OPTIMUM STIFFNESS STUDY FOR A PARALLEL LINK ROBOT CRANE UNDER HORIZONTAL FORCE

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ABSTRACT

A new type of crane suspension mechanism is described. This mechanism can provide a significant increase in the payload stiffness to external and inertial loads compared to the suspension of conventional cranes, thus making cranes more suitable for robotic applications. An optimization study was conducted to determine the best choice of the design parameters of this suspension mechanism which maximizes its stiffness.

The stiffness functions of the robot crane suspension to various types of external loads common to robot crane applications were determined. Their optimization properties were studied using theoretical and numerical analysis techniques. It was found that feasible optimal designs which maximize stiffness are possible, but they are dependent on the type of the assumed external load and height. In this paper we report the optimization results for the case of a single external horizontal force.

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INTRODUCTION

A new crane suspension mechanism was proposed in [1,2] which results in a significant increase in the payload stiffness to external and inertial loads compared to the suspension of conventional cranes, thus making cranes more suitable for robotic application. The mechanism concept is shown by the schematic drawing of Figure 1. It consists of an equilateral triangular platform, called the lower platform here, which is suspended by six wireropes, two at each vertex of the triangle, from an overhead carriage of the same shape, called the upper platform. The carriage includes a single winch onto which all six wireropes attach and rope guides which guide the six wireropes away from the winch in three pairs equidistantly spaced. If desired, it is possible to adjust the lengths of the individual wireropes with actuators or additional winches.The carriage can be attached to the overhead, gantry or boom crane depending on the application.

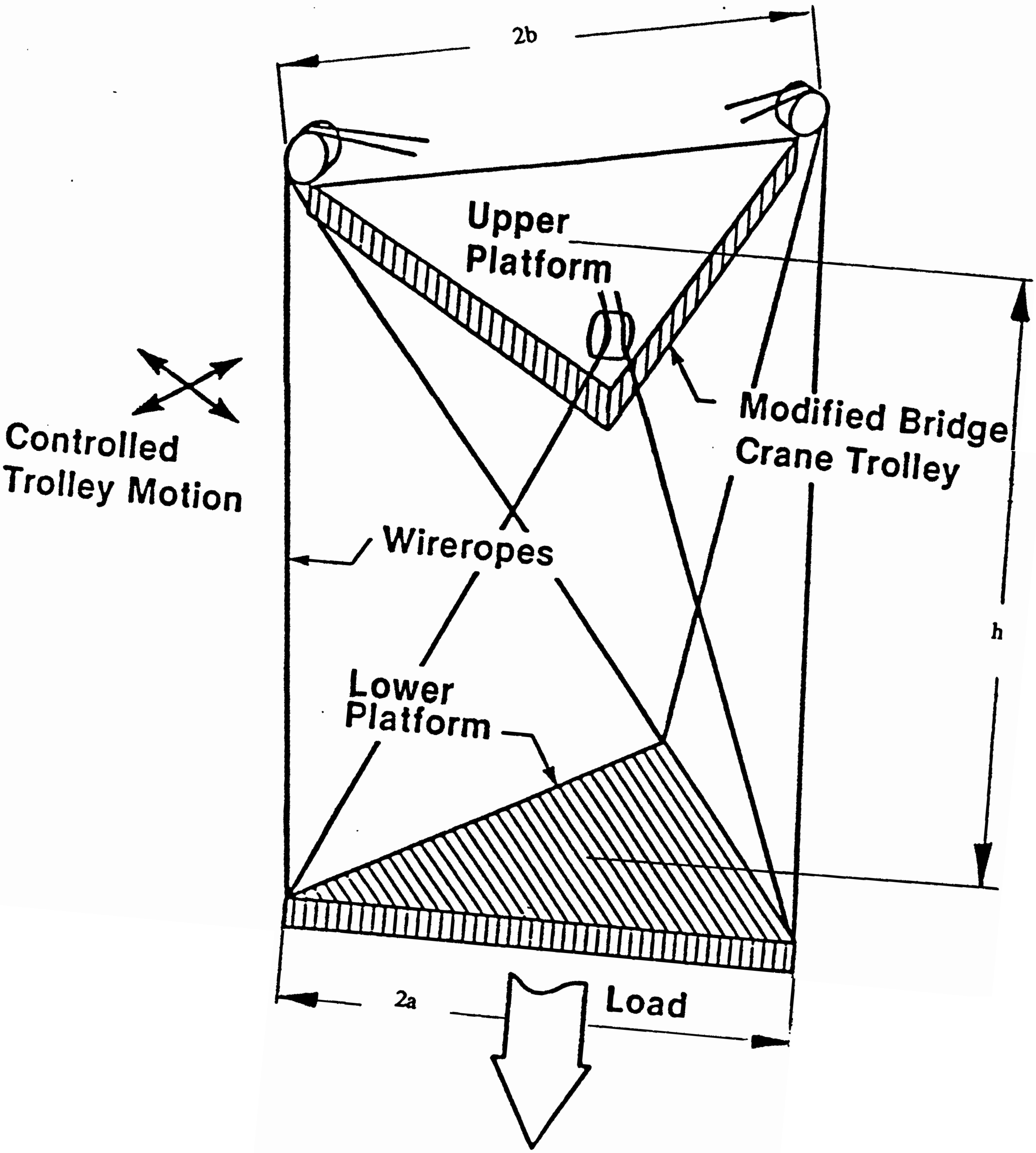


Fig. 1 Mechanism concept

The suspended lower platform behaves as if the six wireropes were an extensible single beam with a spring constant dependent on the magnitude of the suspended weight and the height of the crane, for a given size of the two platforms and wirerope type and diameter. This results in a significant improvement in stiffness over a conventional crane. This enables the load to be accurately positioned and also provides a stable platform which can be used to exert torques and side forces on objects being positioned. The suspended lower platform can be used as a stabilized base for the direct mounting of conventional manipulator arms, or for the support of special substructures for specific crane applications [3].

The proposed robot crane suspension mechanism takes advantage of the suspended load to maintain the wireropes in tension and thus with their spatial orientation, oppose any horizontal displacement of the payload. The stiffness created by this geometric orientation is superimposed onto the pendulum stiffness of conventional cranes.

The objective of this work is to determine the optimum combinations of the dimensions of the upper and lower platforms which maximize this stiffness for practical values of the total suspended weight, crane height, and diameter of the steel wireropes.

THE STIFFNESS MATRIX

If the rigid body motion displacements of the lower platform are small, it is possible to linearize the equations of its quasi–static motion to derive the following [2]:

$$\underline{P} = [\,K\,]\,\underline{\delta u} \qquad (1)$$

Where the stiffness matrix K is given by :

$$K = \begin{vmatrix} K_1 & 0 & 0 & 0 & -K_2 & 0 \\ 0 & K_1 & 0 & K_2 & 0 & 0 \\ 0 & 0 & K_3 & 0 & 0 & 0 \\ 0 & K_2 & 0 & K_4 & 0 & 0 \\ -K_2 & 0 & 0 & 0 & K_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_5 \end{vmatrix} \qquad (2)$$

$\underline{P} = [\,f_x,\ f_y,\ f_z,\ m_x,\ m_y,\ m_z\,]^T$ is the vector of the external load of force $\underline{f}$ and moment $\underline{m}$, applied to the center of gravity of the lower platform, assumed to be located at the centroid of the triangle, formed by connecting the three suspension points.

$\underline{\delta u} = [\,\delta u_x,\ \delta u_y,\ \delta u_z,\ \delta\theta,\ \delta\psi,\ \delta\phi\,]^T$ is the vector of the resulting displacements, of the center of gravity.

The elements of the stiffness matrix are :

$$K_1 = 4k(a^2 + b^2 - ab)l_0/l^3 + w/h$$

$$K_2 = 2ka(2a - b)l_0/l^3$$

$$K_3 = 6kh^2 l_0/l^3 + w/h$$

$K_4 = 4ka^2h^2l_0/l^3 + abw/3h$

$K_5 = 8ka^2b^2l_0/l^3 + 2abw/3h$

$l = \sqrt{h^2 + 4(a^2 + b^2 - ab)/3}$

$$h = wl/[6k(l - l_0)] \qquad (3)$$

Where :

l, h – are the reference steady state position wirerope length and the height of the lower platform from the overhead carriage, respectively, after the application of the weight w with no external load.

l_0 – is the reference steady state position wirerope length for no weight and no external load.

k – is the wirerope stiffness assumed to be the same for all six wireropes.

a – is equal to one–half the length of the side of the equilateral triangle formed by the three suspension points of the lower platform.

b – is equal to one–half of the length of the side of the corresponding upper platform triangle.

w – is the total suspended weight, (force of gravity).

THE MODIFIED STIFFNESS MATRIX

To get the elements of the stiffness matrix in a more convenient form for our investigation, we will replace k with AE/l_0 in equations (3), where A is the metallic area of the wirerope [5], and E is the Young's modulus of elasticity.
After substitution we get :

$K_1 = 2F(a^2 + b^2 - ab) + g$

$K_2 = Fah(2a - b)$

$K_3 = 3Fh^2 + g$

$K_4 = 2Fa^2h^2 + abg/3$

$$K_5 = 4Fa^2b^2 + 2abg/3 \qquad (4)$$

Where :

$F = 2AE/f^{3/2}$

$f = h^2 + 4(a^2 + b^2 - ab)/3$

$g = w/h$

Now, we have $K_i = f(w, h, A, E, a, b)$, the elements of the stiffness matrix are a function of the total weight, the height after the total weight is suspended, the metal cross section of the wirerope, Young's modulus of elasticity of the wirerope, and the one–half side length of the lower and the upper platform triangles respectively.

STIFFNESS FUNCTIONS WHICH ARE IMPORTANT FOR COMMON ROBOT CRANE LOADING

For robot crane applications the main stiffness functions of interest are the following:

1. $K_5 = m_z/\delta\phi$, which is the stiffness of the system to an external moment about the vertical axis and is given by equation (4).
2. $K_6 = K_x = f_x/\delta u_x$, which is the stiffness of the system to a single external horizontal force load.

Solving equation (1) for $\underline{P} = [f_x,\ 0,\ 0,\ 0,\ 0,\ 0]^T$ gives,

$$f_x = (K_1 - K^2_2/K_4)\ \delta u_x \tag{5}$$

Then $K_6 = K_x = (K_1 - K^2_2/K_4)$.

3. $K_7 = K_\theta = m_x/\delta\theta$, which is the stiffness of the system to a single external moment about a horizontal axis.

Solving equation (1) for $\underline{P} = [0,\ 0,\ 0,\ m_x,\ 0,\ 0\ ]^T$ gives,

$$m_x = (K_4 - K^2_2/K_1)\ \delta\theta \tag{6}$$

Then $K_7 = K_\theta = (K_4 - K^2_2/K_1)$.

THE OPTIMIZATION PROBLEM

We want to investigate the optimization properties of seven stiffness functions,
$K_i = f(w,\ h,\ A,\ E,\ a,\ b)$, $i = 1,\ 2,\ \ldots,7$

The optimization problem considered was the following. Assuming that a weight w has to be delivered to a location of height h, determine the design of the robot crane suspension which maximizes the stiffness function K_i. It is assumed here that the lower platform will be suspended by steel wireropes of a given composite Young's modulus of elasticity E.

As can be seen from equations (4), (5),(6), the stiffnesses K_i for $i = 1,\ 2,\ldots,7$ are polynomial functions of A and thus continually increase as A increases. Consequantly, the only optimization design variables which need to be considered are : a and b.

The necessary conditions for K_i to reach an optimum are :

$$\begin{aligned} &\partial K_i/\partial a = 0\ ; \\ &\partial K_i/\partial b = 0 \end{aligned} \tag{7}$$

The partial derivatives expressions for $i = 1, 2,\ldots,7$ are given in appendix A.

In order for the stiffness optimum to be maximum at a point (a,b) the following conditions must be satisfied [4] :

$$B^2_0 - A_0C_0 < 0 \ \text{ and } \ A_0 + C_0 < 0 \tag{8}$$

Where :

$$A_0 = \partial^2 K_i / \partial a^2 |_{(a,b)} \ ;$$
$$C_0 = \partial^2 K_i / \partial b^2 |_{(a,b)} \ ;$$
$$B_0 = \partial^2 K_i / \partial a \, \partial b |_{(a,b)}.$$

GLOBAL MAXIMUM ANALYSIS

A general optimization search was conducted for $-5h \le a \le 5h$; $-5h \le b \le 5h$. Any solution of equations (7),within this range which satisfies the neccesary condition (8) is a relative maximum stiffness solution for the specified values of w and h. The objective of this search was to identify these maxima within a wide feasible range of a and b, in order to better understand the behavior of the stiffness functions. The values of the stiffness functions at the boundaries were not considered during this analysis.

In practice of course both the weight w and the delivery height h will vary. For this reason a grid of w versus h was created, for w varying from 10,000 to 100,000 lbf. in steps of 10,000 lbf., and h from 10 to 100 ft. in steps of 10 ft..

The relative maximum search optimization problem was solved for all w, h combinations defined by that grid, for three diameters of wirerope: D=0.375 in., 0.5625 in., 0.75 in..

Since this relative maximum optimization search covered the whole range of values of interest of the optimization variables a b the operating conditions w h and the design parameter D, it is called the global maximum analysis.

LOCAL MAXIMUM ANALYSIS

In order to obtain results which could be useful to practical robot crane designs we investigated the local maximum properties of the stiffness functions for the case where the size of the upper platform b is fixed.Three different values of b were selected. These are : b = 1 ft., 3 ft., 6 ft.

The values of w, h and D used for this search were the same used for the global maximum analysis search.

Thus, we were left with only one optimization variable to search. This variable is: a, the one – half side length of the lower platform.

The necessary and sufficient conditions for a maximum are :

$$\partial K_i / \partial a = 0 \ ; \quad \partial^2 K_i / \partial a^2 < 0 \quad \text{For } i = 1,2,...,7 \tag{9}$$

The optimization search was constrained by 2 in. $\le a \le$ 36 in.
The boundry values of the stiffness functions were determind and compared with those obtained from the solution of (9).

RESULTS

Due to the page limit on this paper, only the results of the optimization study of stiffness function $K_6 = K_x = (K_1 - K^2_2/K_4)$ are reported. This represents the stiffness of the crane suspension system to a single external horizontal force.

Stiffness function K_x

Due to the complexity of the K_x function we had to use numerical techniques to solve equations (7). The global maximum analysis results are:

$$a^* = 0.707\ h$$
$$b^* = 1.414\ h \qquad (10)$$

where * indicates solution of equations (7), which satisfies inequalities (8).
The local maximum analysis gave the following results:
From equation (9) we find that

$$(2a-b)\ f(w,h,A,E,a,b) = 0 \qquad (11)$$

Where the function $f(\cdot)$ is derived in Appendix B.

Equation (11) indicates that K_x has always an optimum, either a maximum or a minimum, at $a = b/2$. Indeed plotting K_x versus a (see Fig. 2) reveals the presence of this optimum, in this case a local maximum. By varying the values of w, h, D it is possible to result in the movement of the maximum towards higher values of a and its replacement by a minimum at $a = b/2$ (see Fig. 3). In this case the maximum resulted from a solution of $f(\cdot) = 0$.

From Figures 2 and 3 it can be seen that as a approaches zero, K_x can take very high values. The value of K_x for the smallest possible a will have to be compared with that for a_* and the largest be declared the local maximum, where a_* indicates a solution of equation (9), which satisfies the inequality condition.

CONCLUSION

The global maximum analysis indicated that the dimensions of the two platforms must satisfy the relationship $a^* = 0.707$ h, $b^* = 1.414$ h in order to maximize the stiffness of the proposed suspension mechanism to a single external horizontal force, without considering boundary conditions.

This relationship would result in unwieldy sized platforms for most crane application ; therefore the rule developed by the local maximum analysis which considers boundary conditions should be followed. That is, the value of K_x for the smallest possible a will have to be compared with that for a_* and the largest be declared the local maximum. Where a_* indicates a solution of equation (9), which satisfies the inequality condition.

Fig. 2 : Stiffness K_x plot

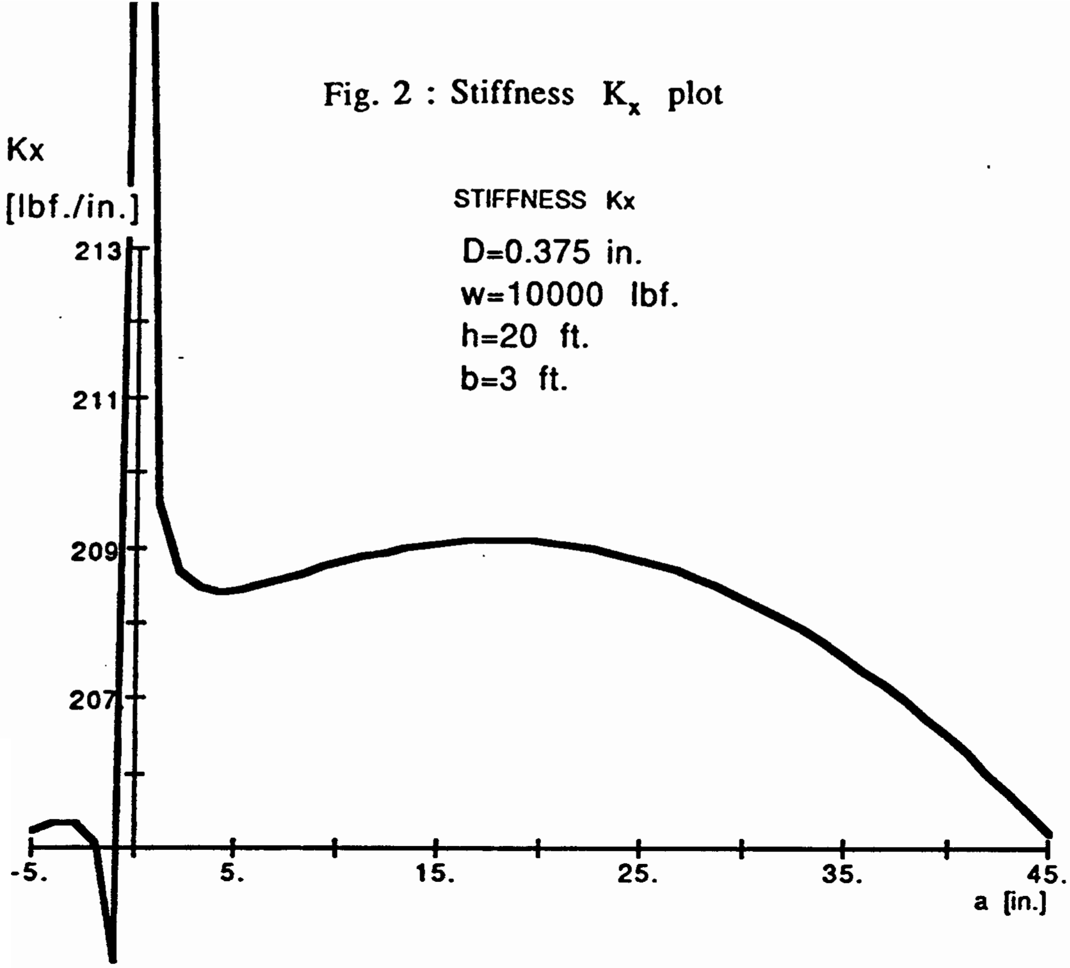
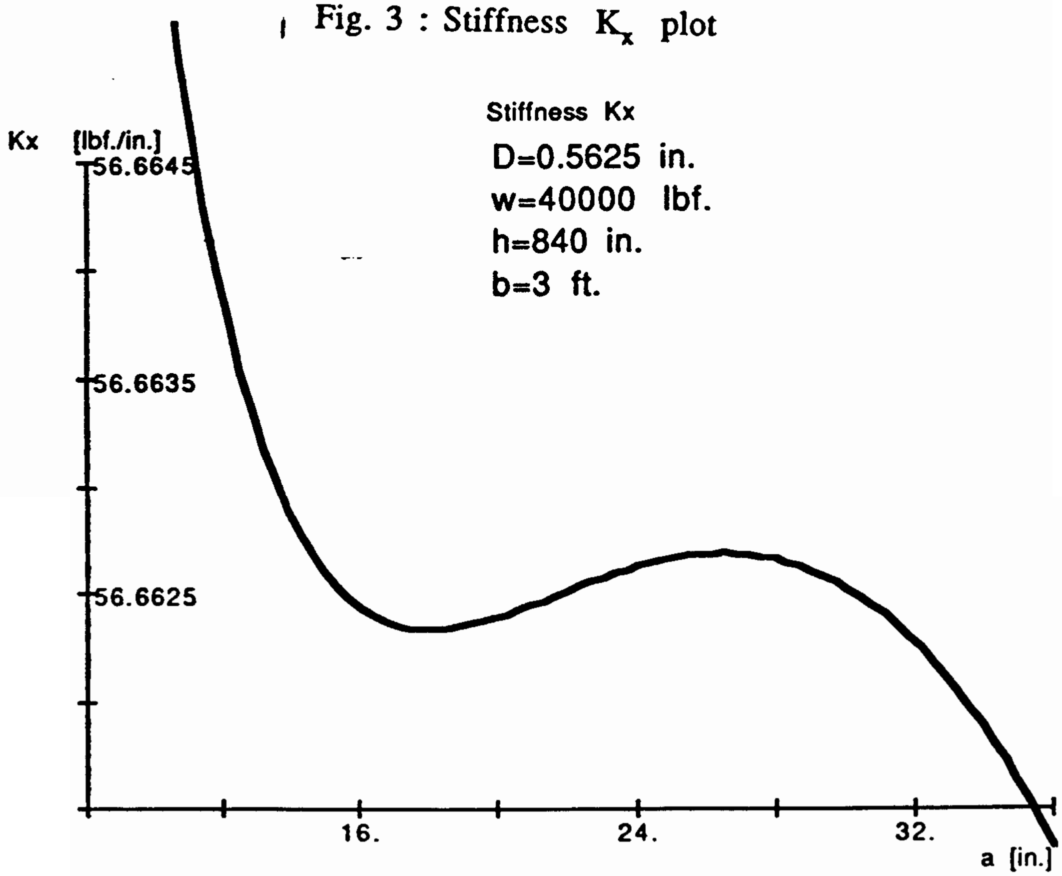


Fig. 3 : Stiffness K_x plot



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APPENDIX A

Using equations (4), (6), we can write the partial derivatives for K_1 to K_7.

$$\partial K_1/\partial a = 2(a^2 + b^2 - ab)(\partial F/\partial a) + 2F(2a - b)$$
$$\partial K_1/\partial b = 2(a^2 + b^2 - ab)(\partial F/\partial b) + 2F(2b - a)$$
$$\partial K_2/\partial a = h[a(2a - b)(\partial F/\partial a) + F(4a - b)]$$
$$\partial K_2/\partial b = h[a(2a - b)(\partial F/\partial b) - Fa]$$
$$\partial K_3/\partial a = 3h^2(\partial F/\partial a)$$
$$\partial K_3/\partial b = 3h^2(\partial F/\partial b)$$
$$\partial K_4/\partial a = 2h^2a[a(\partial F/\partial a) + 2F] + bg/3$$
$$\partial K_4/\partial b = 2h^2a^2(\partial F/\partial b) + ag/3$$
$$\partial K_5/\partial a = 4ab^2[a(\partial F/\partial a) + 2F] + 2bg/3$$
$$\partial K_5/\partial b = 4ba^2[b(\partial F/\partial b) + 2F] + 2ag/3$$
$$\partial K_6/\partial a = (\partial K_1/\partial a) - 2(K_2/K_4)(\partial K_2/\partial a) + (K_2/K_4)^2(\partial K_4/\partial a)$$
$$\partial K_6/\partial b = (\partial K_1/\partial b) - 2(K_2/K_4)(\partial K_2/\partial b) + (K_2/K_4)^2(\partial K_4/\partial b)$$
$$\partial K_7/\partial a = (\partial K_4/\partial a) - 2(K_2/K_1)(\partial K_2/\partial a) + (K_2/K_1)^2(\partial K_1/\partial a)$$
$$\partial K_7/\partial b = (\partial K_4/\partial b) - 2(K_2/K_1)(\partial K_2/\partial b) + (K_2/K_1)^2(\partial K_1/\partial b)$$
$$\partial F/\partial a = 4AE(b - 2a)/f^{5/2}$$
$$\partial F/\partial b = 4AE(a - 2b)/f^{5/2}$$

APPENDIX B

The stiffness function for K_x is :

$$K_x = K_1 - K^2{}_2/K_4 \quad (1)$$

The partial derivative with respect to a is :

$$\partial K_x/\partial a = \partial K_1/\partial a + K_2[(K_2/K^2{}_4)(\partial K_4/\partial a) - 2(1/K_4)(\partial K_2/\partial a)] \quad (2)$$

Substituting $\partial K_1/\partial a$ from appendix A and K_2 from equation (4) into equation (2) we get :

$$\partial K_x/\partial a = (2a - b)[G + L\, f(\cdot)] \quad (3)$$

Where :

$$G = 4AE/f^{3/2}[1 - 2(a^2 + b^2 - ab)/f] \quad (4)$$

$$L = F\,h\,a \quad (5)$$

$$f(\cdot) = (K_2/K^2{}_4)(\partial K_4/\partial a) - 2(1/K_4)(\partial K_2/\partial a) \quad (6)$$

The necessary condition in order to optimize K_6 is :

$$\partial K_x/\partial a = 0 \quad (7)$$

From equation (7) we get two solutions :

$$2a = b \quad (8)$$

$$G + L\, f(\cdot) = 0 \quad (9)$$

From equations (8),(9) a must be chosen so that K_x will be maximum.