# DEVELOPMENT OF THE NIST ROBOT CRANE TELEOPERATION CONTROLLER

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**ABSTRACT** An inexpensive and easy to operate controller has been developed for the teleoperation control of the NIST Robot Crane. This robot crane has the Stewart platform parallel link manipulator design, with cables as parallel links and winches as actuators. The controller is currently capable of rate-position control and force control of the crane platform. The control action can take place with respect to the base-frame coordinate axes, the moving platform coordinate axes, or the tool-tip coordinate axes.

The front panel display of the controller is designed to interact with the operator in simple, easy to understand, menu selection type icons. Critical parameters are displayed in real time plots and digital displays. A 3-D animation window of the workspace and the moving crane platform can help the operator with command planning and premotion detecting of collisions.

The operator can interact with the controller without the need for any computer programming knowledge and without having to type any commands. The rate-position movement commands are input from a spaceball or a Stewart mechanism joy-stick. Critical control mode selection choices can be made from a set of nine buttons attached to the spaceball joystick support.

## **INTRODUCTION**

A new type robot crane has been developed by the Robot Systems Division of the National Institute of Standards and Technology. A schematic drawing of this robot, called the NIST Robot Crane, is shown in Figure 1. It consists of two Stewart mechanism structures one inside the other. The outside mechanism is made of metal tubes of fixed length and serves as a structural support to a moving crane platform supported by the inner mechanism. This platform is suspended by six braided steel cables which form the inner mechanism. As long as the cables are in tension they form a kinematically constrained mechanism which controls the position and orientation of the moving platform. The lengths of the cables are controlled by six winches thus allowing the control of the moving platform motions.

Two functioning prototype models of the NIST Robot Crane have been constructed, a large model made up of 6 meter long Aluminum tubes and a small mobile one. A picture of the large model is shown in Figure 2. A detail description of this model is given in [Albus, J., et al., 92]. The first teleoperation controller built was a simple analog joystick connected directly to the power amplifiers driving the winches. The joystick was another small size Stewart platform mechanism shown in Figure 3. The moving platform was fitted with a handle which allowed the operator to control its position and orientation with respect to the base plate. The change in the lengths of the links was measured with six spring loaded potentiometers. The output voltages were used as the inputs to the power amplifiers, following a sequence of connections which is dictated by the relative orientation between the joystick and the controlled robot crane mechanism. The power amplifiers were set to operate in their armature feedback mode, thus controlling the velocity of the winches for low speeds. This simple controller worked well and produced adequate rate control of the moving platform in the central section of the work-space. Close to the edges of the workspace the performance was not so good. Furthermore this controller lacked the flexibility which is necessary in order to perform certain difficult crane operations. For these reasons it was decided to develop a digital NIST Robot Crane controller. This paper describes the development and current state of this second controller.

## **CONTROLLER HARDWARE**

Whenever possible standard commercially available components were used for the construction of the controller. Figure 4 shows the schematic diagram of the controller hardware. It consists of the computer, the power amplifiers, the joysticks and the sensors. <u>Computer and Power Amplifiers</u> -- A Macintosh IIfx\* computer was used as the central processor controller. Several specialized boards were added to the Nu-bus\* of the computer to handle the data acquisition and servo control needs of the controller. These include two servo control boards (M2 and M3 in Fig. 4), a DMA (Direct Memory Access) board, a D/A (Digital-to-Analog) converter board\* and an A/D (Analog-to-Digital) converter board (M6 and M5 in Fig. 4).

The spaceball joystick communicates with the controller through the serial port of the computer. The two servo boards accept the input signals from the six cable encoders (three per board) and translate them to cable length information from a reference home pose (moving platform position and orientation). The cable lengths information is compared to the commanded lengths and the appropriate power amplifiers command signals are produced. The servo boards use a PID control algorithm and are capable of either trapezoidal profile position control, velocity, or velocity profiling control operation. The servo boards outputs are input commands to six power amplifiers which provide power to the six cable winches.

The D/A converter board can also be used to generate input commands to the six power amplifiers. The main advantage of using this board, instead of the servo boards, is the freedom to use any desired control algorithm and even torque control, which is necessary for such operations as weight and friction compensation and stiffness control. The disadvantage is that the servo cycle time might increase and that this is time added to the operating time of the central controller processor. The A/D converter board has sixteen input channels available and is used to monitor all the analog sensors, like load-cells and potentiometers, as well as the Stewart joystick whenever it is used instead of the spaceball joystick.

The power amplifiers used are capable of either velocity or torque control mode. To keep the winch spooling mechanism simple no tachometers are used. Thus in the case when the velocity control mode is selected the power amplifiers are operating under armature feedback.

<u>Joysticks and Sensors</u> -- Two different types of joysticks have been tested for sending the move rate commands to the controller, a spaceball and a Stewart mechanism joystick. The main part of the spaceball joystick (model 2003 SPACEBALL TECHNOLOGIES Inc., Lowell, MA)\* consists of a hand ball mounted on top of a six degree-offreedom force/torque sensor (see Fig. 4). When the operator holding the ball applies a force and/or moment, it is measured, digitized and its Cartesian coordinate frame X, Y, Z axes components are sent to the central controller processor through a serial port line. The base of the joystick has nine buttons which allow the operator to signal the central controller processor when he/she wants to change to a new control mode.

The Stewart mechanism joystick is a home made device very similar to that of the robot crane itself (see Fig. 3). It consists of two small size equilateral triangles connected by six spring loaded linear potentiometers forming the legs of a Stewart mechanism. A handle is mounted on top of the smaller of the two triangles allowing the operator to move it with respect to the larger triangle. Any changes of the position and orientation of the small triangle result in changes to the outputs of the potentiometers. The potentiometers outputs are measured, digitized and interpreted by the central controller processor. It determines the desired position and orientation changes of the robot crane moving platform. The main disadvantages found with the use of this joystick were, the non-linearities and unreliability of the potentiometer sensors, and the time required to interpret the potentiometers outputs. There are no mode selection buttons on this joystick although they can be added if desired.

The six cable encoders, one for each winch spooling mechanism, are mounted on the shafts of rollers which are pressed against the cable, but are free to rotate [Bostelman, R., et al., 92]. Each encoder generates 3600 phase quadrature counts per rotation. This corresponds to approximately 43 counts/mm (1100 counts/1"). The force control sensor is a simple linear potentiometer mounted between the lower side of the robot crane moving platform and a leaf-spring steel plate. When the plate comes in contact with an object it deforms. The potentiometer measures this deformation which corresponds to the contact force. The axis of the sensor is normal to the plane of the platform. This is the only direction in which force can be currently controlled.

# **CONTROLLER SOFTWARE**

Figure 5 shows the diagram of the software processes and interfaces used by the digital controller. The software consists of modules written in C (resource codes) interacting with the robot crane operator through a front panel which was developed using Lab View (National Instruments Corp., Austin, TX)\*. Figure 6 shows the video monitor front panel which allows the operator to interact with the controller in real time and in a very simple intuitive fashion.

<u>Controller software architecture</u> -- The software architecture was based on [Albus, J., et al., 87] and [Fiala,

<sup>\*</sup> National Institute of Standards and Technology (NIST) does not endorse the products mentioned in this paper. These products are used for illustration purposes only and are not mentioned because they are better than another similar product.

J., et al., 92]. The rectangular boxes in Figure 5 represent software processes; the ovals represent the data which are transmitted from one process to the other. The processes are labeled according to their function as Sensory Processing (SP), World Modeling (WM), or Task Decomposition (TD), and Operator Interface (OI). The symbols used in the figure are the following:

1: The vector of the lengths of the six cables measured from the crane platform home pose.

v: The vector of the linear velocities of the six cables of the crane platform.

F: The vector of the tension forces of the six cables of the crane platform.

T: The vector of the torques applied to the six winches of the crane platform.

x: A 4x4 transformation matrix between the Cartesian coordinate frame attached to the crane platform and the base Cartesian coordinate frame.

Tcomp: The torque vector which must be applied to the six winches of the crane platform in order to compensate the gravity of the total suspended load and the friction of the cables driving mechanism.

Tstif: The torque vector which must be applied to the six winches of the crane platform as it is determined by the stiffness algorithm.

Fz: The contact force applied to a steel leaf spring plate mounted underneath the crane moving platform. This force is measured by a potentiometer mounted between the platform and the plate, so that its axis is normal to the plane of the platform (Z-axis of the platform Cartesian coordinate frame).

dx: The vector of three translations and three rotations along the X, Y, Z, axes of a Cartesian coordinate frame sent by either of the two joysticks to the controller.

cm: A number, specifying the control mode selected by the operator. The availability of a large number of control modes and the easiness of selecting them is a significant advantage offered by this controller. In the next sections there will be a more detailed discussion about this subject.

cp: A control parameter, for example: the rate of movement, selected by the operator.

The main processes currently used by the controller are the following:

Fwd. Kinematics : The forward kinematics processes of a Stewart mechanism is a time consuming iterative algorithm. The position and orientation of the moving platform are determined, with respect to a base-frame, if the cable lengths are known. The sizes of the two platforms and the coordinates of the suspension points, with respect to the same frame have to be known too. If the Stewart joystick is used to control the robot crane then two "Fwd. Kinematics" processes are needed, one for the crane and one for the joystick.

Gravity-Friction : This process determines the amount of torque necessary for each of the six winches to balance its cable tension and the friction of the driving mechanism. The cable tension can be estimated based on

the current pose of the crane platform, if the total suspended weight is known. The torque necessary to compensate for friction has to be determined experimentally. The controller can be programmed to perform this experiment without the assistance of the operator.

Inv. Kinematics : The inverse kinematics process of a Stewart mechanism is a relatively simple and fast Using equations (7) of the Appendix this algorithm. process determines the cable lengths which correspond to a certain desired dx. The process has to know the control mode cm desired by the operator. Another function served by this process is a simple safety check to determine whether the commanded position of the crane platform suspension points would not result in a significant increase of the cables tension. This will happen if a suspension point of the crane platform comes too close to the plane defined by the suspension points of the upper platform. When this happens one winch pulls against the other, in order to balance the weight, and the corresponding cable tensions can rise to unsafe levels.

Stiffness : This process determines the amount of torque necessary for each of the six winches in order to control the stiffness of the crane platform. This process has not been implemented to the controller yet.

The operator can choose either to use the two Servo: servo boards operating under their own processors or a Macintosh processor servo algorithm. The servo boards perform a trapezoidal profile position control, while the Macintosh processor servo algorithm uses no profiling. The cycle time of the servo boards is approximately 340 µs (appr. 3 KHz), while the Macintosh processor servo algorithm cycle time is approximately 45 ms. For crane platform speeds lower than 100 mm/s a simple PD (Proportional-Derivative) control algorithm is used. The Macintosh processor servo algorithm, after tuning, gives very good performance. The servo boards are currently being tested and performance information is not available For higher speeds overshoot appears and more vet. sophisticated control algorithms are necessary. Torque control can only be implemented with the D/A converter board and is currently being tested, too.

Joystick teleoperation software -- In the case of the spaceball joystick this software consists of the spaceball controller process and the teleoperation process. The spaceball controller process is part of the spaceball unit and is responsible for the initialization and the reading of the load-cell signals and buttons and the serial communication with the controller when there is an input from the operator. The ball force/moment and the button settings are written to the serial port buffer. This activates the teleoperation process which interprets the ball force/moment reading as desired rates of crane platform motion dx, and the button settings as control mode (cm), or control parameters (cp) settings. For example, the pressing of button #6 is interpreted to mean that the desired control mode (cm=6) is force control. The pressing of button #5 is interpreted to mean that the desired control mode (cm=5) is

translation in a certain direction, etc. The pressing of buttons #1, #2, #3, results in the change of a control parameter (cp), in this case the rate of movement. Button #1 sets it to zero, button #2 decrements it by a certain amount and button #3 increments it by the same amount. The rate in this case is similar to an automobile throttle. Once the teleoperation process is activated, all the related processes become activated too and execute sequentially based on the flow indicated by the arrows. If the servo boards are used they operate concurrently.

The Stewart joystick has no controller process and no buttons. This necessitates the continuous monitoring of its output by the Stewart joystick Fwd. Kinematics process, which is rather time consuming. The main advantage of this joystick is its low cost, but due to its present design limitations it is not used anymore.

Front panel display software -- An intuitive, easy to interact in real time, iconic front panel operator interface was developed. Figure 6 shows an example of this interface. On the left there is a column of control modes (cm) or control parameters (cp) selection buttons. These are activated or deactivated by the computer mouse. When selected the first will initiate servo action, the second will set the rate to zero, the third will initiate force control, the fourth will force the commanded motions to take place with respect to the crane platform frame, the fifth will take the crane platform back to the home pose, and the sixth will set the command signals to zero and go through a program termination procedure. The two digital scopes in the middle plot the values of the commanded and measured cable lengths. The dynamic response of the crane can be monitored from these plots. On the right side of the panel, the various gains and the selected rate are displayed. The gains will not be changeable by the operator in the final version of this controller. Below the gain icons is an animation window is used to check for platform collisions or other constrain violations. In its final form the window will display the crane workspace with all the objects located in it in three dimensional space. The crane platform will be displayed as a triangle moving in this space. When a move command is issued, the platform will move from the current pose to the commanded pose immediately and warn the controller and the operator of any constrain violation before the platform has a chance to move to that location.

# TESTING

Two applications were chosen for the preliminary testing of this controller. One, lifting and positioning a heavy pipe in three dimensional space so that its axis and flange line up with another similar pipe and two, the grinding of a Steel I-beam. Figures 7 and 8 show the crane platform performing these two operations. Both tests have been repeated many times very successfully. To facilitate the lining up of the two pipes another control mode was created which allows motion about a frame located on the flange of the manipulated pipe. This is activated by button #7 of the spaceball joystick.

#### SUMMARY

This paper describes the digital teleoperation controller of the NIST Robot Crane. The robot crane has the Stewart platform parallel link manipulator design, with cables as parallel links and winches as actuators. The controller was developed using relatively inexpensive commercially available hardware and software. The controller is currently capable of rate-position control and force control of the crane platform. The architecture of the controller software was based on the NIST RCS, with a user-friendly, easy to understand, front panel interface. Two different types of joysticks have been tested. Low speed tests of robot grinding and heavy load manipulation gave very good results.

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### APPENDIX

## Simplified Mathematical Model

A simplified model of the basic structure of the robot crane, which consists of the wireropes suspension system, is shown in Figure A1 for the initial home position. The overhead support and the suspended moving platform are represented by two equilateral triangles. The overhead triangle is assumed to be fixed in space and has three vertices located at

$$\begin{array}{l} \mathbf{A} : (-\mathbf{b}, -\mathbf{b}\sqrt{3}/3, 0), \\ \overline{\mathbf{B}} : (\mathbf{b}, -\mathbf{b}\sqrt{3}/3, 0), \\ \overline{\mathbf{C}} : (0, 2\mathbf{b}\sqrt{3}/3, 0). \end{array}$$
(1)

with respect to a fixed Cartesian coordinate frame (X,Y,Z), based on the fixed upper triangle (see Figure A1). The origin of this coordinate frame is located at the centroid of the triangle and it will be called the base-frame. 2b is the length of the side of the overhead triangle and 2a is the length of the side of the lower triangle. The height h is the distance between the two triangles, which are parallel to each other at the home position.

The lower triangle has three vertices located at

$$\begin{array}{l} D: (0, -2a\sqrt{3}/3, 0),\\ \overline{E}: (a, a\sqrt{3}/3, 0),\\ \overline{F}: (-a, a\sqrt{3}/3, 0). \end{array}$$

with respect to coordinate frame (Xp,Yp,Zp), based on the suspended moving platform triangle (see Figure A1). The origin of this coordinate frame is located at the centroid of the triangle and it will be called the moving-platformframe. The coordinate transformation between the baseframe and the moving-platform-frame for the relative position and orientation selected here is given by

$$\overline{\mathbf{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(3)

Let the suspended moving platform triangle undergo a rigid body motion characterized by three displacements,  $u_X$ ,  $u_y$ ,  $u_z$ , with respect to coordinate axes Xp,Yp,Zp and three successive rotations performed in the following sequence : first, rotation by an angle  $\phi_X$  about Xp-axis, then  $\phi_Y$  about Yp-axis, and then  $\phi_Z$  about Zp-axis. The corresponding moving platform differential transformation is given by

$\overline{\Delta T} =$	$\cos\phi_z \cos\phi_y$	$cos \phi_z \ sin \phi_y \ sin \phi_x \text{-} sin \phi_z \ cos \phi_x$	$cos\varphi_z  sin\varphi_y  cos\varphi_x + sin\varphi_z  cos\varphi_x$	ux	(4)
	$sin\phi_z \cos\phi_y$	$sin\phi_z sin\phi_y sin\phi_x + cos\phi_z cos\phi_x$	$sin\phi_z sin\phi_y cos\phi_x$ - $cos\phi_z sin\phi_x$	uy	
	$-\sin\phi_y$	$\cos\phi_y \sin\phi_x$	$\cos\phi_y \cos\phi_x$	uz	
	0	0	0	1 _	

The new coordinate transformation between the base-frame and the moving-platform-frame is given by

$$\overline{T}_{n} = \overline{T} \overline{\Delta T} = \begin{bmatrix} \cos\phi_{z} \cos\phi_{y} & \cos\phi_{z} \sin\phi_{y} \sin\phi_{x} - \sin\phi_{z} \cos\phi_{x} & \cos\phi_{z} \sin\phi_{y} \cos\phi_{x} + \sin\phi_{z} \cos\phi_{x} & u_{x} \\ \sin\phi_{z} \cos\phi_{y} & \sin\phi_{z} \sin\phi_{y} \sin\phi_{x} + \cos\phi_{z} \cos\phi_{x} & \sin\phi_{z} \sin\phi_{y} \cos\phi_{x} - \cos\phi_{z} \sin\phi_{x} & u_{y} \\ -\sin\phi_{y} & \cos\phi_{y} \sin\phi_{x} & \cos\phi_{y} \cos\phi_{x} & u_{z} + h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(5)

After the end of the motion the new coordinates of the vertices of the moving platform with respect to the base-frame are

$$\begin{split} &\overline{D}': (-2aQ_{12}\sqrt{3}/3 + u_x, -2aQ_{22}\sqrt{3}/3 + u_y, -2aQ_{32}\sqrt{3}/3 + u_z + h), \\ &\overline{E}': (aQ_{11} + aQ_{12}\sqrt{3}/3 + u_x, aQ_{21} + aQ_{22}\sqrt{3}/3 + u_y, aQ_{31} + aQ_{32}\sqrt{3}/3 + u_z + h), \\ &\overline{F}': (-aQ_{11} + aQ_{12}\sqrt{3}/3 + u_x, -aQ_{21} + aQ_{22}\sqrt{3}/3 + u_y, -aQ_{31} + aQ_{32}\sqrt{3}/3 + u_z + h). \end{split}$$

Where  $Q_{ij}$  is the ith row, jth column element of matrix  $\overline{T}_n$ .

To simplify the equations it is assumed here that the wires are suspended from a single point (the robot crane controller does not make that assumption), in that case the vectors  $\bar{l}_i$ ' (i = 1,...,6) defining the new position of the wires can be found from (1) and (6) and are expressed as

$$\bar{l}_{1}' = \bar{A} - \bar{D}' = (-b + 2aQ_{12}\sqrt{3}/3 - u_{x}, -b\sqrt{3}/3 + 2aQ_{22}\sqrt{3}/3 - u_{y}, -h + 2aQ_{32}\sqrt{3}/3 - u_{z})$$

$$\bar{l}_{2}' = \bar{B} - \bar{D}' = (b + 2aQ_{12}\sqrt{3}/3 - u_{x}, -b\sqrt{3}/3 + 2aQ_{22}\sqrt{3}/3 - u_{y}, -h + 2aQ_{32}\sqrt{3}/3 - u_{z})$$

$$\bar{l}_{3}' = \bar{B} - \bar{E}' = (b - aQ_{11} - aQ_{12}\sqrt{3}/3 - u_{x}, -b\sqrt{3}/3 - aQ_{21} - aQ_{22}\sqrt{3}/3 - u_{y}, -h - aQ_{31} - aQ_{32}\sqrt{3}/3 - u_{z})$$

$$\bar{l}_{4}' = \bar{C} - \bar{E}' = (-aQ_{11} - aQ_{12}\sqrt{3}/3 - u_{x}, 2b\sqrt{3}/3 - aQ_{21} - aQ_{22}\sqrt{3}/3 - u_{y}, -h - aQ_{31} - aQ_{32}\sqrt{3}/3 - u_{z})$$

$$\bar{l}_{5}' = \bar{C} - \bar{F}' = (aQ_{11} - aQ_{12}\sqrt{3}/3 - u_{x}, 2b\sqrt{3}/3 + aQ_{21} - aQ_{22}\sqrt{3}/3 - u_{y}, -h + aQ_{31} - aQ_{32}\sqrt{3}/3 - u_{z})$$

$$\bar{l}_{6}' = \bar{A} - \bar{F}' = (-b + aQ_{11} - aQ_{12}\sqrt{3}/3 - u_{x}, -b\sqrt{3}/3 + aQ_{21} - aQ_{22}\sqrt{3}/3 - u_{y}, -h + aQ_{31} - aQ_{32}\sqrt{3}/3 - u_{z})$$

 $I_6 = A-F = (-b+aQ_{11}-aQ_{12}\sqrt{5/5-u_x}, -b\sqrt{5/5+aQ_{21}-aQ_{22}\sqrt{5/5-u_y}}, -h+aQ_{31}-aQ_{32}\sqrt{5/5-u_z})$ This mathematical analysis is used by the "Moving Platform Frame" control mode (see Fig. 6) algorithm in order to determine the lengths of the wire-ropes necessary to force the commanded motions to take place with respect to the moving platform frame. This analysis is valid for any coordinate transformation  $\overline{T}$ , it can even be extended to the transformation between a tool mounted on the moving platform and the base-frame. Thus to line-up the flange holes of the two pipes in the case of the pipe positioning test (see Fig. 7) motion about a coordinate frame located on the flange of the moving pipe is used, selected from a joystick button. The default control mode is motion about the base-frame, which requires a modified equation (5), since in this case  $\overline{T}_n = \Delta T \overline{T}$ .

The balance of forces acting on the lower platform requires that

$$\bar{f} + \sum_{n=1}^{6} \bar{f}_n = 0$$
 (8)

where  $\bar{f}$  is the external force applied at the center of gravity of the platform. The directions of the wire tensions  $\bar{f}_n$  are given by the corresponding vectors of equations (7).

The balance of moments acting on the lower platform requires that

$$\overline{\mathbf{m}} + \overline{\mathbf{Q}} \,\overline{\mathbf{D}} \,\mathbf{x} \,(\overline{\mathbf{f}}_1 + \overline{\mathbf{f}}_2) + \overline{\mathbf{Q}} \,\overline{\mathbf{E}} \,\mathbf{x} \,(\overline{\mathbf{f}}_3 + \overline{\mathbf{f}}_4) + \overline{\mathbf{Q}} \,\overline{\mathbf{F}} \,\mathbf{x} \,(\overline{\mathbf{f}}_5 + \overline{\mathbf{f}}_6) = 0 \tag{9}$$

where  $\overline{m}$  is the external moment applied upon the lower platform.  $\overline{Q}$  is the directional cosines part of matrix  $\overline{T}_n$  (the first 3x3 elements).

- Fig. 1. Schematic drawing of the NIST Robot Crane.
- Fig. 2. Picture of the large NIST Robot Crane.
- Fig. 3. Stewart platform joystick.
- Fig. A1. Robot crane cable support structure.
- Fig. 7 Pipe positioning test
- Fig. 8 I beam grinding test