

RECONSTRUCTION DURING CAMERA FIXATION

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ABSTRACT

This paper deals with quantitative aspects of camera fixation for reconstruction of a static scene. In general, when the camera undergoes translation and rotation, there is an infinite number of points that produce equal optical flow for any instantaneous point in time. For the case where the rotation axis of the camera is perpendicular to the instantaneous translation vector, these points form a circle (called the Equal Flow Circle or simply EFC) and a line. A special case of the EFCs is the Zero Flow Circle (ZFC) where both components of the optical flow are equal to zero. A fixation point is the intersection of all the ZFCs. Points inside and outside the ZFC are quantitatively mapped using the EFCs. We show how to find the exact location of points in space during fixation.

1. INTRODUCTION

Camera fixation is defined as actively controlling the camera motion so that a given visible point in 3-D space (a fixated point) is constantly imaged to the same point in the image plane. There are several advantages offered by fixation (for reconstruction purposes):

1. Determination of relative range. The optical flow arising from static points in a 3-D neighborhood of the fixation point can be used to easily determine whether these points are in front of or behind the fixation point [4]. Also, these points will have relatively small optical flow values, allowing the use of gradient-based flow extraction methods [9],[12],[16]. Similarly, range can be verified. If the range of a static object is hypothesized, then this range can be verified by pointing the camera optical axis at the object and then fixating on the object at the given range. If the optical flow of points on the object are near zero, then the range is verified.
2. Detailed analysis of objects. If a moving camera is fixated on an object of interest, then this object will be kept in the camera field of view for a long period of time, thus allowing detailed analysis of the object's properties.
3. Increasing resolution. If a moving camera is fixated on a region of interest, then the camera field of view can be quite narrow thus allowing detailed analysis of the region.

In this paper we show how to quantitatively find the location of points in space during camera fixation. The paper begins by describing the coordinate systems that are used, followed by deriving expressions for the optical flow in spherical coordinates for six-degree-of-freedom camera motion. These equations are solved to find sets of points in 3-D space that result in equal flow values for instantaneous camera motion. In the case where the rotation axis is perpendicular to the translation vector, the equal flow points form circles and lines at each instant of time. If the camera motion is further restricted to continuously fixate on a point, we show how the equal-flow circles can be used to quantitatively analyze the space.

Previous work in the area of camera fixation has been mainly in the following categories: (1) fixation for qualitative depth estimation [4], (2) fixating on a moving target for tracking applications [2], and (3) stereo fixation for vergence control [4],[6]. Surprisingly, little previous work that leads to a quantitative analysis of single camera fixation has been done [1],[7]. Cutting [7] did an analysis of camera fixation. We use a different analysis approach than he used, and our results are an extension and elaboration of the results he obtained.

2. EQUATIONS OF MOTION AND OPTICAL FLOW

This section describes the equations that relate a point in 3-D space to the projection of that point in the image for general six-degree-of-freedom motion of the camera. Some of the equations can be found in many books, e.g., see [10].

In the following analysis, we assume a moving camera in a stationary environment. Suppose the coordinate system is fixed with respect to the camera as shown in Figure 1. Assume a pinhole camera model and that the pinhole point of the camera is at the origin of the coordinate system. We derive the optical flow components in the spherical coordinates (R, θ, ϕ) . In this frame, angular velocities $(\dot{\theta}$ and $\dot{\phi})$ of any point in space, say P , are identical to the optical flow values at P' in the image domain. Figure 2 illustrates this concept: $\dot{\theta}$ and $\dot{\phi}$ of a point in space are the same as $\dot{\theta}$ and $\dot{\phi}$ of the projected point P' in the image domain, and therefore there is no need to convert angular velocities of points in 3D space to optical flow. In Figure 2 the image domain is a sphere. However, for practical purposes the surface of the image sphere can be mapped onto an image plane (or other surface).

We start with the derivation of the velocity of a 3-D point in the XYZ coordinates (Figure 1). Let the instantaneous coordinates of the point P be $R = (X, Y, Z)^T$ (where the superscript T denotes transpose). If the instantaneous translational velocity of the camera is $t = (U, V, W)^T$ and the instantaneous angular velocity is $\omega = (A, B, C)^T$ then the velocity vector V of the point P with respect to the XYZ coordinate system is:

$$V = -t - \omega \times R \quad (1)$$

or:

$$V_x = -U - BZ + CY \quad (2)$$

$$V_y = -V - CX + AZ \quad (3)$$

$$V_z = -W - AY + BX \quad (4)$$

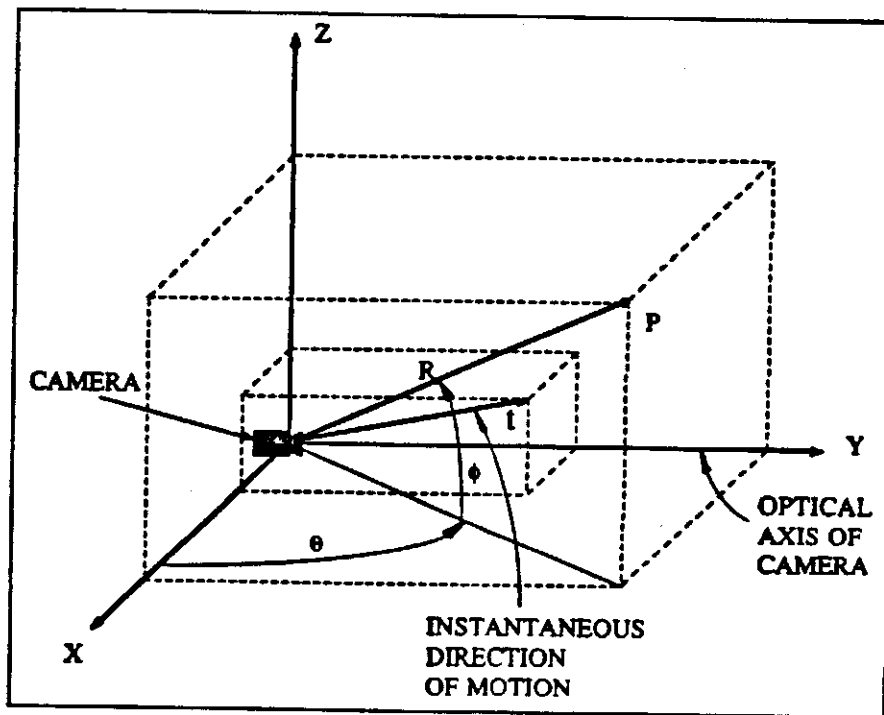


Figure 1: Coordinate system fixed to camera

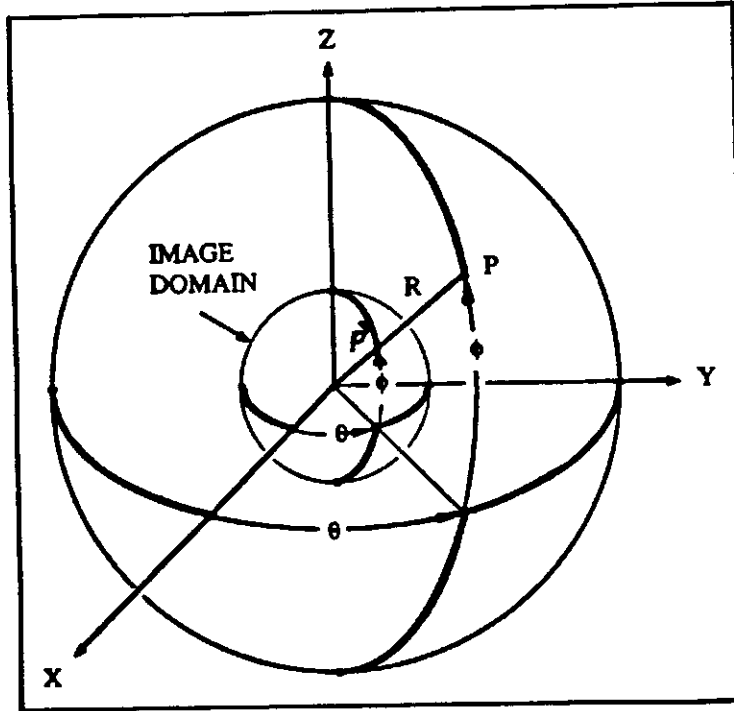


Figure 2: Image domain

where V_X , V_Y , and V_Z are the components of the velocity vector V along the X , Y , and Z directions respectively.

To convert from $R \theta \phi$ to XYZ coordinates we use the relations:

$$X = R \cos\phi \cos\theta \quad (5)$$

$$Y = R \cos\phi \sin\theta \quad (6)$$

$$Z = R \sin\phi. \quad (7)$$

Meriam [11] describes the relations between V_R, V_θ, V_ϕ and V_X, V_Y, V_Z (where V_R, V_θ, V_ϕ are the components of the vector V in $R \theta \phi$ coordinates), and the relations between V_R, V_θ, V_ϕ and $\dot{\theta}, \dot{\phi}$ (dot denotes derivative with respect to time). In order to find the optical flow of a 3-D point in $R \theta \phi$ coordinates (which is also the angular velocity of the point) in terms of the components of the vector V , we use equations (2)-(7) and the relations that are described in [11] and [15] to get:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{-Y}{X^2+Y^2} & \frac{X}{X^2+Y^2} & 0 \\ \frac{-XZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{-YZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{\sqrt{X^2+Y^2}}{X^2+Y^2+Z^2} \end{bmatrix} \begin{bmatrix} -U-BZ+CY \\ -V-CX+AZ \\ -W-AY+BX \end{bmatrix} \quad (8)$$

Let the camera motion vectors t and ω be given as follows:

$$t = (U, V, 0)^T \quad (9)$$

$$\omega = (0, 0, C)^T. \quad (10)$$

This means that the translation vector may lie anywhere in the instantaneous XY plane while the rotation is about the Z axis. Substituting these motion vectors into equation set (8) yields:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{-Y}{X^2+Y^2} & \frac{X}{X^2+Y^2} & 0 \\ \frac{-XZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{-YZ}{\sqrt{X^2+Y^2}(X^2+Y^2+Z^2)} & \frac{\sqrt{X^2+Y^2}}{X^2+Y^2+Z^2} \end{bmatrix} \begin{bmatrix} -U+CY \\ -V-CX \\ 0 \end{bmatrix} \quad (11)$$

Setting $\dot{\theta}$ and $\dot{\phi}$ in equation set (11) to constants will result in a set of equal flow points for this specific motion.

3. EQUAL FLOW CIRCLES

For visualization purposes, we decided to examine the case where the optical flow value of $\dot{\theta}$ is constant and the optical flow value of $\dot{\phi}$ is zero. From equation set (11) the following solutions are obtained:

$$Z = 0 \quad \text{and} \quad \left[X + \frac{V}{2(C+\dot{\theta})} \right]^2 + \left[Y - \frac{U}{2(C+\dot{\theta})} \right]^2 = \left[\frac{V}{2(C+\dot{\theta})} \right]^2 + \left[\frac{U}{2(C+\dot{\theta})} \right]^2 \quad (12)$$

$$X = -\frac{V}{C+\dot{\theta}} \quad \text{and} \quad Y = \frac{U}{C+\dot{\theta}} \quad (13)$$

These solutions are drawn in Figure 3. Solution (12) is an equation of a circle that lies in the XY plane. Solution (13) is a straight line perpendicular to the XY plane.

The meaning of these solutions is the following: all points in 3-D space that lie on the circle or the line described by solutions (12) and (13) and which are visible (i.e., unoccluded and in the field of view of the camera) produce the same instantaneous optical flow $\dot{\theta}$ and zero instantaneous optical flow $\dot{\phi}$. We call the circle on which equal flow points lie the Equal Flow Circle (EFC). A set of EFCs is illustrated in Figure 4. The label of each circle represents the optical flow $\dot{\theta}$ in the image that corresponds to points on this circle. Figure 4 shows EFCs for the case where the camera undergoes instantaneous translation and rotation. Here, there is a circle with finite radius that produces zero flow ($\dot{\theta} = 0$ in the image domain). We call this circle Zero Flow Circle (ZFC) [13].

4. EQUAL FLOW CIRCLES AND FIXATION

If the point on which the camera fixates is visible in the image, then the corresponding image point will have zero optical flow during fixation. However, at a specific time instant, a fixation point is one out of many that may produce zero optical flow. For motion in an instantaneous XY plane, as described previously, these points lie on a circle and a line. Since points inside the circle produce flow values of opposite sign to those outside the circle, a point in 3-D space which is not the fixation point may produce different flow values at different instants of time.

By definition, a fixation point (if visible) produces zero optical flow at all instants of time during the motion. If the fixation point lies in the instantaneous XY plane, then the ZFC at each instant of time must contain the fixation point. (There are special cases in which more than one fixation point exists.) The fixation point is the intersection of all the ZFCs. Figure 5 shows sets of EFCs during fixation at different instants of time.

5. MAPPING THE SPACE WHILE FIXATING

The EFCs can be described in a more generalized form, using ratios between the $\dot{\theta}$ of each circle to the rotation parameter C , i.e., $\frac{\dot{\theta}}{C}$. Figure 6 shows EFCs with normalized values of $\frac{\dot{\theta}}{C}$. Obviously, the normalized value of the ZFC is 0. Figure 7 shows normalized EFCs during fixation.

Using the EFC (or the normalized EFC) concept it is possible to quantitatively map the space in such a way that any point on the XY plane can be located. The method is based on relative mapping. Given (only) the ratio between the optical flow $\dot{\theta}$ of a point and the camera rotation parameter C , $\frac{\dot{\theta}}{C}$ (assuming $C \neq 0$), and the projection of the point in the image (i.e., θ and ϕ), then the location of the point relative to the fixation point (or relative to the camera) can be obtained. The instantaneous direction of motion of the camera should be known. Refer to Figure 8: After locating the instantaneous direction of motion θ_i , the location of normalized EFCs in camera coordinates can be determined. $\frac{\theta_p}{C}$

determines the location of the normalized EFC on which the point P is located relative to the ZFC. The angle $(\theta_p - \theta_i)$ determines the exact location of the point on that EFC relative to the instantaneous direction of motion.

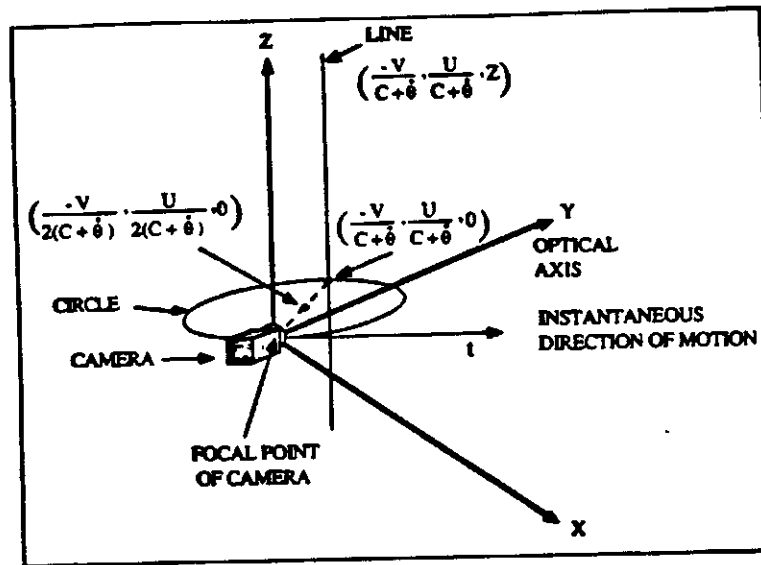


Figure 3: Pictorial description of solutions (12) and (13)

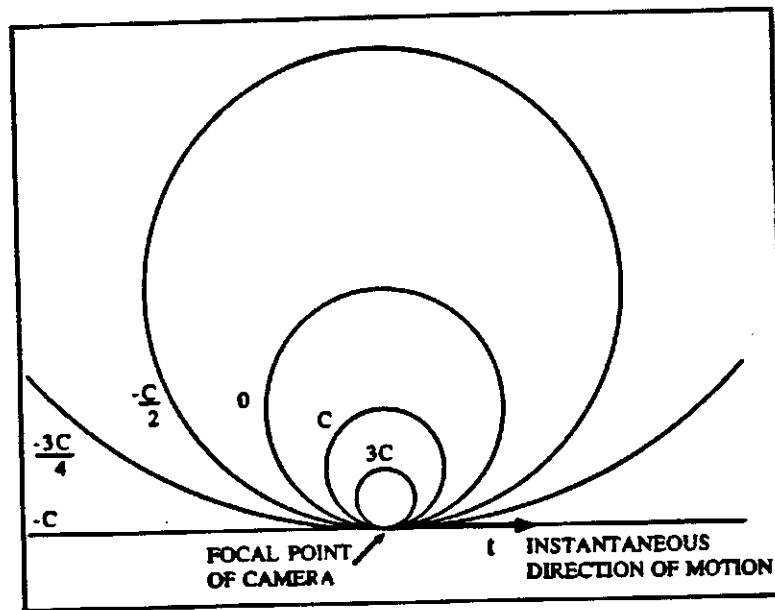


Figure 4: Optical flow values due to camera translation and rotation

Figure 9 is a suggestion for a more elegant mapping. This mapping consists of two orthogonal families of circles. One family is the EFCs. A point can be mapped relative to the fixation point by specifying two numbers that are derived from the optical flow (they specify two orthogonal circles) [14].

6. DISCUSSION

In this paper we presented a quantitative way for analyzing fixation. We show how to relate optical flow of points near the fixation point to their locations relative to the camera or the fixation point. For explanation purposes we analyzed a special case of motion. However, a similar approach (though it could be more difficult to visualize the results) can be taken for a more general motion of the camera. The analysis for the current motion can also be extended to find equal flow curves, i.e., curves that correspond to constant $\dot{\theta}$ and constant $\dot{\phi}$, or curves that correspond to constant $f(\dot{\theta}, \dot{\phi})$ where $f(\cdot)$ is a function of $\dot{\theta}$ and $\dot{\phi}$.

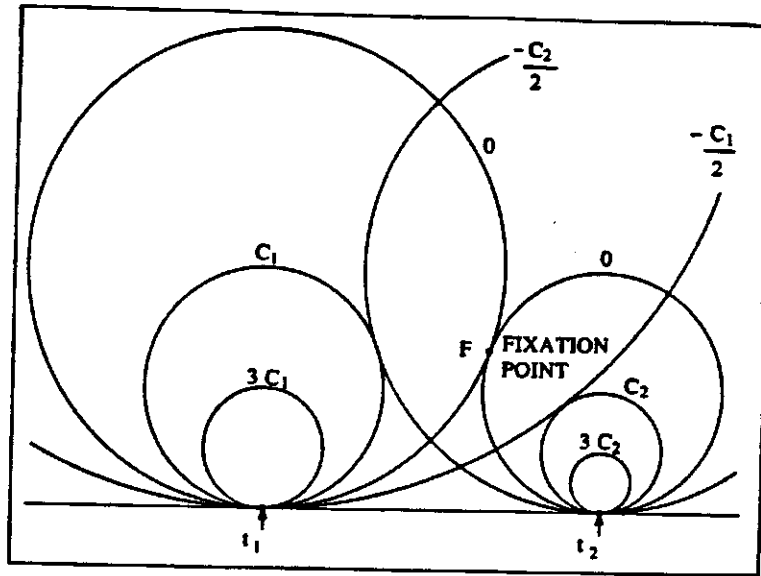


Figure 5: EFCs as a function of time

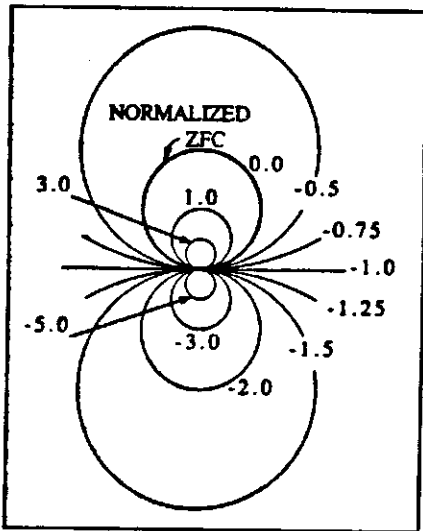


Figure 6: Normalized EFCs

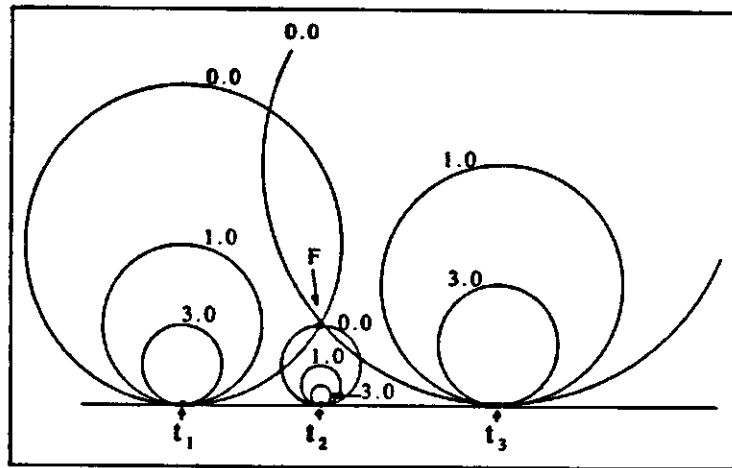


Figure 7: Normalized EFCs during fixation

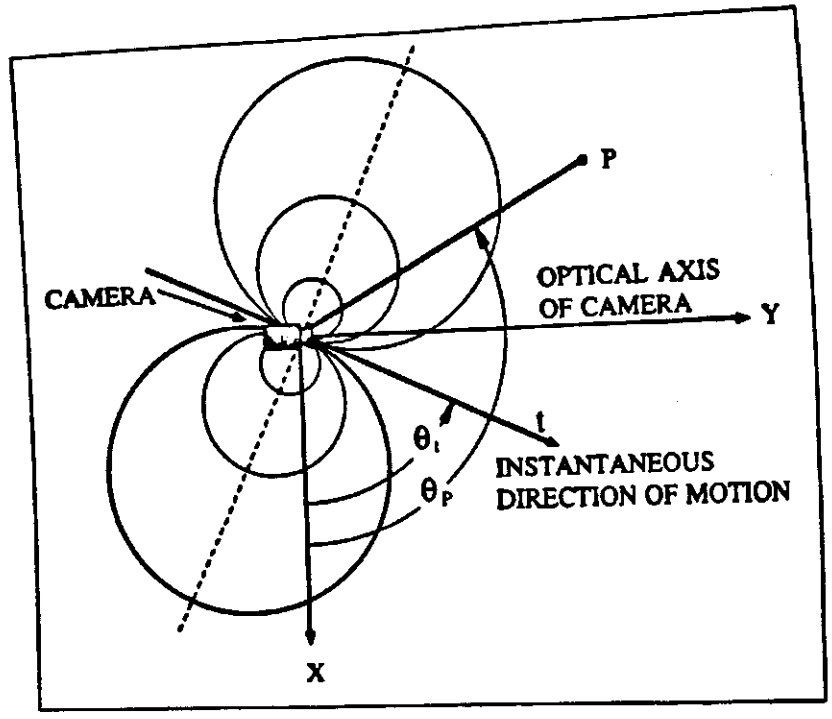


Figure 8: Mapping the XY plane using EFCs and angles

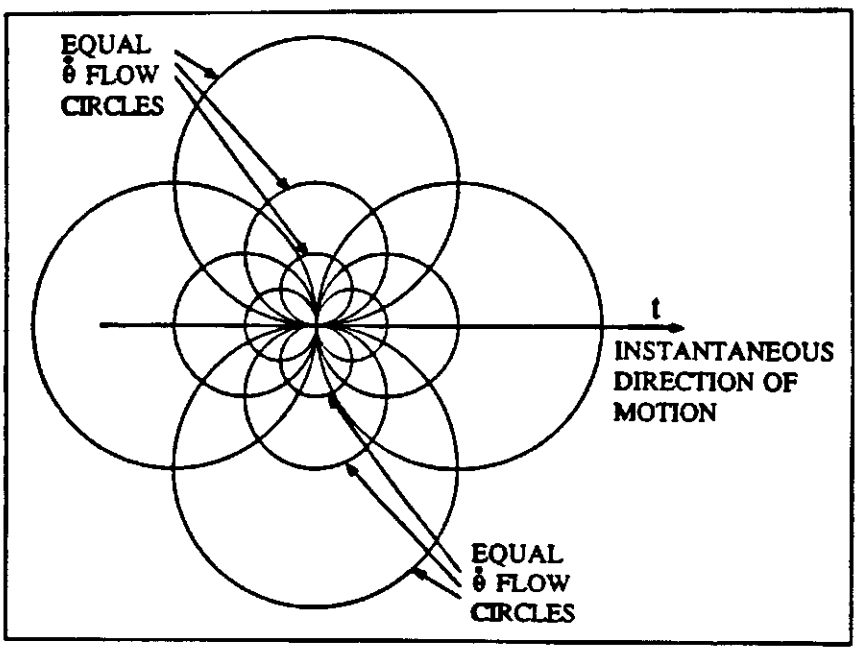


Figure 9: Mapping the XY plane using EFCs and their orthogonal circles

7. ACKNOWLEDGEMENTS

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