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THE GEOMETRIC ELEMENTS FOR PIPE ROUTING USING A COMPUTER

by

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ABSTRACT

Algorithms have been developed for the design of arbitrary three-dimensional pipe (duct) routing.The pipe center line is defined by a polygon of line segments and an array of elbow radius values. A two-dimensional cross section is positioned and scaled along the length of the center line.

1. Introduction

Much time is devoted to arranging equipment and designing its connecting piping (or duct work). A two-dimensional representation of a complex piping system is a cumbersome design tool. When all the piping is shown on paper, the result is often an unintelligible maze [See Chapter 8 of Reference 1].

Computers can easily store and track large amounts of data. For this reason they seem ideal to handle and track data that will require frequent changes. For larger piping systems local changes will be the order of the day, hence it is natural to consider the development of the pipe (duct) routing on a computer. Aided by existing display software of solid modeling or wireframe modeling systems, our task of developing a pipe (duct) routing system is not an overwhelming effort.

A three-dimensional pipe routing system could be based on the algorithms which have been developed and will be presented in this paper. A system of this type could have the following desirable properties:

Store the outside dimensions.
Be able (when desired) to store the inside dimension.
Supply scaling capability to the pipe diameter (or diameters).
Display (and hence plot) views of any plan or elevation.

And with the use of existing (commercial or otherwise) modeling software:

Detect interference between components.
Plot views with hidden lines removed.

2. The Type Of Pipe Routing To Be Considered

The pipe (duct) center line is defined by a polygon with each knot point (each defining point of the polygon except the polygon end points) being flagged as a location for a circular arc to be constructed, or a location for a mitered type elbow. Upon its completion the center line will consist of straight line segments connected by circular arcs or points where two line segments meet (for a mitered type elbow location). The pipe duct will be straight pipe sections connected by elbows. The elbows are either circular sections or of the mitered type.

The input pipe section (the cross section) need not be circular, but for convenience we worked with a circular pipe section. The pipe definition at storage time will be directly applicable for wire frame or faceted type modeling.

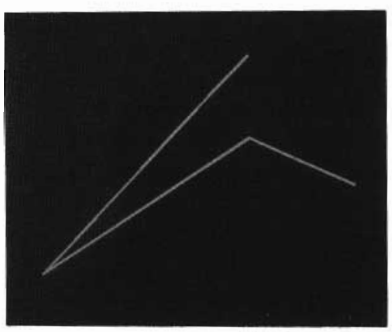

Figure 1A : Input polygon

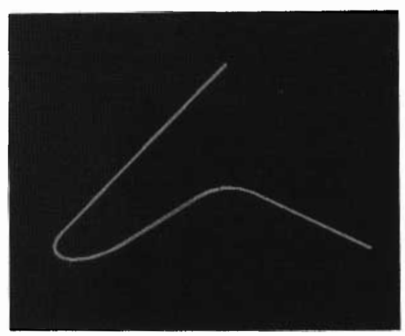

Figure 1B : Pipe (duct) center line with two circular elbows

3. Mathematical (Routines) Tools

Five mathematical procedures are described that will be used in the pipe geometry construction process.

(A) At the intersection point (which is also the end point of both) of line segment L_1 and L_2 (both in the $z = 0$ plane), determine points on a circular arc of a given radius that is tangent to both lines. The center of the circular arc, PC, is determined by the intersection point of the lines determined by offsetting lines L_1 and L_2 (if L_1 and L_2 have a common point). Next, the start (end) point, P_1, of the circular arc is determined by the line that is perpendicular to L_1 and passes through PC. Similarly, the end (start) point, P_2, on L_2 is determined. The arc's angle can now be determined and the points on the arc between P_1 and P_2 easily computed.

(B) Given a plane and a ray (straight line) determine the intersection point of the ray and the plane. We will assume that the ray does not lie in a plane that is parallel to the given plane. Let (x_0, y_0, z_0) be a point on the ray (line), (DX, DY, DZ) the direction vector for the ray (line) and $x*A + y*B + z*C + D = 0$ be the equation for the given plane. Then the equation for the ray (line) is

$$x = x_0 + s*DX$$

$$y = y_0 + s*DY$$

$$z = z_0 + s*DZ$$

and the intersection point is

$$IX = x_0 + t^*DX$$

$$IY = y_0 + t^*DY$$

$$IZ = z_0 + t^*DZ$$

where $t = -(x_0{}^*A + y_0{}^*B + z_0{}^*C + D)/DIV$, with $DIV = DX^*A + DY^*B + DZ^*C$.

(C) Determine the equation of a plane containing three noncolinear given points. We will use the fact that if $P_0(x_0, y_0, z_0)$ is a given point, and let L be a given line with direction numbers A, B, C, then the equation of the plane passing through P_0 and perpendicular to L is

$$A^*(x - x_0) + B^*(y - y_0) + C^*(z - z_0) = 0 \text{ ,or}$$

$$x^*A + y^*B + z^*C = D\text{, where } D = x_0{}^*A + y_0{}^*B + z_0{}^*C.$$

Using this fact our problem is reduced to finding the direction numbers for a line that is perpendicular to the plane defined by the three points. If the three points are noncolinear, they can be used to define two vectors in their plane whose vector (or cross) product defines a vector (nonzero) perpendicular to the plane. That is (A, B, C) = U * V, where U and V are two vectors (the cross product can be zero only if U or V is zero, or if U is proportional to V).

(D) Determine the transformation matrix and its inverse that maps a plane into the $z = 0$ plane. Let $x^*A + y^*B + z^*C + D = 0$, be the equation of the plane, and let $p_0 = (x_0, y_0, z_0)$ be a point on the plane. Set L = A/F, M = B/F, and N = C/F, where $F = \sqrt{A^2 + B^2 + C^2}$, $E = x_0{}^*L + y_0 + z_0{}^*N$, and $R = \sqrt{M^2 + N^2}$. Then the transformation matrix that maps the given plane (points in this plane) into the $z = 0$ plane is

$$\begin{bmatrix} R & 0 & L & 0 \\ -L^*M/R & N/R & M & 0 \\ -L^*N/R & -M/R & N & 0 \\ 0 & 0 & -E & 1 \end{bmatrix}$$

and its inverse is

$$\begin{bmatrix} R & -L^*M/R & -L^*N/R & 0 \\ 0 & N/R & -M/R & 0 \\ L & M & N & 0 \\ E^*L & E^*M & E^*N & 1 \end{bmatrix}$$

[See the appendix of reference 2.]

(E) Given the transformation that maps an object from a local coordinate system into its corresponding world (global) space coordinates, let the point $P = (px, py, pz)$ locate the origin of the local system in world space, and (x_1, x_2, x_3), (y_1, y_2, y_3), and (z_1, z_2, z_3) be unit vectors that give the x, y, and z axis directions for the local system in world space. Then a point (X, Y, Z) in the local system is mapped into its world space coordinates by use of the following rule:

$$(x, y, z) = X^*(x_1, x_2, x_3) + Y^*(y_1, y_2, y_3) + Z^*(z_1, z_2, z_3) + (px, py, pz).$$

4. Construction Of A Pipe Run With A Single Elbow In The x-y (z=0) Plane

The polygon we are considering has two straight line segments, with each line being in the $x - y$ $(z = 0)$

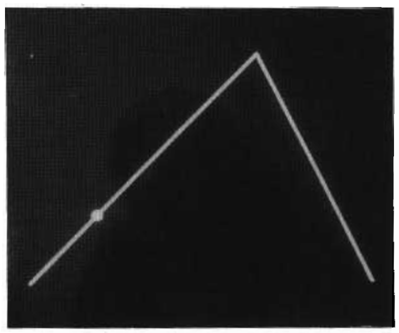

Figure 2 : A two line segment polygon in the z = 0 plane. A start point for the pipe run has been added.The end point is assumed to be the end of the second segment.

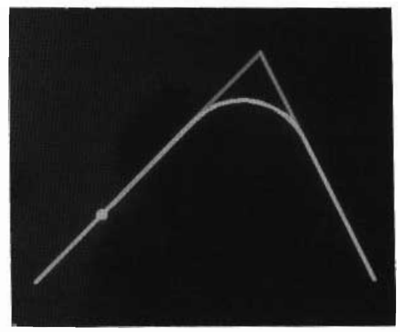

Figure 3 : The polygon of Figure 2 with the center line for a circular elbow added.

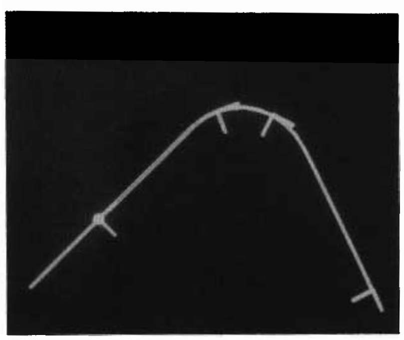

Figure 4 : The center line curve for a circular elbow (starting with a polygon of two line segments) with a few orientation vectors and tangent vectors being displayed.

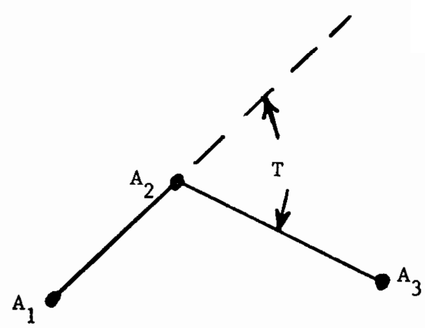

Figure 5 : The angle, T, used to compute the effective orientation vector at the mitered elbow.

plane (Figure 2). We will consider the steps required in defining a pipe run using the given polygon and a given cross section (the pipe cross section need not be circular). First we determine the center line information, that is the coordinate points along the center line and a local coordinate system at each point which will be used to position the cross section in place. Next, we position the center of the cross section (the [0,0] point for the cross section) at the computed points and then generate a wire frame or faceted model.

The following steps are taken in determining the center line information:

(i) Select a point on the initial straght line segments of the polygon where the pipe (duct) definition is to start (Figure 2 and Figure 3). (It is usual to use the first point of the line segment as a default value.) At this point a local coordinate system is defined and used to position the cross section during a later operation. One axis of this system is determined by the tangent vector of the center line curve. The next axis is defined by an input vector normal to the tangent vector or a default value. This will be referred to as the orientation vector. This orientation vector is used to position the cross section in the plane defined by the tangent vector. The positioning of the cross section becomes important for noncircular cross sections (and to avoid twisting of the pipe during constructing in the general 3-dimensional case in the following section).

The tangent vector is determined by the difference of the second and first points of the polygon. That is, the unit tangent vector is:

$$T(1) = (A_2 - A_1) / \| A_2 - A_1 \| ,$$

where A_1, A_2, and A_3 are the sequence of defining points for the polygon.

If the orientation vector is not supplied then it is defaulted to the vector

$$OR = (YT, -XT, 0) ,$$

where $T(1) = (XT, YT, 0)$.

(ii) Points that will determine the elbow are now determined. Two cases are considered, the circular elbow and the mitered elbow.

(a) For the circular elbow the procedure (A) from the preceding section is used to compute the center of the arc, the start point of the arc and the end point of the arc. A predetermined number of intermediate points are computed. The tangent vectors to the circular arc are computed at the time the points are computed.

At the start of the circular arc is assigned the orientation vector of the preceding point (in this case the first point). The other orientation vectors are obtained by rotating this orientation vector as the points along the circular arc are computed (Figure 4).

(b) For the mitered case the only center line point at the elbow is the second point of the polygon. If A_1, A_2, and A_3 are the successive points of the polygon, then the unit normal to the projection (miter) plane at point A_2 is given by [3]:

$$N_2 = (V_1 + V_2) / \| V_1 + V_2 \| , \textit{ where}$$
$$V_1 = (A_2 - A_1) / \| A_2 - A_1 \| , \text{ and}$$
$$V_2 = (A_3 - A_2) / \| A_3 - A_2 \| .$$

What we call the effective orientation vector at the corner point A_2 is determined. This orientation vector is not needed to orient the cross section at the above mentioned projection (mitered) plane (as seen below), but rather to define an orientation vector at the next point on the straight line segment.

We proceed to determine the angle, T, between the extended line segment defined by points A_1 and A_2 and the line segment defined by the points A_2 and A_3 (Figure 5). The sign of the magnitude is determined by looking from point A_1 to A_2 and observing on which side of this ray point A_3 lies. If the point is on the right side, it is assigned a positive value and if it is on the left, it is assigned a negative value.

The effective orientation vector is now computed by the orientation vector of the preceeding point and rotating it about the z-axis by T-degrees.

(iii) We now select a point on the second straight line segment where the pipe definition is to end (usually this will be the end pont of the line segment). The tangent vector is that of the line segment, and the orientation vector is obtained by copying the orientation vector of the preceding point (i.e., the last point on the circular arc or the effective orientation vector for the mitered case).

We now proceed to position the cross section at center line points or project an existing positioned section onto a projection (mitered) plane. A given point of the collection of center line points will be a positioning point, that is a point where we position the cross section with the aid of the orientation vector, or a point where we have a projection plane and the preceding positioned section is projected into the projection plane. These two cases are considered separately:

(iv) First the positioning of the cross section at a given point $P = (Px, Py, Pz)$ using the local coordinate system mentioned in (i) above is considered. The center line tangent vector Tp at this point determines the local Z-axis, the orientation vector $OR = (x_1, x_2, x_3)$ determines the local X-axis, and the vector cross product $Tp \ x \ OR = (y_1, y_2, y_3)$ corresponds to the local Y-axis. Using procedure (E), a given point (X, Y) of the cross section is properly oriented by

$$(x, y, z) = X(x_1, x_2, x_3) + Y(y_1, y_2, y_3) + (Px, Py, Pz) .$$

Thusly, all points where positioning is required are handled (Figure 6).

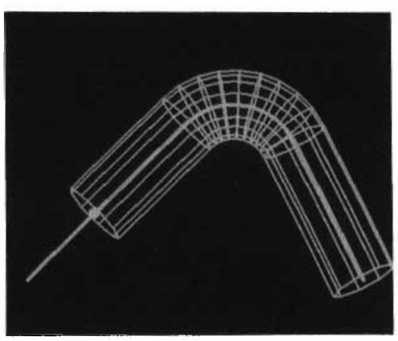

Figure 6 : Pipe run with a circular elbow for a polygon of two line segments.

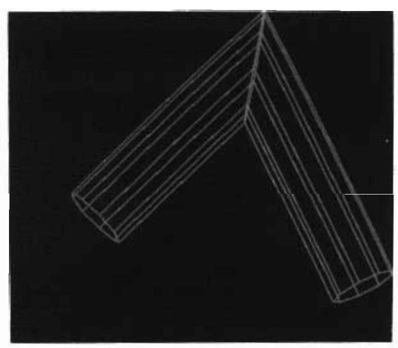

Figure 7 : A pipe run for a polgon of two line segments (in a plane) with a mitered type elbow.

(v) Now we consider that the given point is at a projection (mitered) plane. The normal to the projection plane was given in part (b) of (ii) as $N_2 = (xN, yN, zN)$. The tangent vector at the first point considered along the straight line segment had tangent vector $T(1) = (Tx, Ty, Tz)$ which will be the direction for each ray (line) considered. We also assume that at the point selected in the above procedure (i) a cross section has been positioned by procedure (iv). Each defining point of the positioned cross section (there is a finite number) is projected in direction T(1) into the projection plane to define a section outline. Procedure (B) is used to accomplish this (Figure 7).

(vi) With all sections in place, corresponding points of adjacent cross sections are attached by straight lines. That is the i-th point of section j is attached to the i-th point of section j+1.

5. Using A General 3-Dimensional Defining Polygon

The general 3-dimensional pipe (duct) run is easy to discuss with the use of the tools of the preceding two sections. Using (C) and (D) of section 3 we reduce the 3-dimensional pipe (duct) run problem to one of 2-dimensions (in the x-y [z=0] plane) sequentially and apply the discussion of section 4. This approach will seem reasonable when we consider the defining sequence of points P_1, P_2,...,' P_n for the polygon. Any successive three points will be assumed to be noncolinear and hence define a plane. Let P_I, P_{I+1}, and P_{I+2} be three such points, then we can proceed to use procedures (C) and (D) to transform the plane these points define into the z = 0 plane and use a slightly modified version of the discussion of the preceding section. We will:

(a) discuss the placement of an initial point on the line segment defined by points P_1 and P_2, this point's orientation vector, and the placement of the cross section at this point;

(b) discuss the definition of elbows at the points $P_2, P_3, \ldots, P_{n-1}$;

(c) discuss the center line end point on the pipe (duct) run which is on the line segment defined by the points P_{n-1} and P_n.

An initial point on the line segment defined by the points P_1 and P_2 must be supplied. This point is usually P_1 itself. An orientation vector can be supplied or a default value computed for this initial point. We use the points P_1, P_2 and P_3, the operations of (C) and the transform of (D), and then follow parts of the procedure (i) where needed. Now the inverse transform of (D) is applied to the orientation arm and the initial point. Subsequently follow (iv) to position the cross section at this initial point. This locates the cross section and the orientation arm in 3-D world space for the initial point.

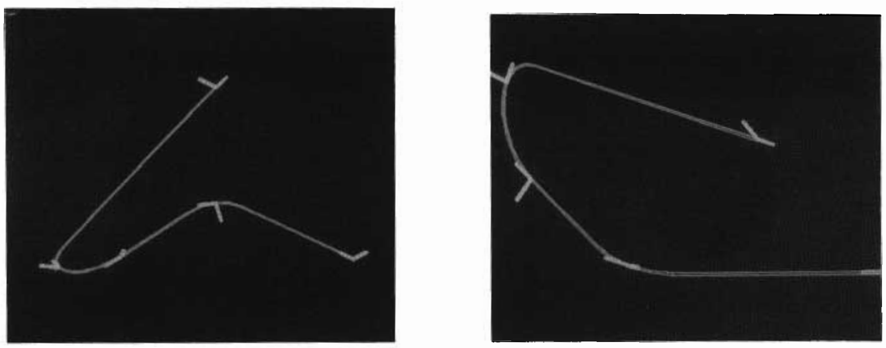

Figure 8 : Two views of a center line for a nonplaner pipe run. Note: the orientation vectors are at selected locations. The polygon for this center line can be viewed in Figure 1.

The end point on the line segment defined by points P_{n-1} and P_n is input or defaulted to the end point of the line segment. Using point P_{n-2}, P_{n-1} and P_n, we tranform the end point and the orientation vector of the preceding point of the center line into the z = 0 plane. The appropriate operations are carried out and the inverse transformation of (D) is applied to the cross section and procedure (iv) carried out.

Finally procedure (vi) of section 4 is carried out. That is corresponding points of adjacent cross sections are attached by straight lines.

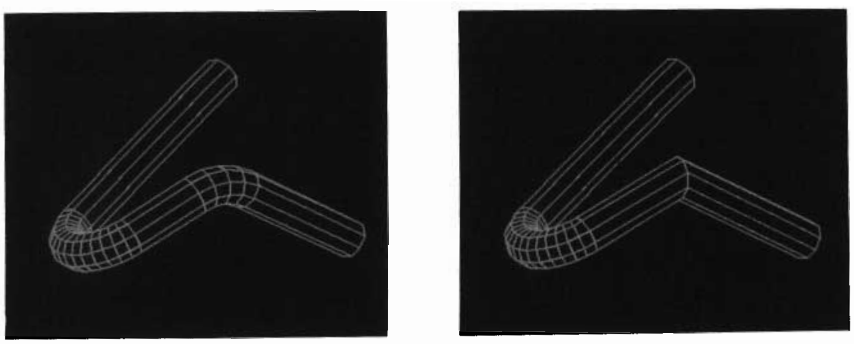

Figure 9 : A pipe run with a constant circular cross section. Mitered and circular elbow displayed.

6. Scaling The Cross Sections At Key Points

Up to this point the discussion has been restricted to a constant pipe (duct) cross section. It seems reasonable to consider a procedure to vary the duct cross section by scaling a copy of the original cross section just prior to the placement of this scaled cross section at a given point along the center line. To carry this out, each point in the above generated center line array is a default scale factor of one. A procedure to add scale factors to circular elbows and to selected locations along the straight line segments has been implemented.

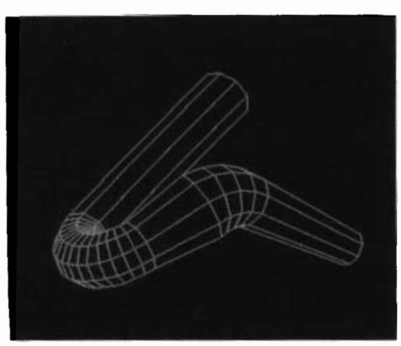

Figure 10 : Example of scale factors being added to the begining and end of an elbow.

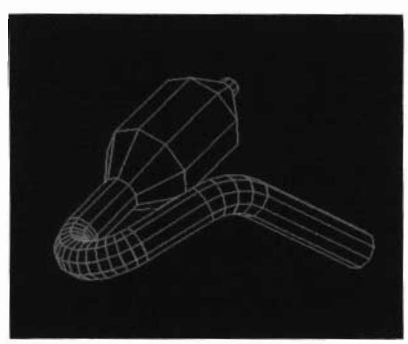

Figure 11 : Example of scale factors being assigned along the straight line portion of a pipe run.

One should be allowed to input scale factors at the beginning of a circular elbow and at its end. Then intermediate locations of the elbow are assigned scale factors in a linear fashion with respect to the angular sweep of the elbow. For example for a 90-degree elbow the scale factor at position of 30-degrees from the beginning of the elbow will be two-thirds the scale factor at the end point of the elbow.

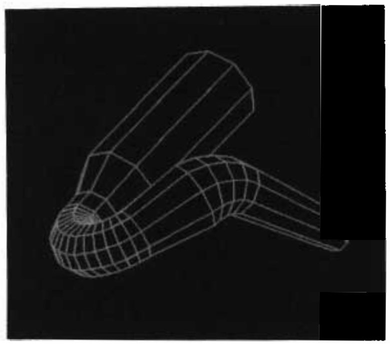

Figure 12 : Example of scale factors being assigned at an elbow and also along a straight line portion of a pipe run.

Figure 13 : The outer and inner walls at any location are being scaled by the same scale factor.

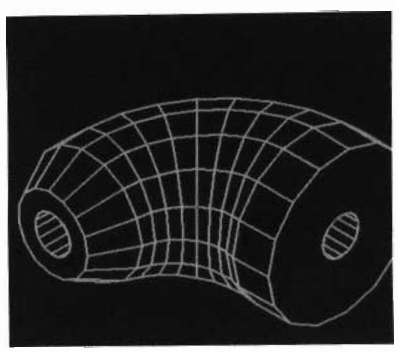

Figure 14 : The outer wall is scaled while the inner wall maintains a constant diameter.

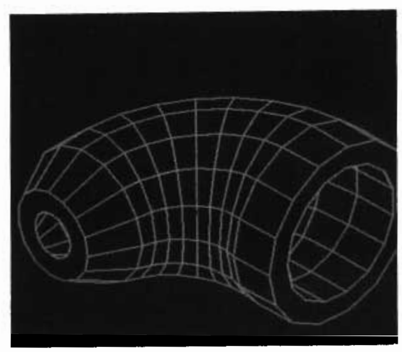

Figure 15 : the outer wall is a scaled and pipe thickness is kept a constant.

Scale factors should be allowed at any point along a line segment. If the point on the linear segment is not in the center line array, then it will be added to the center line array along with a copy of the tangent and orientation vector of the adjacent point in the center line array (that is, the added point is assigned the tangent value and orientation value of the adjacent point).

At the time the cross section is to be positioned at a point on the center line, the scale factor is checked. If the scale factor is different from unity, a copy of the cross section is scaled and this scaled copy is positioned at the given point (Figure 10, 11, and 12).

7. Pipes With Outer And Inner Walls

One should be able to model a pipe (duct) with not only an outer wall but an inner wall (Figure 13). The following special operation on the inner and outer walls can be made available:

(i) One is able to scale both the inner and outer with the same scale factor (Figure 13).

(ii) One is able to vary the scale of the outer wall while maintaining a constant scale factor for the inner wall (Figure 14).

(iii) One is able to scale the outer wall but maintain a constant pipe wall thickness (Figure 15).

For case number (iii), for a circular pipe with a constant wall thickness WT and an outer wall scale factor given at a point, the inner wall scale factor must be computed. The inner wall radius is the outer wall radius minus the wall thickness WT. Let RO be the outer wall radius and RI be the inner wall radius before scaling the section. Then, with SO being the scale factor for the outer wall and SI the scale factor for the inner wall to be determined we have

$$RO * SO - WT = \textit{inner wall radius after scaling} = RI * SI \text{ , or}$$

$$SI = (RO * SO - WT) / RI$$

8. Conclusions

The method of representation of a pipe (duct) run that has been described can be easily interfaced to display routines, and associated analysis routines. This method can be applied to both design and representation applications, whether interactive or batch oriented. The key to success of this method is based on five mathematical routines that have been discussed in section 3. Via the use of these math procedures, a pipe routing system is a reasonable project to undertake.

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