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Finite-Element Analysis of Flexible Fixturing System

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A computer software has been developed for the analysis and design of fixtures. This software can lead the designer to the optimal design of the fixturing system which minimizes the total work done on the workpiece, the fixturing force, the deformation index, or the maximum effective stress. The workpiece is modeled as a linear isotropic elastic solid. The machining forces are simulated by specifying applied forces acting on part of the surface of the workpiece. The fixturing system consists of a number of fixture elements, each in contact with the workpiece with specified location and area of contact. At the interface of contact, Coulomb's law of friction is employed. The boundary conditions at the interface of contact are treated exactly. This computer software system is composed of a finite-element program and a computer graphic program which displays the undeformed and deformed workpiece with hidden lines removed. Three sample problems have been solved and the numerical results are presented in this paper.

Introduction

An important factor in flexible manufacturing is workpiece positioning and constraint. Fixturing a workpiece is basically a process to guide and locate the workpiece with appropriate geometrical constraints provided by the fixture elements. Kinematic analysis is essential for understanding the workpiece motion under geometrical constraints, as discussed by Lozano-Perez, Mason, and Taylor [1] for automatic motion synthesis and by Mason [2] for analyzing pushing motions. Recently, Asada and By [3] presented a complete theoretical study based on kinematic analysis to derive the condition for the fixturing system to provide total constraint on the workpiece inhibiting all motion. They also derived the necessary condition for the fixture-workpiece combination to be accessible and detachable in a reliable manner.

However, in kinematic analysis, the workpiece is assumed to be a rigid body; hence, it can only have six degrees of freedom in rigid-body motion. For this assumption, the corresponding fixturing system is the one which eliminates the rigid-body motion. Also, in the work of Asada and By [3], the friction force between the workpiece and the fixture element is not taken into consideration. By assuming the workpiece to be rigid, there can be six equations of motion. If the fixturing system provides more than six constraints, then the fixture-workpiece combination is statically indeterminate. Also, it is unrealistic to ignore friction between the workpiece and the fixture elements. Actually, friction is the predominant mechanism for workpiece holding in most fixturing applications.

In this work, the workpiece is modeled as a deformable body based on linear elasticity, and Coulomb friction is included. After finite-element analysis is performed for the fixturing system, the deformation of the workpiece, the clamping forces of the fixture elements, the stress distribution, and any other quantities derivable from the displacements and stresses can be calculated. As optimal design of the fixturing system is concerned, one may want to minimize the clamping forces, the maximum effective stress, or the deformation. Because, large clamping forces indicate that excessive amount of work is done on the workpiece; high stress may cause failure of the workpiece; and large deformation may cause the loss of accuracy in precision machining. In order to analyze and optimally design the flexible fixturing system, a computer software has been developed.

It is worthwhile to mention that the boundary conditions at the interface of contact are treated exactly in this work (Section 5). In some commercially available finite-element programs, those kinds of boundary conditions are approximated by using the GAP element. The exact treatment of the boundary conditions for fixturing is equivalent to using GAP elements with infinite stiffness – of course, this won't work. Even if one is willing to approximate the stiffness by a very large, not infinite, value, the rate of convergence decreases as the stiffness increases, which means one has to sacrifice the accuracy of the solutions for the sake of computational economy.

Throughout this paper, the standard notations for vector, matrix, and tensor are used; the subscripts indicate the components in a vector, matrix, or tensor; the superscript is just a part of the symbol. It is noticed that, in this paper, each variable (cf. Nomenclature) is either dimensionless or involves with the unit of length, the unit of force, or the combination of both. For example, if the unit of length is inch and the unit of force is pound, then the unit for Young's modulus and stress is psi and the unit for work (energy) is in.-lb.

Problem Description

Let a three-dimensional general-shaped workpiece occupy a

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region V with its surrounding surface denoted by S. The finite-element mesh of the workpiece consists of N^* elements with N^P nodal points.

The machining forces exerted on the workpiece are simulated by specifying forces acting on those nodal points where the machining is occurring.

The workpiece is held in position by the fixture elements, which are assumed to be rigid. Only nodal points on S may be in contact with the fixture elements. There are N^f fixture elements; and the αth fixture element ($\alpha = 1, 2, 3, \ldots . N^f$) is in contact with surface $S(\alpha)$ which consists of $m(\alpha)$ nodal points. After the clamping actions of the fixture elements are actuated, each fixture-element induces an inward displacement $\Delta(\alpha)$ normal to the surface $S(\alpha)$.

Coordinate Transformation

The global coordinate system employed in this work is a rectangular coordinate system (x, y, z). However, for those nodal points on S and in contact with fixture elements, it is convenient to use local coordinate systems (n, r, t) where the n-axis is normal to the surface at the nodal point in question, the r-axis and the t-axis are mutually perpendicular and on the tangent plane. Suppose the unit outward normal (to the surface S) of the ith nodal point is denoted by a vector

$$\mathbf{n} = (n_x, n_y, n_z) = (a_1, a_2, a_3), \tag{1}$$

then $\mathbf{r}$ and $\mathbf{t}$ may be constructed as follows:

(*a*) If $a_1 = 0$

$$\mathbf{r} = (r_x, r_y, r_z) = (b_1, b_2, b_3) = (1,0,0) \tag{2}$$

$$\mathbf{t} = (t_x, t_y, t_z) = (c_1, c_2, c_3) = (0, a_3, -a_2) \tag{3}$$

(*b*) If $a_1 \neq 0$

$$\mathbf{r} = (r_x, r_y, r_z) = (b_1, b_2, b_3) = (-a_1 a_2,\ 1 - a_2^2,\ -a_2 a_3)/\sqrt{1 - a_2^2}, \tag{4}$$

$$t = (t_x, t_y, t_z) = (c_1, c_2, c_3) = (-a_3,\ 0,\ a_1)/\sqrt{1 - a_2^2}. \tag{5}$$

The transformation matrix for the ith nodal point can then be constructed as

$$[Q] = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, \tag{6}$$

and the global displacements (u_x, u_y, u_z) and global forces (f_x, f_y, f_z) at the ith nodal point can be transformed to their corresponding local values according to

$$\begin{bmatrix} u_n & f_n \\ u_r & f_r \\ u_t & f_t \end{bmatrix} = [Q] \begin{bmatrix} u_x & f_x \\ u_y & f_y \\ u_z & f_z \end{bmatrix}. \tag{7}$$

It is straightforward to transform the governing equilibrium equations to the following form

$$K_{ij}^* u_j^* = f_i^*, \tag{8}$$

where $[K^*]$ is the transformed stiffness matrix, and the transformed nodal point displacement and force vectors are

$$\mathbf{u}^* = \{u_x(1), u_y(1), u_z(1), \ldots, u_n(i), u_r(i), u_t(i), \ldots\}, \tag{9}$$

$$\mathbf{f}^* = \{f_x(1) f_y(1) f_z(1), \ldots f_n(i) f_r(i) f_t(i), \ldots\}. \tag{10}$$

Nomenclature

- c = coefficient of friction
- D = deformation index, see equation (25); unit = [Length] = $[L]$
- E = Young's modulus; unit = [Force]/$[L]^2$ = $[F]/[L]^2$
- F = amount of fixturing force; unit = $[F]$
- $\mathbf{f}^*$ = transformed nodal point force vector, see equation (10); unit = $[F]$
- f = amount of tangential force; unit = $[F]$
- $\bar{f}_n, \bar{f}_r, \bar{f}_t, \bar{f}, \bar{u}_r, \bar{u}_t, \bar{u}^s$ = values of $f_n, f_r, f_t, f, u_r, u_t, u^s$ obtained in the previous iterative step
- (f_x, f_y, f_z) = nodal point forces in (x, y, z) coordinate; unit = $[F]$
- (f_n, f_r, f_t) = nodal point forces in (n, r, t) coordinate; unit = $[F]$
- $[K^*]$ = transformed stiffness matrix, see equation (8); unit = $[F]/[L]$
- $m(\alpha)$ = number of nodal points on $S(\alpha)$
- (n,r,t) = local coordinate system, see Section 3; unit = $[L]$
- N^* = number of finite element in the workpiece
- N^P = number of nodal points in the finite-element mesh
- N^f = number of fixturing elements
- $[Q]$ = transformation matrix between (x, y, z) and (n, r, t)
- S = surface surrounding the workpiece
- $S(\alpha)$ = the interface between the workpiece and the αth fixturing element
- u^s = amount of tangential displacement; unit = $[L]$
- $\mathbf{u}^*$ = transformed nodal point displacement vector, see equation (9); unit = $[L]$
- (u_x, u_y, u_z) = nodal point displacements in (x, y, z)-coordinate; unit = $[L]$
- (u_n, u_r, u_t) = nodal point displacements in (n, r, t)-coordinate; unit = $[L]$
- V = region occupied by the workpiece
- W = total work done on the workpiece; unit = $[F]$ $[L]$
- W^m = work done by the machining forces; unit = $[F]$ $[L]$
- W^f = work done by the fixturing elements; unit = $[F]$ $[L]$
- (x, y, z) = global coordinate system; unit = $[L]$
- $\Delta(\alpha)$ = the inward displacement normal to $S(\alpha)$ induced by the αth fixturing element; unit = $[L]$
- Δ_c = cutoff tightness, unit = $[L]$
- ν = Poisson's ratio
- σ = maximum effective stress; unit = $[F]/[L]^2$
- θ = angle between the tangent force vector and the tangent displacement vector

Law of Friction

At the interface beween the workpiece and the fixture element, it is assumed that friction force exists. Without friction force, any axis-symmetric workpiece, such as a cylinder or a sphere, could not be held in position because the normal constraints induced by the fixture elements cannot prevent rigid-body rotation of the workpiece about its axis of symmetry.

In this work, Coulomb's law of friction is expressed as either one of the following:

(*a*) *Sticking Case*

$$f \equiv \sqrt{f_r^2 + f_t^2} < c\,|f_n|, \tag{11}$$

$$u_r = u_t = 0, \tag{12}$$

(*b*) *Sliding Case*

$$f = c\,|f_n|, \tag{13}$$

$$\cos(\theta) \equiv (f_r u_r + f_t u_t)/f u^s = -1, \tag{14}$$

where, in equations (11)–(14), c is the coefficient of friction; f is the shear force on the tangent plane; u^s defined as

$$u^s \equiv \sqrt{u_r^2 + u_t^2} \tag{15}$$

is the tangential displacement; $\cos(\theta)$ equal to -1 means the shear force is parallel to the tangential displacement but opposite in direction. Also, it is noticed that the fixture element can only exert compression on the workpiece, i.e., $f_n \le 0$, otherwise, that nodal point in question should be regarded as being detached from the fixture element.

Boundary Conditions

A mechanical system represented by equations (8)–(10) has $3N^P$ degrees of freedom. For each degree of freedom, either the nodal point force or the nodal point displacement has to be specified, and its counterpart will be obtained after equation (8) is solved. The machining forces are specified as the applied forces. The applied forces at the interior points as well as those points on S which are not in contact with any fixture element, are zero. For a typical point which is in contact with the αth fixture element, the boundary conditions, to begin with, are specified as

$$u_n = -\Delta(\alpha), \qquad u_r = u_t = 0. \tag{16}$$

After equation (8) is solved, one obtains f_n, f_r, f_t, however, it is then necessary to check whether the followings are satisfied

$$f_n \le 0, \tag{17}$$

$$f < -cf_n. \tag{18}$$

If conditions (17) and (18) are satisfied, it means boundary conditions (16) are correct; otherwise, one has to change the boundary conditions and solve equation (8) again. The iterative process may be represented by the following cases:

Case 1. This is the case to begin with. It is the sticking case in which the friction force is less than the coefficient of friction times the normal force and hence there is no tangential displacement.

Specify:

$$u_n = -\Delta(\alpha), \qquad u_r = u_t = 0. \tag{16*}$$

Check:

If $f_n > 0$, go to Case 3.

If $f \ge -cf_n$, go to Case 2.

Case 2. This is the beginning of the sliding case in which the point in contact is allowed to move on the tangential plane and a friction force equals to cf_n is applied as a resistance to the motion.

Specify:

$$u_n = -\Delta(\alpha), \quad f_r = A\bar{f}_r, \quad f_t = A\bar{f}_t, \tag{19}$$

where

$$A \equiv -c\bar{f}_n/\bar{f}, \tag{20}$$

and $\bar{f}_n, \bar{f}_r, \bar{f}_t, \bar{f}$ are the quantities obtained in previous case.

Check:

If $f_n > 0$, go to Case 3.

If $f \ne -cf_n$ or $\cos(\theta) \ne -1$, go to Case 4.

Case 3. This is the case in which the workpiece is detached from the fixture element.

Specify:

$$f_n = f_r = f_t = 0. \tag{21}$$

Check:

If $u_n \ge -\Delta(\alpha)$, go to Case 1.

Case 4. This is the sliding case in which the amount of the friction force should be equal to cf_n and the direction of the friction force should be parallel but opposite to the tangential displacement.

Specify:

$$u_n = -\Delta(\alpha), \quad f_r = -B\bar{u}_r, \quad f_t = -B\bar{u}_t, \tag{22}$$

where

$$B \equiv -c\bar{f}_n/\bar{u}^s, \tag{23}$$

and $\bar{f}_n$, $\bar{u}_r$, $\bar{u}_t$, $\bar{u}^s$ are the quantities obtained in previous case.

Check:

If $f_n > 0$, go to Case 3.

If $f \ne -cf_n$ or $\cos(\theta) \ne -1$, go the Case 4 again.

It is noticed that in Cases 1, 2, 4 the normal displacement instead of the normal force is specified as the boundary condition. This is because one fixture element is in contact with several nodal points and if, say, equal amount of normal forces are applied at those nodal points, unequal normal displacements may be induced at the interface of contact, which is undesired. Besides, after equation (8) is solved, one may calculate the distribution of nodal forces and stresses at the interface.

In classical finite-element analysis, the boundary conditions are known in advance; however, in this analysis, the boundary conditions may be expressed as in any one of the four cases. The validity of specified boundary conditions needs to be verified based on the solutions which are the consequences of the specified boundary conditions. Therefore, although this is a linear system, it involves iterations. The iterative process continues until all the boundary conditions are correctly specified or the discrepancies (observed in those checks) are smaller than a specified error tolerance. After the iterative process stops, one obtains the nodal point forces and displacements from which the deformed shape of the workpiece, and the stresses at each Gauss point within every element can be calculated. Also, the total strain energy, which is equal to the total work done by the machining forces, W^m, and by the clamping forces of the fixture elements, W^f, can be calculated as

$$W = \frac{1}{2} u_i^* f_i^* = W^m + W^f. \tag{24}$$

If it is so desired, W^m and W^f can also be calculated individually and independently.

A quantity, named the deformation index, is defined as

$$D \equiv \sqrt{\mathbf{u}^* \cdot \mathbf{u}^* / 3N^P} \tag{25}$$

It is seen that the deformation index is a first-order measure of

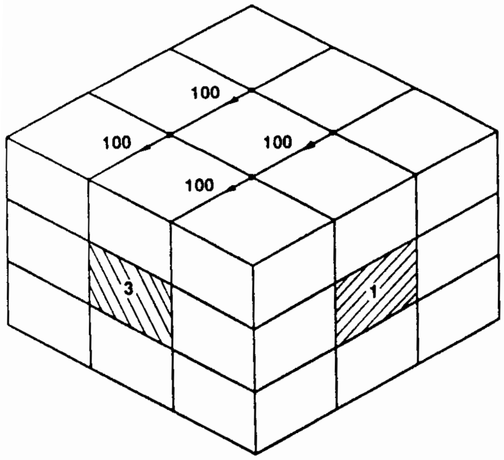

Fig. 1 Undeformed shape of the workpart; $N^f = 3$

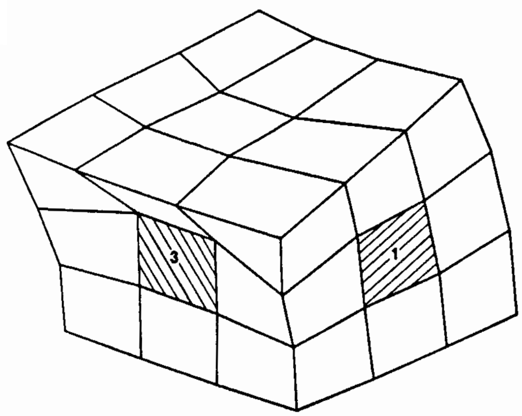

Fig. 2 Typical deformed shape of the workpart; $N^f = 3$

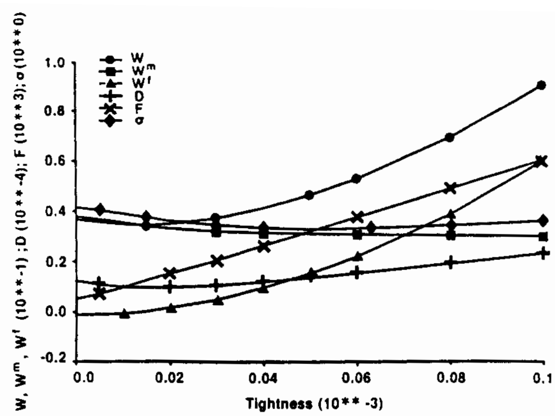

Fig. 3 Various parameters as functions of tightness; $c = 1.0$; $N^f = 3$

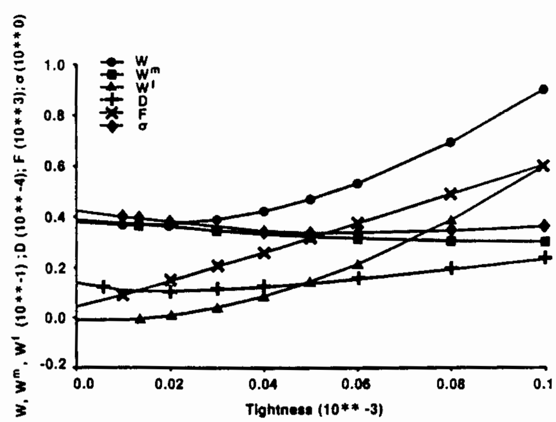

Fig. 4 Various parameters as functions of tightness; $c = 0.5$; $N^f = 3$

the extent of deformation of the workpiece. Also, if it is desired, dividing D by the characteristic length of the workpiece will make it nondimensional. Of course, there are many factors to be considered in the design of a fixturing system. Ideally, one may regard the optimal fixturing system as the one which minimizes either the total work done, W, the deformation index, D, the fixturing force, F (defined as the magnitude of total forces exerted by a fixturing element), or the maximum effective stress, σ, which is defined as

$$\sigma \equiv \sqrt{1.5[\sigma_{ij}\sigma_{ij} - \sigma_{mm}\sigma_{nn}/3]}$$

Numerical Results

In this section, for illustrative purposes, the numerical results of three sample problems are presented. Common to all three problems, the Young's modulus, E, and the Poisson's ratio, ν, are set to be 1.0E5 and 0.3, respectively. The input and output data of the three problems are described and shown as follows:

Problem 1. The workpiece occupies

$$V = \{x,y,z \mid 0 \le x \le 60,\ 0 \le x \le 60,\ 0 \le z \le 60\}$$

which is divided into 27 8-node solid elements and 64 nodal points, as shown in Fig. 1. The machining forces are simulated by specifying $f_x = -100$ acting on four points, (20,20,60), (20,40,60), (40,20,60), (40,40,60), on the top surface of the workpiece. It should be mentioned that the measurement and analysis of machining forces certainly is an important issue. Although it has not been addressed here in this paper, the software developed, even at this stage, can be used to study the sensitivity of the fixturing system due to the change of the amount, direction, and location of the machining force. The bottom surface of the workpiece is constrainted against motion in the z-direction. There are three fixture elements: the first one has a contact surface $S(1)$ against the workpiece, shown as the shaded area in Fig. 1, and $S(1)$ can be described as

$$S(1) = \{x,y,z \mid y = 0,\ 20 \le x \le 40,\ 20 \le z \le 40\} \quad (26)$$

and, for the second and the third fixture elements,

$$S(2) = \{x,y,z \mid y = 60,\ 20 \le x \le 40,\ 20 \le z \le 40\}, \quad (27)$$

$$S(3) = \{x,y,z \mid x = 0,\ 20 \le y \le 40,\ 20 \le z \le 40\}. \quad (28)$$

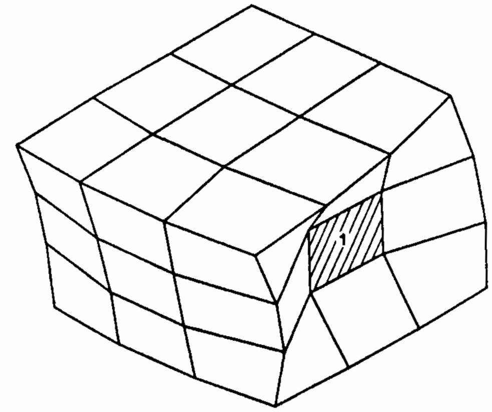

Fig. 5 Typical deformed shape of the workpart; $N^f = 2$

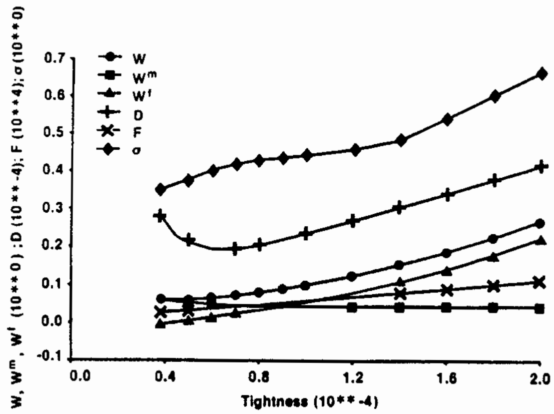

Fig. 6 Various parameters as functions of tightness; $c = 1.0$; $N^f = 2$

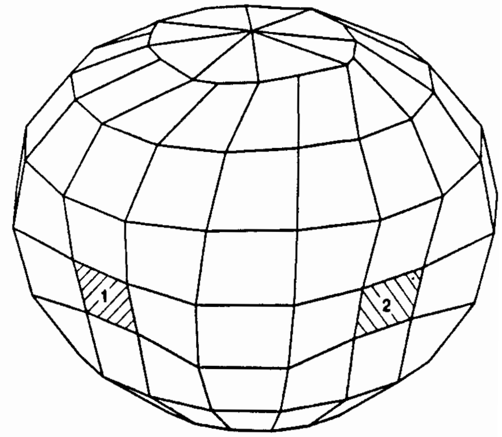

Fig. 9 Typical deformed shape of the workpart; $N^f = 4$

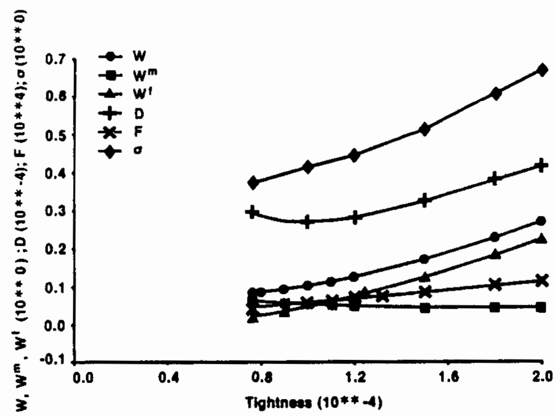

Fig. 7 Various parameters as functions of tightness; $c = 0.5$; $N^f = 2$

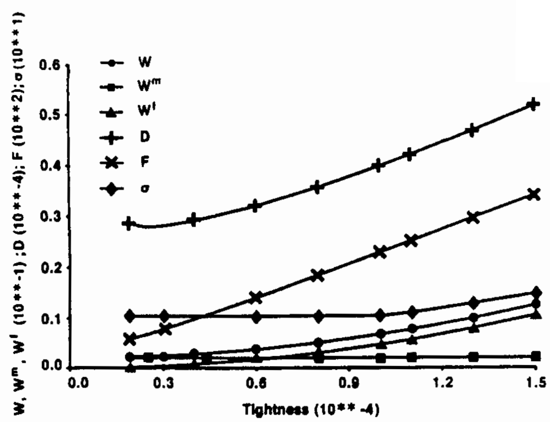

Fig. 10 Various parameters as functions of tightness; $c = 1.0$; $N^f = 4$

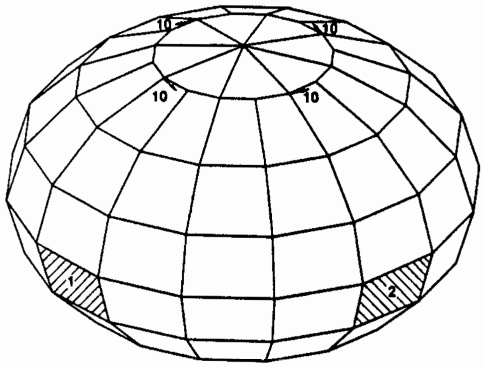

Fig. 8 Undeformed shape of the workpart; $N^f = 4$

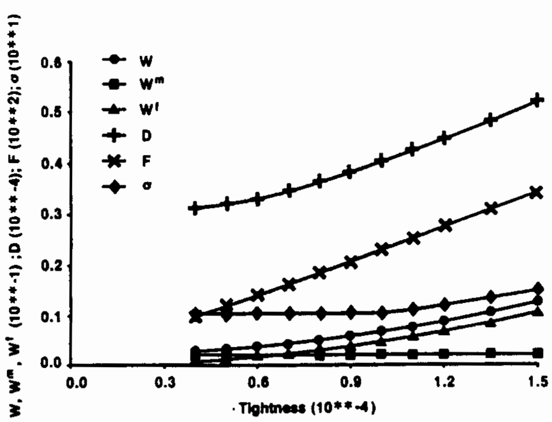

Fig. 11 Various parameters as functions of tightness; $c = 0.5$; $N^f = 4$

The contact surface $S(3)$ is also shown as the shaded area in Fig. 1. The inward normal displacement induced by the fixture element, Δ, may be called the tightness and, in this problem, $\Delta(1) = \Delta(2) = \Delta$, $\Delta(3) = 0$, which means $S(1)$ and $S(2)$ are clamps and $S(3)$ is a stop.

It is seen that this fixturing system works even if the friction force is neglected or the tightness, Δ, is set at zero. For a given Δ and coefficient of friction, c, the finite-element solutions are obtained after a few iterations. The typical and exaggerated deformed shape of the workpiece is shown in Fig. 2. In Figs. 3 and 4, the total work done, W, the work done by machining force, W^m, the work done by fixture elements, W^f, the amount of fixturing force, F, acting on $S(1)$, the deformation index, D, and the maximum effective stress, σ, are shown as functions of the tightness, Δ, for $c = 1.0$ and $c = 0.5$, respectively.

Problem 2. This is the same problem as the previous one except that the fixture element No. 3 is taken away from the system. It is seen that this fixturing system depends solely on the friction forces, acting on $S(1)$ and $S(2)$, to prevent the motion of the workpiece in the negative x-direction. Therefore, there is a cutoff tightness, Δ_c, such that if $\Delta < \Delta_c$, the workpiece is *unstable* – this is indicated, in the finite-element program, by a singular stiffness matrix, $[K^*]$, and the solution does not exist. Associated with the cutoff tightness, there is a cutoff fixturing force which is the lower bound of the clamping force of that fixture element. The typical deformed shape of the workpiece is shown in Fig. 5. The numerical results of this problem are shown in Figs. 6 and 7.

Problem 3. The workpiece is a ball which occupies

$$V = \{r, \theta, \phi \mid 8 \le r \le 10, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}.$$

V is divided into 112 8-node solid elements and 228 nodal points, as shown in Fig. 8. There are four machining forces, each of 10 units acting on (x, y, z) = (3.827, 0.0, 9.239), (0.0, 3.827, 9.239), (−3.827, 0.0, 9.239), (0.0, −3.827, 9.239), as shown in Fig. 8. There are four fixture elements, located 90 deg apart in the x–y plane, two of which can be seen in Fig. 8. This problem is seen to have a periodicity of 90 deg about the z-axis.

It is obvious that if there is no friction force, there is no way to prevent the rigid-body rotation of the ball. There is a cutoff tightness, Δ_c, associated with this problem, below which part motion cannot be prevented. A typical deformed shape of the ball is shown in Fig. 9. The numerical results are shown in Figs. 10 and 11.

Discussions

It is seen that the fixturing systems fall into two major categories: the first one (represented by Problem 1) does not solely depend on the friction forces to provide the constraints for the workpiece and the second one (represented by Problems 2 and 3) solely depends on the friction forces to prevent the workpiece from rigid-body motion. Therefore, the tightness (or the coefficient of friction) in the first category can equal to zero as shown in Figs. 3, 4; for the second category, there is a cutoff tightness associated with the fixturing system as shown in Figs. 6, 7, 10, and 11. Moreover, the cutoff tightness is almost inversely proportional to the coefficient of friction.

As indicated in Figs. 3, 4 and 6, at smaller tightness, the work done by the fixture elements is negative – this is because, usually at small tightness, the work done due to the friction forces at the interfaces dominates and there the friction forces are opposite in direction to the tangential displacements as dictated by the friction law. On the other hand, as the tightness becomes larger, the solutions are the same for different values of the coefficient of friction, c, because all the nodal points at the interfaces are in the sticking case equations (11) and (12) and the boundary conditions are independent of c.

Common to all three problems, it is noticed that the fixturing force is increasing monotonically, almost linearly, with the tightness; the work done by the fixture elements is also increasing monotonically with the tightness; the work done by the machining forces decreases and then levels off as the tightness increases; therefore, the total work done either has a relative minimum, as shown in Figs. 3 and 4 or an absolute minimum, as shown in Figs. 6, 7, 10, and 11 – the lack of relative minimum in these cases is due to the existence of the cutoff tightness.

It is also seen that the deformation index has a relative minimum in Figs. 3, 4, 6, 7 and 10, and the maximum effective stress has a relative minimum in Figs. 3 and 4. In the first category, there are relative minima for the total work done, W, the deformation index, D, and the maximum effective stress, σ. However, the locations of those relative minimums may not coincide with each other, hence it is up to the designer to make the decision as to which parameter to optimize. Certainly, the designer should avoid excessive tightness. In the second category, at the cutoff tightness, W, F, and σ are at their absolute minimum. Of course, the designer may not want to have a fixturing system for which the tightness is too close to the cutoff tightness. For precision machining, part deformation results in part-dimensional errors, so the deformation index may often be the parameter to optimize.

Another observation is that, for both categories, the fixturing system with the larger coefficient of friction is better in the sense that, at the optimal design, it results in smaller W, D, F and σ. For example, for Problem 2 at cutoff tighness, $W=0.085$, $D=0.297\text{E-}4$, $F=468$, $\sigma=0.372$ at $c=0.5$ are reduced to $W=0.062$, $D=0.282\text{E-}4$, $F=290$, $\sigma=0.348$ at $c=1.0$.

In this work, the fixture elements, which hold the workpiece in position, are assumed to be rigid. Of course, it would be more realistic and interesting to model the fixture elements as deformable bodies. However, that would make the analysis much more difficult.

Another limitation of this work is that the friction force is assumed to obey the simplest Coulomb's law. When the fixturing force becomes so great that local yielding occurs at the interface, the Coulomb's law of friction is no longer valid.

In conclusion, it is admitted that this work is only a theoretical analysis of the fixturing system. Further research needs to be done on several areas, e.g., the measurement of machining forces, the determination of coefficient of friction (even the investigation of the friction law itself), the integration of software and hardware, etc., before one can have a practical and automated tool for fixturing.

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