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Towards an Understanding of Camera Fixation

by

D. Raviv and M. Herman

TOWARDS AN UNDERSTANDING OF CAMERA FIXATION

Daniel Raviv* and **Martin Herman****

*The Robotics Center and The Electrical Engineering Department
Florida Atlantic University, Boca Raton, FL 33431; and
Sensory Intelligence Group, Robot Systems Division
National Institute of Standards and Technology (NIST)
Bldg 220, Room B124, Gaithersburg, MD 20899

**Sensory Intelligence Group, Robot Systems Division
National Institute of Standards and Technology (NIST)
Bldg 220, Room B124, Gaithersburg, MD 20899

ABSTRACT

A fixation point is a point in 3-D space that projects to zero optical flow in an image over some period of time while the camera is moving. This paper deals with quantitative aspects of fixation for a static scene. For the case where the rotation axis of the camera is perpendicular to the instantaneous translation vector we show that there is an infinite number of points that produce zero instantaneous optical flow. These points lie on a circle (called the Zero Flow Circle or simply ZFC) and a line. The ZFC changes its location and radius as a function of time, and the intersection of all the ZFCs is a fixation point. Points inside the ZFC produce optical flow that is opposite in sign to those that are outside the ZFC. This fact explains in a more quantitative way phenomena due to fixation. In particular, points in the neighborhood of the fixation point may change the sign of their optical flow as the camera moves. In a set of experiments, we show how the concept of the ZFC can be used to explain the optical flow produced by 3-D points near the fixation point.

1. INTRODUCTION

Camera fixation is an important concept in active vision that makes the image interpretation easier. Camera fixation is defined as actively controlling the camera motion so that a given visible point in 3-D space (a fixated point) is constantly imaged to the same point in the image plane. There are several advantages offered by fixation:

1. Determination of relative range. Because the imaged position of the 3-D point remains constant, the point results in zero optical flow. However, the optical flow arising from static points in a 3-D neighborhood of the fixation point can be used to easily determine whether these points are in front of or behind the fixation point [2]. Also, these points will have relatively small optical flow values, allowing the use of gradient-based flow extraction methods [4],[6].
2. Verifying range. If the range of a static object is hypothesized using some methods, then this range can be verified by pointing the camera optical axis at the object and then fixating on the object at the given range. If the optical flow of points on the object are near zero, then the range is verified.
3. Detailed analysis of objects. If a moving camera is fixated on an object of interest, then this object will be kept in the camera field of view for a long period of time, thus allowing detailed analysis of the object's properties. Also, multiple views of this object will be obtained, resulting in a more complete understanding of the object.
4. Increasing resolution. If a moving camera is fixated on a region of interest, then the camera field of view can be quite narrow. This results in high resolution imagery and allows detailed analysis of the region.
5. Motion compensation. If a camera with pan/tilt mechanism is mounted on a platform that is moving, then by fixating the camera at a very distant point, the camera will have no rotation in an inertial coordinate system. This is a way for maintaining camera orientation in an inertial frame without using an inertial navigation system.

The most general form of fixation arises from a six degree of freedom camera motion. A specialized and simple form of fixation is where a camera undergoes translation and no rotation. In this case, the camera can be thought of as fixating on a distant point lying on the line through the camera focal point in the direction of the velocity vector.

In this paper we quantitatively analyze the case where a camera fixates on a point in the 3-D space. The paper begins by deriving expressions for the optical flow for a six degree of freedom motion of the camera. We solve these equations to find the set of points in 3-D space that result in zero optical flow for instantaneous camera motion, where the rotation axis is perpendicular to the translation vector. These points form a circle and a line at each instant of time. If the camera motion is further restricted to continuously fixate on a point, we show how the zero-flow circles, one for each instant of time, can be used to elegantly analyze the optical flow that arises from points in a 2-D neighborhood around the fixation point. We finally show results taken during fixation experiments which can now be understood from the zero-flow circle analysis.

Previous work in the area of camera fixation has been mainly in the following three areas: (1) fixation for qualitative depth estimation [2], (2) fixating on a moving target for tracking applications [1], and (3) stereo fixation for vergence control [2],[3],[7]. Little previous work has been done that leads to a quantitative analysis of single camera fixation [4].

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2. SIX DEGREE OF FREEDOM MOTION EQUATIONS

This section describes the equations that relate a point in 3-D space to the motion of that point in the image for general six degree of freedom motion of the camera. These equations can be found in many books, e.g., see [5].

In the following analysis we assume a moving camera in a stationary environment. Suppose the coordinate system is fixed with respect to the camera as shown in Figure 1. Let the instantaneous coordinates of a point P in space be $\mathbf{r} = (X, Y, Z)$. (Note that here $Z>0$ for points in front of the imaging system.) If the instantaneous translational velocity of the camera is $\mathbf{t} = (U, V, W)^t$ and the instantaneous angular velocity is $\omega = (A, B, C)^t$ (where the superscript t denotes transpose) then the velocity vector $\mathbf{V}$ of the point P with respect to the XYZ coordinate system is:

$$\mathbf{V} = -\mathbf{t} - \omega \times \mathbf{r} \qquad (1)$$

or:

$$\dot{X} = -U - BZ + CY \qquad (2)$$

$$\dot{Y} = -V - CX + AZ \qquad (3)$$

$$\dot{Z} = -W - AY + BX \qquad (4)$$

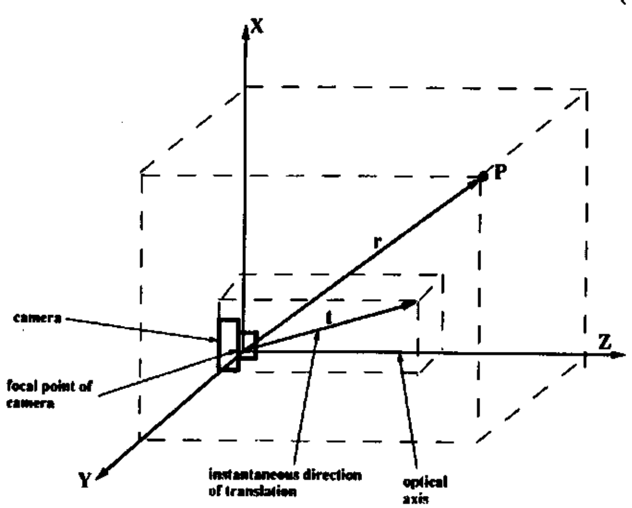

Figure 1: Coordinate system fixed to camera

The dot denotes derivative with respect to time. If f is the focal length of the camera, and using the standard pinhole camera model, the 3-D coordinates X and Y are related to their projected coordinates x and y by:

$$x = \frac{X}{Z} f \qquad (5)$$

and

$$y = \frac{Y}{Z} f \qquad (6)$$

The optical flow (u, v) is defined by (assuming $Z>0$):

$$u = \dot{x}$$

$$v = \dot{y}$$

The values u and v may be obtained by differentiating equations (5) and (6) with respect to time (for fixed, non-zero f), and substituting into equations (2)-(4):

$$\frac{u}{f} = \frac{\dot{X}}{Z} - \frac{X\dot{Z}}{Z^2} = \left[-\frac{U}{Z} - B + C\frac{Y}{Z}\right] - \frac{X}{Z}\left[-\frac{W}{Z} - A\frac{Y}{Z} + B\frac{X}{Z}\right] \qquad (7)$$

$$\frac{v}{f} = \frac{\dot{Y}}{Z} - \frac{Y\dot{Z}}{Z^2} = \left[-\frac{V}{Z} - C\frac{X}{Z} + A\right] - \frac{Y}{Z}\left[-\frac{W}{Z} - A\frac{Y}{Z} + B\frac{X}{Z}\right] \qquad (8)$$

Given the optical flow values u and v and the motion parameters, then the general simultaneous solution for (X, Y, Z) of equations (7) and (8) is an intersection of two quadratic surfaces.

3. POINTS OF ZERO FLOW

Suppose that we want to determine the locus of points in 3D space that produce zero optical flow in the image for a given arbitrary six degree of freedom camera motion. To do so we simply set $u=0$ and $v=0$ in equations (7) and (8) and solve for X, Y, and Z ($Z \neq 0$). All points in 3-D space that satisfy this solution are called Zero Flow Points (ZFPs). However, the solution to these two equations is not unique. In general, there is an infinite number of solutions.

In this paper we analyze a specific motion in the instantaneous $Y-Z$ plane of the camera coordinate system. Later we use this analysis for the case where the camera fixates on a single point in space.

Let the camera motion vectors $\mathbf{t}$ and ω be defined as follows:

$$\mathbf{t} = (0, V, W)^t$$

$$\omega = (A, 0, 0)^t$$

This means that the translational vector may lie anywhere in the instantanous $Y-Z$ plane while the rotation is about the X axis. Substituting these motion vectors into equations (7) and (8) gives:

$$\frac{u}{f} = -\frac{X}{Z}\left[-\frac{W}{Z} - A\frac{Y}{Z}\right] \qquad (9)$$

$$\frac{v}{f} = \left[-\frac{V}{Z} + A\right] - \frac{Y}{Z}\left[-\frac{W}{Z} - A\frac{Y}{Z}\right] \qquad (10)$$

To determine the ZFPs for this motion, we set $u=0$ and $v=0$ in equations (9) and (10) to get the solutions (assuming $Z \neq 0$):

$$X = 0 \text{ and } \left[Z - \frac{V}{2A}\right]^2 + \left[Y + \frac{W}{2A}\right]^2 = \left[\frac{V}{2A}\right]^2 + \left[\frac{W}{2A}\right]^2 \qquad (11)$$

$$Y = -\frac{W}{A} \text{ and } Z = \frac{V}{A} \qquad (12)$$

These solutions are drawn in Figure 2. Solution (11) is an equation of a circle that lies in the $Y-Z$ plane. The radius of the circle is $\left[\left[\frac{V}{2A}\right]^2 + \left[\frac{W}{2A}\right]^2\right]^{\frac{1}{2}}$ and its center is at $\left[0, -\frac{W}{2A}, \frac{V}{2A}\right]$. The circle is tangent to the camera translation vector at the origin. This can be shown as follows: the slope $\frac{dZ}{dY}$ of a tangent to the circle at any point is

$$\frac{dZ}{dY} = \frac{-\left(Y+\frac{W}{2A}\right)}{\left(Z-\frac{V}{2A}\right)}.$$ At the origin, this takes the value $\frac{W}{V}$, which is the same as the slope of the translation vector t.

Solution (12) is a straight line perpendicular to the $Y-Z$ plane and intersecting this plane at the point $\left(-\frac{W}{A},\frac{V}{A}\right)$. This intersection point also lies on the circle defined in solution (11).

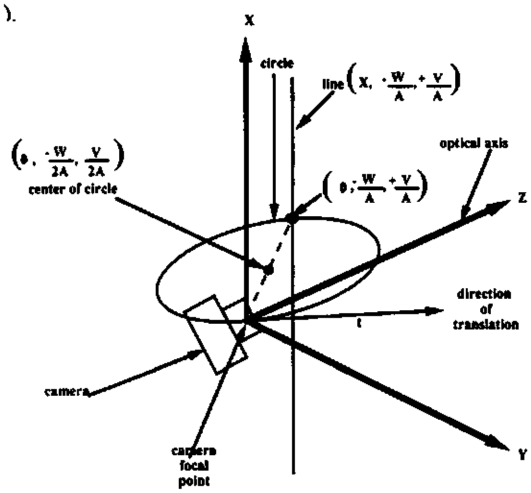

Figure 2: Pictorial description of solutions (11) and (12)

The meaning of these solutions is the following: any point in 3-D space that lies on the circle or the line described by solutions (11) and (12) and which is visible (i.e., unoccluded and in the field of view of the camera) will result in zero optical flow. We call the circle on which ZFPs lie the Zero Flow Circle (ZFC). If the X component of the camera rotation vector ω is positive (i.e., $A>0$), then visible points in $Y-Z$ plane that are inside the ZFC produce optical flow $\frac{v}{f}<0$, while visible points outside the ZFC produce optical flow $\frac{v}{f}>0$ in the image plane (see Figure 3). This can be shown as follows. The points that are inside the ZFC satisfy

$$\left(Z-\frac{V}{2A}\right)^2+\left(Y+\frac{W}{2A}\right)^2<\left(\frac{V}{2A}\right)^2+\left(\frac{W}{2A}\right)^2 \qquad (13)$$

Given the constraint in (13), then for $A>0$, equation (10) is satisfied if and only if $\frac{v}{f}<0$. A similar discussion holds for $A<0$.

4. THE ZFCs AS A FUNCTION OF TIME

As the camera moves through 3-D space, the ZFC moves with it. Figure 4 is an example of a camera path with some ZFCs. At each instant of time, the radius of the ZFC is a function of the instantaneous motion parameters **t** and ω. The location of the ZFC is such that it always contains the origin of the camera coordinate system (the same as the camera focal point), is tangent to the instantaneous translation vector **t**, and is perpendicular to the instantaneous rotation vector. Furthermore, it lies to the left or to the right of the translation vector depending on whether the instantanous rotation is positive or negative.

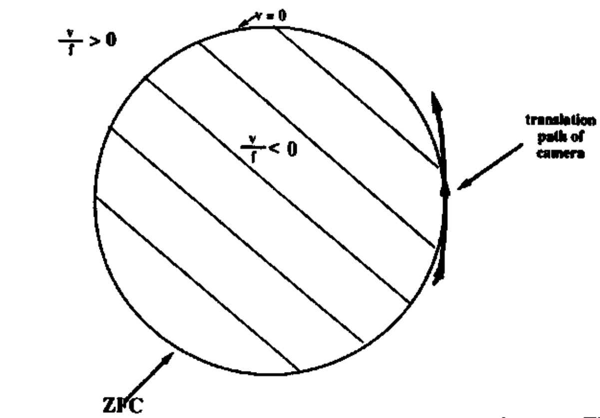

Figure 3: Optical flow signs inside and outside the ZFC

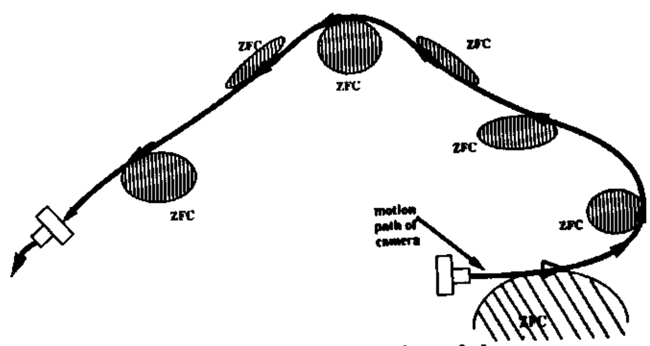

Figure 4: ZFCs as a function of time

5. FIXATION

If the point on which the camera fixates is visible in the image, then the corresponding image point will have zero optical flow during fixation. However, at a specific time instant, a fixation point is one out of many that may produce zero optical flow. For motion in an instantaneous plane, as described previously, these points lie on a circle and a line. Since points inside the circle produce flow values of opposite sign to those that are outside the circle, a point in 3-D space which is not the fixation point may produce different flow values at different instants of time.

Consider Figure 5 which shows fixation during rectilinear translation. By definition, a fixation point (assume it is visible) produces zero optical flow at all instants of time during the motion. If the fixation point lies in the instantaneous $Y-Z$ plane, then the ZFC at each instant of time must contain the fixation point. (There are cases where there are more than one fixation points.)

Now consider the point A in Figure 5. We qualitatively describe how the optical flow due to A changes throughout the camera motion. At time instant t1, point A lies outside the instantaneous ZFC; at time t2, point A lies on the ZFC; at time t3, point A lies inside the ZFC; at time t4, point A lies again on the ZFC; and at time t5, point A lies again outside the ZFC. This means that the optical flow due to

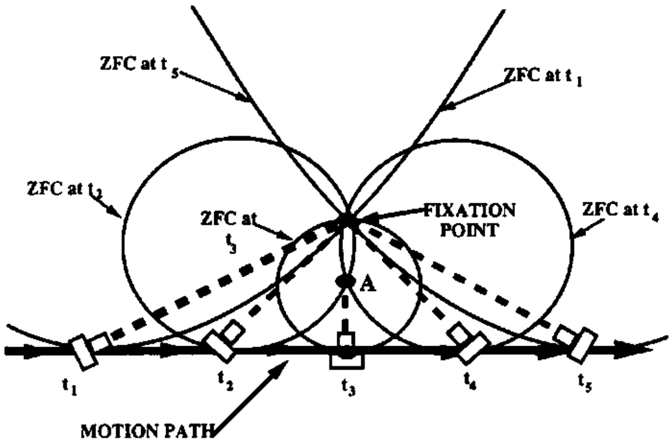

Figure 5: Fixation and the ZFCs

point A changes sign twice from $\frac{v}{f}>0$ to $\frac{v}{f}<0$ and back to $\frac{v}{f}>0$. For $t\to-\infty$ or $t\to\infty$ the optical flow asymptotically approaches zero.

Figure 6a is a sketch of the qualitative distance $\Delta\frac{y}{f}$ as a function of time, i.e., the difference in the image plane between the y coordinate of the point A and the y coordinate of the fixation point as a function of time. Figure 6b shows a qualitative sketch of the optical flow $\frac{v}{f}$ due to a point A as a function of time. This flow is the time derivative of $\Delta\frac{y}{f}$. At $t\to-\infty$, t_3, and at $t\to\infty$ the image of point A is at the same location as the image of the fixation point; thus $\Delta\frac{y}{f}=0$ at these times.

For $t<t_2$, $\Delta\frac{y}{f}$ is a strictly increasing function, thus the optical flow $\frac{v}{f}$ is positive (and point A lies outside the ZFC). At $t=t_2$, $\Delta\frac{y}{f}$ gets its maximum value and the optical flow is zero (point A lies on the ZFC). For $t_2<t<t_4$, $\Delta\frac{y}{f}$ is a strictly decreasing function, and the optical flow is negative (point A lies inside the ZFC). At $t=t_4$, $\Delta\frac{y}{f}$ gets its minimum value, and the optical flow is zero (point A lies on the ZFC). For $t>t_4$, $\Delta\frac{y}{f}$ is a strictly increasing function, and the optical flow is positive (point A lies outside the ZFC).

6. EXPERIMENTAL RESULTS

We performed some experiments to test the theory of the ZFCs. These experiments involved camera motion under fixation. We predicted qualitative values for optical flow based on ZFC analysis, and compared it with actual flow values obtained during experimentation. In all cases, the actual flow values are consistent with the predicted values.

We examined three fixation scenarios. All three experiments involved the same camera motion and the same objects in the scene. However, the configuration of the objects relative to the camera translation motion path was different in the three experiments. Three different objects were placed colinearly on a table. The camera motion involved a constant velocity translation with fixation on the middle object. The experimental scenarios are shown in figure 7 and will be described in more detail below.

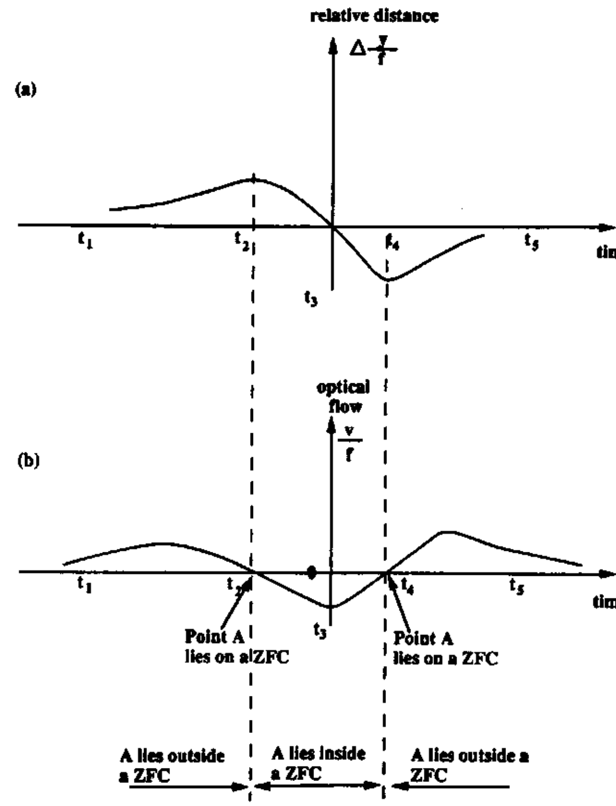

Figure 6: (a) Relative distance between point A and the fixation point, (b) Optical flow due to point A

For each experiment, the imagery at a single scan line $x=0$ of the camera was recorded as a function of time. (The camera scanline was parallel to the table top at all times.) Each experiment thus resulted in a spatio-temporal image for the given scan line. The horizontal coordinate of the figure represents the pixels spatial position on the scan line (the y coordinate of the image as in Figure 1), and the vertical coordinate represents time for successive frames of the same scan line. These images are shown in Figure 8, and will be described in more detail below. In each case the fixated object is the darker vertical line in each image, while the other two curves belong to the other objects. The optical flow is the time derivative of each curve, or the slope of the curves in Figure 8.

<u>Experiment 1:</u> (Figures 7(a) and 8(a))

In this experiment the three objects (A1, A2, and the fixation point) lie on a line parallel to the direction of camera translation path (Figure 7a). The points A1 and A2 produce optical flow that is changing as a function of time (as seen by the slope $\frac{dy}{dt}$ in Figure 8a). There is one place for each point where there is a change in the optical flow sign. This is due to a change in relative location between the points and the ZFCs. First ($t=t_1$), the point A1 is inside the ZFC (producing negative optical flow) and the point A2 is outside the ZFC (positive flow). Then ($t=t_2$), the point A1 is on the ZFC (zero flow) and A2 is still outside the ZFC. When the camera is perpendicular to the direction of translation ($t=t_3$) A1 and A2 are outside the ZFC (positive flow). Later ($t=t_4$), The point A1 is outside the ZFC (positive flow) and the point A2 is on the ZFC (zero flow). Finally ($t=t_5$) A1 is outside the ZFC (positive flow) and A2 is inside the ZFC (negative flow).

Experiment 2: (Figures 7(b) and 8(b))

In this experiment the three objects (A3, A4, and the fixated object) lie on a line perpendicular to the direction of camera translation. First ($t=t_1$), A4 is inside the ZFC (negative flow) and A3 is outside it (positive flow). Then ($t=t_2$), A4 is on the ZFC (zero flow) and A3 is outside the ZFC (positive flow). At time instant $t=t_3$, A3 and A4 are outside the ZFC (positive flow). Then ($t=t_4$) A3 is on the ZFC (zero flow) and A4 is outside the ZFC (positive flow).

Later ($t=t_5$), A3 stays inside the ZFC (negative flow) while A4 is outside the ZFC (positive flow). Similarly, we can continue to analyze the time instants after t_5. Note that in this case each point changes the direction of its corresponding optical flow twice due to two passes through ZFCs. In Figure 8b we marked the zero flow points (two for A3, and two for A4).

Experiment 3: (Figures 7(c) and 8(c))

In this case we examine a more general case. The points A5 and A6 lie on a line that is not parallel nor perpendicular to the direction of the camera direction. Here, as in experiment 2, for each point there are two changes in the sign of the optical flow that can be explained using the ZFCs. Due to experimental limitations only one change for each point is seen in Figure 8c.

7. DISCUSSION

In this paper we present a quantitative way for analyzing fixation. Using the new concept of ZFCs it is possible to explain the behavior, e.g., direction of optical flow, of points near the fixation point as a function of time. The instantaneous camera's direction of translation and the fixation point determines the plane on which the ZFCs can be found. We show that points inside the ZFC produce optical flow that is opposite in sign to that produced by points outside the ZFC. (When a point in 3-D space crosses a ZFC it produces Zero flow.) This analysis complements the qualitative understanding of fixation: It shows that a point that is not the fixation point may change its optical flow sign, and is not restricted to "moves to the right (or to the left)" as has commonly been assumed.

The extension of the ZFC concept is currently being investigated at NIST. We plan to exploit this concept in a vision based navigation algorithm for a real-time robot system.

8. ACKNOWLEDGEMENTS

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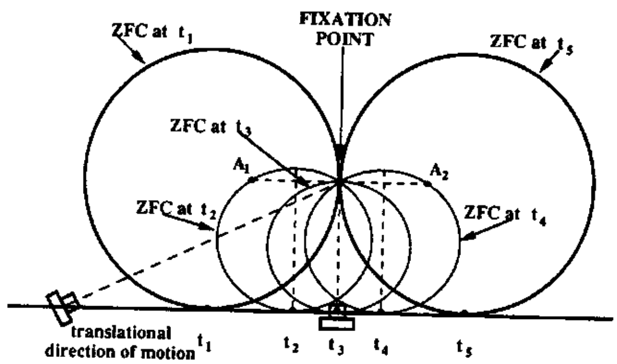

Figure 7(a): Points and ZFCs in Experiment 1

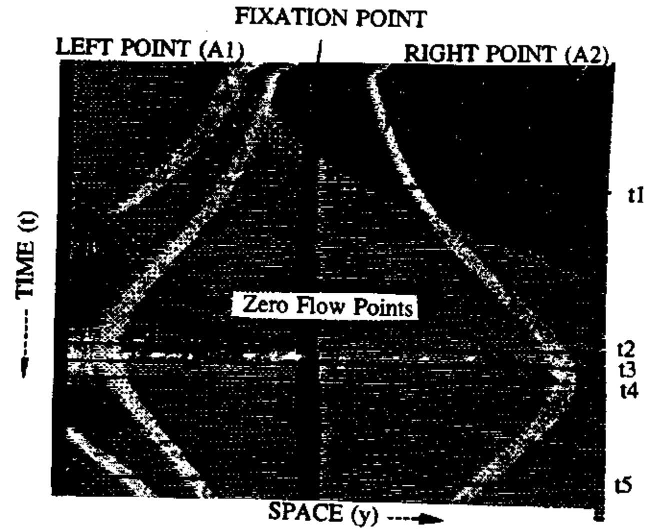

Figure 8(a): Experiment 1: Results in time-space domain

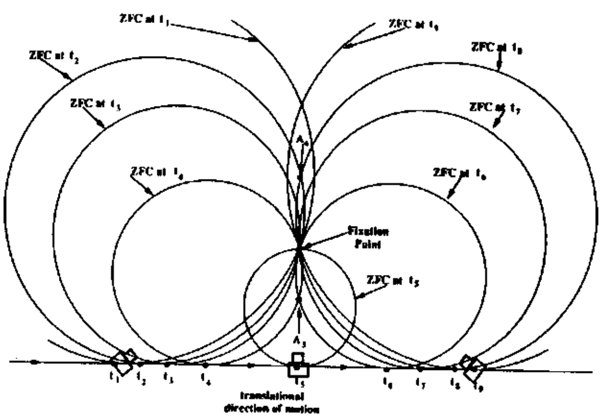

Figure 7(b): Points and ZFCs in Experiment 2

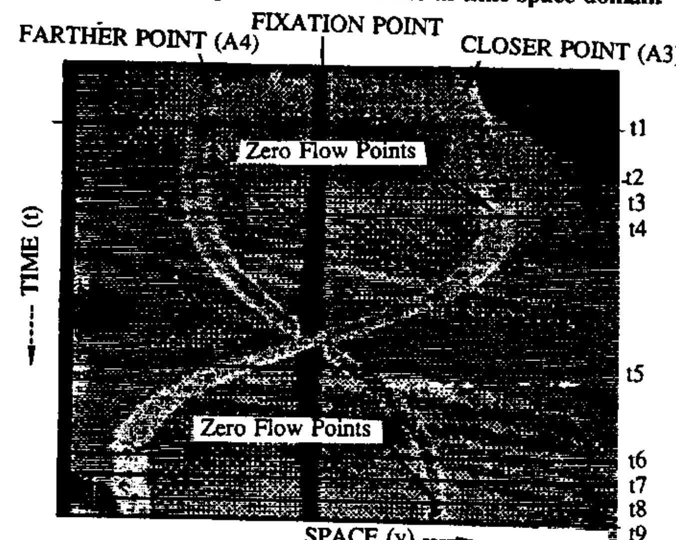

Figure 8(b): Experiment 2: Results in time-space domain

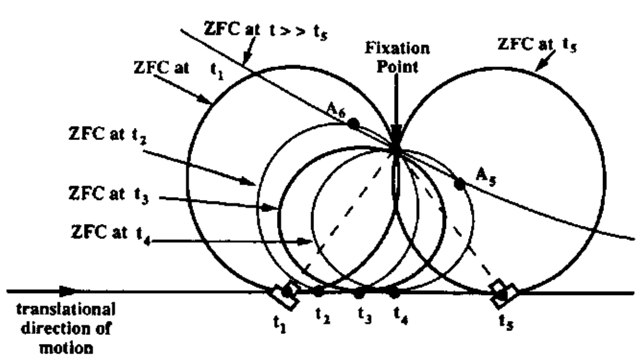

Figure 7(c): Points and ZFCs in Experiment 3

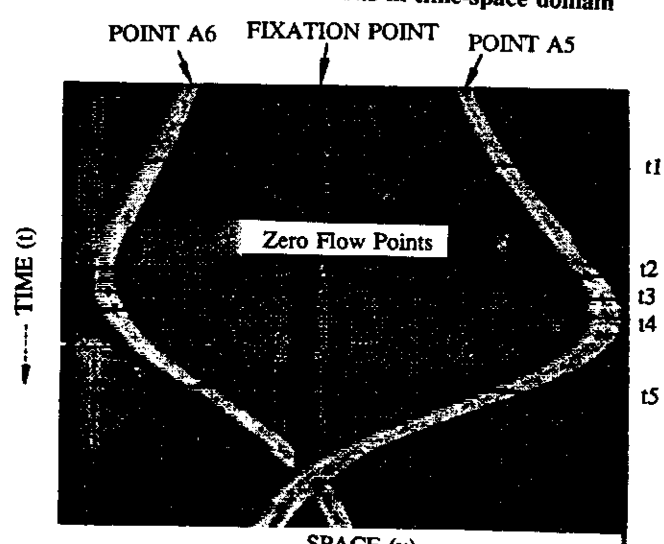

Figure 8(c): Experiment 3: Results in time-space domain