

Generation of Smooth Trajectories Without Planning

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Abstract

A technique for generating smooth trajectories of a system is presented. The approach is to find a dynamical equation with the desired transient behavior and use it as the basis of the control algorithm. It is shown that an appropriate dynamical equation can be obtained by varying the position and velocity gains of a proportional-derivative control loop over the duration of the movement. This dynamical equation can be used to generate trajectories on-line with minimal planning. The resulting manipulator control system responds to path errors in a more reasonable fashion than traditional approaches to trajectory control since explicit replanning is not required.

Keywords:

robot control, trajectory generation, smoothing, dynamical systems, proportional-derivative control, articulated mechanisms

1. Introduction

Most manipulator control systems move the robotic manipulator by periodically updating the attractor of a simple control loop. (The “attractor” is commonly called the “reference” or “command” signal, however the language of [4] will be adopted here to provide a more geometric interpretation.) The attractor typically consists of joint positions, velocities, and accelerations, but may also involve force or position in Cartesian coordinates [1,2]. The approach of periodically updating the attractor suffers from the fact that every state through which the manipulator is to pass must be planned well in advance. Thus, with planning, the manipulator can only take the one preplanned path from the current state to the goal state. There is no allowance for alternatives based on the situation at the time of movement.

Researchers have tried many variations on Proportional-Derivative (PD) control to improve its usefulness for controlling robotic manipulators [2,3]. This paper, in Section 2, reexamines the simple PD control loop’s transient properties when used without preplanned paths. The technique of using planned trajectories to modify the behavior of the simple control loop is discussed in Section 3. A new approach to modifying PD controller behavior by using time-varying gains is developed in Section 4. This approach allows the manipulator to move toward the goal state along a trajectory that maintains the overall smoothness of the motion, as discussed in Section 5. Section 6 demonstrates the algorithm on a two-link planar manipulator by simulation.

2. Manipulator Motion Control

Motion control of robots is commonly achieved by Proportional-Derivative (PD) servo control loops on each joint [1]. The form of this control is given by

$$\tau_{act} = K_p (x_d - x) - K_v \dot{x} \quad (1)$$

where x is the joint position, \dot{x} is the joint velocity, and $K_p, K_v > 0$ are the position and velocity gains. The control output of the PD equation, τ_{act} , is the amount of torque to be produced by the actuator. (This value may be converted into a level of current to be sent to the motor.) The control equation and the plant (motor and mechanism) form a dynamical system with a point attractor at the joint position x_d . This means that the system, from any initial position, will approach a neighborhood of the point $x = x_d$, eventually reaching the neighborhood and staying there [4,5].

The manner in which the attractor is approached is determined by the transient behavior of the dynamical system. The transient behavior of equation (1) is less than ideal for manipulator control. Figure 1 shows simulations of the position and velocity profiles of a motion generated by equation (1) when the plant is a double integrator, i.e. the closed loop equation is

$$\ddot{x} = K_p (x_d - x) - K_v \dot{x}$$

The velocity profile for this system is grossly skewed to the left, showing that the majority of the motion occurs early, but the system slows down and approaches the attractor very slowly at the end. Thus, to complete a motion within a specified time interval, a much larger than necessary peak velocity must be generated.

Khatib [2] presented a modification of this dynamical system that would limit the maximum

velocity. For a desired maximum velocity of V_{max} , the modified system is given by

$$\tau_{act} = K_p v (x_d - x) - K_v \dot{x} \quad (2)$$

where

$$v = \min \left(1, \frac{V_{max}}{\left| \frac{K_p}{K_v} (x_d - x) \right|} \right)$$

This system still has a point attractor at $x = x_d$, but the transient behavior has been modified. The position and velocity profiles of equation (2) are shown in Figure 2. Note that when V_{max} is large, the transient behavior of equation (2) will revert to that of equation (1). When V_{max} is small enough to influence the behavior of the system, as depicted in the figure, the velocity profile takes on a somewhat trapezoidal shape with a significantly smaller peak velocity at the expense of a longer motion duration.

The motions generated by equations (1) and (2) both suffer from a lack of “smoothness”. Hogan and Flash [6] propose the use of the mean squared magnitude of the rate of change of acceleration (jerk) as a measure of the smoothness of a movement.

$$smoothness = \int_0^{t_f} |\ddot{x}|^2 dt \quad (3)$$

By applying a measure such as this, the relative smoothness of different motions can be compared. For example, for the two motions shown in Figures 1 and 2, the smoothness measures calculated from (3) are 1,817,214 and 111,838, respectively, over the interval from zero to two seconds. This implies that the velocity-limited motion is smoother, in effect trading movement duration for smoothness. Both of the motions are smooth near the end of the trajectory and “jerky” at the beginning. The amount of jerk is related to the magnitude of the gains. For the example motions shown in the figures, gains of $K_p = 64$ and $K_v = 16$ were used.

3. Planned Motion Trajectories

Large initial jerk and slow final movement make PD controller motions less than ideal for generation of manipulator trajectories. When a robotic manipulator is moved over a large distance, the motion should be controlled so that the velocity peak occurs in the center of the motion and is not unreasonably large. To achieve this with a PD controller, planned trajectories are normally used to guide the system’s transient motion. Thus, for an underlying control algorithm of

$$\tau_{act} = K_p (x_d - x) + K_v (\dot{x}_d - \dot{x}) + \ddot{x}_d \quad (4)$$

a planned sequence of desired trajectory points $(x_d, \dot{x}_d, \ddot{x}_d)$ is used to move the system in a desired motion. By periodically updating $(x_d, \dot{x}_d, \ddot{x}_d)$ in the control equation, a desired transient behavior

can be achieved. Thus, a dynamical system with undesirable characteristics is made to behave in a desirable manner by continuously adjusting its attractor.

One type of planned trajectory used for manipulator control is that of quintic polynomials in time [1]. The desired motion of the system is given by

$$\begin{aligned}x_d(t) &= a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \\ \dot{x}_d(t) &= a_1 + 2a_2t + 3a_3t^2 + 4a_4t^3 + 5a_5t^4 \\ \ddot{x}_d(t) &= 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3\end{aligned}\tag{5}$$

for $0 < t < t_f$. The coefficients a_i are computed to satisfy the boundary conditions at $t=0$ and $t=t_f$ for position, velocity, and acceleration. For example, suppose the boundary conditions are $x(0)=x_0$, $\dot{x}(0)=\ddot{x}(0)=0$, and $x_d(t_f)=\dot{x}_d(t_f)=\ddot{x}_d(t_f)=0$. Then the coefficients for the quintic polynomial trajectory are given by [1]

$$\begin{aligned}a_0 &= x_0 \\ a_1 &= 0 \\ a_2 &= 0 \\ a_3 &= -\frac{10x_0}{t_f^3} \\ a_4 &= \frac{15x_0}{t_f^4} \\ a_5 &= -\frac{6x_0}{t_f^5}\end{aligned}\tag{6}$$

Figure 3 shows the position and velocity profiles for a quintic polynomial trajectory with a duration approximately the same as that of Figure 1. A quintic trajectory will be very smooth according to the smoothness measure (3), having a value of 814 for the motion of Figure 3. The symmetry of the velocity profile is an indication of the smoothness of the motion, i.e. the motion is evenly distributed over time. A double integrator plant and the control algorithm (4) were used to produce the motion of Figure 3. The control gains were increased to $K_p = 1024$ and $K_v = 64$, to improve the tracking of the quintic trajectory. The gains must be increased to a level appropriate for the speed of the planned motion if the quintic trajectory is to be tracked accurately.

Thus, the control system for a planned trajectory will involve a high-gain PD controller. Such a control algorithm can create problems. For example, suppose the actual initial position and ve-

locity of the arm are different from the initial conditions used in computing the coefficients of the planned trajectory. This situation is depicted in Figure 4. Note that the high-gain PD controller forces the system quickly back to the planned trajectory. This motion is not smooth. The smoothness measure for the ideal quintic (with initial conditions $x(0)= 2.0$ and $\dot{x}(0) = 0.0$) is 88, while the smoothness measure for the quintic with incorrect initial conditions of $x(0)= 2.2$ and $\dot{x}(0)= 0.5$ is 8,763,714. Thus, if the actual initial conditions deviate from those used in planning, the system will not be able to produce a smooth movement. This problem arises because the system has the unsmooth behavior of (1) whenever it is far from the desired attractor.

4. Dynamically Generated Trajectories

Rather than settle for a fixed-gain PD controller of the form

$$\tau_{act} = K_p (x_d(t) - x) + K_v (\dot{x}_d(t) - \dot{x}) + \ddot{x}_d(t)$$

as the basis of manipulator motion, one can try to find an alternative to (1) that provides better transient behavior. One such alternative is the dynamical system of the form

$$\ddot{x}_d = K_v(t) \dot{x} + K_p(t) (x - x_d) \quad (7)$$

Here, \ddot{x} is the plant and $K_v(t) \dot{x} + K_p(t) (x - x_d)$ represents the control algorithm. Such a system can be devised with quintic polynomial trajectories as its “natural” transient behavior.

Consider a dynamical system of the form

$$K_1(t) \ddot{x} + K_2(t) \dot{x} + K_3(t) x = 0 \quad (8)$$

that is to have a quintic polynomial trajectory from any state $x = x_0$ to the final state $x = 0$. (Assume that the desired initial and final velocities and accelerations are zero.) From (5) and (6), we have

$$\begin{aligned} x(t) &= x_0 + \frac{-10x_0}{t_f^3} t^3 + \frac{15x_0}{t_f^4} t^4 + \frac{-6x_0}{t_f^5} t^5 \\ \dot{x}(t) &= \frac{-30x_0}{t_f^3} t^2 + \frac{60x_0}{t_f^4} t^3 + \frac{-30x_0}{t_f^5} t^4 \\ \ddot{x}(t) &= \frac{-60x_0}{t_f^3} t + \frac{180x_0}{t_f^4} t^2 + \frac{-120x_0}{t_f^5} t^3 \end{aligned} \quad (9)$$

Choose

$$\begin{aligned} K_1(t) &= b_0 + b_1 t \\ K_2(t) &= c_0 + c_1 t + c_2 t^2 \end{aligned}$$

$$K_3(t) = d_0 t \quad (10)$$

Substituting (9) and (10) into (8) and solving for the unknown coefficients yields

$$b_0 = \frac{t_f}{8}$$

$$b_1 = \frac{-1}{8}$$

$$c_0 = 1$$

$$c_1 = \frac{3}{4t_f}$$

$$c_2 = \frac{-3}{2t_f^2}$$

$$d_0 = \frac{15}{2t_f^2}$$

Thus, the dynamical system

$$\ddot{x} = \left(\frac{8 + \frac{6}{t_f}t - \frac{12}{t_f^2}t^2}{t - t_f} \right) \dot{x} + \left(\frac{\frac{60}{t_f^2}t}{t - t_f} \right) (x - x_d) \quad (11)$$

is obtained, which can also be written as

$$\ddot{x} = \frac{1}{t_f} \left(\frac{-12\lambda^2 + 6\lambda + 8}{\lambda - 1} \right) \dot{x} + \frac{1}{t_f} \left(\frac{60\lambda}{\lambda - 1} \right) (x - x_d) \quad (12)$$

where $\lambda = t/t_f$. There is a problem with the denominators of gains $K_v(t)$ and $K_p(t)$ in these equations, however. The function $(t - t_f)$ is zero at $t = t_f$. The gain functions grow toward minus infinity as t approaches t_f but they can be limited to some reasonable steady state values and held there for $t \geq t_f$. Therefore, a system valid for all $t > 0$ is

$$\ddot{x} = \begin{cases} \max(K_v(t), -K_v^{max})\dot{x} + \max(K_p(t), -K_p^{max})(x - x_d), & 0 \leq t < t_f \\ -K_v^{max}\dot{x} - K_p^{max}(x - x_d), & t_f \leq t \end{cases} \quad (13)$$

where $K_p(t)$ and $K_v(t)$ are given in equation (11) above.

Position and velocity profiles of this dynamical system (13) in approaching the point attractor $x = x_d$ are shown in Figure 5. Here, gain limits are $K_p^{\max} = 1024$ and $K_v^{\max} = 64$. Note that the transient behavior is a quintic polynomial trajectory, (compare with Figure 3,) and therefore has the same smoothness properties as the trajectories described in the previous section. This system also has PD controller steady state behavior.

There are many ways to create time-varying gains that produce quintic polynomial behavior, although equation (11) may be the simplest. In addition, many different types of transient responses can be created using the basic system of equation (7). Bullock and Grossberg [7] propose a dynamical system as a model of human motor behavior. This system can be written in the form

$$\dot{x} = \left(\frac{2}{t(1+t^2)} - \alpha \right) \dot{x} - \alpha G_0 \left(\frac{t^2}{1+t^2} \right) (x - x_d) \quad (14)$$

Here, α and G_0 are constants which determine the shape of the transient response [7], and the velocity gain must be limited near $t=0$. The transient response of this system can be symmetrical, like the quintic polynomial system, but for many movements the transient response looks more like $x(t) = e^{-at^3}$ than a quintic polynomial. This type of response is somewhere in between the perfectly symmetrical quintic polynomial motion and the highly skewed response of the PD controller equation (1). A dynamical system providing a response with the shape e^{-at^3} is given by

$$\ddot{x} = -3at^2\dot{x} - 6at(x - x_d) \quad (15)$$

where appropriate limits must be placed on the gains in order for the system to be valid for all t . The position and velocity profiles of (15) with $a=4$ are shown in Figure 6. Compare this figure with Figures 1 and 5. The smoothness measure for the trajectory in Figure 6 over the two-second interval is 2025.

5. Smoothness in Dynamically Generated Trajectories

The dynamical system described in equation (13) has quintic polynomial transient behavior. This system will be referred to here as a *dynamic quintic generator*, since it produces quintic polynomial-like trajectories without having an explicit representation of the quintic polynomial. The dynamic quintic generator is a PD controller that uses time-varying gains to smooth out the motion instead of periodically updating the attractor. This means that the system is not forced to be in a certain state at a certain time, but rather can choose an arbitrary state that is appropriate for the situation.

Note that the equation parameters do not depend on the initial position of the system, unlike the planned trajectory described by equations (5) and (6). The system (13) only depends on the desired final time t_f . In other words, only the duration of the movement need be specified in computing the coefficients of $K_p(t)$ and $K_v(t)$. The resulting gains will provide the correct transient response for movements over any distance! Figure 7 depicts movements of three different distances using the functions $K_p(t)$ and $K_v(t)$ computed for $t_f = 2$ s. Note that all the movements finish at time $t = 2$ s and have quintic polynomial profiles.

The dynamic quintic generator finishes on time even when there is a non-zero initial velocity. A movement from the initial conditions $x(0) = 2.2$ rad, $\dot{x}(0) = 0.5$ rad/s, is shown in Figure 8. These initial conditions are the same as those used for the quintic simulation depicted in Figure 4. The value of smoothness measure for the dynamic quintic generator is 499 over this movement, while it is 8,763,714 for the planned quintic polynomial movement. The dynamic quintic generator maintains its smoothness properties even when there is variation in the initial conditions.

Another problem area for planned trajectories is switching the goal point during the middle of the movement. Once it is determined that a new goal point is to be attained, the planner must try to predict the position, velocity, and acceleration of the system at some time t_s . Then the planner must compute a new plan for the system using the initial conditions at t_s and start the execution of the new plan at time t_s . If the prediction of conditions at t_s is correct then the transition to the new trajectory should be smooth. If the prediction is not correct and the low-level controller is a high-gain PD controller, then the transition will definitely be jerky.

Since the dynamic quintic generator is not so particular about initial conditions, the transition to a new trajectory can be done while maintaining motion smoothness. The switch is effected by simply changing x_d and resetting t to zero. If a finish time for the entire motion other than $t_s + t_f$ is desired, then new coefficients of the gain functions can be determined. Computing new coefficients only takes four multiplies when using the form of equation (12). Figure 9 depicts a motion of the dynamic quintic generator with a switch at time $t_s = 1.5$ s. The original goal was $x_d = 2$ rad which was to be attained 2.0 s from the start of the motion. The switched motion finishes at the new goal $x_d = 1$ rad at time $1.5 + 2.0 = 3.5$ s. The motion remains smooth even with the switch in the middle. The value of the smoothness measure for the entire motion is 288.

6. Simulation of Dynamic Trajectory Generation on Two-Link Manipulator

A simulation of a two-link planar manipulator was performed to demonstrate the applicability of these ideas to robot manipulator control. The dynamic model for the two-link device was taken from [1]. The link lengths used are 1.0 m for link 1 and 0.8 m for link 2. The link masses used are 5 kg for link 1 and 2 kg for link 2. The dynamic simulation included all inertia, gravity, centrifugal, and Coriolis terms, but no frictional disturbances.

To provide an accurate comparison between the PD controller with planned trajectory and the dynamic quintic generator given by (13), both control algorithms included gravity and inertia compensation. The PD algorithm is

$$\tau_{act} = M [K_p (\theta_d(t) - \theta) + K_v (\dot{\theta}_d(t) - \dot{\theta}) + \ddot{\theta}_d(t)] + G \quad (16)$$

where M is the inertia matrix, G is the gravity vector, and the vectors $\theta_d(t)$, $\dot{\theta}_d(t)$, and $\ddot{\theta}_d(t)$ are the planned trajectory inputs (which are updated every control cycle.)

The dynamic quintic control algorithm is given by

$$\tau_{act} = \begin{cases} M [\max(K_v(t), -K_v^{max})\dot{\theta} + \max(K_p(t), -K_p^{max})(\theta - \theta_d)] + G, & 0 \leq t < t_f \\ M [-K_v^{max}\dot{\theta} - K_p^{max}(\theta - \theta_d)] + G, & t_f \leq t \end{cases} \quad (17)$$

where M is the inertia matrix, G is the gravity vector, and the variable gain functions $K_p(t)$ and $K_v(t)$ are as in (11) or (12).

Note that the centrifugal and Coriolis terms of the dynamic equation are not compensated in the control algorithms. Thus, these terms will tend to act as a disturbance on the control. If the centrifugal and Coriolis terms were included in the control we would expect the systems to behave as simulated in the previous sections.

Figure 10 shows the positions of the two manipulator joints during a motion from $(0^\circ, 0^\circ)$ to $(120^\circ, 45^\circ)$ in one second. The motions of the PD controller with the planned quintic polynomial trajectory and the dynamic quintic generator have comparable smoothness. For joint 2, the PD control with planning has a smoothness measure of 1649, while the dynamic quintic generator's smoothness is 1718. Also, the PD controller rejects the disturbances caused by centrifugal and Coriolis effects. This is due, in part, to the acceleration feedforward term of (16). Without this term, (16) would cause the joint positions to overshoot badly and not reach steady state within one second. In general for the two-link simulations it is found that the dynamic quintic generator does not exactly produce a quintic polynomial trajectory, but maintains its smoothness and finishes within the specified time t_f .

7. Conclusions

A dynamical system that corresponds to a PD controller with time-varying gains has been described. This system produces smooth motion to point attractors which is different from the natural motion of PD controllers. The system also has advantages over preplanned trajectories fed into fixed-gain PD controllers, namely, more smoothness over varied initial conditions. The system is not meant to replace standard PD control for all aspects of movement — there may be many applications where tracking plans as accurately as possible is the best solution, e.g. welding, machining.

There is some indication that biological motor control systems formulate dynamical systems for movement based on the task [7,8]. It is not difficult to believe that a dynamical system as complex as a mammalian nervous system would be capable of emulating a variety of dynamical systems, thereby providing motor control schemes appropriate to the task. An approach to robot motion which has as its most fundamental basis fixed-gain PD control makes a limiting assumption about the nature of robot tasks. The uses of new and different control schemes should be considered with this in mind, rather than evaluating all schemes against the industrial task. Thus, the dynamical systems examined in this paper represent a first step in exploring the more complex bases of control required for more intelligent behavior.

8. Acknowledgments

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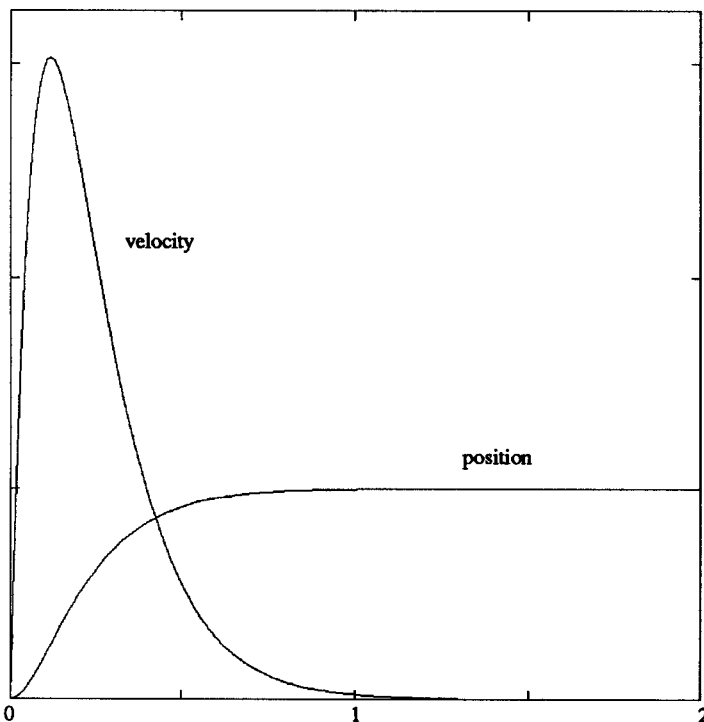


Figure 1. PD Controller Natural Motion

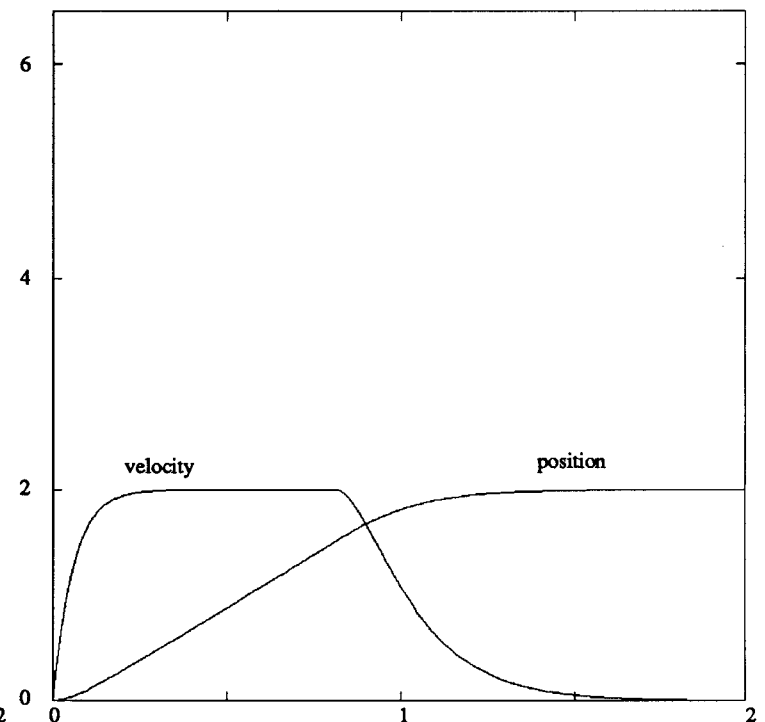


Figure 2. PD Controller Velocity-Limited Motion

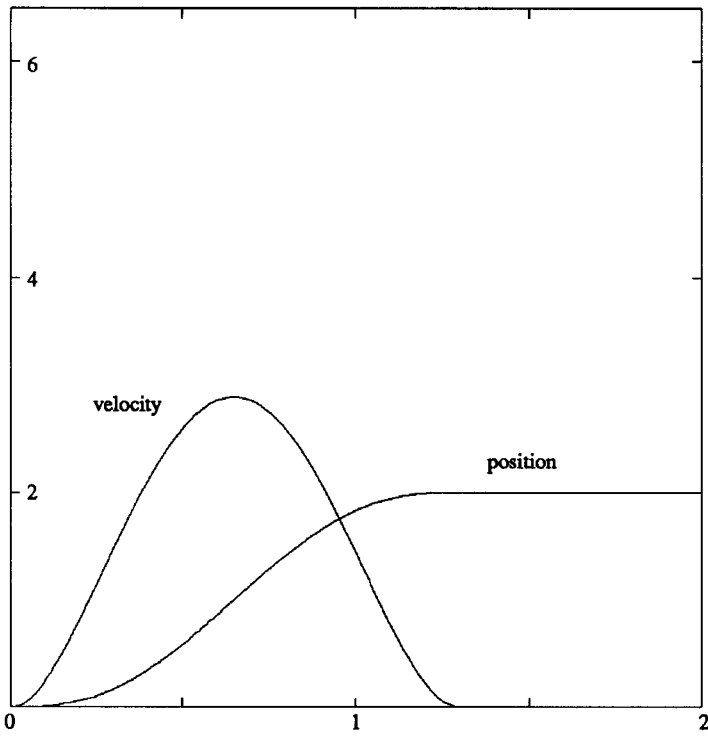


Figure 3. Quintic Polynomial Trajectory

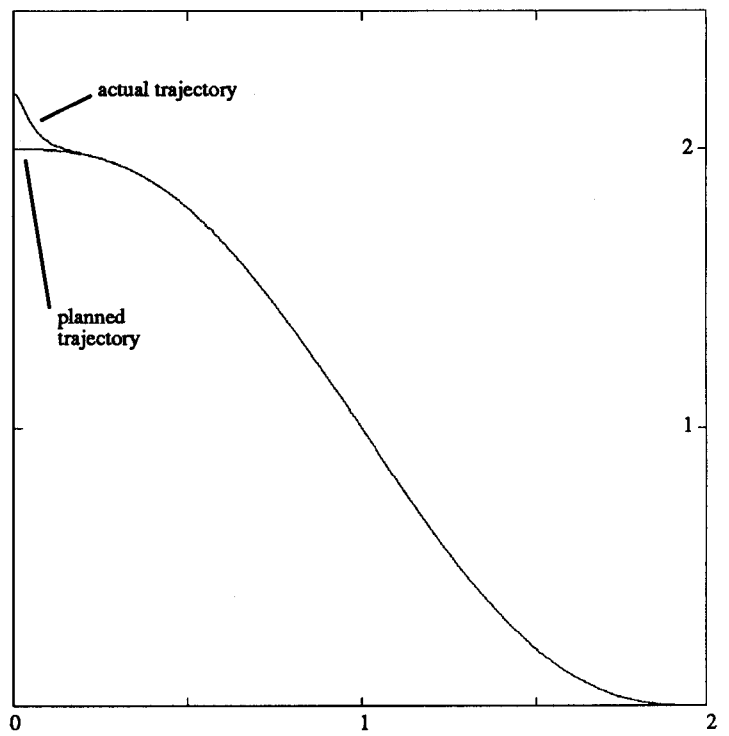


Figure 4. PD Controller and Planned Quintic with Inexact Initial Conditions

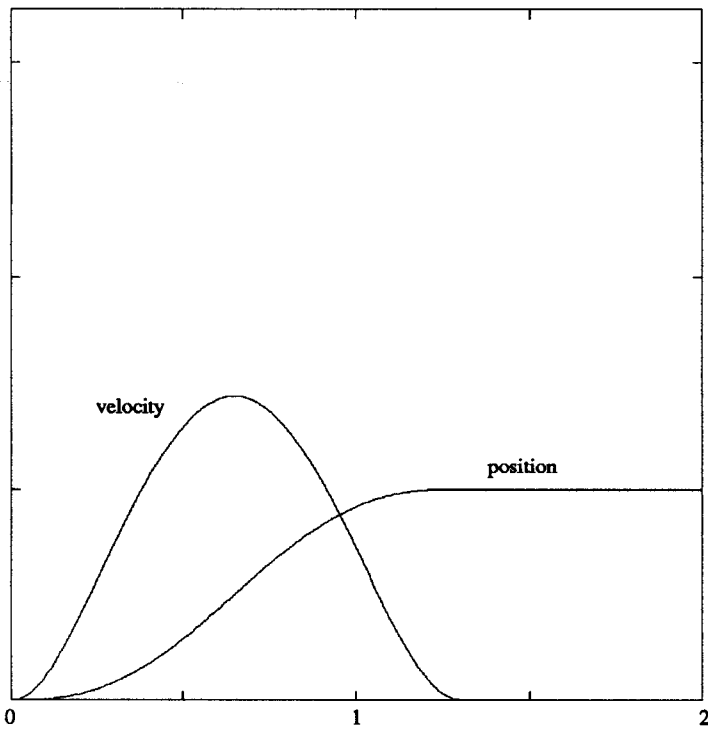


Figure 5. Dynamic Quintic Generator Motion

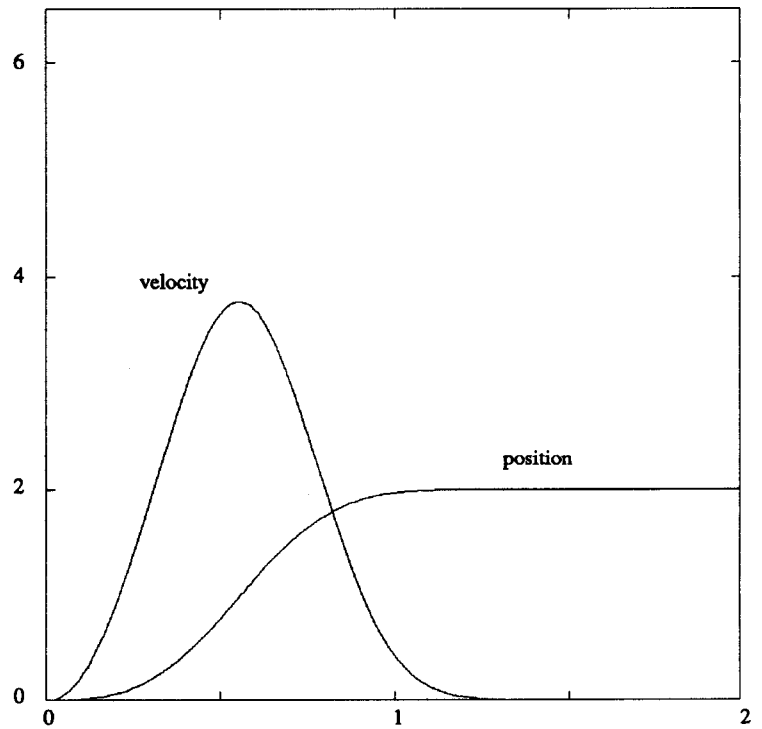


Figure 6. Motion of e^{-at^3} System

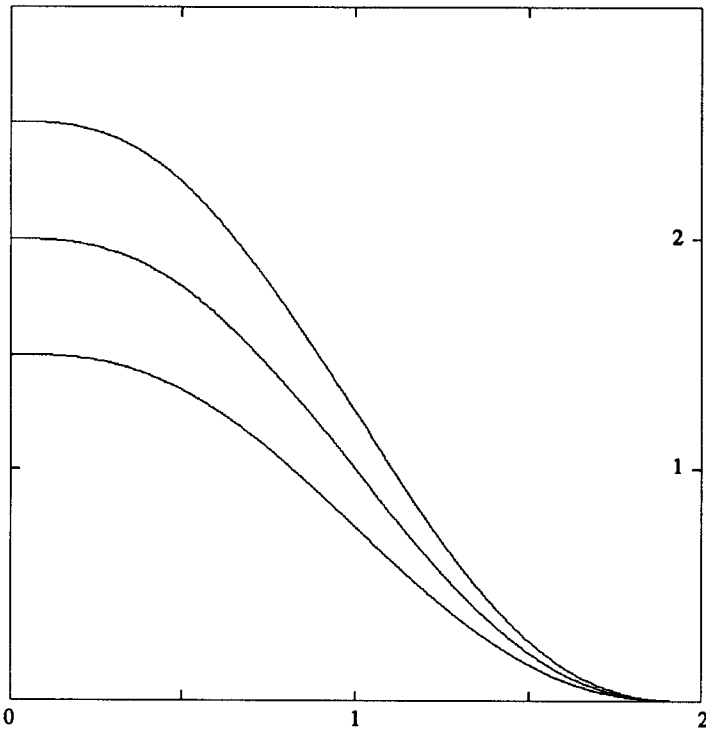


Figure 7. Dynamic Quintic Generator Motions

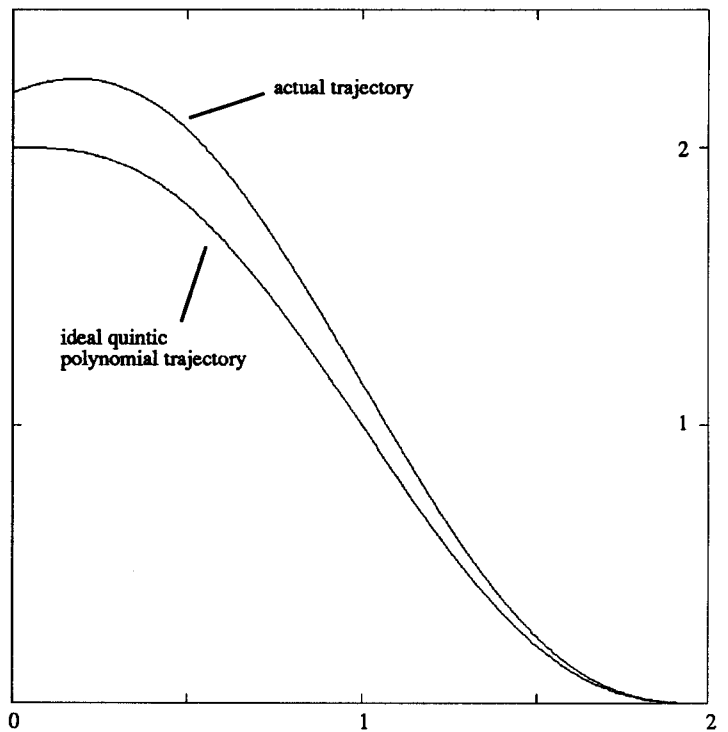


Figure 8. Dynamic Quintic Generator with Inexact Initial Conditions

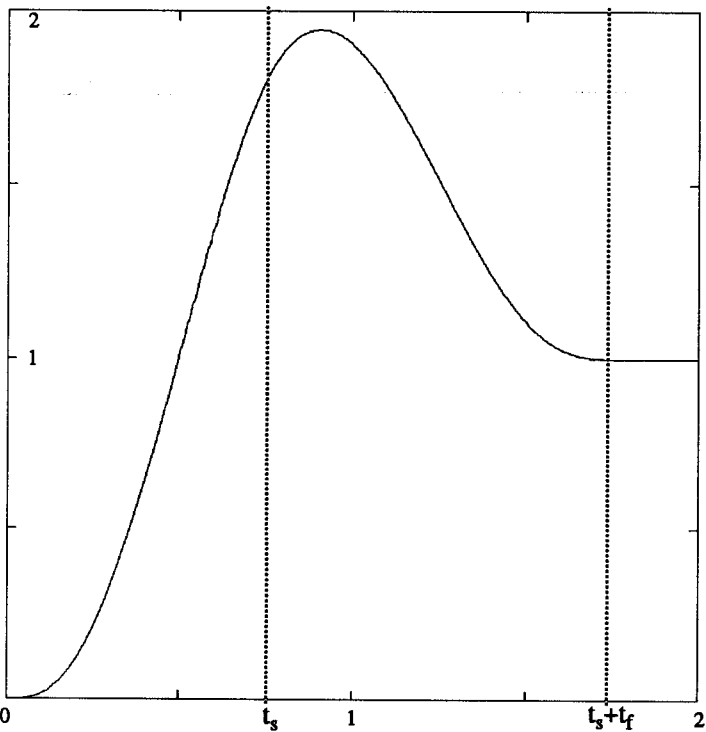


Figure 9. Quintic Generator with Goal Switch

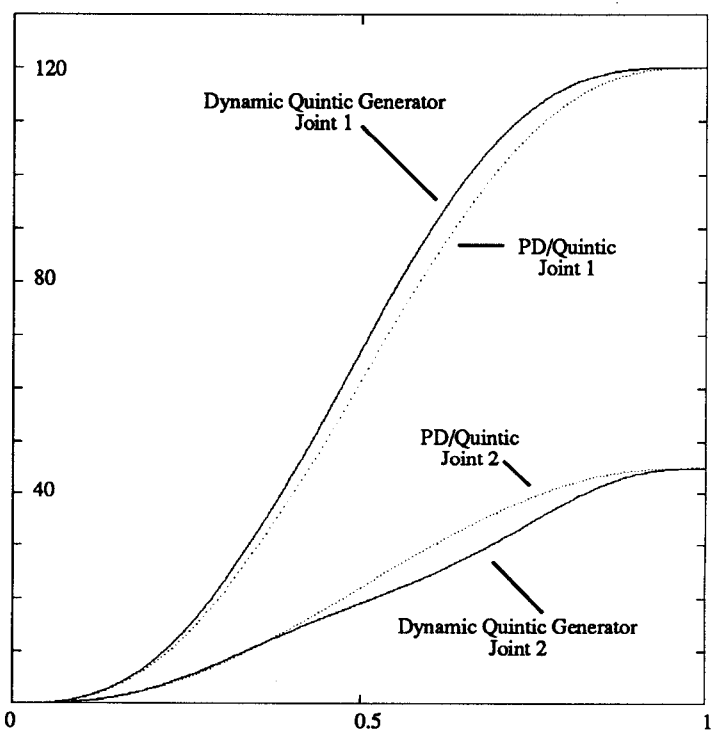


Figure 10. Two-Link Simulation Comparison