrate these calculations.

AN EXAMPLE

Let $x_5 = (2.8250, 2.4000, 4.0395)$, $x_6 = (3.9020, 5.2000, 4.0353)$, and $x_7 = (2.0480, 5.2000, 4.7853)$ be three points on the fixture's planar surface. Using these points, $\vec{v}_1 = \langle 1.0770, 2.8000, -0.0042 \rangle$, $\vec{v}_2 = \langle -0.7770, 2.8000, 0.7458 \rangle$, and $\vec{n} = \langle 2.1000, -0.8000, 5.1913 \rangle$. The equation for the planar surface created by these points is,

$$2.1000(x - 2.8250) - 0.8000(y - 2.4000) + 5.1913(z - 4.0395) = 0. \quad (7)$$

Let $x_1 = (2.9270, 2.0000, 1.6250)$, $x_2 = (1.7750, 2.8000, 1.4438)$, $x_3 = (1.3730, 2.6000, 3.1166)$, and $x_4 = (3.5270, 3.2000, 3.1082)$ be four points on the cylindrical surface. These points correspond to a cylindrical surface of radius one with an axis (defined using a vector and point) of $\vec{s} = \langle 0.3000, 0.6000, 0.7416 \rangle$ and point $q = (2.0000, 2.0000, 2.0000)$. Since the answer is

known, the results of the example can be verified. Using points x_1 through x_4 and equation 1, we obtain the following cubic equation,

$$\begin{aligned} & -1.7854 - 5.2577s_x + 3.1244s_x^2 - 0.5044s_x^3 \\ & + 0.4890s_y + 16.1190s_x s_y + 0.8550s_x^2 s_y \\ & - 3.7012s_y^2 + 4.3710s_x s_y^2 - 2.0480s_y^3 = 0. \end{aligned} \quad (8)$$

Using the normal from the planar surface \vec{n} and equation 5, we obtain the following quadratic equation,

$$\begin{aligned} & 0.6844 + 1.3627s_x - 0.7244s_x^2 \\ & - 0.5191s_y - 0.21s_x s_y - 0.96s_y^2 = 0. \end{aligned} \quad (9)$$

Using an equation solver, equations 8 and 9 are solved for \vec{s} . Six solutions exist, four of which are real. The four real solutions are: $\vec{s} = \langle -0.3801, 0.1127, 1.0000 \rangle$, $\vec{s} = \langle -0.2223, -0.8946, 1.0000 \rangle$, $\vec{s} = \langle 0.4045, 0.8090, 1.0000 \rangle$, and $\vec{s} = \langle 2.2456, 0.0855, 1.0000 \rangle$.

Using the values of \vec{s} , we can calculate the values for the radius of the surface and a point on the axis of the cylindrical surface using methods from Nederbragt and Ravani (1997c). We know the radius for this particular case is exactly one. Hence, the vector \vec{s} with an associated radius of one is the correct solution. The case with $\vec{s} = \langle 0.4045, 0.8090, 1.0000 \rangle$ has a mathematically determined radius of 1.0000. Therefore, it is the correct cylindrical surface. This matches the expected results (note: \vec{s} needs to be normalized to match the expected result). The calculated point on the axis using the correct value of \vec{s} is (2.0000, 2.0000, 2.0000), which also matches the expected results.

With the location of both surfaces known, we can now create a reference frame. Our three mutually perpendicular reference frame vectors are $\vec{Z} = \vec{s}$, $\vec{Y} = \vec{n} \times \vec{Z}$, and $\vec{X} = \vec{Y} \times \vec{Z}$. We now need to find the origin which is the intersection of the axis and planar surface. We can write the axis of the cylindrical surface as,

$$\begin{aligned} & (2.000, 2.000, 2.000) \\ & + l \langle 0.3000, 0.6000, 0.7416 \rangle = (o_x, o_y, o_z). \end{aligned} \quad (10)$$

Substituting equation 10 into equation 7, we find that $l = 3.0000$. Substituting this result back into equation 10, we find the origin is (2.9000, 3.8000, 4.2249).

DESIGN OF A CYLINDER-PLANE FIXTURE

We now have the mathematical means to find a coordinate frame using the geometric elements that compose the cylinder-plane fixture. However, this does not describe how to design and

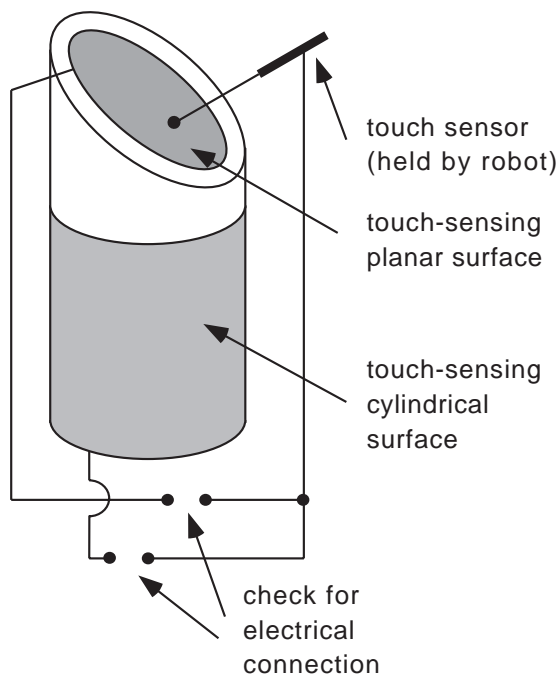


Figure 3. THE TOUCH SENSING CIRCUIT FOR THE CYLINDER-PLANE FIXTURE

build a working fixture. In this section, the design of a fixture in terms of its electrical and mechanical components is discussed.

Our fixture needs several touches to its surfaces, four to the cylindrical surface and three to the planar surface, to have sufficient data to create a reference frame. In general, the point locations will be obtained by a robot holding a touch-sensing device that makes contact with the fixture. At each contact instance, the location of the contact will be stored in the robot's computer for later use. This procedure will continue until all seven points are recorded.

In order to use the mathematical procedure presented in the earlier section, it is necessary to know which of the touch contacts are to the cylindrical surface and which ones are to the planar surface. The idea behind building a reference fixture is that the robot can determine the location of the fixture and, using this information, know the location of everything that the fixture is attached to. Therefore, the robot is not going to know if it is touching the cylindrical surface or the planar surface. Actually it will not know if it is even touching the fixture at all. Therefore, this information needs to be relayed to the computer during a touch contact. This could be accomplished by a person controlling the robot. This person could inform the control system of the robot of what it made contact with during every contact. This would require human assistance, which would limit the usability of the fixture.

Instead, the fixture could be designed to automatically indi-

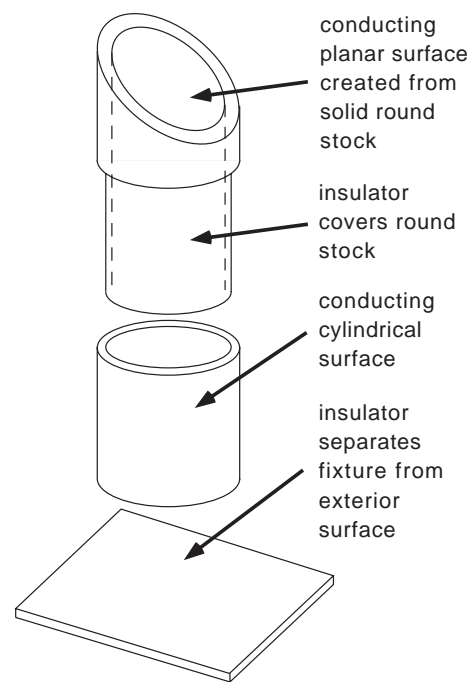


Figure 4. A CYLINDER-PLANE FIXTURE DESIGN

cate that a contact has been made to the cylindrical surface or planar surface. One easy way to do this is to do a conductivity test. If each surface is conductive, each surface is isolated from conductive materials, and the tip of the touch probe held by the robot is conductive, then during a contact to the surface the conductivity between the touch probe and each surface can be checked to see if contact has been made. This idea is illustrated in Figure 3.

For this design to work each surface must be conductive and isolated from other conductive material. Figure 4 illustrates a simple way to accomplish this. It is also important to eliminate any conductive edges that could be touched that would give incorrect results. For example, the conducting planar surface in Figure 4 has its edges covered with an insulator so that the touch probe cannot touch at those points. If the edge was not covered, then the probe could make a conducting contact at a point that is not on the planar surface (an edge point).

Our laboratory demonstration fixture is shown in Figure 5. The unit is capable of sensing contacts to its tactile surfaces. Contact to any other part of the fixture and everything except the fixture are ignored.

CONCLUSION

In this paper we presented a novel design for a practical tactile-sensing fixture. The mathematical procedure needed for determination of a reference frame was given and demonstrated.



Figure 5. CYLINDER-PLANE FIXTURE PROTOTYPE

Finally, the design of the fixture was described and a picture of the demonstration fixture was shown. In the future, the authors plan to investigate the effect of measurement errors on the mathematically determined reference frame.

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