Design of Tactile Fixtures for Robotics and Manufacturing

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April 7, 1997

Abstract

This paper presents a theoretical framework for the design of tactile sensing fixtures for robotics and manufacturing. The framework presented uses group theory to analyze the symmetry of contact conditions on a fixture to evaluate a fixture design for referencing the sensor frame with respect to the fixture frame. Mechanical fixtures consisting of planar, spherical, and cylindrical surfaces are studied for their usefulness as part of referencing fixtures. The theory developed is used in guiding the design of a simple yet novel touch sensing fixture for part referencing and calibration in manufacturing and robotics.

1 Introduction

Part referencing is the process of determining the relative location of a part with respect to a tool (such as a machine tool, a robot, or a material handling system) or with respect to a world coordinate system. Part location data is necessary for automated machine tool programming and part processing. In manufacturing, mechanical fixtures have been designed (see, for example, Duffie et. al. [3] or Slocum [17]) that would allow repeatable positioning of a pallet with respect to a machine tool at a pre-determined location. In robot calibration, the position of the end-effector is usually measured at a set of pre-determined locations using some form of a sensing system. This data is then combined with joint encoder readings from the same set of locations to update the kinematic parameters of the robot in its programming system (see, for example, Roth, Mooring and Ravani [16] or Hollerbach [6]) to improve its positioning accuracy. Since both part referencing and calibration require measurement of relative locations between two objects, mechanical fixtures are usually used to simplify the sensing function and to improve repeatability. There are also approaches that have relied on directly measuring elements of feature surfaces of the parts eliminating the need for mechanical fixtures. These approaches have usually been based on the use of non-contact type sensing systems such as theodolites used in robot calibration (Whitney, Lozinski, and Rourke [20]) or laser interferometry (see for example, Hasegawa, Suehiro, and Ogasawara [4] or Lau, Hocken and Haight [8]).

Mechanical fixtures, however, are used (most of the time) in conjunction with touch or tactile sensing. In this paper, we are only concerned with this type of sensing systems. Much of the existing work related to tactile sensing fixtures have reported one of a kind and ad hoc systems. There has been very little effort on developing a broader method or theory for design of such fixtures or for better understanding of key design parameters. An exception to this is the work of McCallion and Pham [10] in relationship to their studies of robotic assembly. These authors have used kinematic mobility criterion to systematically determine the number of touches necessary for different sensing arrangements using faces of a cubical fixture to determine the location of an end-effector with respect to the fixture. Such cube shaped tactile sensing fixtures have also been used in robot calibration by Mooring and Pack [12]. Other common shapes used for the mechanical fixtures are three spheres (see Duffie et. al. [3] or Slocum [17]).

In this paper, we develop a general theoretical foundation that can aid the design of tactile sensing fixtures using group theory by exploiting the symmetry of different measuring arrangements. The use of group theory is appropriate since the idea behind part referencing is to determine the relative displacement between two parts which forms the well known Euclidean group or one of its sub-groups. The use of the Euclidean group in mechanical system analysis is not new. Hervé [5] was the first to use group theory in terms of Euclidean group and its sub-groups to study mechanisms. Group theory has also been applied in robotics for assembly (see, for example, Popplestone [15], Thomas and Torras [18], Lau and Popplestone [9], and Nnaji [13]). In this paper we use the Euclidean group in Mechanical Design for the purpose of designing mechanical touch sensing fixtures. This seems to be a new and practical development utilizing the well known principles of the Euclidean group.

The organization of the paper is as follows. First, we discuss part referencing based on touch sensing. We then discuss a few relevant aspects of symmetry groups and introduce and prove several propositions that form the basis of our new theory that can aid the design of touch sensing fixture systems. We then consider mechanical fixtures with surfaces consisting of spheres, planes (this will include cubes with planar faces), right cylinders and their combinations and use group theory and the propositions developed to study their use in developing touch sensing fixtures. Finally, learning from our study of different fixture arrangements, we present a simple yet novel touch sensing fixture for part referencing and calibration in manufacturing and robotics. In the appendix, we show the application of Lie algebras as a computational tool for enumeration of subgroups used in our design theory.

2 Part Referencing Based on Touch Sensing

Part referencing using tactile sensing involves bringing a sensing element and a surface of the part into contact with one another, activating the touch sensor, and measuring the location of the touch point in the sensor coordinate system. This was the case, for example, in the system described by Duffie et al. [3] where a touch sensor was attached to the end of a robot and it was moved until it contacted the spherical surface of a fixture. In such a system the location of the touch is only known in the robot manipulator frame. The shape of the touch surface on the fixture is, however, completely known. (In this case it is a sphere.)

If several touches are made to the surface, then enough information may be obtained to determine the relative location of the two frames. Duffie et al. [3] used a fixture consisting of three separate spheres of known radii. They found that four separate touches to each of the spheres made it possible to determine the location of the fixture with respect to the robot. McCallion and Pham [10] used three non-collinear touches to a plane to determine its location in space, and, using three perpendicular planes of a cube, the relative location of the robot to the fixture was found.

In a tactile fixture-sensor system, the number of necessary touches can be reduced if bilateral position sensing is used. This means that there are sensing elements on both parts that are referenced with respect to one another. In the case of a touch sensing finger on a robot touching faces of a cube, only three non-collinear touches will be sufficient if the planar faces are equipped with touch sensitive pads measuring the location of the touch also in the coordinate system of the fixture.

In design of a tactile sensing fixture, one has to combine appropriate feature surfaces such as planes or spheres with proper number and arrangement of touches to determine the relative location of the two frames. The Euclidean group provides a mathematical basis to do this. Using Euclidean groups, for example, it can be shown that a fixture with only one spherical surface is not sufficient for part referencing.

3 Classification of the Continuous Subgroups of $SE(3)$

Before introducing the propositions necessary for analysis of fixtures, the continuous subgroups of the Euclidean group need to be discussed. The subgroups of the Euclidean group are useful for analysis because their characteristics are well known, and they can represent any real solid in Euclidean space. Table 1 lists all of the continuous subgroup classes for the special Euclidean group. The table also gives the dimension, the notation, and the corresponding lower pair kinematic joint (assuming one exists) for each subgroup class.

Table 1 is a complete listing of all of the continuous subgroups of $SE(3)$. This can be shown using the Lie algebra associated with the Euclidean group. The first step in finding the subgroups is to split the Lie algebra for $SE(3)$ using the

Classes of Subgroups of the Displacement Group			
Constraint Description	Notation	d.o.f.	associated lower pair
identity element	$\{I\}$	0	none
rectilinear translation	$\{T_u\}$	1	prismatic
rotation about an axis	$\{R_u\}$	1	revolute
helicoidal motion	$\{H_{u,p}\}$	1	screw
planar translation	$\{T_P\}$	2	none
cylindrical motion	$\{C_u\}$	2	cylindrical
spatial translation	$\{T\}$	3	none
planar motion	$\{G_P\}$	3	plane
spherical motion	$\{S_o\}$	3	spherical
Y movement	$\{Y_{u,p}\}$	3	none
X movement	$\{X_u\}$	4	none
general motion	$\{D\}$	6	none

Table 1: Subgroups of the Euclidean group – displacement group.

Translations T in $SE(3)$ to form an ideal. This is proven in Proposition 1, and makes it possible to split the algebra into two groups, as explained next.

Proposition 1 *Translations T form an ideal in $se(3)$.*
proof: *Let X_1 and Y_2 represent two displacements where $X_1 = (x_1; y_1) \in se(3)$ and $X_2 = (0; y_2) \in T$ (this is a screw representation for displacements [2] [11]). $[X_1, X_2] = X_1 \times X_2 = (x_1; y_1) \times (0; y_2) = (0; x_1 \times y_2) \in T$. Therefore, T is an ideal from the definition of ideal.*

If given a Lie algebra V and an ideal H of V, then V/H is also a Lie algebra [19]. Since $se(3)$ is a Lie algebra and T is an ideal, then $se(3)/T$ must also be a Lie algebra. From [7] it is known that $se(3)/T = so(3)$. Figure 1 shows the mapping $\pi : se(3) \mapsto se(3)/T = so(3)$. The Lie algebra $so(3)$ is of dimension three, therefore it could have subalgebras of dimension three, two, one, or zero(trivial).

Proposition 2 *The Lie algebra $so(3)$ doesn't have a subalgebra of dimension two.*
proof: *Let V be a subalgebra of $so(3)$ of dimension 2. Then there exists vectors $u, v \in V$, where u, v are independent. But $u \times v \in V$, and $u, v, u \times v$ are independent, hence V must be of dimension three. Therefore, we have a contradiction, and V cannot be a subalgebra of dimension two.*

With the subalgebra $so(3)$ split into its subalgebras, the complete list of corresponding continuous subgroups can be found. The details are discussed in Appendix A. With the subgroups known, we now prove several propositions for design analysis of fixtures.

4 Theoretical Basis For Design

In this section, we develop a formal theory for design and evaluation of touch sensitive fixtures based on the symmetries associated with the primitive surfaces of a fixture. We only

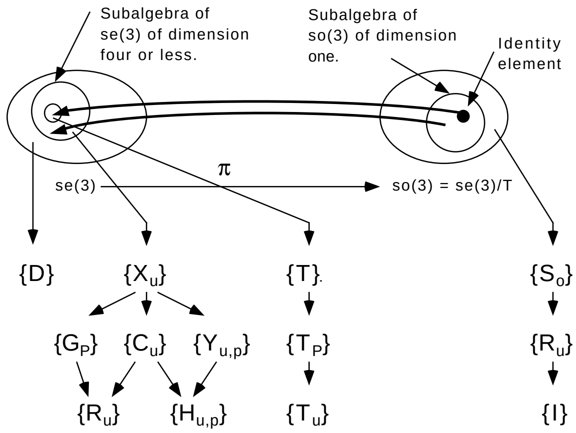

Figure 1: The mapping : $se(3) \rightarrow so(3) = se(3)/T$.

consider touch sensing involving point contacts between the touching element and the fixture surface. We introduce and prove three propositions that would provide the basis for the design procedure developed in the next section. We start with an introduction to a few relevant aspects of symmetry groups and proceed with a definition of a primitive surface.

All solids and surfaces have a group describing their symmetry. In the case where the object has no symmetry at all, the object's symmetry group is only the identity element $\{I\}$. Objects can have a symmetry group of finite order or infinite order. If an object has a symmetry group of finite order then the object can only be rotated into a finite number of positions without changing its location in space. For more details on group theory and symmetries see Yale [21].

Definition 1 *A* **primitive surface** *of a solid is defined as an algebraic surface that locally coincides with a bounded face of the solid. The primitive features of a cube, for example, are the six infinite planes that bound the solid volume.*

The reason for treating a surface as a primitive surface is understandable when you consider that a set of touches is being made to the surface in order to find its location in space. It would be very difficult for a robot to touch the edge of a surface using a touch sensing probe because the edge has no thickness. Therefore by treating the surface as infinite, the edges do not become involved.

Proposition 3 *Let S be a set of primitive surfaces and let G be the symmetry group for the set S. If G contains all of possible rotation elements about an axis, L-L', then the set S cannot be used to uniquely determine the relative location of the frame associated with S to the frame of the touch sensor in three dimensional Euclidean space.*

Proof*: Assume that S can uniquely determine the relative location of the frame associated with S to the robot's frame in three dimensional Euclidean space. Rotate S about the axis L-L' more than zero degrees but less one complete revolution. Since all rotations about L-L' are in the group G, then the set S after the rotation will "look" the same as it did prior to the rotation. However, the frame associated with the set S will no longer be the same frame as it was prior to the rotation. Therefore, the set S cannot be used to uniquely determine the relative location of the frame associated with S to the frame of the sensor in three dimensional Euclidean space.*

Proposition 4 *Let S be a set of primitive surfaces and let G be the symmetry group of the set S. If G contains any translations then the set S cannot be used to uniquely determine the relative location of the frame associated with S to the frame of the sensor in three dimensional Euclidean space.*

Proof*: Assume that S can uniquely determine the relative location of the frame associated with S to the frame of the sensor in three dimensional Euclidean space. Translate S using any element translation element in G. Since the translation element is in the group G, then the set S after the translation will "look" the same as it did prior to the translation However, the frame associated with the set S will no longer be the same frame as it was prior to the translation. Therefore, the set S cannot be used to uniquely determine the relative location of the frame associated with S to the frame of the sensor in three dimensional Euclidean space.*

Proposition 5 *Let S_1 and S_2 be two sets of primitive surfaces, and let G_1 and G_2 be the symmetry groups for S_1 and S_2. Let S_3 represent the combination of S_1 and S_2, and let G_3 represent the symmetry group associated with S_3. All finite symmetries in G_3 were also finite symmetries in either G_1 or G_2. No new finite symmetries can be created from the combination of surfaces.*

Proof*: It has known that the combination of two symmetry groups results in a group that is either equal in size to the intersection of the two original groups or smaller. Therefore, the new group has no new elements in it that were not in the original groups. Therefore, the only way to get new finite symmetries is by the intersection of continuous groups. The only way to get an intersection of two continuous groups that is not the identity element is by having the continuous groups be the same, resulting in another continuous group. Therefore, no new finite symmetries are created.*

Propositions 3, 4,and 5 are powerful tools for the analysis of any geometric referencing fixture. In most cases, a referencing fixture's primitive surfaces can be represented using the simple group notation introduced in the previous section. If it is possible to represent the primitive surfaces using the group notation then the complete fixture can be analyzed by taking the intersections of the group representations of the primitive surfaces. Let $G_1, \cdots, G_n$ represent the group notation for n primitive surfaces that form a referencing fixture. The fixture is a "useful" fixture if:

$$G_1 \cap G_2 \cap G_3 \cap \cdots \cap G_n = \{I\} \tag{1}$$

where "useful" means that it can uniquely determine the relative position between the reference frame and the robot end effector.

Equation 1 is a very powerful tool, however, the mathematical intersection of two or more groups usually requires some geometric insight that equation 1 cannot provide. In addition, equation 1 will not always give perfect results for an actual fixture when it comes to finite symmetries. This

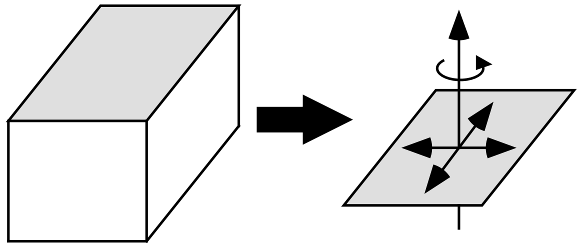

Figure 2: A cube with only one touchable surface is treated as an infinite plane.

is due to the fact that the actual fixture may not have finite symmetries that the primitive surface model does have. This may cause a "useful" fixture not to pass equation 1 because of the remainder of finite symmetries after the intersection of all group representations. It is, in general, a good idea to use both the propositions and equation 1 when analyzing a fixture design to be sure that the fixture will work. The use of both methods is discussed in the next section.

5 Tactile Sensing Fixtures with Primitive Features Consisting of Planes, Spheres, and Right Cylinders.

Using the above three propositions, fixtures consisting of planes, right cylinders, spheres, and combinations of these elements are analyzed. In order to do this, each fixture must be treated as a primitive surface or group of primitive surfaces. For example, given a cube where only one side can be touched, that side is treated as if it where an infinite plane and the other sides are ignored (Figure 2).

Once a fixture is broken down into a primitive surface or group of primitive surfaces, then it should not contain any continuous rotation or translation groups. Propositions 3 and 4 are used to check for these continuous groups. If the fixture does have continuous groups then it cannot be used to uniquely determine the relative location of the fixture to the tool. Finally, finite symmetries must be checked for their effect on the design of the fixture.

Using Proposition 5, it is known that all finite symmetries in the final fixture originate from each individual surface. Therefore, a possible and useful way to check for finite symmetries is to look at the finite symmetries of each surfaces of a fixture on a one-by-one basis. Among a sphere, cylinder, and plane, only a sphere has no finite symmetries which makes it easy to work with (Figure 3). The cylinder, when treated as a primitive surface, has an infinite number of finite symmetries. Every axis perpendicular to the center line and intersecting the center line of the cylinder has a finite symmetry about it (Figure 3). The plane, when treated as a primitive surface, also has an infinite number of finite symmetries. Any axis through the plane has a finite rotational symmetry about it (Figure 3). Table 2 shows the continuous and finite group notation for the sphere, plane, and right cylinder.

After breaking the fixture down into individual primitive surfaces and knowing the finite symmetries of these primitive surfaces, it is time to see if the finite symmetries are still

Group Notation for Primitive Surfaces	
Primitive Surface	Notation
Sphere	$\{S_o\}$ where o is the center of the sphere.
Right Cylinder	$\{C_u\}\{rot(v, n\pi)\}$ where u is the axis of the cylinder, $n \in N$, and $v \perp u$.
Plane	$\{G_P\}\{rot(v, n\pi)\}$ where v is in the plane P and $n \in N$.

Table 2: Group notation for a sphere, plane, and right cylinder.

present after the addition of the other primitive surfaces to the fixture. Each primitive surface should be judged relative to the other surfaces to see if the finite symmetries go away. If there are no finite symmetries left, then the fixture is "useful." If the fixture still has some finite symmetries left, then the real shape of the fixture(not the combination of the primitive surfaces) may or may not eliminate the finite symmetry. For example, a fixture may have a planar surface that can only be reached on one side, this eliminates the finite rotation of the primitive planar surface associated with the real planar surface (Note: the group representation would simply be $\{G_P\}$ for this case). If there are finite symmetries left after completely analyzing the fixture then the fixture will not uniquely determine the location of the fixture to the sensor – it will not be "useful." This is similar to getting the result $\{I\}$ using equation 1. Several examples are given to better illustrate this step.

Of the three surfaces being used for the example, the sphere, cylinder, and plane, none can be used by themselves to uniquely determine the relative position of the fixture frame to the frame of the sensor in $SE(3)$. Therefore, a combination of these surfaces must be used to make a proper fixture. However, it is useful to show the problems with each of these surfaces when used alone.

A sphere (Figure 3) has no finite rotational symmetries and no continuous translational symmetries, however, it does have an infinite number of continuous rotational symmetries. Any axis through the center of the sphere can be used to create a continuous rotational symmetry. Obviously, one sphere can not be used for a complete fixture. This can all be seen in the group notation for the sphere $\{S_o\}$.

A cylinder (Figure 3) has finite rotational symmetries, a continuous rotational symmetry, and a continuous translational symmetry. The continuous translational symmetry comes from the fact that the cylinder, treated as a primitive surface, can be translated in the direction of the center line of the cylinder and the cylinder will look the same. The continuous rotational symmetry comes from the fact that any rotation about the center line of the cylinder returns the cylinder to itself. The finite rotational symmetries come from flipping the cylinder on an axis perpendicular to the center line of the cylinder. Because the cylinder is treated as a primitive surface, the cylinder when rotated 180 degrees returns to itself. This can all be seen in the group notation for the cylinder $\{C_u\}\{rot(v, n\pi)\}$ where u is the axis of the cylinder, $n \in N$,

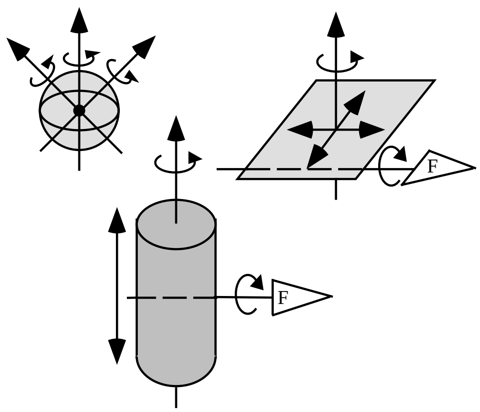

Figure 3: A sphere fixture, a plane fixture, and a cylinder fixture

and $v \perp u$.

A plane (Figure 3) has finite rotational symmetries, continuous rotational symmetries, and continuous translational symmetries. The translational symmetries are due to the fact that any movement of the plane in a direction contained in the plane, returns the plane to itself. The finite rotational symmetries, like the cylinder, are 180 degree flipping symmetries. The continuous rotational symmetries come from any rotation about an axis perpendicular to the plane. When the plane is rotated by any of these axes, the plane returns to itself. This can all be seen in the group notation for the plane $\{G_P\}\{rot(v, n\pi)\}$ where v is in the plane P and $n \in N$.

Now that the basic surfaces have been covered, combinations of these surfaces should be judged for the usefulness in a fixture design. The simplest combinations to start with are combinations of spheres because spheres do not have finite rotational symmetries. A fixture containing two spheres, Figure 4a, still will not be a complete fixture because a continuous rotational symmetry exists. The axis for this symmetry is through the center of both spheres. If three spheres are used, Figure 4, a complete fixture will exist as long as the centers of the three spheres are non-collinear. If the spheres are collinear, a continuous rotational symmetry through the center of the three spheres exists. From equation 1 the combination of two spheres results in

$$\{S_o\} \cap \{S_{o'}\} = \{R_u\} \tag{2}$$

where u is the axis through centers o and o'. The combination of three spheres results in

$$\{S_o\} \cap \{S_{o'}\} \cap \{S_{o''}\} = \{I\}. \tag{3}$$

If the centers of the spheres are collinear then the result will again be $\{R_u\}$ where u is now the axis through all three sphere centers.

Fixtures containing just planes are a little more difficult to judge than spheres because they may contain finite rotation groups. A fixture containing just two planes will not be a "useful" fixture because there will be continuous groups for any configuration of two planes in addition to finite symmetries (Figure 5). If the two planes intersect then a continuous

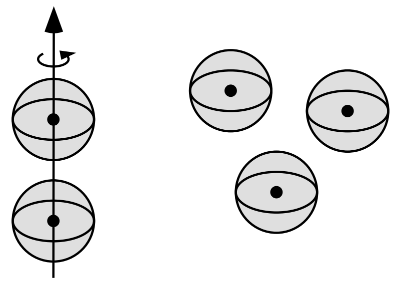

Figure 4: A two sphere fixture and a three sphere fixture

translational group exists in the direction of the line formed by the intersection of the two planes. If the planes are parallel then there are rotational and translational continuous groups in the same directions as the one plane case. If the two planes are perpendicular (see Figure 5) notice that there are two finite symmetries that are created in addition to the finite symmetry along the line of intersection. These two finite symmetries can be eliminated by making the two planes intersect at an angle other than 90 degrees. From equation 1 the combination of two planes results in

$$\{G_P\}\{rot(v, n\pi)\} \cap \{G_{P'}\}\{rot(w, n\pi)\} = \{T_u\}\{rot(u, n\pi)\} \tag{4}$$

where u is the line created by the intersection of the two planes. Equation 4 is valid when the angle between the planes is not zero nor 90 degrees. If the angle of intersection is 90 degrees then the result contains two more finite symmetries. If the planes are parallel then the result is general planar motion without the finite symmetries. If the two planes are coincident then nothing is obtained from the combination of the two planes.

If three planes are used and none of the planes or lines formed by the intersection of the planes are parallel to each other then no continuous groups exist for that fixture (Figure 5). However, finite groups can exist. They will exist when any of the planes are perpendicular to any of the other planes. When all three planes are mutually perpendicular, the fixture has many finite symmetries. These finite symmetries can be eliminated by the design of the real fixture. If the fixture's sides do not extend past the edges of the other sides then there will be no finite symmetries. This is the cubical fixture design used by McCallion and Pham [10]. From equation 1 the combination of three planes results in

$$\begin{aligned}\{G_P\}\{rot(v, n\pi)\} \cap \{G_{P'}\}\{rot(w, n\pi)\} \\ \cap \{G_{P''}\}\{rot(x, n\pi)\} = \{I\}.\end{aligned} \tag{5}$$

Equation 5 is valid when none of the planes are perpendicular nor parallel to each other.

Spheres and planes can be used together to form fixtures. If one plane is used with one sphere to from a fixture, that fixture will have a continuous rotational group about the axis through the center of the sphere and perpendicular to the plane (Figure 6). If the sphere has its center located in the plane a finite symmetry will exist in addition to the continuous rotation symmetry. From equation 1 the combination of

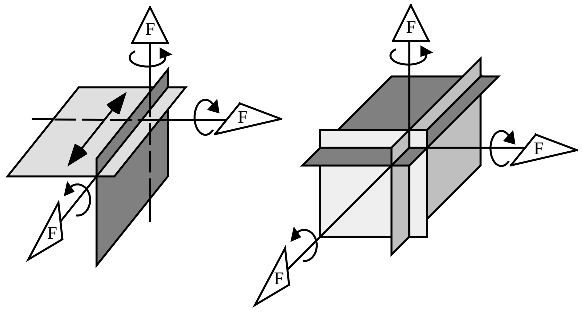

Figure 5: A two plane fixture and a three plane fixture

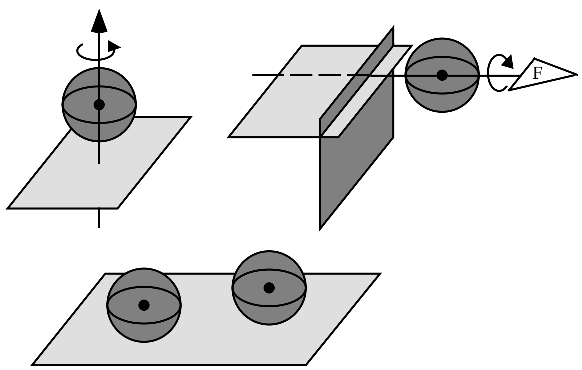

Figure 6: A one plane - one sphere fixture, a two plane - one sphere fixture, and a one plane - two sphere fixture

a sphere and plane where the center of the sphere is located outside of the plane results in

$$\{G_P\}\{rot(v, n\pi)\} \cap \{S_o\} = \{R_u\} \tag{6}$$

where u goes through o and is perpendicular to P. It is obvious that a one plane, one sphere fixture doesn't satisfy the requirements.

If two planes are used with one sphere, the fixture will not have any continuous groups as long as the two planes are not parallel (Figure 6). If the center of the sphere is located outside of these two planes then the fixture will not have any finite symmetries also. If the sphere is located with its center on one or both planes then finite symmetries may exist. If two spheres are used with one plane to form a fixture, the fixture will not have any symmetry problems as long as at least one sphere is located outside of the plane and the axis through the center of the two spheres is not perpendicular to the plane (Figure 6).

Two cylinders can also be used to form a fixture. If the cylinders are parallel then a continuous translational group will exist in the direction of the center line of both cylinders. However, placing the cylinders at an angle to each other will solve this problem (Figure 7). If this is done, finite symmetries may still exist. The relative placement of the two cylinders and the finite length of the cylinders can be used to eliminate this problem in the design of a fixture consisting of two cylinders. From equation 1 a two cylinder fixture where the center lines of the cylinders are not parallel result in

$$\{C_u\}\{rot(v, n\pi)\} \cap \{C_{u'}\}\{rot(v, n\pi)\} = \{rot(v, n\pi)\} \tag{7}$$

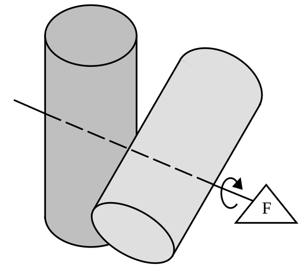

Figure 7: A two cylinder fixture

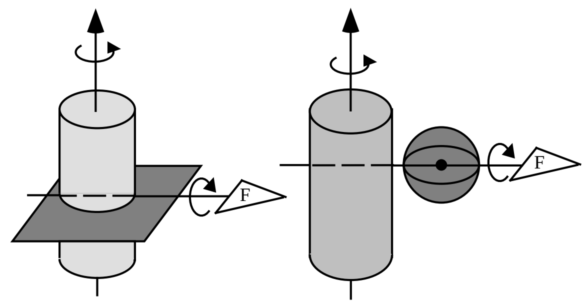

Figure 8: A one cylinder - one sphere fixture and a one cylinder - one plane fixture

where $v \perp u$ and $v \perp u'$. As stated earlier, the finite symmetry created in the intersection of the two groups can be eliminated by properly designing the actual fixture.

Fixtures can also be made using cylinders and other objects. For example, a sphere-cylinder fixture can be made that will not have any continuous groups as long as the sphere's center is not located on the center line of the cylinder (Figure 8). This fixture will, however, always have a finite group associated with it when using the primitive feature representations for the cylinder and the sphere. The finite symmetry is due to the rotation about an axis through the center of the sphere and perpendicular to the center line of the cylinder. From equation 1,

$$\{S_o\} \cap \{C_u\}\{rot(v, n\pi)\} = \{rot(w, n\pi)\} \tag{8}$$

where $w \perp u$ and w goes through o. The actual fixture can be designed to eliminate this finite symmetry by placing the sphere outside of the actual range of the cylinder's center line.

A cylinder can also be combined with a plane to form a useful fixture. If the plane is not parallel nor perpendicular to the center line of the cylinder then there will be no continuous symmetries. Again, there will be a finite symmetry problem, however, the actual fixture will not have this finite symmetry because of mechanical constraints in the design of such a fixture. If the plane is perpendicular to the center line of the cylinder, then there is a continuous rotation symmetry about the center line of the cylinder (Figure 8). However, this rotation symmetry can be eliminated by the addition of another primitive surface.

6 Design of a simple Touch Sensing Fixture for Part Referencing and Calibration.

As discussed in section two, bilateral tactile sensing can reduce the number of touch points necessary to determine the relative location of the sensor frame to the frame of the fixture. In the case of a touch sensing finger on a robot that touches faces of a cube, only three non-collinear touches will be needed if the planar faces are equipped with touch sensitive pads that measure the location of the touch in the coordinate system of the fixture. There are several different ways of determining the location of a touch on a planar surface. Bicchi, Salisbury, and Brock [1] used a force-moment sensor in the base of an object to determine the location of a touch to the surface of that object. Moreover, touch sensitive computer screens are currently being used to give the location of a touch to the surface of a screen, Ormond [14].

Touch sensitive screens and force-moment sensors do have a problem, they can only handle one touch to their surface at a time to properly work. Digitizers, however, sense a energized coil's magnetic field to determine the location of the "touch." The coil is usually located at the end of a pen or puck. Hence, three coils could simultaneously touch the digitizer and then be activated in sequence until the locations of all three coils are known.

Using the idea of a three coil/digitizer combination, we have designed a touch sensing tripod/digitizer fixture. This fixture incorporates a three finger touch sensor where each finger is composed of a digital indicator with a coil at its tip. When this sensor, or tripod, comes in contact with the digitizer each digital indicator moves in until all of three digital indicator tips come into contact with the digitizer. Once in contact, each coil is energized and the location of each indicator tip is found in the frame of the digitizer. The location of the three tips is also known in the frame of the touch sensing tripod because the displacement of each digital indicator is known. Therefore, the location of the three points is known in both frames and the relative location of the tripod to the digitizer can be found. This touch sensing tripod/digitizer fixture is built in our laboratory and is presently being tested. Figure 13 shows the unit being test on a milling machine. This fixture, in addition to its simplicity, has the advantage of being able to measure the location of the object with one touching motion.

Since the tripod/digitizer fixture will be used for calibration and referencing, it is critical for the components used in the design to be as accurate as possible. The accuracy of the tripod/digitizer fixture is limited by the accuracy of the digitizer and the digital indicators used. The design developed here and shown in Figure 9 uses relatively inexpensive components. The digital indicators being used have a stroke of one inch (25.4 mm) with an accuracy of 0.001 inches (0.0254 mm) over that range. The digitizer used for the prototype fixture is relatively old and was found to be the limiting part for the accuracy of the prototype system developed.

We performed tests that indicated the system has an accuracy of 0.030 inches (0.762 mm) over the 11.7 inch by 11.7 inch surface (300 mm x 300 mm). It should be pointed out, however, that the accuracy of the system can be easily improved by using a more accurate digitizer. Digitizers are available with accuracies of plus or minus 0.005 inches (0.127 mm) and sizes up to 44 inches by 60 inches (1100 mm x 1500 mm).

Figure 9: Touch sensing tripod/digitizer fixture

7 Conclusion

In this paper we presented a theoretical foundation for design of tactile sensing fixtures using theory of continuous groups. We used this theory to aid us in design of a very simple but novel fixture design with several advantages over other reported tactile fixture systems.

8 Acknowledgment

The authors would like to thank Professor Karger of Charles University of Progue for his discussion on the use of Lie algebras for group classification.

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A Finding the Continuous Subgroup Classes of $SE(3)$

With the subalgebras of $so(3)$ known, we can now break down each possible subalgebra in $se(3)$ using combinations of translations and the $so(3)$ subalgebras. We first begin by examining the subalgebras of $se(3)$ that contain a subalgebra of $so(3)$ of dimension one.

If we let V_1 be a subalgebra of $se(3)$, and we let $\pi(V_1)$ be one dimensional, then all screws of V_1 are of the form $(\lambda x, y)$, where x is fixed. Let $X_1, X_2, \cdots, X_n$ be a basis of V_1. For simplicity, let $X_1 = (x_1; y_1)$, $X_2 = (0; y_2)$, $X_3 = (0; y_3)$, and $X_4 = (0; y_4)$. The dimension of V_1 can, at most, be four because $\pi(V_1)$ is of dimension one out of a possibility of three.

It is now known that V_1 is of dimension four or less. The next step is to look at all the possible dimensions for V_1. We will begin with dimension one and proceed to dimension four.

If the dimension of V_1 is one then the basis must be of dimension one. We know $X_1 = (x_1; y_1)$, $X_2 = (0; y_2)$, $X_3 = (0; y_3)$, and $X_4 = (0; y_4)$, however, three of these basis vectors must be dependent on the remaining one for V_1 to be of dimension one. Moreover, we know that X_1 cannot be zero because $\pi(V_1) = 1$, therefore X_2. X_3, and X_4 must be dependent on X_1. If this basis is going to be dimension one, then $X_1 \times aX_1$, where a is a constant, must be dependent on X_1. This will only be true if y_1 is dependent on x_1. Hence, the basis must be of the form $X_1 = (x_1; \lambda x_1)$. For this case, if λ is not zero then we have a helicoidal motion along a vector, or if λ is equal to zero then we have a revolute motion. These two cases correspond to two of the lower mechanical joints.

If the dimension of V_1 is two then the basis must be of dimension two. Therefore, $X_1, \cdots, X_4$ cannot be all independent. For simplicity, let X_3 and X_4 be zero. If X_1 and X_2 are our basis vectors then the cross product between them must be zero for the basis to be of dimension two. Therefore,

$$X_1 \times X_2 = (x_1; y_1) \times (0; y_2) = (0; x_1 \times y_2) = 0 \tag{9}$$

$$\Rightarrow x_1 \times y_2 = 0 \rightarrow y_2 = \lambda x_1. \tag{10}$$

Also, y_1 must be either zero or dependent on y_2 for the basis to be of dimension two because of the same reason as explained in the dimension one case. Hence, the canonical basis is: $X_1 = (x_1; 0)$ and $X_2 = (0; x_1)$. This case is very similar to the case of dimension one except that the rotation in the x_1 direction and the translation in the x_1 direction are independent. This type of motion is called cylindrical motion which is also a lower mechanical joint.

If the dimension of V_1 is three then the basis must also be of dimension three. Therefore, one of the basis vectors $X_1, \cdots, X_4$ is dependent on the other three. Moreover, the cross products between the basis vectors must also be dependent. Let X_4 be the dependent basis vector. Now we know that X_4, $X_1 \times X_2 = (0; x_1 \times y_2)$, and $X_1 \times X_3 = (0; x_1 \times y_3)$ must be dependent on X_1, X_2, and X_3. This will be the case if $X_3 = a(X_1 \times X_2)$ where a is a constant and $X_2 = c(X_1 \times X_3)$ where c is a constant (see Figure 10). We can write this basis for this case as $X_1 = (x_1; \lambda x_1)$, $X_2 = (0; y_1)$, and $X_3 = (0; y_2)$. This represents two subgroup classes. If λ is zero then we have planar motion, a lower mechanical pair. If λ is not

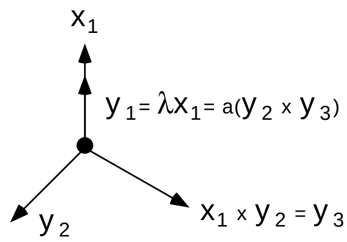

Figure 10: The basis of V_1 if $dim(V_1) = 3$ and $dim(\pi(V_1)) = 1$.

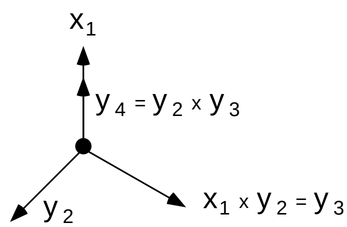

Figure 11: The basis of V_1 if $dim(V_1) = 4$ and $dim(\pi(V_1)) = 1$.

zero then we have planar translation with a helicoidal motion perpendicular to the planar translation, we will call this a "Y-movement."

If the dimension of V_1 is four then the basis must also be of dimension four. Hence the basis vectors $X_1, \cdots, X_4$ must be all independent, however, the cross product between them should be dependent. This result leads to another subgroup classification that we will call "X-movement." This subgroup class corresponds to general translation and one axis rotation (see Figure 11). Note that the vectors X_1 and X_2 do not need to line up for this class of group, but the basis is easier to visualize if they are.

This takes care of the subgroup class associated with $\pi(V_1)$ being of dimension one. Now let $\pi(V_1)$ be of dimension three. In other words let $\pi(V_1) = so(3)$. Then the basis can be written in the canonical form $X_1 = (x_1; 0)$, $X_2 = (x_2; 0)$, $X_3 = (x_3; 0)$, $X_4 = (0; y_1)$, $X_5 = (0; y_2)$, and $X_6 = (0; y_3)$.

Proposition 6 *There is only one case of dimension four or greater for V_1 given the basis $X_1 = (x_1; 0)$, $X_2 = (x_2; 0)$, $X_3 = (x_3; 0)$, $X_4 = (0; y_1)$, $X_5 = (0; y_2)$, and $X_6 = (0; y_3)$ and $\pi(V_1) = so(3)$. It is the Euclidean group of dimension six.*

proof: *Let the dimension of V_1 be four, then the basis can be written as $X_1 = (x_1; 0)$, $X_2 = (x_2; 0)$, $X_3 = (x_3; 0)$, $X_4 = (0; y_1)$. The first three basis vectors make $se(3)$ and the last basis vector is for the fourth dimension and corresponds to a translation. For V_1 to be of dimension four the cross product between the basis vectors must be dependent. $X_1 \times X_4 = (0; x_1 \times y_1)$, and $X_2 \times X_4 = (0; x_2 \times y_1)$. The result of the two cross products must be independent of the basis vectors and each other, therefore, V_1 is of dimension six, which is $se(3)$. The same result occurs if a basis of five independent basis vectors is used. Hence, the proposition is true.*

From Proposition 6 we know that there is only one case of dimension four or greater for V_1 given the basis $X_1, \cdots, X_6$ and $\pi(V_1) = so(3)$; it is the Euclidean group of dimension six. If the dimension of three is considered then the canonical basis would be $X_1 = (x_1; 0)$, $X_2 = (x_2; 0)$, and $X_3 = (x_3; 0)$. This is the basis for the subalgebra $so(3)$, therefore V_1 for $\pi(V_1) = 0$ where V_1 is of dimension three corresponds to the subgroup class $SO(3)$. This is the class of spherical rotations, a lower mechanical joint.

If $\pi(V_1)$ is of dimension zero, then we have a eliminated rotations from the Euclidean group. This only leaves translations. The basis of V_1 in canonical form for this case is $X_1 = (0; y_1)$, $X_2 = (0; y_2)$, and $X_3 = (0; y_3)$. This basis corresponds to general translation if the three basis vectors are independent. If only two of the basis vectors are independent, then V_1 is of dimension two, and we have planar translation. We do not have to worry about the cross product between X_1 and X_2 being independent for this case because the cross product between two translations is zero. If only one basis vector is independent then V_1 is of dimension one, and we have rectilinear translation. This case corresponds to rectilinear motion, a lower mechanical joint – prismatic joint.

This only leaves the trivial case of dimension zero which corresponds to no motion at all.