The Effect of Time-Delay and Discrete Control on the Contact Stability of Simple Position Controllers

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Abstract

By analysis of the driving-point admittance, it is shown how time-delays and discrete control can create instabilities for a simple position controller in contact with the environment. The lowest frequency of contact instability due to time-delay or sampling is determined analytically. It is shown how mechanical compliance between the motor and point of contact can eliminate these instabilities.To achieve the best relative stability when contacting arbitrary environments, the mechanical/control design of manipulators should maintain a critical relationship between the frequency of the compliant mode and a frequency associated with contact instability.

1. Introduction

The simple proportional-derivative (PD) controllers used for controlling most robots show a remarkable robustness in a number of tasks including those which involve contact with the environment. Recently, some authors have noted that the time-delays and sampling in these controllers should have a detrimental effect on stability during contact with certain environments [1,2]. However, since such instabilities are not often observed in practice, these authors conclude that inherent mechanical compliance ultimately stabilizes these systems. The idea that the robustness of simple position controllers in contact is due to mechanical compliance is analyzed in detail in this paper.

First, consider a force generator (motor) u acting on a damped mass, as shown in Figure 1. An ideal proportional-derivative position control law

$$u = K_p(x_d - x) - K_v v \qquad (1)$$

with K_p, $K_v > 0$, would produce a stable system, both in free-space and in contact, since it has a passive physical equivalent [1]. That is, the same control could be achieved by attaching a physical spring and damper to the mass, and physical systems are always passive, or energy dissipating. Alternatively, one could examine the passivity of the driving-point admittance of the mechanical system. If the driving-point admittance does not have more than 90° of phase shift, then there is a passive physical equivalent and the system will be stable in contact with any passive environment [1,2]. To investigate stability it is only necessary to look at the admittance due to feedback [2], so

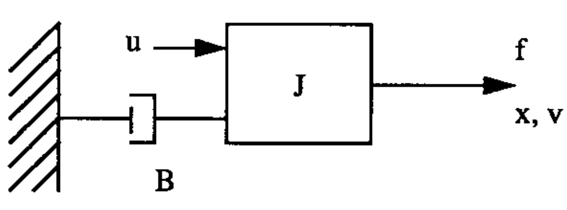

Figure 1. Damped mass system.

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for (1), the driving-point admittance is

$$Y(s) = \frac{-v(s)}{f(s)} = \frac{s}{(Js+B)s + (K_p + K_v s)} \tag{2}$$

The phase of Y(jω) for this system is between 90° and -90° for all $\omega > 0$, and the system is stable in contact with arbitrary environments.

2. Effect of Time-Delay

For the controller (1) with a time-delay T, the driving-point admittance is

$$Y(s) = \frac{s}{(Js+B)s + [K_p + K_v s]\, e^{-Ts}} \tag{3}$$

The real part of Y(jω) is

$$\frac{K_v\omega^2\cos(T\omega) - K_p\omega\sin(T\omega) + B\omega^2}{[-J\omega^2\sin(T\omega) + B\omega\cos(T\omega) + K_v\omega]^2 + [-B\omega\sin(T\omega) - J\omega^2\cos(T\omega) + K_p]^2}$$

The positive real condition, Real{ Y(jω) } ≥ 0, can be used to test the passivity of the admittance [1]. As also derived in [1], this yields that the system presents a passive admittance as long as

$$K_v\omega\cos(T\omega) - K_p\sin(T\omega) + B\omega \geq 0 \tag{4}$$

which indicates that the system may exhibit passivity violations for frequencies above a certain value. These violations are characterized by phase in excess of -90° over specific frequency intervals, which means that the system will exhibit instability in contact with springs of certain stiffnesses.

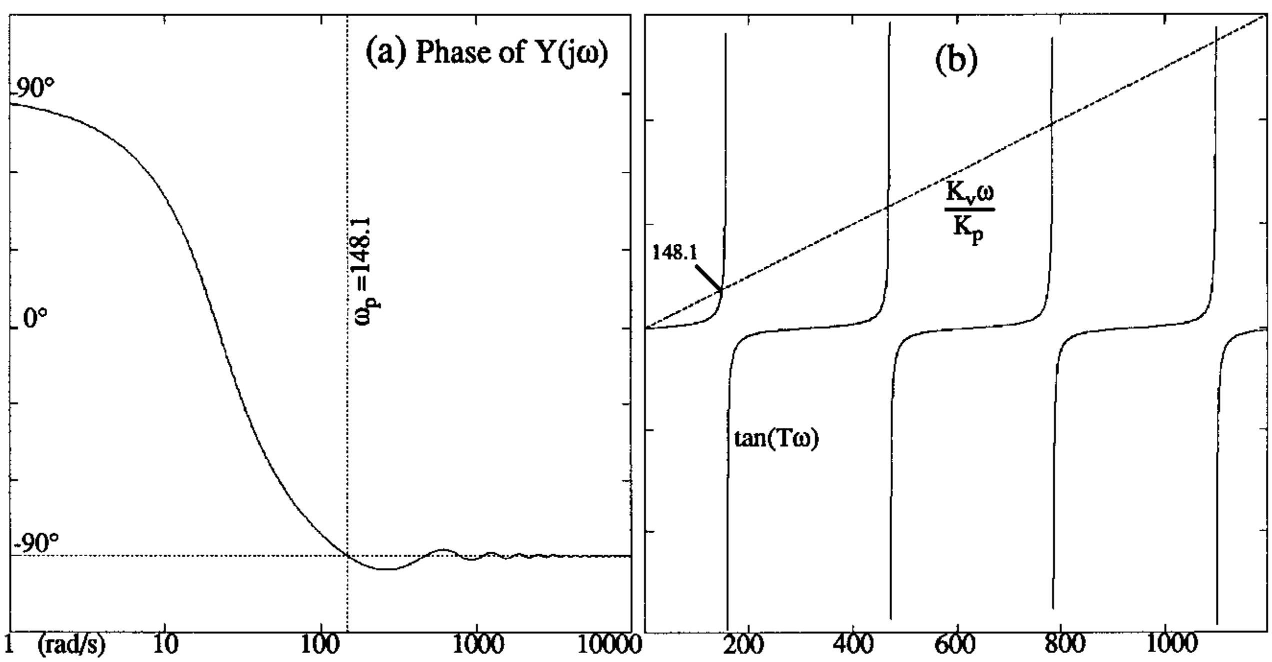


Figure 2. Effect of time-delay on the passivity of the damped mass system.

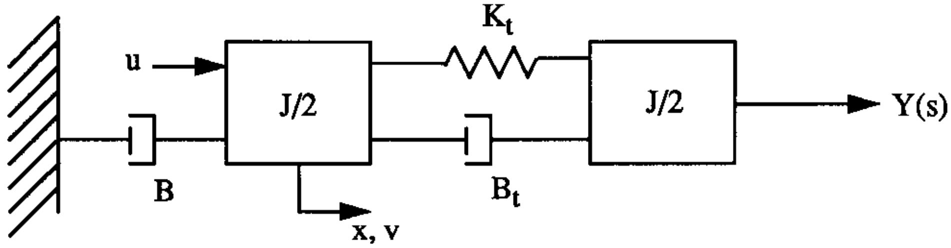


Figure 3. System with compliance.

Figure 2(a) shows the phase of $Y(j\omega)$ for J=1, B=0, T=0.01, K_p=360, and K_v=27. (Any consistent set of units may be chosen for these numbers.)

Without friction, the frequencies delimiting the passivity violations are the intersections of the line $K_v\omega/K_p$ with $tan(T\omega)$, as depicted in Figure 2(b). The frequency of the first such intersection is the frequency ω_p at which passivity is first violated. The presence of friction may stabilize the system. The amount of damping required to completely stabilize the contact instability can be used as a measure of relative stability. For the system of (3), damping can stabilize the system only at unreasonable levels. Suppose substantial friction of $B=K_v$ exists. Then (4) can be written

$$\frac{K_v\omega}{K_p} \geq \frac{\sin(T\omega)}{1+\cos(T\omega)} = \tan\left(\frac{T\omega}{2}\right) \qquad (5)$$

which indicates that contact instabilities still exist but for stiffer environments. Note also that as B increases, so does ω_p. Only when $B \gg K_v$ can the system be completely stabilized. Such high levels of friction clearly limit performance.

For most robots there is some compliance between the motor and the point of contact [3]. The presence of this compliance may help stabilize the system during contact. The damped mass model can be modified to include a compliant transmission modeled as a damped spring [1], as shown in Figure 3. To simplify initially, let B_t=0. Computing Real{ $Y(j\omega)$ } = $N(\omega)/D(\omega)$, yields

$$N(\omega) = 16K_t^2K_v\omega^2\cos(T\omega) - 16K_t^2K_p\omega\sin(T\omega) + 16K_t^2B\omega^2 \qquad (6)$$

Since $D(\omega)>0$ for $\omega>0$, as before, the system will be stable in contact with arbitrary passive environments provided $N(\omega)\geq 0$. This produces condition (4) again, and apparently the system with compliance will suffer the same type of passivity violations as before. However, the effect of the compliance is to improve the relative stability of the system by requiring less damping at certain frequencies.

The undamped natural frequencies of the system are

$$\omega_{n1} = \sqrt{\frac{2K_t}{J}} \quad , \quad \omega_{n2} = \sqrt{\frac{4K_t}{J}}$$

At contact frequencies below ω_{n1} the masses move together with the spring unstretched [4], such that the transmission damping has little effect. At these frequencies the situation of (4) applies, which requires significant motor-side friction B to stabilize. Above ω_{n2} the masses move out of phase and the system may be stabilized by a smaller amount of friction B_t in the transmission. In order to have a system with the best relative stability, i.e., which can be stabilized by the least amount of friction, the requirement is

$$\omega_r < \omega_p \tag{7}$$

where

$$\omega_r \equiv \sqrt{\frac{4K_t}{J}} \tag{8}$$

This relation can be achieved by decreasing the time-delay, which increases ω_p, or by decreasing the stiffness of the transmission K_t. Although the presence of friction will also affect these values, a system design maintaining $\omega_r < \omega_p$, assuming no friction, will retain good relative stability in the actual system.

As an example, Figure 4 shows the phase of $Y(j\omega)$ for the system of Figure 3 with J=1, controlled by (1) with a time-delay of T=0.01, and K_p=360, K_v=27. When $B=B_t=0$, ω_p=148.1 rad/s, as obtained from the graph of Figure 2(b). When K_t=3000, $\omega_r < \omega_p$ and the system is made completely passive by transmission damping of B_t=0.55. On the other hand, when K_t=60000, the system requires B_t>100 (B=0) or B>100 (B_t=0) for passivity.

3. Effect of Discrete Control

A computer implementation of (1) will introduce sample and hold elements into the control law. For a discrete controller with a zero-order hold

$$u = \frac{1-e^{-hs}}{hs}\left(K_p(x_d - x) - K_v v\right) \tag{9}$$

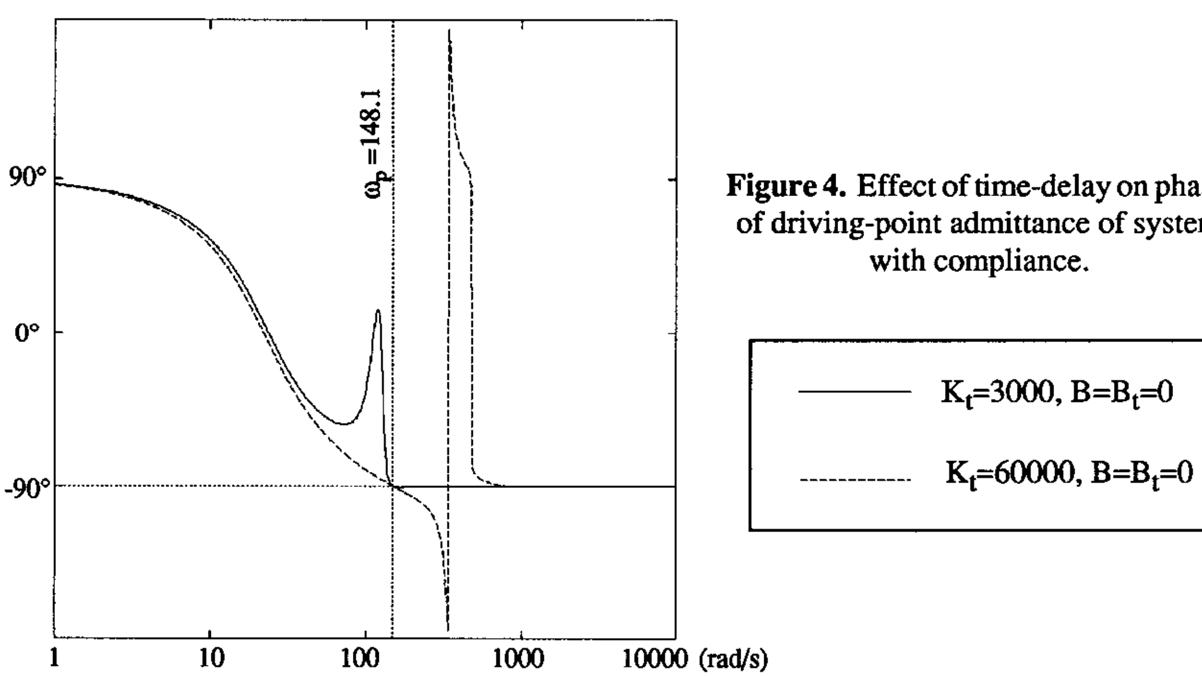


Figure 4. Effect of time-delay on phase of driving-point admittance of system with compliance.

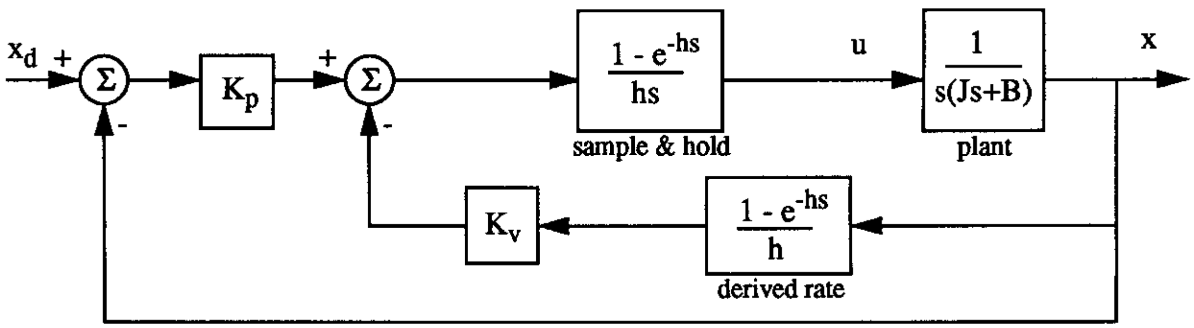


Figure 5. Discrete controller with derived rate.

Here it is assumed that any computational delays are small compared to h. Following the same reasoning as before, the positive real condition for this controller reduces to

$$K_v\omega \sin(h\omega) + K_p \cos(h\omega) + Bh\omega^2 - K_p \geq 0 \qquad (h \neq 0) \tag{10}$$

which, when B=0 and $\omega < \pi/h$, can be written

$$\frac{K_v\omega}{K_p} \geq \frac{1 - \cos(h\omega)}{\sin(h\omega)} = \tan\left(\frac{h\omega}{2}\right) \tag{11}$$

This shows that the effect of discrete control on system passivity is the same as the effect of time-delay. Likewise, the effect of compliance between the motor and the point of contact is to improve the relative stability provided the condition (7) is maintained.

A position controller with an analog velocity loop

$$u = \frac{1 - e^{-hs}}{hs}\left(K_p(x_d - x)\right) - K_v v \tag{12}$$

will generally not exhibit contact stability problems since the positive real condition is

$$K_p(\cos(h\omega) - 1) + hK_v\omega^2 \geq 0 \qquad (h \neq 0) \tag{13}$$

and this condition is only violated when $hK_p > 2K_v$. Contact instabilities in simple position controllers are principally due to delays in the velocity feedback loop.

As another example, consider the controller depicted in Figure 5 applied to the damped mass system. The positive real condition yields

$$2K_v\cos^2(h\omega) - (2K_v + hK_p)\cos(h\omega) + (hK_p - Bh^2\omega^2) \leq 0 \qquad (h \neq 0) \tag{14}$$

From which we obtain the condition

$$\frac{(2K_v + hK_p) - \Gamma}{4K_v} \leq \cos(h\omega) \leq \frac{(2K_v + hK_p) + \Gamma}{4K_v} \tag{15}$$

$$\Gamma = \sqrt{8Bh^2\omega^2K_v + (2K_v - hK_p)^2}$$

As with the previous discrete controllers, there are multiple frequency intervals of phase in excess

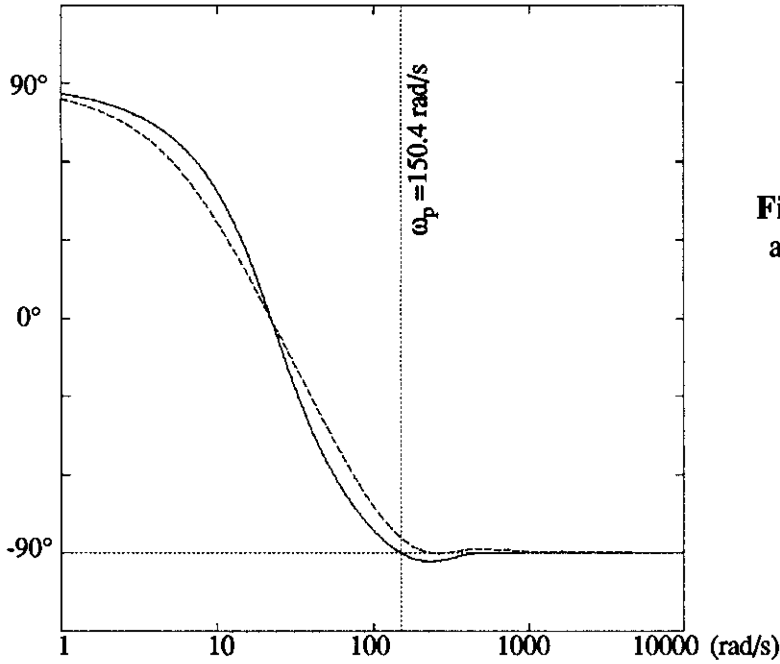


Figure 6. Phase of the driving-point admittance of the derived rate controller.

of -90°. The intervals above the first interval of passivity violation, however, have relatively little phase excess, as shown in Figure 6. The figure shows the phase for h=0.01, K_p=360, and K_v=27. From (15) with B=0, the first interval of passivity violation begins at the frequency

$$\omega_p = \frac{1}{h}\cos^{-1}\left(\frac{hK_p}{2K_v}\right) \tag{16}$$

The minimum damping B required for passivity can be obtained from (15) as well. Large amounts of damping may be required depending on the gains and sampling rate chosen. For the example of Figure 6, B>13 is required to make the system completely passive. As with the time-delay problem, the controller of Figure 5 has the same positive real condition (14) when a transmission compliance is inserted as in Figure 3. Thus, as before, the presence of transmission compliance will improve the relative stability when $\omega_r < \omega_p$.

4. Effect of Discrete Control with Torque Loop

The effect of compliance external to the controller was examined in prior sections. Some modern manipulators, however, use drive transmissions surrounded by torque loops [5]. This approach places the major mechanical compliance of the device inside the controller. To see how this affects stability, consider the model of Figure 7. A spring representing the drive compliance has been inserted between a motor mass and a damped load. The position and velocity at the load are controlled using the equivalent of a torque loop, which attempts to control the force in the spring. The analog force control law is given by

$$\gamma = K(f_d - f) \tag{17}$$

where the desired force f_d is the input u from the position controller in Figure 5, and f is the sensed force produced by the transmission spring K_t. The resonant frequency of the force loop is

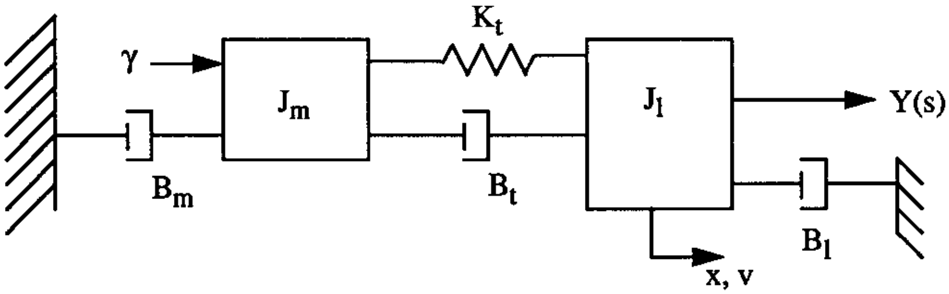


Figure 7. System with compliance internal to control.

$$\omega_r \equiv \sqrt{\frac{K_t(K+1)}{J_m}} \tag{18}$$

Applying the positive real condition when there is no damping, B_m=0, B_t=0, and B_l=0,

$$h^2 K K_t \omega^2 \left[J_m \omega^2 - (K+1) K_t\right] \left[2K_v \cos(h\omega) - hK_p\right] \left[\cos(h\omega) - 1\right] \geq 0$$

The frequency (16) is still a determining boundary of the first instability interval, but the actual frequency interval is given by

$$\omega_p \leq \omega \leq \omega_r \quad \text{when} \quad (\omega_p < \omega_r) \tag{19}$$

or

$$\omega_r \leq \omega \leq \omega_p \quad \text{when} \quad (\omega_r < \omega_p) \tag{20}$$

Thus, for the torque loop model, there is a prominent first region of contact instability. For the first case (19), the system may be passive in the upper part of the region provided that ω_r is sufficiently far from ω_p to hold the entire first instability interval starting at ω_p. In this case, the relative stability of the system in contact is similar to that of the damped mass system—a large amount of friction is needed to stabilize the system. Damping requirements will also be large when the system is unstable at ω_r, which can be a problem for (20) as well. This is due to the resonant frequency term

$$J_m \omega^2 - (K+1) K_t \tag{21}$$

in the positive real condition. For a given ω_r, this is most problematic when J_m is small, since J_m determines the steepness of (21) around ω_r.

Again, B_t is most effective in stabilizing the system for (20), rather than (19). In general, some damping is needed to stabilize the higher frequency non-passive intervals. B_l will work as well as B_t for this. However, there is another way to stabilize the prominent first interval, provided J_m is not too small, as is illustrated in Figure 8. The following base set of parameters is chosen for the figure: B_m=0, B_t=0, B_l=0, J_l=1, J_m=2, K_t=12000, K_p=360, K_v=27, h=0.01. With K=7, the situation is that of (19), and the phase of the driving-point admittance is shown as the solid line in Figure 8. Only with damping of B_m>19 and B_l=0.1, can this admittance be made completely passive. An example of (20) can be obtained by using K=1. The result is shown in Figure 8 as the dashed line. Note that the passivity violation for this case is due to phase in excess of 90°, rather than -90°. Thus, the system can be made passive by the addition of phase lag in the PD loop. Since the controller

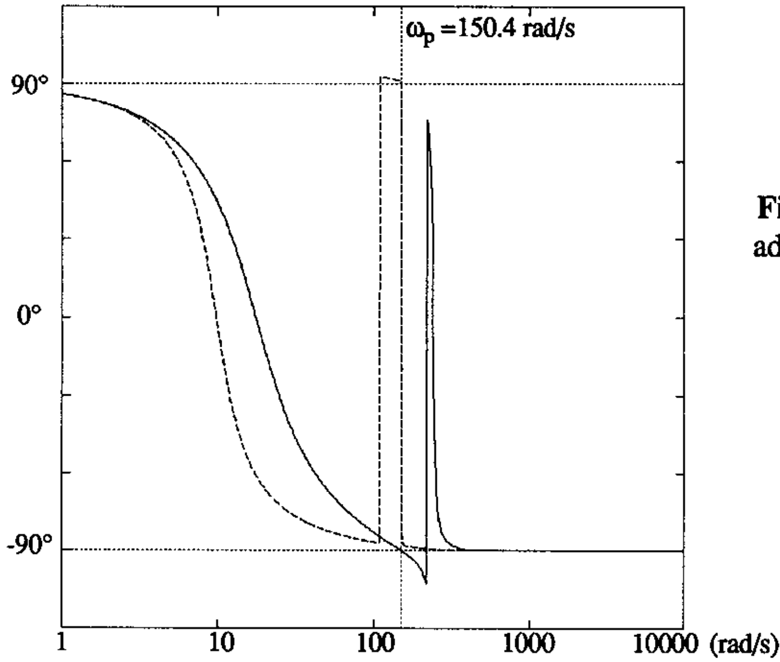
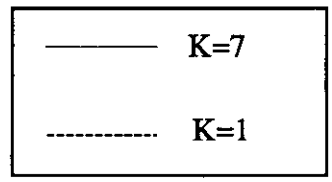


Figure 8. Phase of the driving-point admittance of torque loop model controlled by derived rate controller.



will typically contain some phase lag due to computational delay, it is possible to design the system so that it is completely passive without excessive damping provided $\omega_r < \omega_p$. For this example, lag due to a computational delay of T=0.0035 and B_l=0.15 makes the system stable in contact with arbitrary environments.

5. Conclusions

Simple position controllers may exhibit contact instabilities, particularly when low sampling rates and long time-delays are involved. Most commercial robot controllers are stable in contact because they are typically controlled with analog PD controllers. When commercial systems use discrete control, they use sampling rates as high as possible. This generally ensures that the frequencies of passivity violations are above the resonant mode of the mechanical structure, $\omega_p > \omega_r$. As shown in this paper, achieving this relationship for these two critical frequencies makes contact stability much easier to achieve.

6. References

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