EVALUATION OF ONE- AND TWO-SIDED GEOMETRIC FITTING ALGORITHMS IN INDUSTRIAL SOFTWARE

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Recent work in testing and comparing maximum-inscribed, minimum-circumscribed, and minimum-zone (Chebyshev) fitting algorithms indicates that serious problems can exist in present commercial software packages.

1. Introduction

The historical motivation for this work came as a result of the 1988 GIDEP alert [1] which warned of possible significant measurement uncertainty arising from least-squares fitting software embedded in coordinate measuring machines (CMMs). The National Institute of Standards and Technology (NIST) responded with the Algorithm Testing and Evaluation Program for Coordinate Measuring Systems (ATEP-CMS). This NIST special test service helps quantify the uncertainty of measurement due to least-squares fitting software for lines, planes, circles, spheres, cylinders, cones, and tori. The ATEP-CMS relies on internal least-squares reference fitting algorithms developed for all these geometries [2]. The test service proved successful, as it has revealed weak points in several least-squares fitting packages, which have been subsequently revised.

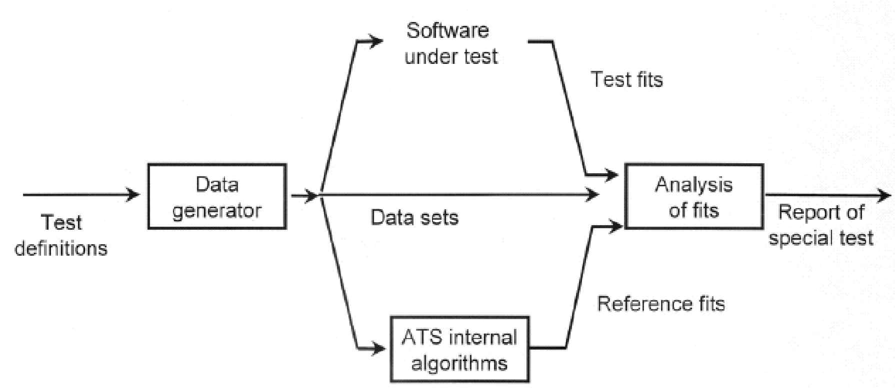

Fig. 1. Architecture of the NIST algorithm testing system.

As helpful as that work has been, the scope of the alert and the initial ATEP-CMS response was limited to least-squares fitting algorithms. Efforts have now been made to address software that performs feature fitting according to objectives that more closely mirror some of the language of geometric dimensioning and tolerancing —namely, fitting using a minimum-zone or (when applicable) a maximum-inscribed or minimum-circumscribed criterion.

2. New Reference Algorithms

NIST recently developed reference algorithms for these cases (Fig. 2). The reference algorithms were developed with emphasis on the criterion of generating a correct result; matters of speed, for example, were considered secondary. When results were compared with all these features and with some industrial vendor software, the results for the industrial non-least-squares cases were alarming. In several cases the reported fits erred to an unacceptably large degree. In some cases the reported maximum-inscribed fit was not even close to being an inscribed feature.

The choice of an optimization algorithm for a fitting problem depends on the nature of the problem. For instance, while the Levenburg-Marquardt method (or other "downhill only" approaches) was found to be suitable for a least-squares objective [2], it would be unacceptable for these new fits. This is primarily because these new objective functions often have their absolute minima hidden among several nearby local minima, causing a downhill-only approach to generally miss finding the true solution. For this reason the use of a seemingly straightforward minimization algorithm would be a naïve and unsatisfactory solution. Additionally, the minimization algorithm must be able to work with non-differentiable objective functions, which is often the case with these fits.

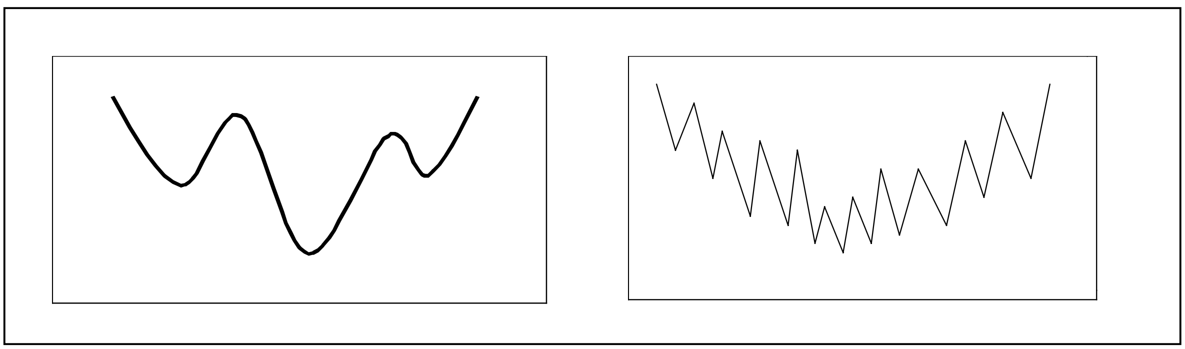

Fig. 2. An "artist's rendition" of objective functions. The left picture represents a least-squares objective function—smooth and having its global minimum away from other local minima. The picture on the right represents the objective function for fits in this paper where the function is non-differentiable and has its global minimum hidden among several nearby local minima. The actual objective functions are in several dimensions.

The minimization algorithm chosen is based on simulated annealing—a method known for finding a global minimum in the presence of local minima. We implemented a version of the algorithm that requires only function values and not any derivative information. The implementation was put through a battery of tests giving us confidence that it works extremely well. Additionally, in our comparisons with industrial software, when there are discrepancies the better fit can be easily determined based on a simple comparison of objective functions. For instance, if the problem is finding the maximum-inscribed circle, differing solutions can be compared by first ensuring the reported solutions are both inscribed circles and if they are, then the better solution is the one of larger radius.

	Lines	Planes	Circles	Spheres	Cylinders	Cones
Minimum-zone	x	x	x	x	x	x
Minimum-circumscribed			x	x	x	
Maximum-inscribed			x	x	x	

Fig. 3. New reference algorithms. An "x" indicates NIST has developed a reference algorithm for the indicated fit objective. Least-squares reference algorithms for ATEP-CMS already exist for all these geometries.

3. Industrial Intercomparison and Results

Our comparison with industrial software involves only two packages. It is acknowledged that a greater number of packages, say five, would be preferable. However, although resources of time limited the number, it is evident that even these are of substantial importance. As the results show, there are alarming problems in both packages. If five comparisons were performed and the additional three all agreed completely, there would still be sufficient cause for concern regarding industrial software. Hence, even this limited endeavor is enough to warrant a concern in these areas of fitting algorithms.

Each test data set was constructed using less than 200 points and simulated measurement and form errors were introduced totaling between 0.1% and 1.0% of the size of its nominal feature. Ten test data sets were used for each geometry in most cases.

Categories for the results are broken down as "acceptable" when the reported solution deviates from the reference by less than 10% of the size of the errors introduced into the part, "bad" when the deviation is between 10% and 50%, and "failure" (marked with an 'x') when deviation was greater than 50% or when the algorithm did not produce a result.

Maximum-Inscribed Fit Results

		Industrial Software A	Industrial Software B
Circles	Good	●●●●●●●●●●	●●●●●●●●●
	Poor		●
	Failure		
Spheres	Good	●●●●●●	
	Poor		
	Failure	x	xxxxxxxx
Cylinders	Good	●●●	●●●●●●●●●
	Poor	●	●
	Failure	xxxxx	

Table 1. Summary of fit results for the maximum-inscribed case

Minimum-Circumscribed Fit Results

		Industrial Software A	Industrial Software B
Circles	Good	●●●●●●●●●●	●●●●●●●●●●
	Poor		
	Failure		
Spheres	Good	●●●●●●	●
	Poor		
	Failure	x	xxxxxxxxx
Cylinders	Good	●●●	●●●●●●●●●●
	Poor	●	
	Failure	xxxxx	

Table 2. Summary of fit results for the minimum-circumscribed case

Minimum-Zone Fit Results

		Industrial Software A	Industrial Software B
Lines	Good	●●●●●●●	●●●●●
	Poor	●	
	Failure	xx	xxxxx
Planes	Good	●●●●●●●●●●	●●●●●●●●●
	Poor		
	Failure		x
Circles	Good	●●●●●●●●●●	●●●●●●●●●●
	Poor		
	Failure		
Spheres	Good	●●●●●●●●●	●●●●●●●●●●
	Poor		
	Failure		
Cylinders	Good	●●●●●●●●●	(data not available)
	Poor	●	
	Failure		
Cones	Good		
	Poor	●●	
	Failure	xxxxxxxx	

Table 3. Summary of fit results for the minimum-zone case

4. Conclusions

The tables show alarming results. It is the opinion of the author that there is no reason why every block should show only good results. As a result of these findings, the ATEP-CMS service at NIST has planned a priority item to create a collection of reference pairs for industry—reference pairs being test data sets along with their correct fits according to the fit objectives studied here. These will be available for download from the Internet for industrial use.

5. References

[1] Walker, R., *CMM Form Tolerance Algorithm Testing*, GIDEP Alert X1-A-88-01, Government-Industry Data Exchange Program, DOD, Washington, D.C., 1988.

[2] C. M. Shakarji, *Least-Squares Fitting Algorithms of the NIST Algorithm Testing System*, J. Res. Natl. Inst. Stand. Technol. **103** (6), 633-641 (1998).