# Stability of Precision Diamond Turning Processes That Use Round Nosed Tools* 

D. E. Gilsinn<br>Mem. ASME

M. A. Davies<br>Assoc. Mem. ASME

National Institute of Standards and Technology, Gaithersburg, MD 20899

## B. Balachandran

Assoc. Mem. ASME Department of Mechanical Engineering, University of Maryland,
College Park, MD 20742-3035

A system of retarded differential equations model is developed for a diamond-turning process with chip area cut by a round nosed tool. The results are presented in the form of stability charts that are found to be perturbed forms of those obtained for orthogonal cutting tools. [DOI: 10.1115/1.1373648]

## 1 Introduction

In orthogonal cutting the overlap factor, $\mu$, is one. For $\mu<1$ Özdoganlar and Endress [1] developed an iterative procedure to compute the stability limit. For a large set of frequencies, defining the stability boundaries, this becomes numerically prohibitive. We introduce an analytic algorithm for the stability chart of a tool with a round nosed tip with an overlap factor $\mu<1$.

## 2 Model and Characteristic Equation

The tool model is an approximation to a round boring bar as a uniform, homogeneous, cantilever beam. We consider depths of cut from 1 to $3 \mu \mathrm{~m}$, assuming tangential oscillations are small, with circular tool tip profile approximated by a parabolic tool tip profile with depth of cut much smaller than the tool tip radius [2]. The chip area $A_{c}$ is a function of the feed rate, $f$, depth of cut, $d$, tool tip radius, $R$, tool displacements at the previous cut, $q_{\tau}$ $=q(t-\tau)$, and the present cut, $q(t)$, where $\tau$ is the delay time for one spindle revolution. $A_{c} \approx \alpha_{0}+\alpha_{1} q+\alpha_{2} q_{\tau}$ where $\alpha_{0}=(24 d f R$

[^0]$\left.-f^{3}\right) /(24 R), \quad \alpha_{1}=(f+2 \sqrt{2 R d}) / 2, \quad \alpha_{2}=(f-2 \sqrt{2 R d}) / 2 . \quad$ Half width for the cut is $z_{0} \approx \sqrt{2 R d}$. Asai and Kobayashi [3] show $\alpha_{1}$ approximates the chip width for a round nosed tool.

The dynamic equations are a system of time delay differential equations, linearized about zero,
$\ddot{q}_{i}+2 \xi_{i} \omega_{i} \dot{q}_{i}+\omega_{i}^{2} q_{i}=-W_{i} K \alpha_{1}\left(\sum_{n=1}^{N} q_{n} W_{n}+\mu \sum_{n=1}^{N} q_{n \tau} W_{n}\right)$
where $\mu=\alpha_{2} / \alpha_{1}$. The characteristic equation is

$$
\begin{align*}
\prod_{m=1}^{N} & \left(s^{2}+2 \xi_{m} \omega_{m} s+\omega_{m}^{2}\right)+K \alpha_{1}\left(1+\mu e^{-s \tau}\right) \\
& \times \sum_{m=1}^{N}\left(\prod_{\substack{k=1 \\
k \neq m}}^{N}\left(s^{2}+2 \xi_{k} \omega_{k} s+\omega_{k}^{2}\right)\right) W_{m}^{2}=0 \tag{2}
\end{align*}
$$

## 3 Perturbation Analysis

Write (2) as

$$
\begin{equation*}
1+K \alpha_{1}\left(1+\mu e^{-s \tau}\right) \sum_{m=1}^{N} \Phi_{m}(s) W_{m}^{2}=0 \tag{3}
\end{equation*}
$$

where $\quad \Phi_{m}(s)=1 /\left(s^{2}+2 \xi_{m} \omega_{m} s+\omega_{m}^{2}\right)$. Let $s=i \omega, \quad \Phi_{m}(i \omega)$ $=G_{m}(\omega)+i H_{m}(\omega), \quad G(\omega)=\sum_{m=1}^{N} W_{m}^{2} G_{m}(\omega) \quad$ and $\quad H(\omega)$ $=\sum_{m=1}^{N} W_{m}^{2} H_{m}(\omega)$. Equating real and imaginary parts of (3) to zero

$$
\begin{gather*}
1+K \alpha_{1} G(\omega)(1+\mu \cos \omega \tau)+K \alpha_{1} H(\omega) \mu \sin \omega \tau=0  \tag{4}\\
-H(\omega)(1+\mu \cos \omega \tau)+G(\omega) \mu \sin \omega \tau=0 \tag{5}
\end{gather*}
$$

From the definition of $\mu, \alpha_{1}$, and $\alpha_{2} \mu=(-1+\epsilon) /(1+\epsilon)$ for $\epsilon=f / 2 z_{0}$. In diamond turning $\epsilon$ is small. Since $1 /(1+\epsilon)=1-\epsilon$ $+\epsilon^{2}-\ldots, \mu \approx-1+2 \epsilon$.
From (5)

$$
\begin{equation*}
\frac{H(\omega)}{G(\omega)}=\frac{\mu \sin \omega \tau}{1+\mu \cos \omega \tau} \approx \frac{(-1+2 \epsilon) \sin \omega \tau}{1+(-1+2 \epsilon) \cos \omega \tau} \tag{6}
\end{equation*}
$$

Let $\omega(\boldsymbol{\epsilon})=w_{0}+\epsilon w_{1}+\epsilon^{2} w_{2}+\ldots$ then from (6) solve for the coefficients $w_{i}$ for $i=0,1,2, \ldots$ so that to a first order $\epsilon$

$$
\begin{align*}
\frac{H(\omega)}{G(\omega)} & \approx \frac{H\left(w_{0}\right)+\epsilon w_{1} H^{\prime}\left(w_{0}\right)}{G\left(w_{0}\right)+\epsilon w_{1} G^{\prime}\left(w_{0}\right)} \\
& \approx \frac{-\sin w_{0} \tau_{0}+\epsilon\left(2 \sin w_{0} \tau_{0}-w_{1} \tau_{0} \cos w_{0} \tau_{0}\right)}{\left(1-\cos w_{o} \tau_{0}\right)+\epsilon\left(2 \cos w_{0} \tau_{0}-w_{1} \tau_{0} \sin w_{o} \tau_{0}\right)} \tag{7}
\end{align*}
$$

where $w_{0}$ and $\tau_{0}$ are selected to satisfy (7) when $\epsilon=0$. By cross multiplying the second part of (7) compute $w_{1}$.
Find an implicit function $\tau(\epsilon)$, defined on an interval about $\epsilon=0$, with $\quad \tau(0)=\tau_{0} \quad$ and $\quad\left((-1+\epsilon) \sin \omega(\epsilon) \tau_{0}\right) /((1+\epsilon)+(-1$ $\left.+\epsilon) \cos \omega(\epsilon) \tau_{0}\right)=\left(-\sin w_{0} \tau(\epsilon)\right) /\left(1-\cos w_{0} \tau(\epsilon)\right)$. The left side of (6) becomes $\quad H(\omega(\epsilon)) / G(\omega(\epsilon))=\left(-\sin w_{0} \tau(\epsilon)\right) /\left(1-\cos w_{0} \tau(\epsilon)\right)$ $=\tan \psi(\epsilon)$ where $\psi(\epsilon)$ is the phase.


Fig. 1 Stability chart for a 101.6 mm (5/8in) bar during a $1 \mu \mathrm{~m}$ depth of cut showing the second mode lobes overlapping portions of the stability region between first mode lobes at high rotation rates. Solid and dashed lines correspond to first and second modes, respectively.

Solve for $w_{0} \tau(\epsilon)$ using half angle formulas to show $\tan \psi(\epsilon)$ $=\tan \left(\pi / 2+w_{0} \tau(\epsilon) / 2 \pm n \pi\right)$. Then $\psi(\epsilon)=\pi / 2+w_{0} \tau(\epsilon) / 2 \pm n \pi$. In order to maintain $\tau(\epsilon)>0$, select the negative sign and $n=2$ $+p$ for $p=0,1,2, \ldots$ Then $w_{0} \tau(\epsilon)=2(\psi(\epsilon)+p \pi)+3 \pi$ for $p$ $=0,1,2, \ldots$ The spindle rotation rate is

$$
\begin{equation*}
\Omega(\boldsymbol{\epsilon})=\frac{1}{\tau(\boldsymbol{\epsilon})}=\frac{w_{0}}{2(\psi(\boldsymbol{\epsilon})+p \pi)+3 \pi} \tag{8}
\end{equation*}
$$

Solve (4) for $\alpha_{1}$, using (7) and keeping first order terms in $\epsilon$

$$
\begin{equation*}
\hat{\alpha}_{1}\left(\epsilon, \omega_{0}\right) \approx \frac{-1}{2 K G\left(w_{0}\right)+\epsilon K P\left(w_{0}\right)} \tag{9}
\end{equation*}
$$

where

$$
\begin{align*}
P\left(w_{0}\right)= & G\left(w_{0}\right)\left[2 \cos w_{0} \tau_{0}+\left(\frac{w_{1}}{w_{0}}\right)\left(w_{0} \tau_{0}\right) \sin w_{0} \tau_{0}\right] \\
& +H\left(w_{0}\right)\left[2 \sin w_{0} \tau_{0}-\left(\frac{w_{1}}{w_{0}}\right)\left(w_{0} \tau_{0}\right) \cos w_{0} \tau_{0}\right] \\
& +w_{1} G^{\prime}\left(\omega_{0}\right)\left(1-\cos w_{0} \tau_{0}\right)-w_{1} H^{\prime}\left(w_{0}\right) \sin w_{0} \tau_{0} \tag{10}
\end{align*}
$$

$\epsilon$ is not an arbitrary parameter. $\alpha_{1}=z_{0}(1+\epsilon)$ so that $\alpha_{1}$ is a function of $\epsilon$. Equation (9) shows that there is another quantity $\hat{\alpha}_{1}\left(\epsilon, \omega_{0}\right)$ that represents the stability limit for $\alpha_{1}$. It is the critical chip width in that if $\alpha_{1}<\hat{\alpha}_{1}\left(\epsilon, \omega_{0}\right)$ then (1) is stable, otherwise unstable.

## 4 Stability Chart Algorithm

(1) Discretize a range of frequencies and form an array, $\omega_{0}$. (2) Form vectors $G\left(\omega_{0}\right), H\left(\omega_{0}\right), G^{\prime}\left(\omega_{0}\right), H^{\prime}\left(\omega_{0}\right)$. (3) Compute vector $\psi_{0}\left(\omega_{0}\right)=\operatorname{atan} 2\left(H\left(\omega_{0}\right), G\left(\omega_{0}\right)\right)$. (4) Compute array $\omega_{0} \tau_{0}$ satisfying (6) with $\mu=-1$. (5) Compute array $\omega_{1}$ from (7) and form the array $\omega(\epsilon)=\omega_{0}+\epsilon \omega_{1}$, where $\epsilon=f_{0} / 2 z_{0}$. (6) Compute $H(\omega(\epsilon)), G(\omega(\epsilon))$ and $\psi(\epsilon)=\operatorname{atan} 2(H(\omega(\epsilon)), G(\omega(\epsilon)))$. (7) Compute array $P\left(\omega_{0}\right)$ from (10) and form the array $2 G\left(\omega_{0}\right)$ $+\epsilon P\left(\omega_{0}\right)$. (8) Find two frequency intervals for each of the solution modes that make $2 G\left(\omega_{0}\right)+\epsilon P\left(\omega_{0}\right)$ negative. First, find the two intervals, $I_{1}$ and $I_{2}$, for which $2 G\left(\omega_{0}\right)$ is negative. (9) Within $I_{1}$ find subinterval, $S_{1}$ for which $2 G\left(\omega_{0}\right)+\epsilon P\left(\omega_{0}\right)$ is negative. (10) Find $S_{2}$ in $I_{2}$. (11) For the frequencies in each of the intervals $S_{1}, S_{2}$ compute arrays $\alpha_{11}(\epsilon), \alpha_{12}(\epsilon)$ from (9). (12) Select a set of lobe multiples, $p$, and compute $\Omega_{p}(\epsilon)$ from (8). (13) Map the array pairs $\left(\Omega_{p}(\boldsymbol{\epsilon}), \alpha_{11}(\boldsymbol{\epsilon})\right)$ and $\left(\Omega_{p}(\boldsymbol{\epsilon}), \alpha_{12}(\boldsymbol{\epsilon})\right)$.

## 5 Computational Results

The length of the steel bar is taken as 101.6 mm (4 in) and the diameter is taken as 6.35 mm ( $1 / 4 \mathrm{in}$ ). The damping factor for the first mode, $\xi_{1}$, was estimated as 0.04 , and the damping factor for the second, $\xi_{2}$, is determined from $\xi_{2}=\xi_{1} \omega_{1} / \omega_{2}$.

For the selected ranges of feed, depth of cut and tool nose radii the minimum critical chip width ranged from about 19 to $23 \mu \mathrm{~m}$. When feeds increased the critical chip width increased. As the depth of cut decreased for a fixed tool nose radius the percentage increased with increased feed. For a fixed depth of cut the percentage increased for a decrease in the tool nose radius.

For low r/min the stability lobes overlap each other, allowing small gaps between them for stable operation. Figure 1 shows the superposition of the lobes for the two modes at high rotation rates. The stable regions between the lobes at high rotation rates allow a much larger chip removal amount than at lower rotation rates. At a rotation rate of $13500 \mathrm{r} / \mathrm{min}$ there is a potential of a gain of approximately $70 \mu \mathrm{~m}$ in critical chip width.

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## Experimentation on the Residual Stresses Generated by Endmilling

## Kurt Jacobus

Mem. ASME, Graduate Research Assistant

S. G. Kapoor<br>Fellow ASME, Professor

R. E. DeVor<br>Fellow ASME, Professor

Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801

Residual stress measurements from endmilling of annealed AISI 4340 have been made at multiple points within the cut geometry to investigate the effects of location on the machining-induced residual stresses from endmilling. In the same experiments the effects of axial depth of cut and feed on the residual stresses induced in the machined surface have been investigated in a design of experiments framework. The experimentation demonstrates that location, feed and axial depth of cut have strong influences on the machining-induced residual stresses from end-milling when expressed in a workpiece coordinate frame. In addition, expression of the residual stresses in a coordinate frame fixed in the tool demonstrates a simplification in the residual stresses from endmilling in that the stresses at multiple locations within the cut geometry show strong similarities when expressed in this coordinate frame. [DOI: 10.1115/1.1392995]

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## 1 Introduction

Residual stresses are an unavoidable consequence of machining processes and can have pronounced effects on component life [1,2] and component geometry [3]. To obtain a full understanding of these effects, the mechanisms of residual stress generation from conventional machining processes including milling must be understood.

The milling process has two fundamental characteristics that distinguish it from other machining processes such as turning and orthogonal/controlled oblique machining. First, as the milling cutter passes through the work to form the new surface, the instantaneous orientation of the cutting edge changes with respect to the workpiece. The change in instantaneous orientation of the cutting edge with respect to the workpiece necessarily means that the chip generation process and thus the plastic deformations imparted to the workpiece are also constantly changing in orientation; hence the residual stresses can be expected to change with cutter rotation and thus location in the cut geometry [4]. Second, as the milling cutter rotates, the undeformed chip thickness continuously changes with rotation of the cutting edge. The change in the undeformed chip thickness with cutter rotation during milling means changes in the proximity of neighboring workpiece material to the cutting process and therefore changes in the workpiece material available to sink the heat generated during machining [5]. The result is variation in workpiece temperature with cutter rotation and therefore variation in the residual stresses due to the strong influence of temperature on residual stress generation (see e.g., [4]).

The current understanding of the residual stresses from milling operations results from a handful of publications [6-9]. These efforts have investigated the effects of processing conditions on the normal residual stresses at the center of the milling cut. Still unknown are the residual stresses across the milling cut geometry (which are expected to vary with location given the characteristics cited above) and the effects of processing conditions on this full biaxial residual stress profile. Because the stresses across the full cutter width will influence component life and component distortion, this information is necessary to determine the full effects of residual stresses on a given component. It is the purpose of this paper to address these needs through thorough experimentation on the residual stresses imparted by endmilling.

## 2 Experimental Details

In order to gain an understanding of the residual stresses generated from endmilling, the residual stresses were measured at four locations within the cut geometry for each of five tests in a slotting cut. Figure 1 presents these measurement locations. Location A (centerline), location B $(6.35 \mathrm{~mm}$ to the right of the centerline), location C ( 12.70 mm to the right of the centerline) and location $\mathrm{D}(12.70 \mathrm{~mm}$ to the left of the centerline) were selected to test the relationship between cutter orientation/cut geometry and the induced residual stresses.

Five tests were conducted in a $2^{2}$ full factorial experimental design plus a center point to determine the main and interaction effects of the axial depth of cut and feed rate on the machininginduced residual stresses. Test 1 was undertaken at a feed of 0.200 $\mathrm{mm} / \mathrm{rev}$ and an axial depth of cut of 0.20 mm , test 2 at feed 0.600 $\mathrm{mm} / \mathrm{rev}$ and an axial depth of cut of 0.20 mm , test 3 at feed 0.200 $\mathrm{mm} / \mathrm{rev}$ and an axial depth of cut of 0.60 mm , test 4 at feed of $0.600 \mathrm{~mm} / \mathrm{rev}$ and an axial depth of cut of 0.60 mm and test 5 at feed of $0.400 \mathrm{~mm} / \mathrm{rev}$ and an axial depth of cut of 0.40 mm . The cutter was fully engaged for all tests in a slotting cut; a cutting speed of $5 \mathrm{~m} / \mathrm{s}$ was chosen to ensure a thermal character in the residual stresses.

AISI 4340 was utilized for workpiece material in annealed condition to ensure a stress-free state prior to machining. After workpiece geometries were prepared, they were soaked at $840^{\circ} \mathrm{C}$ for two hours followed by a furnace cool to complete the anneal.


Fig. 1 Process geometry and measurement locations for endmilling experimentation

Endmilling insert geometry was the SPB-422, a 90 deg square insert with a nose radius of 0.8 mm and a flat rake face. One endmilling insert was mounted in the Kennametal KICR-1.50-SP4-0 inserted endmill, a 0 deg lead cutter with a cutting diameter of 38.1 mm . As mounted, the insert presented a 0 deg axial rake angle, a 0 deg radial rake angle, a 0 deg helix angle and an 11 deg clearance angle. A fresh insert was used for each test.

X-ray diffraction techniques were used to measure the surface and subsurface residual stresses (strains) in the workpieces following machining.

To determine the surface temperatures of the workpiece immediately after the tool pass and their relation to the machininginduced residual stresses, a Raytek Thermalert IV infrared pyrometer was utilized to measure temperatures for two different endmilling passes under each machining condition. The pyrometer was mounted to the machine tool bed to measure surface temperatures of the workpiece at the centerline of the endmilling pass; access to the newly-machined surface was obtained by rapidly reversing the endmill feed following cuts of sufficient length to allow steady state cutting conditions to be achieved.

## 3 Experimental Results and Discussion

Residual stress data for the endmilling experiments are provided in Fig. 2. These stresses are expressed in the workpiece coordinate system described in Fig. 1 with $x_{3}^{w}$ coinciding with the feed direction of the endmill, $x_{2}^{w}$ normal to the newly-generated surface and $x_{1}^{w}$ normal to the feed direction. Each of the rows of plots in Figs. 2 corresponds to a single test condition that includes the full biaxial residual stress profiles in planes parallel to the newly-formed surface at selected depths; for the purposes of the data presentation and later discussion only the centerline measurement location $A$ and the measurement locations most distant from the centerline, locations C and D , are included in each plot. In Fig. 3, the effects of location are provided for each residual stress component $\left(\sigma_{11}^{r, w}, \sigma_{13}^{r, w}\right.$ and $\left.\sigma_{33}^{r, w}\right)$ for all five test conditions at three depths: surface, $x_{2}^{w}=-10 \mu \mathrm{~m}$ and $x_{2}^{w}=-98 \mu \mathrm{~m}$. Finally, Fig. 4 presents the residual stresses from test 1 expressed in the


Fig. 2 Residual stress from endmilling tests 1-5, workpiece coordinate frfme
rotating tool coordinate frame ( $x_{1}-x_{2}-x_{3}$ ) presented in Fig. 1. The purpose of expressing the residual stress data in the tool coordinate frame is to see if simplifications in the residual stress fields are afforded by examining them in and across the primary directions of chip formation.
3.1 Experimental Observations. Figure 2 indicates that the normal stresses expressed in the workpiece coordinate frame ( $\sigma_{11}^{r, w}$ and $\left.\sigma_{33}^{r, w}\right)$ are compressive at subsurface depths between -20 and $-100 \mu \mathrm{~m}$ and assume a more-tensile character closer to the
newly-generated surface; this is a result that follows earlier turning experimentation [4] and also the work of Matsumoto et al. [6] on annealed AISI 4340. Also, as in the previous work the residual stresses are of negligible magnitude by a depth of $-200 \mu \mathrm{~m}$. In addition, it is noted that the shear stress for all tests in the plane of the newly-formed surface, $\sigma_{13}^{r, w}$, assume magnitudes that are small relative to the normal stresses $\left(\sigma_{11}^{r, w}\right.$ and $\left.\sigma_{33}^{r, w}\right)$.
A careful study of the data in Figs. 2-4 reveals the following observations:


Fig. 3 Residual stress from endmilling at surface versus location, workpiece coordinate frame

- At the surface, $x_{2}^{w}=-10 \mu \mathrm{~m}$ and $x_{2}^{w}=-98 \mu \mathrm{~m}$, location is noted to have effects on the residual stresses with the greatest effects evident at the workpiece surface. Also, the magnitudes of $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$, especially at the surface, are quite different, with the normal stress in the feed direction, $\sigma_{33}^{r, w}$, taking on a moretensile character than the normal stress across the feed direction, $\sigma_{11}^{r, w}$ (a similar result has been observed by Matsumoto et al. [6]).
- Feed and axial depth of cut have influences on the machining-induced residual stresses $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$ in the workpiece coordinate frame with the main effects being increases in
the tensile character of the surface residual stresses with increasing feed and to a lesser extent axial depth of cut. Such effects are consistent with prior experimentation on orthogonal machining, controlled oblique machining and turning [4,5].
- Negligible effect of location is noted for the normal stress components $\sigma_{11}^{r}$ and $\sigma_{33}^{r}$ when expressed in the rotating tool coordinate system (Fig. 4).
3.2 Discussion of Experimental Observations. Discussion of the observations cited above from Figs. 2-4 are provided below.


Fig. 4 Residual stress from endmilling test 1, tool coordinate frame

Table 1 Workpiece surface temperature from endmilling tests

| Test | Temp. 1 <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Temp. 2 <br> $\left({ }^{\circ} \mathrm{C}\right)$ |
| :---: | :---: | :---: |
| 1 | 78 | 60 |
| 2 | 112 | 103 |
| 3 | 76 | 83 |
| 4 | 112 | 117 |

- The effects of feed and axial depth of cut on the tensile character of the residual stresses are linked primarily to increases in temperature with increasing feed. Such temperature rises with feed are noted in the temperature data taken at the centerline of cut presented in Table 1. Similar effects of feed have been observed and explained for orthogonal machining, controlled oblique machining and the turning process $[4,5]$.
- The effects of location on the residual stresses expressed in the workpiece coordinate frame may be best understood with the schematic presented in Fig. 5. The schematic represents the relative roles of thermal and mechanical mechanisms in the generation of residual stress and also helps to explain the differences in magnitude of $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$ cited above.

It is well understood that mechanical effects primarily result in compressive residual stresses [10] and thermal effects primarily result in tensile residual stresses [11]. In the schematic, the effect of the thermal mechanism alone leads to tensile values of $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$ with the greatest tensile character occurring at the center of cut (dotted line). There the workpiece temperatures are highest due to the limited workpiece material at close proximity to the cut with the increasing undeformed chip thickness [5]; such a conclusion is supported by the workpiece temperature data in Table 1 which indicates that lower feeds (feed being equal to the undeformed chip thickness at the cut centerline) lead to lower temperatures and so the decreases in chip width as the cutter moves away from the centerline should also lead to lower temperatures. Because the thermal effect results from a scalar temperature field, it is not expected to have orientation effects and is therefore identical for $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$.

The mechanical effects for $\sigma_{11}^{r, w}$ and $\sigma_{33}^{r, w}$ do show an orientation effect due to the change in orientation of the cutting edge and the resulting plastic deformations (dashed lines). Under ideal situation, at the margins of the cut the undeformed chip thickness is zero and therefore the mechanical effect and thus mechanicallyinduced residual stresses are expected to be zero. Conversely, at the center of the cut, the undformed chip thickness is maximum and therefore the mechanical effect is maximum. However, at the centerline the cutting motion is aligned with the $x_{1}^{w}$-direction and


Fig. 5 Schematic interaction of mechanical and thermal effects in endmilling
therefore the maximum mechanical effect and thus the maximum compressive mechanical stresses can be expected to be $\sigma_{11}^{r, w}$. Also at the center of cut little of the motion is aligned with the $x_{3}^{w}$-direction and therefore zero mechanical residual stress is expected for $\sigma_{33}^{r, w}$.

Simultaneous consideration of the thermal and mechanical effects provides an understanding of their interactions. In the case of $\sigma_{11}^{r, w}$, we observe in Fig. 5 that the mechanical and thermal effects compete across the width of the cut with the net result being a final residual stress $\sigma_{11}^{r, w}$ that shows moderate effects of location, with the margins of the cut being more tensile than the center due to the greater influence of the thermal effect. This follows the trend observed near the surface for $\sigma_{11}^{r, w}$ at high values of feed (Fig. 3). In the case of $\sigma_{33}^{r, w}$, Fig. 5 demonstrates that the thermal and mechanical mechanisms interact in a complementary manner leading to maximum tensile stresses near the center of the cut and decreasing stresses towards the margins of the cut. This follows the trend observed near the surface for $\sigma_{33}^{r, w}$ at high values of feed (Fig. 3).
The negligible effect of location on the residual stress components when expressed in the tool coordinate system follows prior experimentation on oblique machining and turning [4]. This experimentation indicates that use of the tool coordinate system leads to simplifications in the interpretations of the residual stresses from these processes in that these tool coordinate directions are principal directions of the machining-induced residual stresses.

This is a significant conclusion and one whose consequence must be explored. From an experimentation standpoint, it suggests that for changes in endmill geometry, the net effect will be only a rotation of the principal residual stresses to a new coordinate frame and thus the effects of tool geometry are immediately known without additional experimentation. From a modeling perspective, the linking of the principal directions of the residual stresses to a coordinate system fixed in the tool motivates the development of a two-dimensional model for the prediction of residual stress which can then be extended by simple means to complex three dimensional cases [5]. Such an approach has been taken in the development of a thermomechanical model for turning, which has subsequently been proven valid [5].

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# Chatter Stability Analysis of the Variable Speed Face-Milling Process 

Sridhar Sastry, Shiv G. Kapoor, Richard E. DeVor, and Geir E. Dullerud<br>Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801

In this study, a solution technique based on a discrete time approach is presented to the stability problem for the variable spindle speed face-milling process. The process dynamics are described by a set of differential-difference equations with time varying periodic coefficients and time delay. A finite difference scheme is used to discretize the system and model it as a linear time varying (LTV) system with multiple time delays. By considering all the states over one period of speed variation, the infinite dimensional periodic time-varying discrete system is converted to a finite dimensional time-varying discrete system. The eigenvalues of the state transition matrix of this finite dimensional system are then used to propose criteria for exponential stability. Predicted stability boundaries are compared with lobes generated by numerical time-domain simulations and experiments performed on an industrial grade variable speed face-milling testbed.
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## 1 Introduction

Chatter, the self excited high amplitude vibration between the workpiece and cutting tool, has been a classic problem in machining processes for several decades. It significantly limits the machining productivity, affects the surface finish quality, causes loss of dimensional accuracy of the workpiece and accelerates the premature failure of cutting tools. The use of spindle speed variation as a technique for chatter suppression has been investigated as early as 1974 by Inamura et al. [1]. However, most of the research in this area was focussed on the turning process (Sexton et al. [2], Jemielniak et al. [3], Zhang et al. [4] and Jayaram [5]) where the cutting forces are constant. The analysis of chatter stability in face-milling is more involved than turning because face-milling is a multi-point interrupted cutting process, which causes the coefficients in the system equation to be periodic in nature as opposed to constant coefficients in the turning process.

Studies on chatter stability analysis of variable speed facemilling have mostly been based on time domain simulation methods (Lin et al. [6], Altintas et al. [7], Devor et al. [8]). These methods, in which the force/vibration signal is observed for onset of instability by repeated simulations using increasing values of depth of cut, are very time consuming. Analytical methods that involve closed-form criteria for evaluation of stability boundary

[^1]are much faster and therefore more desirable. Altintas et al. [9] have recently presented an analytical method for stability analysis of the end-milling process using a variable pitch cutter. However, their method is not applicable to the variable spindle speed machining case as in the former, the time delay between consecutive inserts occupying identical angular position is time invariant but varies from one insert to another as compared to it being time varying but identical for all inserts in the latter.
Tsao et al. [10] presented the first analytical method for chatter stability analysis of the variable speed face-milling process. By using the spindle angular position rather than time as the independent variable, the time varying time delay in the system equations is converted to a constant delay in the angle domain. However, the analysis presented in their work was for a very simple situation of continuous cutting with only one insert in cut at any point of time. Also, their scheme of converting from time to angle domain is applicable only to systems with a single mode of vibration and becomes rather cumbersome if the machining dynamics has multiple modes of vibration. The technique for chatter stability analysis presented in this paper overcomes the above stated limitations and can be applied to systems having multiple modes of vibration and involving interrupted cutting with multiple inserts in cut concurrently. A finite difference scheme is used to discretize the close loop system equations and convert the time varying time delay into multiple discrete delays. The resulting infinite dimensional periodic time-varying discrete system is converted to a finite dimensional time-varying system by considering all the states in one cycle of speed variation. Existing theory of discrete linear time varying (LTV) systems is then used to propose criteria for exponential stability.

## 2 Variable Spindle Speed Trajectory

In all the analysis presented in this paper, the trajectory for spindle speed variation is assumed to be sinusoidal. Mathematically, a sinusoidal speed trajectory can be represented by the expression:

$$
\begin{equation*}
\omega(t)=\omega_{0}+A \sin (2 \pi f t)=\omega_{0}\left(1+R V \quad A \sin \left(R V \quad F \omega_{0} t\right)\right), \tag{1}
\end{equation*}
$$

where $\omega_{0}=$ Nominal rotational speed ( $\mathrm{rad} / \mathrm{s}$ ), $A=$ Amplitude of speed variation ( $\mathrm{rad} / \mathrm{s}$ ), $f=$ Frequency of speed variation ( Hz ), $\mathrm{RVA}=\mathrm{A} / \omega_{0}$ and $\mathrm{RVF}=2 \pi f / \omega_{0}$. For an N insert cutter rotating at a constant speed of $\omega_{0}$, the tooth period, i.e. the time taken by the cutter to rotate by one tooth pitch is a constant $\mathrm{T}=2 \pi /\left(N . \omega_{0}\right)$. However, when spindle speed is varied continuously, the tooth period is time dependent. It has been observed in literature (Canniere et al. [11]) that modulation of spindle speed is roughly equivalent to modulation of the time delay between successive inserts occupying the same angular position. Thus the time delay, $\tau(t)$, can be expressed as:

$$
\begin{equation*}
\tau(t)=T\left(1+R V A \sin \left(R V \quad F \omega_{0} t\right)\right)=T+A_{m} \sin \left(\omega_{m} t\right) \tag{2}
\end{equation*}
$$

## 3 Face-Milling Process Model

The machining process model is a closed loop interaction between the cutting process model and the structural dynamics model which acts as the feedback path. The cutting process model computes the cutting forces, $[\mathrm{F}(\mathrm{t})]$, based on the cutter geometry, workpiece geometry and cutting conditions. These cutting forces act on the machining structure to produce dynamic deflections, $[\mathrm{r}(\mathrm{t})]$, which in turn modulate the cutting forces.
The cutting force model used is the mechanistic $K_{n} K_{f}$ model in which the normal and frictional cutting forces, $F_{n}$ and $F_{f}$, acting on an insert are expressed as the product of the chip load $A_{c}$, and the cutting force coefficients $K_{n}$ and $K_{f}$, respectively. By ignoring the static component of chip load, transforming the normal and frictional components of forces on an insert to the global XY
coordinate frame and summing the forces over all inserts, the dynamic milling force on the cutter, $[\mathrm{F}(\mathrm{t})]$, can be expressed (Sastry [12]) as:

$$
\begin{equation*}
[F(t)]=[A(t)]\{[r(t)]-[r(t-\tau(t))]\} d o c, \tag{3}
\end{equation*}
$$

where $[\mathrm{r}(\mathrm{t})$ ] is the relative cutter-workpiece deflection vector, $[\mathrm{A}(\mathrm{t})]$ is the time varying directional dynamic force coefficients matrix which is a function of the cutting force coefficients and the angular position of the inserts, doc is the depth of cut and $\tau(t)$ is the time dependent time delay (Eq. (2)).

The structural dynamics model captures the system transfer function matrix, $[\mathrm{G}(\mathrm{D})]$, which is the sum of the structural transfer functions of the cutter and the workpiece. The relative cutterworkpiece deflection can then be expressed in terms of the dynamic cutting force as:

$$
\begin{equation*}
[r(t)]=[G(D)][F(t)], \quad\left(D=\frac{d}{d t}\right) . \tag{4}
\end{equation*}
$$

Equation (4) can be written in state space form as:

$$
\begin{gather*}
{[\bar{R}(t)]=[C][\bar{R}(t)]+[E][F(t)]}  \tag{5}\\
{[r(t)]=[H][\bar{R}(t)],} \tag{6}
\end{gather*}
$$

where $[C] \in \mathfrak{R}^{\mathrm{n} \times \mathrm{n}},[\mathrm{E}] \in \mathfrak{R}^{\mathrm{n} \times \mathrm{n}_{\mathrm{F}}}$ and $[\mathrm{H}] \in \mathfrak{R}^{\mathrm{n}_{\mathrm{d}} \times \mathrm{n}}$. Here n refers to the number of states in $[\dot{\bar{R}}(t)], \mathrm{n}_{\mathrm{d}}$ is the number of displacements comprising the vector $[\mathrm{r}(\mathrm{t})]$ and $\mathrm{n}_{\mathrm{F}}$ is the number of force components in the vector $[\mathrm{F}(\mathrm{t})]$. If $[\mathrm{r}(\mathrm{t})]$ and $[\mathrm{F}(\mathrm{t})]$ are taken as $[x(t) y(t)]^{T}$ and $\left[F_{x}(t) F_{y}(t)\right]^{T}$, then $\mathrm{n}_{\mathrm{d}}=2$ and $\mathrm{n}_{\mathrm{F}}=2$. Substituting $[\mathrm{F}(\mathrm{t})]$ from Eq. (3) and $[\mathrm{r}(\mathrm{t})]$ from Eq. (6) in Eq. (5), one obtains the close loop system equation as:

$$
\begin{gather*}
{[\dot{\bar{R}}(t)]=[\bar{A}(t)][\bar{R}(t)]+[\bar{B}(t)][\bar{R}(t-\tau(t))] \text { where, }}  \tag{7}\\
{[\bar{A}(t)]=[C]+[E][A(t)][H] \text { doc and }} \\
{[\bar{B}(t)]=-[E][A(t)][H] d o c .}
\end{gather*}
$$

## 4 Solution Procedure

The continuous independent variable $t$ in Eq. (7) is discretized by choosing a discretization interval $\Delta$. A central finite difference scheme is performed around $n \Delta$ ( n is the discretization index) as follows:

$$
\begin{gather*}
t=n \Delta, \quad[\bar{R}(t)]=[\bar{R}(n \Delta)]=\left[\bar{R}_{n}\right], \quad[\bar{A}(t)]=[\bar{A}(n \Delta)]=\left[\bar{A}_{n}\right] \\
{[\bar{B}(t)]=[\bar{B}(n \Delta)]=\left[\bar{B}_{n}\right], \quad[\dot{\bar{R}}(t)]=\frac{\left[\bar{R}_{n+1}\right]-\left[\bar{R}_{n}\right]}{\Delta} .} \tag{8}
\end{gather*}
$$

Substituting the above relations into Eq. (7), the close loop system equation can be expressed in discrete time domain as:

$$
\begin{gather*}
{\left[\bar{R}_{n+1}\right]=\left[P_{n}\right]\left[\bar{R}_{n}\right]+\left[Q_{n}\right]\left[\bar{R}_{n-\tau_{n}}\right] \text { where, }} \\
{\left[P_{n}\right]=I+\Delta\left[\bar{A}_{n}\right]} \\
{\left[Q_{n}\right]=\Delta\left[\bar{B}_{n}\right] .} \tag{9}
\end{gather*}
$$

The time delay in the discrete domain, $\tau_{n}$, is obtained by rounding $\tau(n \Delta) / \Delta$ to the nearest integer. The smaller the value of $\Delta$, the closer the discretized time delays will be to their corresponding values in the continuous time domain. Figure 1 shows an example of the time delay in the continuous time case for one cycle of spindle speed variation and also the m values $(\mathrm{d}, \mathrm{d}+1, \ldots, \mathrm{~d}+\mathrm{m}-1)$ of the discretized delay, $\tau_{n}$ along with the range of the index, $n$, over which these delays occur. In the figure, $n_{p, s_{i}}$ and $n_{p, e_{i}}(\mathrm{~d} \leqslant \mathrm{p} \leqslant \mathrm{d}+\mathrm{m}-1)$ refer to the starting and ending indices, n , of the $i$ th interval over which the discretized delay is p . The period of the discretized time delay, $N_{p}$, is $2 \pi /\left(\omega_{m} \Delta\right)$ rounded to the closest integer. The infinite dimensional periodic time-varying system in Eq. (9) can be represented in the form of a


Fig. 1 Discretization of time delay
finite dimensional time-varying system by considering all the states over one period of spindle speed variation. The state vector is defined over the period $N_{p}$ as:

$$
\mathbf{X}=\left\{\left[\bar{R}_{0}\right]\left[\bar{R}_{1}\right] \cdots\left[\bar{R}_{N_{P}-1}\right]\right\}^{T}
$$

The finite difference equation in Eq. (9) can then be written in terms of the state vector as:

$$
\begin{align*}
& \mathbf{X}_{l+1}=\left[T_{0}\right] \cdot \mathbf{X}_{l}+\sum_{p=d}^{d+m-1}\left[T_{p}\right] \mathbf{X}_{l-p} \quad \text { where },  \tag{10}\\
& {\left[T_{0}\right]=\left[\begin{array}{cccc}
{\left[P_{0}\right]} & & & 0 \\
& {\left[P_{1}\right]} & & \\
& & \ddots & \\
0 & & & {\left[P_{N_{P}-1}\right]}
\end{array}\right] \text { and }}  \tag{11}\\
& {\left[T_{p}\right]=\left\{\begin{array}{lll}
\sum_{k=1}^{\chi_{p}}[V][Z]^{N_{P-p}}[\mathbf{Q}] & : & (p=d, d+1, \ldots, d+m-1) \\
0 & : & (p-1, \ldots, d-1)
\end{array}\right.}  \tag{12}\\
& {[Z]=\left[\begin{array}{cccc}
0 & \cdots & 0 & I \\
I & \ddots & & 0 \\
& \ddots & & \vdots \\
& & I & 0
\end{array}\right],} \\
& {[\mathbf{Q}]=\left[\left[Q_{0}\right]\left[Q_{1}\right] \cdots\left[Q_{N_{P-1}}\right]\right]^{T} \text { and, }} \\
& v_{i j}=\left\{\begin{array}{l}
I \quad: \text { for } i=j \text { and }\left(n_{p, s_{k}}-p\right) \leqslant j \leqslant\left(n_{p, e_{k}}-p\right) \\
0 \quad \text { :otherwise. }
\end{array}\right.
\end{align*}
$$

In Eq. (12), $\chi_{p}$ is the number of intervals (in one period of the discretized delay, i.e. between 0 and $N_{P}-1$ ) over which the delay is $p,[Z]$ is the cyclic shift matrix which has $N_{P} \times N_{P}$ blocks where each block (either a 0 or an I) is an $\mathrm{n} \times \mathrm{n}$ square matrix. Similarly, $\left[T_{0}\right]$ and [ $T_{p}$ ] also consist of $N_{P} \times N_{P}$ blocks where each block is an $\mathrm{n} \times \mathrm{n}$ square matrix. Equation (10) is the closed loop equation for variable speed milling in the discrete time domain. The system described by Eq. (10) can then be expressed in terms of the state transition matrix as follows:

$$
\begin{gather*}
\widetilde{\mathbf{X}}_{l+1}=\widetilde{\mathbf{T}} \widetilde{\mathbf{X}}_{l} \quad \text { where }, \\
\widetilde{\mathbf{X}}_{l}=\left\{\mathbf{X}_{0} \mathbf{X}_{1} \cdots \mathbf{X}_{d+m-1}\right\}_{l}^{T} \quad \text { and } \tag{13}
\end{gather*}
$$

$$
\widetilde{\mathbf{T}}=\left[\begin{array}{cccccc}
0 & I & 0 & & & 0  \tag{14}\\
0 & 0 & I & & & \\
\vdots & & \ddots & & \ddots & \\
0 & 0 & \cdots & \cdots & 0 & I \\
{\left[T_{d+m-1}\right]} & {\left[T_{d+m-2}\right]} & \cdots & \cdots & \cdots & {\left[T_{0}\right]}
\end{array}\right] .
$$

In the above equation, $\widetilde{\mathbf{T}}$ is the state transition matrix and consists of $(\mathrm{d}+\mathrm{m}) \times(\mathrm{d}+\mathrm{m})$ blocks where each block is an $N_{P} \times N_{P}$ square matrix. The system described by Eq. (13) is exponentially stable if all the eigenvalues of the state transition matrix lie inside a unit disc centered at the origin, i.e.,

$$
\begin{equation*}
\operatorname{eig}(\widetilde{\mathbf{T}}) \subset \overline{\mathcal{D}}, \tag{15}
\end{equation*}
$$

where $\widetilde{\mathcal{D}}$ represents the closed unit disc.

## 5 Model Validation Results

The milling system shown in Fig. 2 is considered to demonstrate the application of the proposed method for stability analysis of the variable speed face-milling process. The 8 insert (Ingersoll GDE323R04: nominal lead angle, $\gamma_{L}=-4.4 \mathrm{deg}$; axial rake angle, $\gamma_{A}=16$ deg; radial rake angle, $\gamma_{R}=13.3 \mathrm{deg}$; insert nose radius $=0.7874 \mathrm{~mm}), 4 \mathrm{in}$. ( 101.6 mm ) diameter face mill performs symmetric milling on a 75 mm wide workpiece with interrupted geometry. The fixture for mounting the workpiece was designed to be flexible in the Y direction having a single vibration mode with modal parameters $\omega_{n}=210 \mathrm{~Hz}, \quad \zeta=0.03, \mathrm{M}$ $=1.895 \mathrm{Kg}$, and to be rigid in the X and Z directions. Stability analysis of this simple one degree-of-freedom system cannot be done using the method proposed by Tsao [10] because of the interrupted nature of the workpiece and owing to multiple inserts being engaged in cut at any given point of time, both of which are common in almost all face-milling applications.

For the one degree-of-freedom case considered here, the relative cutter-workpiece deflection vector, $[\mathrm{r}(\mathrm{t})]$, becomes the scalar $\mathrm{y}(\mathrm{t}) ;[\mathrm{F}(\mathrm{t})]$ reduces to $F_{y}(t)$ and the system state, $[\bar{R}(t)]$ in Eq. (5) is the vector $[y(t) \dot{y}(t)]^{T}$. To determine the limit of stability, the eigenvalues of the state transition matrix, $\widetilde{\mathbf{T}}$ are computed for increasing values of depth of cut till the eigenvalue with the largest magnitude falls on the unit circle centered at the origin. This process is repeated for different values of nominal spindle speeds to generate the stability chart.

The stability chart obtained by the proposed method is compared with that obtained from numerical time-domain simulations. Numerical time-domain simulations were performed by simulating the system equations in Simulink ${ }^{\mathrm{TM}}$ (a product of Mathworks Inc.) using a variable step (Dormand-Prince) solver with a maximum step size of 0.0001 s . Stability charts were generated for the constant feed rate of $600 \mathrm{~mm} / \mathrm{min}$, and for nominal spindle speeds


Fig. 3 Validation of analytical results
in the range $500-3500 \mathrm{RPM}, \mathrm{RVA}=0.1$ and $\mathrm{RVF}=0.5$. The stability charts obtained by the two methods, the proposed method and numerical simulations, are shown in Fig. 3. The average error between the prediction and the results from time domain simulations is less than 10 percent. Although the example presented in this section was for a simple one degree-of-freedom system, the analysis holds true when the system dynamics has multiple vibration modes. However, multiple vibration modes would result in larger number of states in Eq. (5) and thus a proportional increase in the size of the state transition matrix $\widetilde{\mathbf{T}}$.

Experiments were performed by machining an Al2024 workpiece on a specially constructed variable spindle speed facemilling testbed. This industrial grade testbed was designed to have very low spindle inertia ( 0.144 lb .in. $\mathrm{sec}^{2}$ ) to achieve increased angular acceleration required for effective spindle speed variation. The workpiece was mounted on the fixture which was in turn mounted on a KISTLER 9255-A dynamometer to measure the cutting forces. Experimentally observed values of limiting depth of cut for the one degree-of-freedom system are shown as circles in Fig. 3. It is seen that as compared to the experimentally observed values, the proposed method predicts the stability limit with a maximum error of 15 percent. Figures $4(A)$ and $(B)$ show the profile of the face-milled surface for depths of cut of 1.4 mm and 1.0 mm respectively at a nominal spindle speed of 1000 RPM. The higher depth of cut condition which lies in the unstable region, results in a very poor surface finish ( $R_{a}=15.5 \mu \mathrm{~m}$ ) as compared to the surface finish observed when the cut is stable at the lower depth of cut ( $R_{a}=1.19 \mu \mathrm{~m}$ ).


Fig. 4 Profile of face-milled surface

## Conclusions

1 A solution technique based on a discrete time approach was presented for stability analysis of the variable spindle speed face-milling process. The close loop system equation was discretized to convert the time varying time delay to multiple discrete delays. The resultant discrete time linear time varying (LTV) system was analyzed for stability using the eigenvalue approach.
2 The stability limits predicted by the proposed method are validated with results obtained from numerical time-domain simulations for a one degree-of-freedom milling system with interrupted workpiece geometry. The average error between the prediction and the results from time domain simulations is less than 10 percent.
3 Stability limits are also compared with results from experiments performed on an industrial grade variable speed facemilling testbed and it is seen that the obtained results compare favorably to the experimentally observed values.
Although the proposed solution technique is applicable to systems with multiple vibration modes, computational difficulties in evaluating the eigenvalues of the state transition matrix of very large order limit its application to simple low degree-of-freedom systems.

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