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Loop Performance of Digital Controllers
Obtained from the Discretization of
Analog Controllers**

Daniel Raviv

Robotics Center and
Electrical Engineering Department
Florida Atlantic University
Boca Raton, FL 33431
and Intelligent Systems Division

Eddy W. Djaja

Zvi S. Roth

Robotics Center and
Electrical Engineering Department
Florida Atlantic University
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**Technique for Enhancing the Closed-Loop Performance of Digital Controllers
Obtained from the Discretization of Analog Controllers**

Daniel Raviv^{1,2}, Eddy W. Djaja¹ and Zvi S. Roth¹

¹Robotics Center and Electrical Engineering Department
Florida Atlantic University, Boca Raton, FL 33431

² Intelligent Systems Division
National Institute of Standards and Technology
Bldg. 220, Room B 124, Gaithersburg, MD 20899

ABSTRACT

The paper proposes a new simple technique for compensation of Zero Order Hold (ZOH) effects in digital control systems. Given a feedback dynamic system and an analog controller designed in the S-domain, which is discretized by an arbitrary S to Z transformation, an addition of a pole-zero pair in the Z-domain is shown to significantly compensate for the gain and phase changes due to the ZOH.

Applying the proposed method to a variety of examples, including "bench-mark" examples studied by other researchers, the closed-loop performance of the digital control system is compared to the analog system performance in the time and frequency domains.

I. THE PROPOSED METHOD

Digital control design through discretizing an analog controller has been the topic of much controversy [1]. The problem is as follows: given a process $G_p(s)$, sensor $H(s)$ and a presumably well designed analog controller $G_c(s)$ (Fig. 1a), find a digital controller $D(z)$ (Fig. 1b) which produces closed-loop behavior similar to the analog system both in the time and frequency domains.

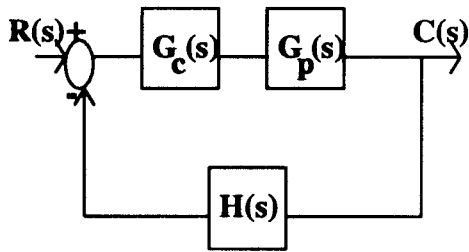


Fig. 1a The analog closed-loop control system

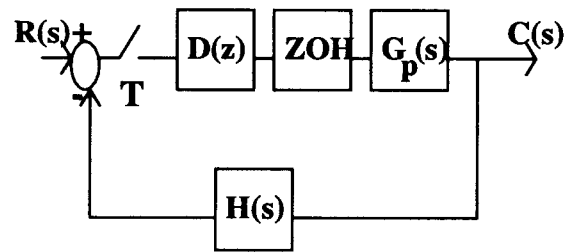


Fig. 1b The digital closed-loop control system

Unquestionably, analog control design followed by controller discretization is far more convenient than direct digital control design for the main reason that the sampling period value T affects the design process only at the final phase and not up front [2]. As observed in [2] and by many practicing engineers, for a sufficiently high sampling rate $\frac{1}{T}$, most discretizing methods produce closed-loop performance which well approximates the analog control performance. In other words, the discretizing problem is meaningful only for relatively low sampling rates. Our goal in this paper is to share with the readers a simple and highly practical Zero Order Hold (ZOH) compensation technique that we have successfully applied to many "bench-mark" problems studied by the above and other researchers, and to other design problems. What we found is that adding this compensation to a discretized controller which maintains closed-loop stability results in a noticeable improved performance.

The method is based on adding a pole-zero pair in the Z-plane to the digital controller obtained through some suitable discretization methods, such as Tustin (bilinear) transformation, with or without pre-warping. The additional pole and zero partially compensate for the low and mid-frequencies phase and gain frequency response effects contributed by the ZOH.

As is well known, the contributions of the ZOH and $G_p(s)$ to the exact discrete time pulse transfer function are not separable. Yet, generally speaking, regardless of the $G_p(s)$ effect, the ZOH causes a delay of approximately $\frac{T}{2}$ as shown intuitively in Figure 2.

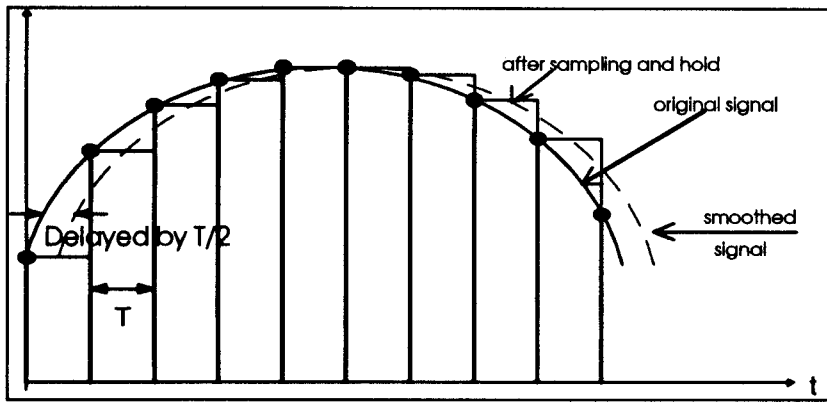


Fig. 2 A reconstructed signal using ZOH and its smoothed approximation

A pole-zero compensation:

$$C(z) = \frac{2z}{z+1} \quad 1)$$

shown in Fig. 3 provides a phase of $\frac{\omega T}{2}$ which exactly cancels the frequency phase response of the ZOH as obtained from $\frac{1-e^{-sT}}{s}$. The ZOH magnitude response is canceled

at frequencies for which $\tan \frac{\omega T}{2} \approx \frac{\omega T}{2}$.

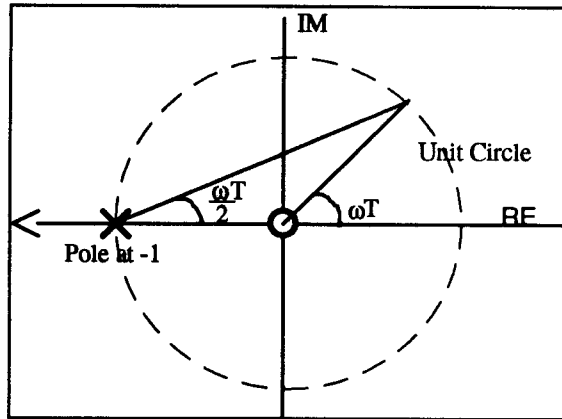


Fig. 3 The location of pole and zero of ZOH compensator in the Z-domain

It is interesting to note that this cancellation (up to a scale factor of $\frac{1}{T}$) is the inverse of the Tustin transformation of the first order Pade approximation to the so-called ZOH transfer function [3]:

$$\frac{1 - e^{-sT}}{s} \approx \frac{T}{1 + \frac{sT}{2}} \quad (2)$$

Then:

$$\frac{T}{1 + \frac{sT}{2}} \Bigg|_{s=\frac{2}{T} \frac{z-1}{z+1}} = \frac{T}{2} \frac{z+1}{z} \quad (3)$$

The method does not guarantee stable closed-loop system since it is independent of the discretization method and the sampling rate. However one can investigate the effects of the sampling rate by applying the polynomial root locus [4]. For a given stable analog closed-loop system, a necessary condition for the proposed method to be used is that the characteristic polynomial of the discretized system:

$$1 + \left(\frac{2z}{z+1} \right) D(z)(1-z^{-1}) Z \left\{ \frac{G_p(s)}{s} \right\} = 0 \quad (4)$$

where $D(z)$ is an arbitrary discretized version of $G_c(s)$, has all the polynomial roots inside the unit circle.

In cases where the proposed compensation (1) causes closed-loop instability, a modified ZOH compensation of the following form is to be considered:

$$C'(z) = \frac{2(z-\epsilon)}{z+1-2\epsilon} \quad (5)$$

where ϵ is a small positive constant. For closed-loop stability, the characteristic polynomial implied by:

$$1 + \left(\frac{2(z-\epsilon)}{z+1-2\epsilon} \right) D(z)(1-z^{-1}) Z \left\{ \frac{G_p(s)}{s} \right\} = 0 \quad (6)$$

must have all roots inside the unit circle. Note that this modified compensation preserves the DC gain of the controller. Figures 4a and 4b illustrate the effect of the new pole-zero pair of the ZOH compensation.

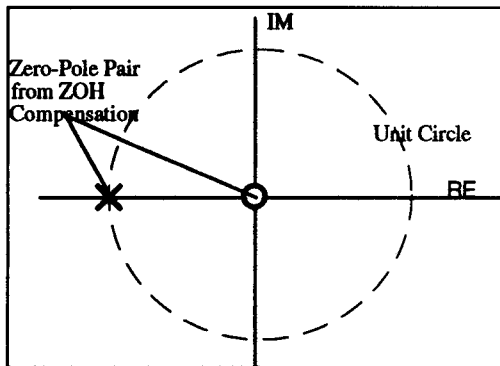


Fig. 4a Unmodified ZOH compensator

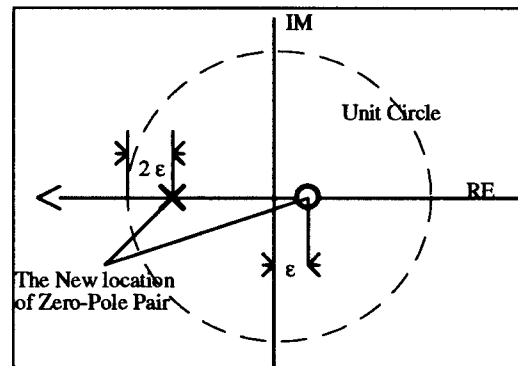


Fig. 4b Modified ZOH compensator

2. SUMMARY OF THE DESIGN PROCEDURE

1. Select a suitable S to Z transformation, i.e., Tustin (bilinear) transformation, and sampling time T to discretize the existing analog controller $G_c(s)$ to obtain $D(z)$.
2. Multiply the result of step (1) by $C(z) = \frac{2z}{z+1}$ to obtain: $\hat{D}(z) = C(z)D(z)$.
- 2a. Match the open-loop DC gain of the digital control system to the analog system.
3. Check the closed-loop stability in Z-domain using equation (4). If stable, observe the closed-loop system performance.
4. If in step (3) the closed-loop system becomes unstable, then:
 - 4a. Try $C'(z) = \frac{2(z-\epsilon)}{z+1-2\epsilon}$ for small $\epsilon > 0$ to obtain $\hat{D}(z) = C'(z)D(z)$; use ϵ as small as possible to guarantee better performance at the higher frequencies.
 - 4b. Match the open-loop DC gain of the analog and discretized systems.
 - 4c. Check the closed-loop stability in Z domain using equation (6). If stable, observe the closed-loop system performance.
5. If the closed-loop system in step (4c) is still unstable, use other methods.

3. EXAMPLES

a. Lag Compensator [5]

Given the process

$$G_p(s) = \frac{4 \times 10^6}{s(s+20)(s+200)} ; H(s)=1.$$

The following analog controller

$G_C(s) = \frac{1}{80} \frac{(s+8)}{(s+0.1)}$ was designed using Bode diagram to satisfy the following design specifications: (a) velocity error constant, K_V at least 1000 s^{-1} , (b) attenuation of all sinusoidal inputs above 400 rad/s by at least 16, (b) steady-state error of (up to) 1% for sinusoidal inputs less than 1 rad/s .

The following table is a summary of the discretization results for three different sampling rates. The discretized controller $D(z)$ using Tustin transformation is listed in the second column. The third column is the ZOH compensation and the fourth column is the complete controller $\hat{D}(z)$ using the proposed ZOH compensation. Note that in the third row, with or without the ZOH compensator, the closed-loop system is unstable. The problem is overcome by using the modified compensator shown in the fourth row.

	$D(z)$	Multiplier	$\hat{D}(z)$
$T = 0.01 \text{ s}$	$\frac{0.0130z - 0.0120}{z - 0.9990}$	$\frac{2z}{z+1}$	$\frac{0.0260z^2 - 0.0240z}{z^2 + 0.0010z - 0.9990}$
$T = 0.05 \text{ s}$	$\frac{0.0150z - 0.0100}{z - 0.9950}$	$\frac{2z}{z+1}$	$\frac{0.0299z^2 - 0.0200z}{z^2 + 0.0050z - 0.9950}$
$T = 0.1 \text{ s}$	Unstable $\frac{0.0174z - 0.0075}{z - 0.9900}$	$\frac{2z}{z+1}$	Unstable $\frac{0.0348z^2 - 0.0150z}{z^2 + 0.0100z - 0.9900}$
$T = 0.1 \text{ s}$	Unstable $\frac{0.0174z - 0.0075}{z - 0.9900}$	$\frac{2(z-0.2)}{(z+0.6)}$	$\frac{0.0348z^2 - 0.0219z + 0.0030}{z^2 - 0.3900z - 0.5940}$

Table 1

Figures 5.1a and b depict the closed-loop step and frequency responses respectively, of the analog system and the digital control system using Tustin with and without ZOH compensation for $T=0.01 \text{ s}$. The ZOH compensation design method performance matches the analog system's performance very well, while without the

compensation, the system has a slightly bigger overshoot as shown in Figures 5.1a. The frequency response with and without ZOH compensation are shown in Figures 5.1b. When reducing the sampling rate to $T=0.05$ s, the system with no compensation has a very poor performance in time and frequency domains as shown in Figures 5.2a and 5.2b. However, whereas with the ZOH compensation, the system performance is close to the analog one.

Reducing the sampling rates even further to $T=0.1$ s results in an unstable closed-loop system with and without compensation as shown in Table 1. Using the modified ZOH compensation with $\epsilon=0.2$ the closed-loop system becomes stable. The step response shown in Fig. 5.3a demonstrates a relatively good performance while Tustin transformation has made the closed-loop system unstable. Figure 5.3b shows the frequency response of the closed-loop system using the modified ZOH compensator.

b. Lead-Lag Compensation [5]

Given

$$G_p(s) = \frac{1000}{s(1 + \frac{s}{10})(1 + \frac{s}{250})} ; H(s)=1.$$

The following analog controller

$$G_c(s) = \frac{(1 + \frac{s}{4.5})}{(1 + \frac{s}{0.1})} \frac{(1 + \frac{s}{10})}{(1 + \frac{s}{110})}$$

has been designed using Bode diagram to meet several specifications: (a) phase margin of at least 50° , (b) velocity error constant, K_v at least 1000 s^{-1} , (c) attenuation of the input noise at 60 Hz and above by a factor of 100, and (d) the steady state error less than 1 rad/s

is less than 1%. Using Tustin transformation applied to $G_c(s)$ provided with $T=0.01$ s, the following digital controller is obtained:

$$D(z) = \frac{0.6597z^2 - 1.2897z + 0.6300}{1.00z^2 - 1.2893z + 0.2900}$$

With the ZOH compensation of $\frac{2z}{z+1}$, the digital controller becomes

$$\hat{D}(z) = \frac{1.3194z^3 - 2.5793z^2 + 1.26z}{z^3 - 0.2893z^2 - 0.9993z + 0.2900}$$

Figures 6a and b depict the closed-loop step and frequency responses respectively, of the analog system, and the digital control system using the Tustin transformation with and without ZOH compensation. The effect of the ZOH compensation is evident.

c. Katz's example [6]

Given

$$G_p(s) = \frac{863.3}{s^2}$$

the following analog controller

$$G_c(s) = 2940 \frac{(s + 29.4)}{(s + 294)^2}$$

has been designed to meet the following closed-loop system specifications: (a) the maximum phase lag at $f=3$ Hz should not be more than 13° , (b) at any given frequency the closed-loop gain should not exceed 5 dB beyond the closed-loop DC gain, and (c) maximum tracking error due to an input disturbance moment of 0.028 N.m should be 0.01 rad. Taking the Tustin transformation with pre-warping* at the sampling time $T=0.03$ s of $G_c(s)$, the following digital controller is obtained by [6]:

$$D(z) = \frac{1.8958z^2 + 1.1685z - 0.7273}{z^2 + 1.1653z + 0.3395}$$

This was the only discretization method that resulted in a stable closed-loop system at such low sampling rate [3]. The same controller with the additional proposed ZOH compensation is:

$$\hat{D}(z) = \frac{3.7916z^3 + 2.3369z^2 - 1.4546z}{z^3 + 2.1653z^2 + 1.5047z + 0.3395}$$

Adding this ZOH compensation to the $D(z)$, the system output at the sampling instants is significantly closer to the original analog step response, shown in Figure 7a. Note also the improvement of the frequency response as shown in Figure 7b. Even though the suggested compensator does not perform better than those obtained by Evans-Kennedy [7] and Keller-Anderson[2] methods, the simplicity of the proposed method is quite attractive.

* Katz 's pre-warping which warps the frequency of both pole and zero (reference [6]) differs from the "regular" pre-warping which warps at one frequency.

d. Rattan's Example [8]

Given:

$$G_P(s) = \frac{10}{s(s+1)}$$

$$G_C(s) = \frac{1+0.416s}{1+0.319s}$$

A digital controller proposed by Rattan (no design specifications are available in his example) is :

$$D_{\text{Rattan}}(z) = \frac{3.436z - 2.191}{z + 0.2390}$$

Using Tustin transformation to discretize the analog controller, the digital controller becomes:

$$D(z) = \frac{2.294z - 1.5935}{z - 0.2991}$$

Applying the modified ZOH compensation with $\varepsilon = 0.1$ to $D(z)$:

$$\hat{D}(z) = \frac{4.5888z^2 - 3.6459z + 0.3187}{z^2 + 0.5009z - 0.2393}$$

In Rattan's example, the closed-loop frequency matching in the W-domain has shown better performance than Tustin and pre-warping methods at $T=0.15$ seconds [8]. Figure 8a shows that the step response of the closed-loop system using the modified ZOH compensation has slightly lower overshoot than Rattan's method and by far better than Tustin transformation without compensation. The frequency response of the analog closed-loop system, digital closed-loop system using Tustin transformation, Tustin with ZOH compensation, and Rattan's method are all plotted in Figure 8b.

4. CONCLUSION

This paper introduced a method to partially compensate for the ZOH effects of a closed-loop digital control system. By multiplying a discretized given controller in Z-domain by a pole-zero pair, a significant closed-loop performance improvement has been achieved in many design examples. We hope that practicing engineers find this method useful.

5. ACKNOWLEDGEMENTS

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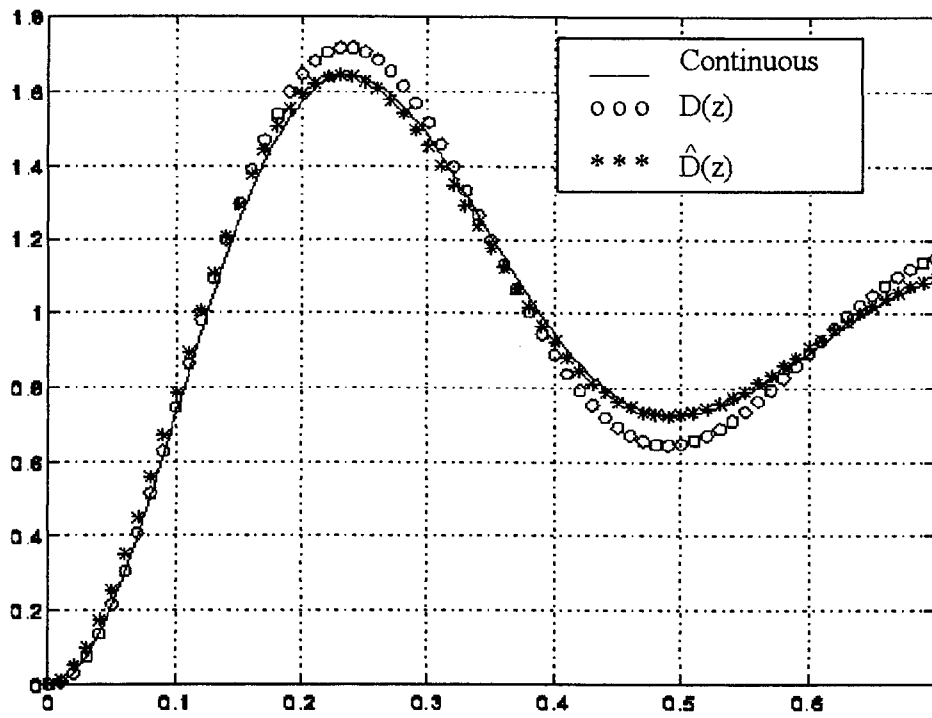


Fig. 5.1a Closed-Loop Step Response of Example Part (a) with $T=0.01$ s

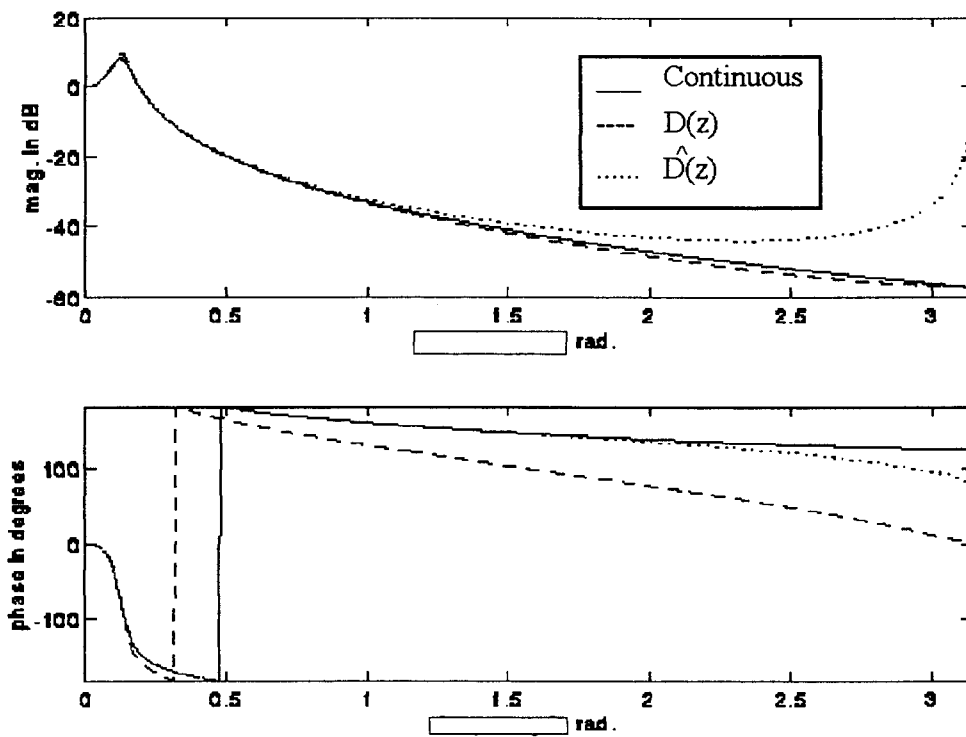


Fig. 5.1b Closed-Loop Frequency Response of Example Part (a) with $T=0.01$ s

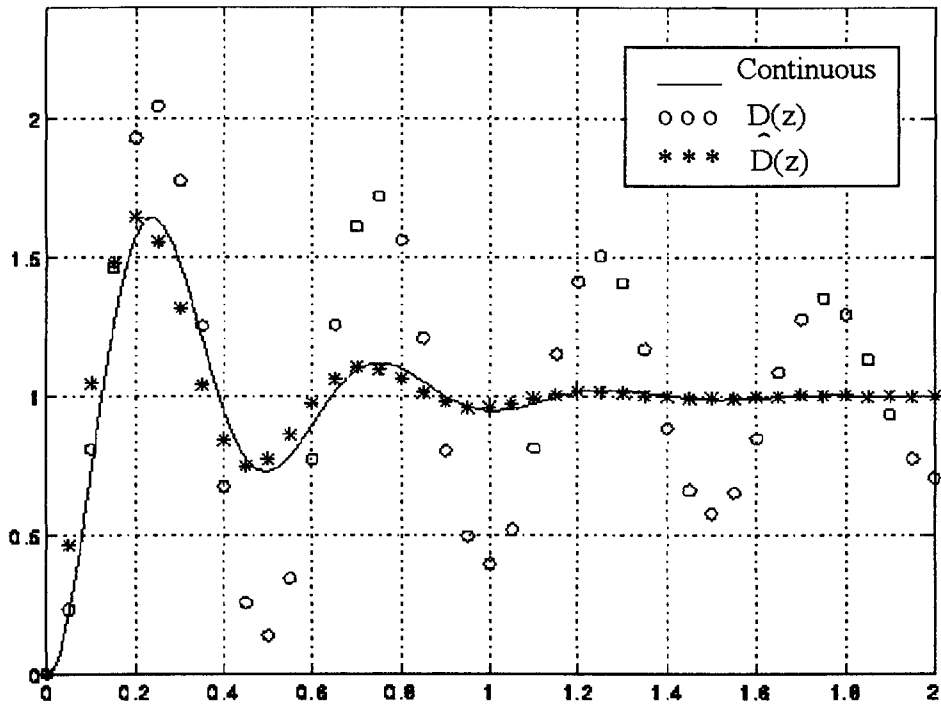


Fig. 5.2a Closed-Loop Step Response of Example Part(a) with $T=0.05$ s

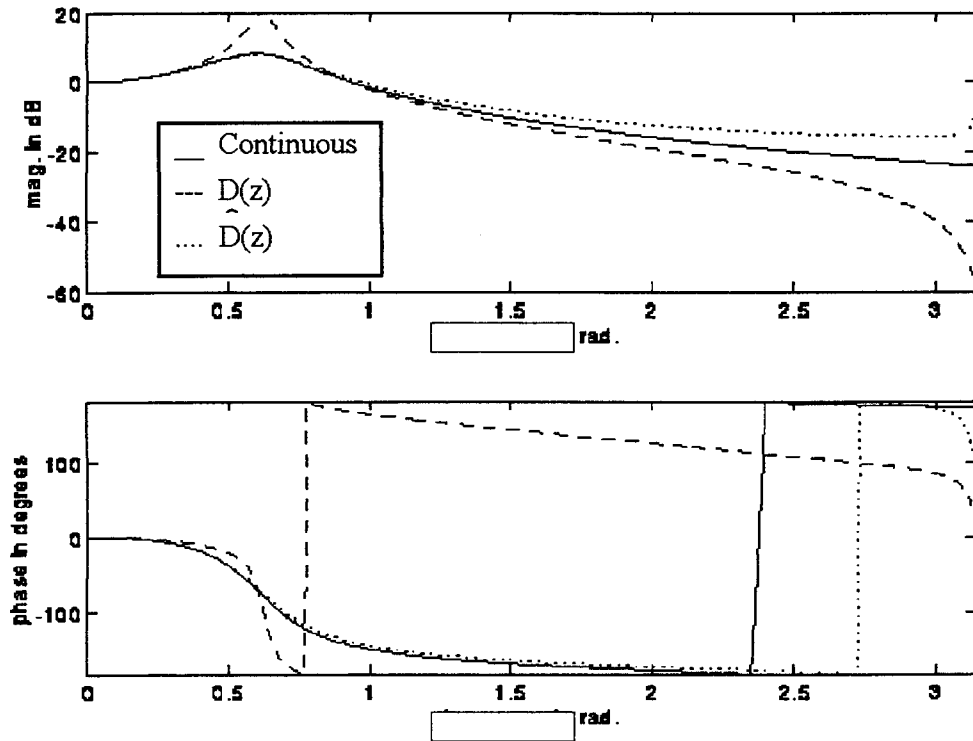


Fig. 5.2b Closed-Loop Frequency Response of Example Part(a) with $T=0.05$

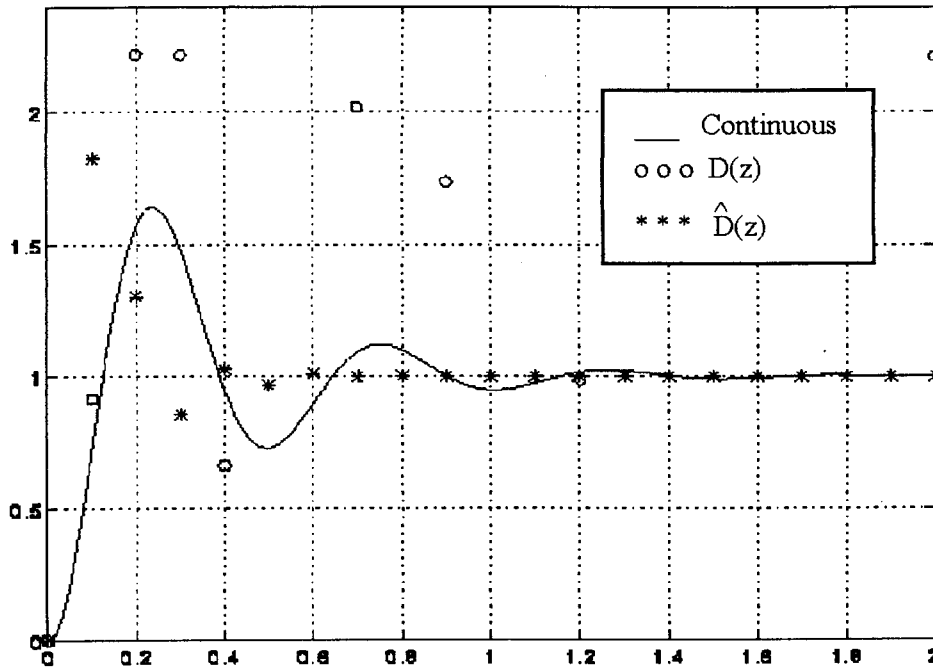


Fig. 5.3a Closed-Loop Step Response of Example Part (a) with $T=0.1$ s

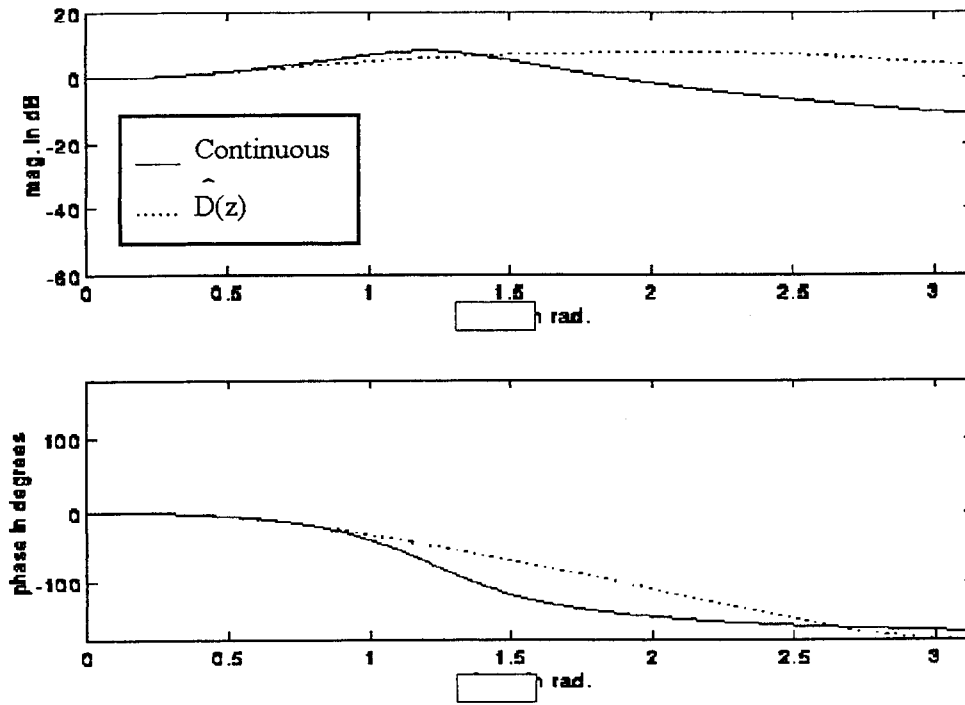


Fig. 5.3b Closed-Loop Frequency Response of Example Part (a) with $T=0.1$ s

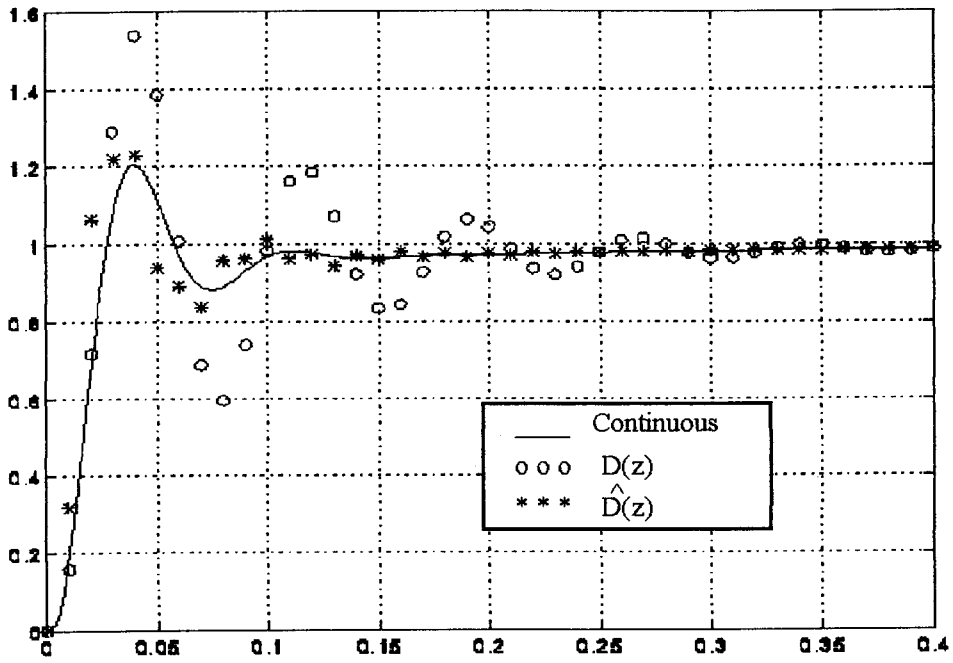


Fig. 6a Closed-Loop Step Response of Example Part(b)

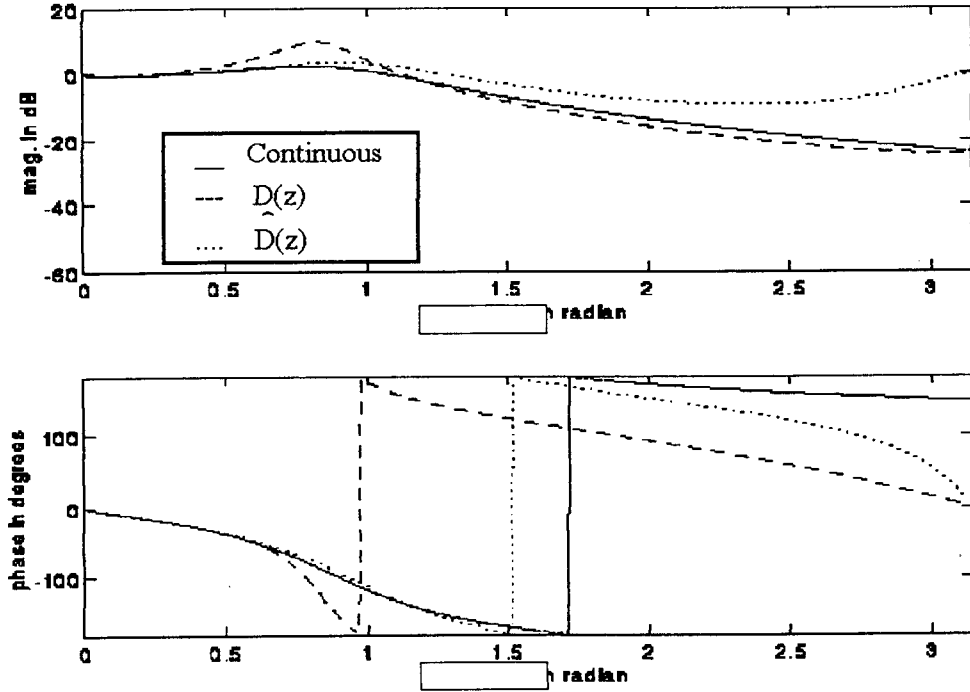


Fig. 6b Closed-Loop Frequency Response of Example Part(b)

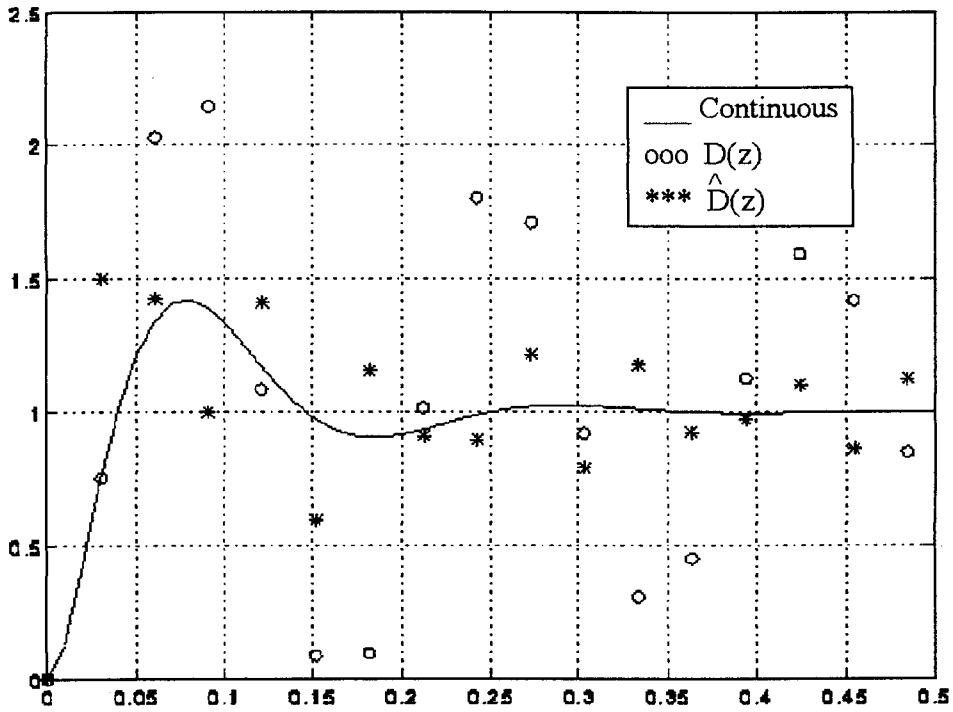


Fig. 7a Closed-Loop Step Response of Example Part(c)

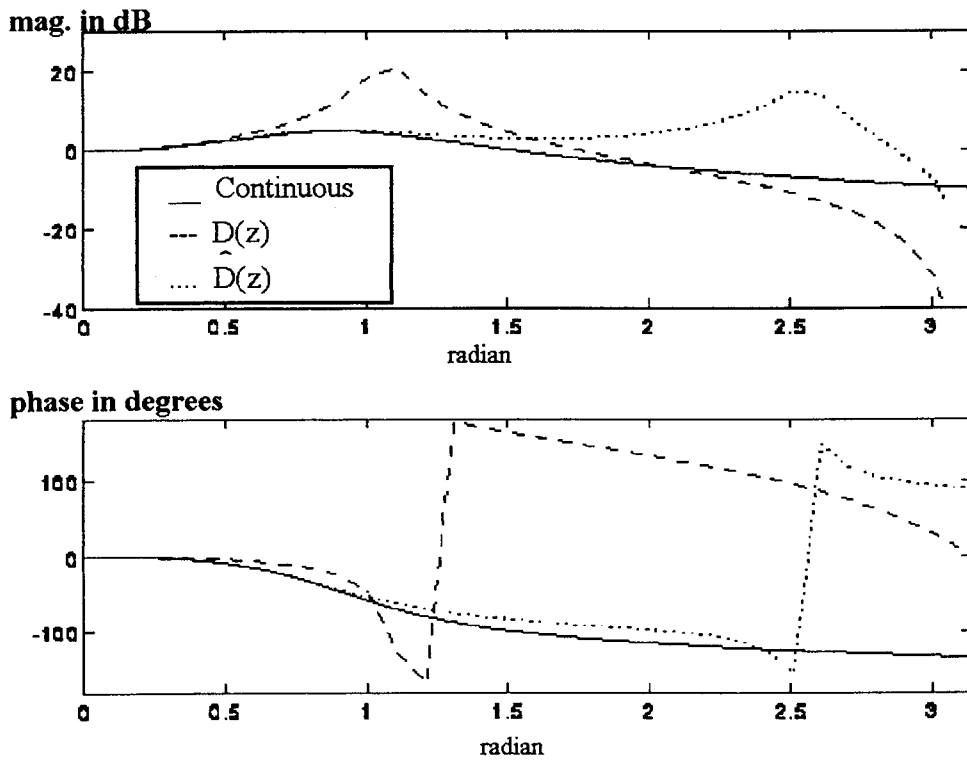


Fig. 7b Closed-Loop Frequency Response of Example Part(c)

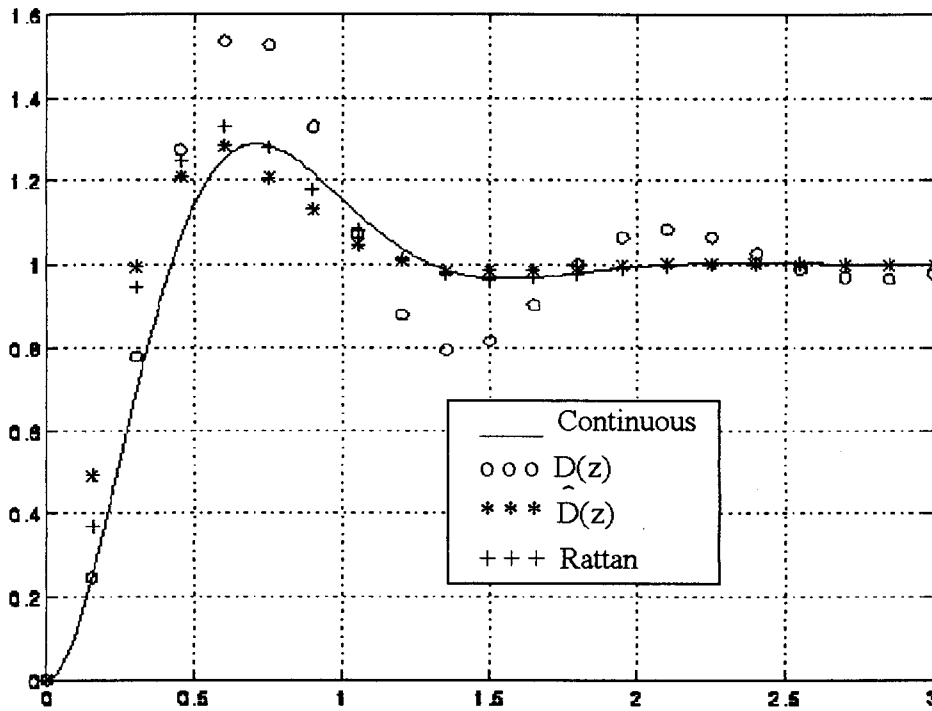


Fig. 8a Closed-Loop Step Response of Example Part(d)

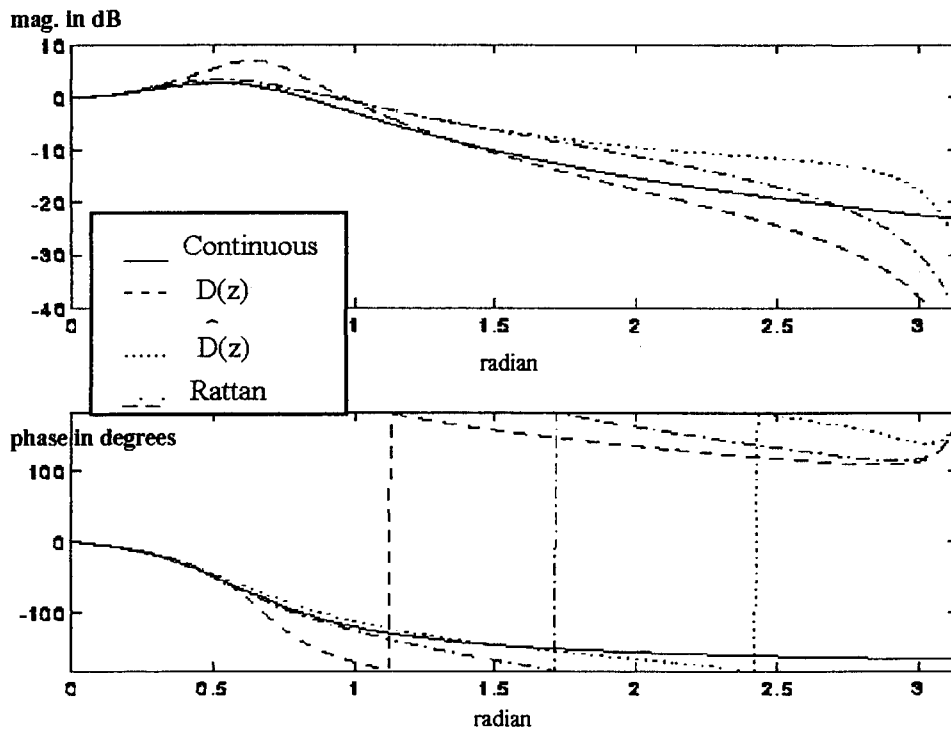


Fig. 8b Closed-Loop Frequency Response of Example Part(d)