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A Review of Current Geometric Tolerancing Theories and Inspection Data Analysis Algorithms

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This report provides an overview of the state of the art in mechanical dimensioning and tolerancing theories and CMM inspection data analysis technology. We expect that the information included in this review will benefit CMM software developers, CMM users, and researchers of new CMM technology.

This document is the results of a survey of published geometric dimensioning and tolerancing theories and post-inspection data analysis algorithms. Both traditional and modern theories have been reviewed. Principles on which current national and international standards are based have been stated. These geometric dimensioning and tolerancing principles are commonly used in mechanical design and part inspection. Post-inspection data analysis algorithms, used for extracting features and evaluating tolerances, have also been reviewed. The effects of using different fitting criteria are discussed. From this theory and algorithm review, we recommend directions for future development in these areas. The bibliography covers activities and accomplishments of the research in advancing inspection technology.

Key words: coordinate measuring machines; data reduction; design; geometric dimensioning and tolerancing; inspection; post-inspection data analysis.

1 INTRODUCTION

In August, 1988, the Government-Industry Data Exchange Program[1] issued an alert [46] on using coordinate measuring machines (CMMs) to inspect mechanical parts. This alert brought to public awareness what has been called a crisis for CMMs. There were problems in using CMMs to check the dimensional correctness of parts: different measurement techniques, using the same data analysis algorithms, resulted in different measurement results; different data analysis algorithms calculated different measurement results using the same measurement technique. These problems are typical of a general problem called *methods divergence*. In the case of CMMs methods divergence due to software problems led to relative errors of up to 50% in accepting bad parts or rejecting good parts [46]. Several factors occurring at the same time resulted in this situation:

— conflicting interpretations of the ANSI Y14.5 National Standard on Dimensioning and Tolerancing [2] by part designers and part inspectors, who frequently misunderstand basic geometric dimensioning and tolerancing principles [23];

[1] The Department of Defense sponsors the Government-Industry Data Exchange Program as a way of effectively disseminating information on manufacturing-related quality problems.

- lack of standard measurement practice, with different measurement techniques yielding different inspection results [39]; and
- performance problem of CMM software, due to both flaws embedded in the software and limitations of post-inspection, data analysis algorithms. [31, 39]

Researchers, software developers, and CMM users have been together dealing with the problem; several research results are now available. Also, the American Society of Mechanical Engineers (ASME) has organized several design- and inspection-related standards committees. Their common objectives are to standardize definitions and practices in using CMMs and other metrology equipment to assure product quality.

This paper provides an overview of current geometric dimensioning and tolerancing theories and post-inspection data analysis algorithms. These theories and algorithms will be the basis of improved CMM technology in the future. As a review paper, we summarize current technology rather than propose solutions to problems in CMM software and engineering metrology.

The fundamental issues that need to be addressed in improving post-inspection, data analysis algorithms are:

- Rigorous definitions of geometric dimensions and tolerances;
- Design of robust, post-inspection, data analysis algorithms;
- Specification of reliable surface-sample measurement practices; and
- Assessment of CMM software performance.

These issues are interrelated: they need to be considered and solved together. This report focuses on developments in the first two areas: theories and algorithms. There has been much recent work on developing tolerancing theories. These theories have been aimed either at providing a theoretical basis for current tolerancing practice or at developing the basis for new tolerancing schemes. Data analysis algorithms have also been the subject of much recent research. Much of this work has been motivated by a greater awareness of the principles of dimensioning and tolerancing. It is becoming more widely accepted that algorithms selected on the basis of mathematical convenience do not always reflect sound metrological principles. The issues of measurement methods and of software assessment are beginning to receive some attention, but there have been very few published results in these areas. We intend to survey measurement practice and performance evaluation results in a future paper.

This report is organized in five sections, including this introductory section. Section 2 describes geometric dimensioning and tolerancing methods and theories. Section 3 presents available post-inspection, data analysis algorithms and techniques. Section 4 contains concluding remarks of this report. All the materials used in this review are listed in alphabetical order by author name in Section 5.

2 A REVIEW OF DIMENSIONING AND TOLERANCING THEORIES

Geometric dimensioning and tolerancing provides a means for specifying the shape requirements of and the interrelationships between part features. Shape requirements include not only functional needs—the suitability for assembly with its designed counterpart(s) and the proper functioning of a mechanical system—but also issues such as manufacturability, esthetics, and conformance to regulations. Because no manufacturing process can make dimensionally perfect parts [18], designers must specify a region to allow dimensional variation in actual parts. This region is called *tolerance zone*. The traditional view of tolerancing is that when the dimensional variation is within the allowable region, the part meets shape requirements; that is, the actual part is functionally acceptable. Taguchi [42] has indicated another point of view about tolerances: any deviation from the ideal geometry is a loss of functional value of the part. The limits of geometric deviation can be decided when the deviation is so far from the nominal geometry such that the part cannot meet shape requirements. These limits are then used as design tolerance limits. Taguchi views the cost of manufacturing imperfect parts as a continuous function, that can be non-zero even though the part may be in tolerance, whereas the traditional view is that the cost is a step function that is zero as long as the part is within tolerance.

Traditionally, engineering drawings consist of two inter-related elements: views [1] showing the shape of a part, and dimensions [38] stating the sizes. Since the Y14.5 standard [2] was first published in 1966, standard methods for specifying feature dimensions and tolerances on mechanical designs have been available to mechanical designers. By specifying geometric dimensions and tolerances, designers can use engineering drawings to convey shape requirements to manufacturing and inspection. These specifications can support automated production systems in addition to human interpretation of drawings. Examples of such data-driven processing can be found in [19, 43].

Major geometric tolerancing theories and methods for mechanical design are usually categorized as the traditional plus/minus tolerancing theory and the modern tolerance zone theory. The traditional plus/minus tolerancing method is used for specifying allowable size variation around the nominal size. The modern tolerance zone method is used not only to specify allowable size variation but allowable variations of feature form and of feature interrelationships. Feature form tolerances are straightness, flatness, roundness, sphericity, and cylindricity. Feature interrelationship tolerances include:

— orientation tolerances such as perpendicularity, angularity, parallelism;
— location tolerances such as position and concentricity;
— runout tolerances such as circular runout and total runout; and
— profile such as line profile and surface profile.

2.1 Requirements for Developing Geometric Dimensioning and Tolerancing Theories

Farmer [14] states several criteria for evaluating dimensioning and tolerancing principles. These criteria can serve as evaluators for existing theories and as foundations for developing new geometric tolerancing theories. By Farmer's criteria, a theory is good if it can:

1) enable designers to specify and analyze designed dimensions and tolerances so the functional requirements of design can be verified;
2) clearly, concisely, and efficiently convey the design information;
3) be unambiguously interpreted during manufacturing processing, assembly and alignment, and inspection;
4) allow parts to be produced by the most efficient methods; and
5) provide the means to return the inspection results for evaluating parts and identifying faults in manufacturing processes.

2.2 Traditional Tolerancing Theory

Traditional plus/minus tolerancing provides a basis for defining the limit of size used in dimensioning mechanical parts. The size tolerance indicates the quantity of the allowable variation of a dimension, either linear or angular. A dimensioning theory developed by Hillyard [17] furnishes a scheme to specify sizes of interrelated features and to check whether a feature is over, under, or exactly defined by a set of specified dimensions on a part.

The advantage of traditional tolerancing is that it is simple for designers to use. It is also simple for inspectors to verify the actual size variation of parts using a micrometer, a caliper, or a protractor. However, there are several shortcomings in this approach. Only size tolerances and simple forms of positional tolerances are supported. There is no specification for form tolerances or complex features interrelationships (including *true position*). As a result, assembly and alignment requirements cannot be represented or verified. Plus/minus tolerancing also lacks the abstraction power in representing tolerances of mechanical parts in Computer-Aided Design/Computer-Aided Manufacturing (CAD/CAM) systems. Requicha [35] discusses how traditional plus/minus tolerancing can be ambiguous in how dimensions vary from nominal.

2.3 Modern Tolerancing Theories

Modern tolerancing theory was developed to overcome shortcomings in traditional tolerancing theory. Modern geometric tolerancing methods are based on two major principles: the Maximum Material Condition (MMC) principle, also called Taylor's principle; and the Independence principle [48]. The MMC requires an envelope which is the boundary surface of a similar perfect form of the nominal feature in the design. The envelope must totally contain the feature and must meet the shape requirements. The similar perfect feature is the feature at the maximum material size limit (the worst case).

The Independence principle makes a clear distinction between size tolerance and form tolerance. It requires tolerancing for size without any reference to form or location tolerances. The latter must be defined separately, when necessary.

ANSI Y14.5 is based on the MMC principle. As stated in ANSI Y14.5, a tolerance zone is a virtual region formed around the true feature. It can be interpreted as regulating the movement of a dial indicator. Requicha [35] proposed mathematical formulations for tolerance zones. In his theory, a tolerance zone is a region bounded by similar perfect geometry, offset from the nominal feature surface. Several techniques have been developed for computing offset surfaces. Approaches developed by Lin [27], Rossignac [37], and Yu [49] are well-suited for constructive solid modeling, while the method of Rogers [36] is based on boundary representations.

Etesami [12] proposed a method for testing the conformity of actual manufactured parts to a tolerance zone. His approach is to construct tolerance zones for a feature and verify whether part boundaries lie entirely within the constructed tolerance zone. The approach uses a boundary representation technique in solid modeling to generate offsets of curves and surfaces, called *constructors*. The constructors are equivalent to Requicha's tolerance zones.

Both of the above theories are aimed at defining part conformance. A different approach was developed by Hoffmann [18]. Hoffmann proposed a set of mathematical models of manufacturing process errors. Traditional plus/minus tolerancing was used to formulate error models that included machining errors (tool wear and machine errors), part setup errors, and alignment errors. The tolerance of a shape dimension was specified by a set of inequality equations for the geometric parameters of the shape. These equations are closely related to the manufacturing error models. The resulting theory is well-suited to developing feedback from inspection to the manufacturing process.

Jayaraman and Srinivasan [40, 21, 41] present a geometric tolerancing theory based on functional gaging concepts. They develop the *virtual boundary* concept. This is a boundary of perfect form, established at a theoretically exact position, that models the fit between two part surfaces in assembly. For a non-interference fit, one surface must lie entirely inside the virtual boundary, while the other surface must lie entirely outside. The virtual boundary also serves as the maximum material envelope for both surfaces. Relating the measurement made of actual features on both parts against the virtual boundary, the type of fit (clearance, transition, or interference [3]) can be calculated using this virtual boundary condition approach.

3 A REVIEW OF ALGORITHMS IN INSPECTION DATA ANALYSIS

The traditional objective of measuring a part is to check whether it conforms to the tolerance specifications shown on engineering drawings. With new advancements in the technology of computerized dimensional measuring machines and in computational geometry, new algorithms are critically needed which are functional, accurate, reliable, and robust for inspection data analysis. Data analysis algorithms are used not only to judge the qualification of the manufactured parts but also to extract information on process errors that can be fed back for process modification, correction, and compensation [11].

Algorithms are coded in data analysis software which takes inspection data points as input and processes them for feature extraction and tolerance evaluation. In a real measurement, the data points collected from a dimensional measuring machine are not perfectly accurate. The accuracy is limited by the errors embedded in hardware such as hysteresis, kinematic error, noise, etc. Very little is known on the sensitivity of data analysis results to point measurement errors.

Data analysis algorithms are primarily fitting algorithms, also called data reduction methods. The purpose of using fitting algorithms is to extract feature parameters that can best represent the actual part dimensions and tolerances. The challenge in designing fitting algorithms is how to best approximate (with certain criteria) the actual geometry of manufactured features. Anthony and Cox [5] proposed a general approach to test inspection data analysis software for reliability.

An important class of algorithms that are not based on fitting fall under the general rubric of soft functional gaging. These algorithms are based on simulating the inspection of parts with functional gages, a process usually carried out with hardware artifacts.

In this section, currently used fitting criteria are first reviewed for their objective functions. The algorithms for calculating minimum tolerance zones are reviewed as well. The use of soft functional gaging technique is also described.

3.1 Inspection Data Fitting Criteria

The purpose of data fitting is to apply an appropriate algorithm to fit a perfect geometric form (line, plane, circle, ellipse, cylinder, sphere, cone, etc.) to sampled data points obtained from the inspection of a manufactured part. The perfect form approximation obtained through fitting is called a *substitute feature*. The substitute feature is represented by a shape vector $\boldsymbol{b}$.[2] (The exact nature of $\boldsymbol{b}$ varies with the geometric form being fit.) The substitute feature is a one-dimensional curve or a two-dimensional surface that we designate as a function $\boldsymbol{f}(\boldsymbol{u}; \boldsymbol{b})$ of a parameter vector $\boldsymbol{u}$. The values of $\boldsymbol{f}$ are points in space (or on a surface, if fitting is being done in two dimensions). As $\boldsymbol{u}$ varies, $\boldsymbol{f}$ moves along the geometry represented by $\boldsymbol{b}$; as $\boldsymbol{b}$ varies, the surface changes shape and location. A particular geometry need not have a single representation. In fact, much of the research on fitting techniques is based on developing clever representations for curves and surfaces.

The fitting problem, generally stated, is to minimize some objective function with respect to $\boldsymbol{b}$. For some kinds of fitting (to be described below) the minimization may be

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[2] Throughout this section, we use the following notation. Numbers are represented by Roman or Greek letters (S, e_i, λ, etc.); if they are specifically integers, they are designated by a FORTRAN-like convention (N, i, etc.). Matrices are represented by bold, upper-case letters ($\boldsymbol{A}$, $\boldsymbol{W}$, ...). Vectors are represented by bold, lower-case, Roman letters ($\boldsymbol{b}$, $\boldsymbol{p}_i$, ...). These conventions carry over to functions, so a vector-valued function of a vector may appear as $\boldsymbol{f}(\boldsymbol{x})$.

subject to certain constraints. The most frequently used fitting algorithms are based on the L_p norm [16]:

$$L_p \doteq \left[\frac{1}{N}\sum_{i=1}^{N} |e_i|^p\right]^{1/p} \tag{3.1}$$

where $0 < p < \infty$, N is the total number of data points, and e_i is the shortest distance between $\boldsymbol{p}_i$, the i^{th} data point, and the considered feature. The best fit feature is the feature that minimizes the L_p norm. Since N is a constant, $1/N$ is usually omitted from the equation in most surface fitting applications. Similarly, since p is fixed, the exponent $1/p$ is often omitted. The resulting objective function is

$$S_p \doteq \sum_{i=1}^{N} |e_i|^p \tag{3.2}$$

S_p represents the same problem as the L_p norm. The value of $\boldsymbol{b}$ that minimizes L_p (and S_p) is called the L_p *estimator* of the feature [16]. When $p = 1$, the fitting problem is least-sum-of-distances fitting, a generalization of finding the median of a data set. When $p = 2$, it is total-least-squares fitting, also called orthogonal distance regression.

L_p fitting can be extended to $p=\infty$ by noting that $\lim_{p\to\infty} L_p = \max_i |e_i|$. Minimizing L_∞ is called the two-sided minimax problem, because the solution minimizes the maximum e_i on both sides of the feature. One-sided minimax fitting is a constrained two-sided minimax fitting. The objective functions for smallest circumscribed features and the largest inscribed features are somewhat different than one-sided minimax objective functions, but the fitting results often appear similar. Lotze [28] describes a set of general fitting criteria. Formulae used in feature fitting are described in [6].

Weckenmann and Heuirichowski [47] and Porta and Waeldele [31] discuss problems concerning CMM fitting software. Issues in the design and testing of dimensional inspection algorithms can be found in [5].

3.1.1 Least-sum-of-distances fitting

This fitting problem is also known as the median-polish fit [16]. The sum of distances can be formulated as:

$$S_1 = \sum_{i=1}^{N} |e_i| \tag{3.3}$$

The objective of this fitting is to minimize the sum of absolute distances, S_1. For both the linear and nonlinear problems, the algorithms of linear and nonlinear programming methods for solving the parameters can be found in [16].

The result of the least sum of distance fitting passes through the median of the distribution of e_i's. This fitting is less sensitive to the data outliers than total least squares fitting, described next.

3.1.2 Total-least-squares fitting

Total-least-squares fitting is by far the most widely used approach in CMM data analysis. The sum of squared distances, S_2, can be formulated as:

$$S_2 = \sum_{i=1}^{N} |e_i|^2 = \sum_{i=1}^{N} e_i^2 \tag{3.4}$$

Each e_i is a function of the data point $\boldsymbol{p}_i$ and the point $\boldsymbol{f}(\boldsymbol{u}_i;\, \boldsymbol{b})$ on the substitute feature closest to $\boldsymbol{p}_i$. Therefore, the least squares fitting can be expressed as:

$$\min_{\boldsymbol{b}} \left(\sum_{i=1}^{N} |\boldsymbol{p}_i - \boldsymbol{f}(\boldsymbol{u}_i;\boldsymbol{b})|^2 \right) \tag{3.5}$$

If each e_i is a nonlinear function of $\boldsymbol{b}$, then the problem is a nonlinear, total least squares fitting, and is usually solved using an iterative process. The commonly used Gauss-Newton and Levenberg-Marquardt iteration algorithms can be applied in finding the minimal S_2. One problem with these methods is that they can easily find only a local minimum. Therefore it is important, although sometimes difficult, to find a good starting value for the iterative process.

If the substitute feature is a line or a plane, then e_i can be expressed as a linear function of $\boldsymbol{b}$ and S_2 can be then formulated as:

$$\begin{aligned} S_2 &= \sum_{i=1}^{N} | e_i |^2 \\ &= \sum_{i=1}^{N} | \boldsymbol{p_i} - \boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{b} |^2 \\ &= | \boldsymbol{Ab} |^2 \\ &= \boldsymbol{b}^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{Ab} \end{aligned} \tag{3.6}$$

where $\boldsymbol{A}$ is a matrix that does not depend on $\boldsymbol{b}$. For the linear total least squares problem, the estimator $\boldsymbol{b}$ can be obtained by using Gaussian elimination, as demonstrated by Forbes [15] and by Dahlquist and Bjorck [9].

The Gauss-Markoff theorem [16] shows that the total least squares estimation is the best linear unbiased estimation. However, this theorem refers only to the class of linear estimations. It does not considered nonlinear alternatives. The result from the total least squares fitting passes through the sample mean of a normal distribution of distances e_i's. The total least squares fitting is more sensitive to data outliers than the least sum of distances fitting.

3.1.3 Two-sided minimax fitting

When p in the L_p-norm formula approaches infinity, the norm becomes the maximum absolute distance. The estimator $\boldsymbol{b}$ is then called the *two-sided minimax* fit for the feature. Minimax fitting minimizes the maximum distance between all the sampled data points and the ideal form is minimal. The problem can be formulated as:

$$\min_{\boldsymbol{b}}\left(\max_{1\le i\le N} |e_i|\right) \tag{3.7}$$

This fitting problem is known as the L_∞-norm estimation problem. A roundness tolerance zone [4], as a result of two-sided fitting, is shown in Figure 1. The resulting fit is strongly affected by data outliers. Lai and Wang [24] and Etesami and Qiao [13] present

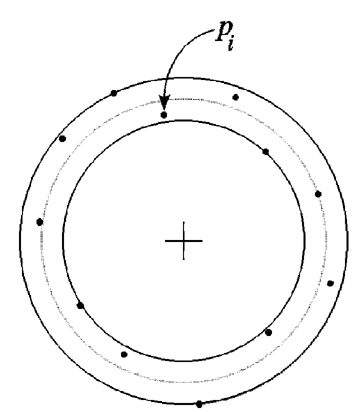

Figure 1 Circles by two-sided fit.

an algorithm for calculating two-sided minimax fit for circles. The algorithm will be described in Section 3.2.2.

3.1.4 One-sided minimax fitting

Two-sided minimax fitting is useful for estimating roundness, cylindricity, and other form deviations. When estimating the size of a feature, however, one is usually interested in having the substitute feature lie entirely outside the material of the part. This can be accomplished using one-sided minimax fitting. The formulation is based on representing the errors e_i so that they are positive on one side of the feature and negative on the other side. (So, for instance, one side of a line or the inside of a circle in a plane, or one side of a plane or the inside of a cylinder, can be chosen as the *positive* side and the other the *negative*.) By properly choosing the representation of the substitute feature, the e_i can always be expressed in this way. One-sided minimax fitting can then be formulated as a constrained optimization problem:

$$\min_{\boldsymbol{b}} \left(\max_{1 \le i \le N} |e_i| \right)$$

$$\text{Subject to} \qquad e_i \le 0, \quad i=1, \ldots, N \tag{3.8}$$

or

$$\min_{\boldsymbol{b}} \left(\max_{1 \le i \le N} |e_i| \right)$$

$$\text{Subject to} \qquad e_i \ge 0, \quad i=1, \ldots, N \tag{3.9}$$

The choice of formulation depends on where the material of the part is with respect to the positive side of the curve or surface.

3.1.5 Smallest circumscribed fitting and largest inscribed fitting

For features of size such as circles, cylinders, and spheres, a common objective is to find the largest inscribed substitute feature or the smallest circumscribed substitute feature. This problem can be formulated as:

$$\min_{\boldsymbol{c}} \quad R$$

$$\text{Subject to} \qquad e_i \le 0, \quad i=1, \ldots, N \tag{3.10}$$

and

$$\max_{\boldsymbol{c}} \; R$$

Subject to

$$\begin{aligned} e_i &\geq 0, \quad i=1, \ldots, N, \\ 0 &= \sum_{i=1}^{N} \lambda_i (\boldsymbol{p_i} - \boldsymbol{c}), \\ 1 &= \sum_{i=1}^{N} \lambda_i, \\ \lambda_i &\geq 0, \quad i=1, \ldots, N \end{aligned} \tag{3.11}$$

where $\boldsymbol{c}$ is the parameter vector which can be the center of a circle or a sphere, or the axis of a cylinder and R is the radius of the circle, sphere, or cylinder.

Equation (3.10) can be used to find the smallest circumscribed circle, while Equation (3.11) can be used to find the largest inscribed circle [4]. The difference between the substitute circle obtained from Equation (3.8) and the smallest circumscribed

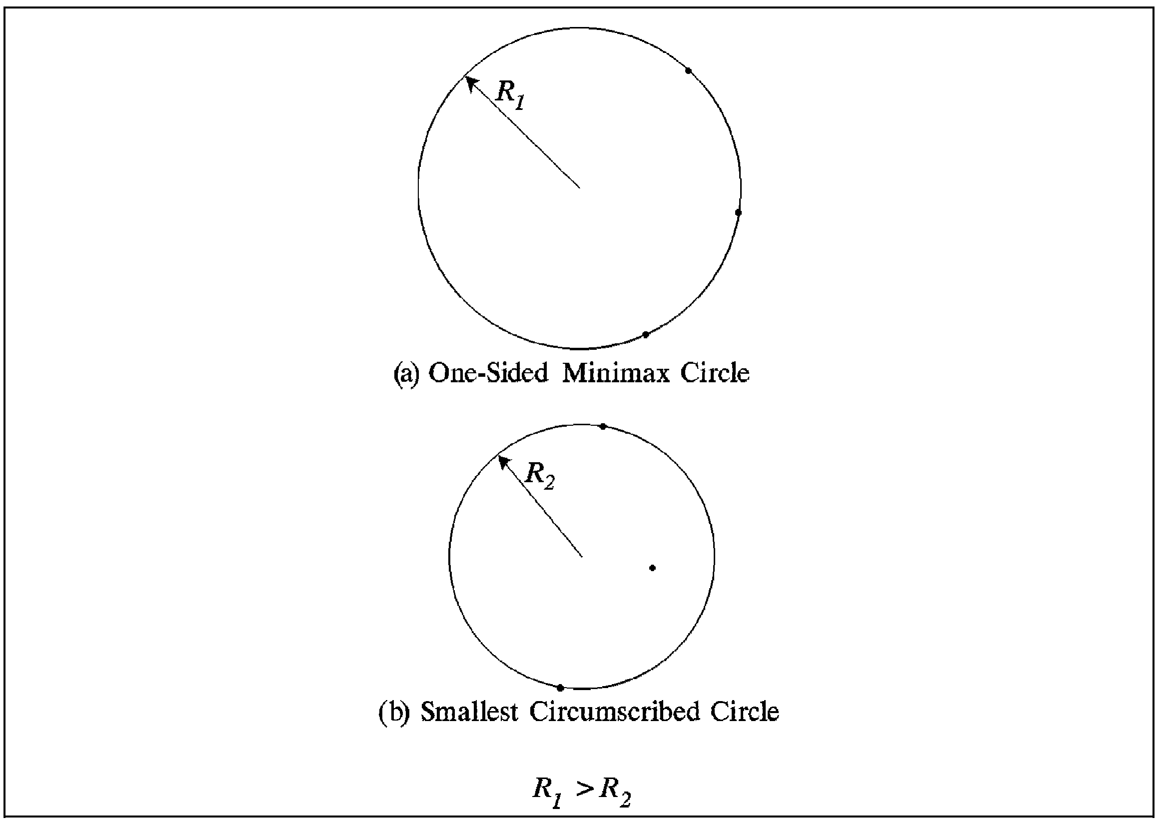

Figure 2 Circles of one-sided (a) and smallest circumscribed (b) fits.

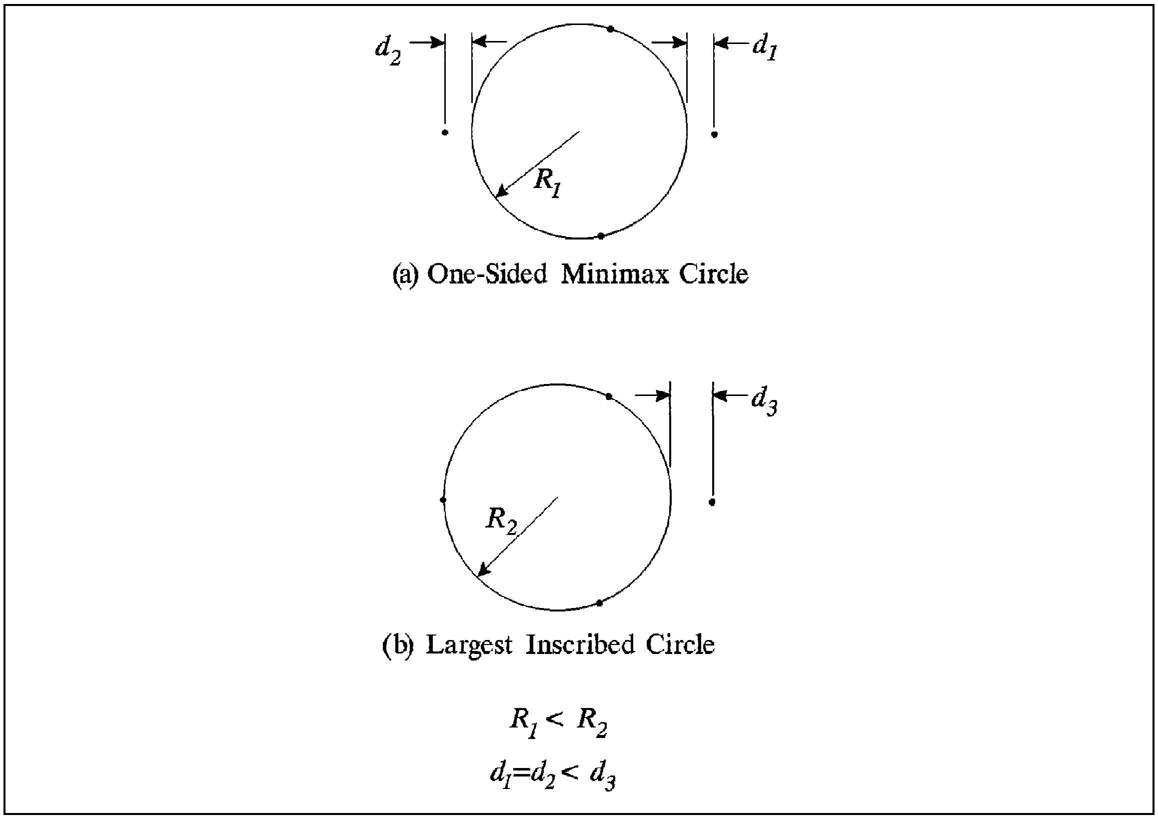

Figure 3 Circles of one-sided (a) and largest inscribed (b) fits.

circle using Equation (3.10) is shown in Figure 2. The Figure shows how the radius of the one-sided, circumscribed minimax circle is larger than the radius of the smallest circumscribed circle. A similar difference can be seen in Figure 3 for a one-sided, inscribed minimax circle, obtained from Equation (3.9), and the largest inscribed circle, obtained from Equation (3.11). The radius of the one-sided, minimax inscribed circle is smaller than the radius of the largest inscribed circle.

Lai and Wang [24] and Etesami and Qiao [13] have presented efficient algorithms in finding these circles. The algorithm is to search the smallest circumscribed circle on the farthest-point Voronoi diagram generated from the convex hull of measured points on a plane. (A *Voronoi diagram* of a set of points is a partition of space into regions. Each region corresponds to a point of the set in that all points of the region are closer to or farther away from the corresponding point than from any other point in the set.) Similarly, the largest inscribed circle can be searched on the closest-point Voronoi diagram. The algorithm for generating two-dimensional convex hulls from a set of points can be found in [32]. Hopp and Reeve [20] have presented an algorithm that solves the smallest circumscribed sphere in arbitrary dimensions. As with two-sided minimax fitting, one-sided minimax fitting is very sensitive to data outliers.

3.2 Algorithms For Calculating Minimum Tolerance Zones

ANSI Y14.5M-1982 [2] specifies standard geometric tolerancing principles and definitions for the purpose of mechanical part design. The Dimensional Measurement Interface Specification [10] specifies standard formats for measured features and measured tolerances as "feature actuals" and tolerance actuals. The above-mentioned fitting algorithms can be applied to calculate the actual values of feature attributes. In this section, the available algorithms for finding the actual tolerance used (the minimum zone [29]) will be discussed. Some minimum-tolerance-zone algorithms have been developed, mostly for two dimensions. These algorithms include straightness, flatness, and roundness. Three-dimensional minimum tolerance zone algorithms still need further research and development. Soft functional gaging, an alternative to minimum zone algorithms for post-inspection data analysis, will also be discussed in this section.

3.2.1 Straightness, flatness, and the medial axis transformation

Algorithms used in the calculation of actual straightness and flatness tolerance zones applying convex hulls were developed by Traband *et al.* [44] and by Cavalier and Joshi [7]. A convex hull needs to be established first; algorithms for constructing three-dimensional convex hull are in [33]. Then, the minimum zone is found by searching the maximum distance between vertex and edges (line or surface) of the established convex hull.

The medial axis transformation method can be used to approximate the median axis (real axis) of a cylindrical feature (hole or shaft). ANSI Y14.5 allows the application of a straightness tolerance to the axis of a feature. This tolerance controls the deviation of the median axis from a straight line.[3] Algorithms for computing the medial axis transformation in two dimensions can be found in [25] and [26]. There are no published algorithms for computing the medial axis in three dimensions. The medial transformation method can also be used to calculate the median axis of a conical feature.

3.2.2 Circularity

Circularity (roundness) is a tolerance zone bounded by two concentric circles with minimum radial separation within which all measurements should lie. The methods proposed by Lai and Wang [24] and by Etesami and Qiao [13] are first, to construct both the near-point Voronoi diagram and the farthest-point Voronoi diagram and then to find intersections of the two diagrams. Each intersection is the center of two concentric circles that form an annular zone within which all the inspection points lie. A search method is then used to find a pair of concentric circles that has the minimum radial separation. This pair of circles forms the actual minimum circularity zone.

[3] Unfortunately, ANSI Y14.5 does not precisely define median axis, so any analysis of a feature must be based on a reasonable assumption regarding the meaning of the tolerance.

Chetwynd [8] presents an iterative process as an alternative to linear programming for calculating circularity.

3.2.3 Cylindricity

Kakino and Kitazawa [22] developed a method for in-process measurements of cylindricity using an extended principle of three-point roundness measurement. A special cylindricity measuring instrument was created for this purpose.

Murthy and Abdin [29] and Murthy [30] compared several algorithms for cylindricity minimum zone calculation. One method Murthy describes used Fourier series and orthogonal polynomials for representing the actual profile of a cylinder. Then the normal least squares method and simplex search method were applied for searching the minimum cylindricity tolerance zone.

3.2.4 Conical form minimum zone

Conicity is a zone bounded by two perfect and coaxial cones some distance apart. Conicity is not a standard tolerance definition in ANSI Y14.5. As a kind of form tolerance, conicity is defined as the region between two similar, perfect, and coaxial cones within which the entire conical part lies. Tsukada *et al.* [45] used this definition as a conical tolerance zone and presented a method of calculating the minimum zone for conical taper form. A simplex search method was applied to search a pair of coaxial cones (inscribed cone and circumscribed cone) that has the minimal distance of separation.

3.3 Soft Functional Gaging

Hard functional gages are used to simulate the assembly parts and to check for interference before the actual assembly process. There are several drawbacks in setup and tooling for hard functional gages. Long lead times in design, high cost to built and certify for use, high cost for maintenance, and inflexibility in design changes have been the difficulties in using hard functional gages.

Soft functional gaging techniques provide a means to achieve the capabilities of hard gages at less cost. A soft functional gage is a particular kind of computer analysis of measured data. Soft functional gaging can be integrated into CMM software. Rasnick and Zurcher [34] propose an approach to soft functional gaging with examples of using and testing soft functional gages. Their method begins with the computer-created perfect geometries of a functional gage. The surfaces of this simulated gage are called tolerance boundaries. The measured points are then overlaid onto the soft gage to determine if the measured points interfere with the tolerance boundaries of the gage. If interference occurs, then the next step is to determine if the measured points can be shifted (translated and rotated) into the gage definition so the interference no longer exists. This can be achieved by designing intelligent algorithms. Intelligent algorithms are especially needed when the part has complicated geometry and topology and many features need to be assembled with their counterparts.

Soft functional gaging and using minimax algorithms to find MMC envelopes are different. Functional gaging, soft or hard, provides an acceptance test of manufactured parts. One-sided fitting algorithms generate the actual MMC envelopes of the parts. These calculated envelopes can then be used not only to test for conformance to tolerances as specified in ANSI Y14.5, but can provide quantitative information usable for process feedback control.

Generally, soft functional gaging appears to be closer to the intent of ANSI Y14.5, at least for checking position tolerances under the MMC principle. Some caveats must be observed, however. First, substitute features must still be used to establish datums, and the shifting algorithm used by a soft functional gage must maintain the geometric constraints imposed by the datum reference frame. Second, the current concept of soft functional gaging tests conformance of the sample points to the tolerance boundaries; it provides no means of interpolating between sample points. Unless the part was measured with a dense sampling of points, this may lead to inaccurate results. Finally, it should be noted that, as with hard functional gaging, soft functional gaging is only applicable to a limited number of tolerance types. Form tolerances, position at other than MMC, and datums must all be handled using substitute feature algorithms.

4 CONCLUDING REMARKS

We have reviewed existing geometric dimensioning and tolerancing theories and methods for representing mechanical designs. The principles specified in ANSI Y14.5 are based on Taylor's principle, while ISO standards have adopted both Taylor's principle and the Independence principle. To better utilize computerized dimensional measurement equipment for part inspection and CAD/CAM systems for part design and manufacturing, rigorous mathematical formulations are needed not only to incorporate dimensions and tolerances into CAD/CAM systems but also to automate the analysis of inspection results.

Several feature-fitting criteria and algorithms were reviewed and their mathematical properties and sensitivity issues were noted. The relationship between algorithm and feature approximation and parameter extraction needs to be further explored and established so users will know how to choose an applicable algorithm. Today, there are few guidelines as to which algorithm to use to fit different features.

Several two-dimensional, minimum-tolerance-zone algorithms have been developed using convex hull and Voronoi diagram methods. These algorithms can be directly applied to calculate straightness, flatness, and roundness. Most three-dimensional minimum tolerance zone algorithms still need to be developed for calculating all actual tolerance zones specified in ANSI Y14.5.

Soft functional gages can be designed by simulating hard functional gaging in computer software. This technique holds advantages in checking interference in assembly. Soft functional gaging is becoming increasingly available on commercial measurement systems.

Further research and development of post-inspection data analysis algorithms are needed. The information on the shape errors of manufactured parts needs to be extracted. This information can then be use to improve the manufacturing process so that higher quality parts will be made.

Computer-based measurement, of which CMM technology is the prime example, has a great potential for improving the overall manufacturing process. The large number of references in the next section demonstrates that research in this area is both active and productive. We expect the next few years to be a period of great growth in this field.

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