

Design Challenges in a Commercial Quantum Hall Effect-Based Resistance Standard

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by

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Executive Summary

Of the electrical measurements that underpin the broad spectrum of modern industry, measurements of voltage, resistance, and impedance are among the most important. In the last few decades, Josephson's effect and the quantum Hall effect have been developed as intrinsic standards for voltage and resistance that provide drift-free values that are, under ideal conditions, related to fundamental constants of nature. These effects have been used for many years by national standards laboratories to maintain fundamental units of voltage and resistance using expensive and complex measurement systems. It is clear, however, that industrial and governmental primary standards laboratories will greatly benefit from having direct access to intrinsic standards. Such standards would provide these laboratories with higher accuracy, drift-free standards, both reducing the time required to perform calibrations and enabling the production of higher quality products. Because of the complexity and the cost of the equipment required to observe the quantum Hall effect, however, it has not yet found wide application outside of national standards laboratories.

The purpose of this manuscript is to assess the possibility of developing a commercially viable quantum Hall effect-based resistance standards system. Since such a system would have to compare favorably with the resistance standards systems currently in use, the first section of this document describes the capabilities of current resistance standards and calibration systems. These systems have two parts: the actual resistance standard, and the measurement system used to compare other resistors to the standard. A quantum Hall effect-based resistance standard would also consist of two separate sub-systems. The quantum Hall resistor would substitute for the artifacts presently used as resistance standards, but the measurement systems required by the quantum Hall resistors are different than those used with present standard resistors. The first section of this document describes the properties of quantum Hall resistors as resistance standards and compares them to those of the standard resistors presently used. Since the commercial measurement systems presently in use lack sufficient accuracy to be used to compare resistors to a quantum Hall resistor, a commercial quantum Hall effect-based resistance standard will require the development of new, low-noise measurement systems. The conclusion of the first section of this document describes the specifications that would have to be met by the new measurement systems. The second section of this document investigates the requirements that would have to be met by the various components of the system, including the sample, cryogenic system, and measurement system, in order to deliver these capabilities. In the last section, several possible commercial quantum Hall effect-based resistance standards systems are described, together with the scientific challenges involved in realizing them.

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1. Capabilities of Resistance Calibration Systems

In order to be commercially viable, a quantum Hall effect-based resistance standard system will have to compete favorably with the resistance calibration systems currently in use. Therefore, before specifications for such a system can be defined, it is necessary to examine the capabilities of current resistance calibration systems. The dissemination of a unit of resistance requires two things: an artifact, which is called the "resistance standard", which has a known value of resistance; and a measurement system capable of comparing this standard to other resistors. The uncertainty of the final resistance calibration is a combination of the uncertainty due to the stability of the standard and the uncertainty that can be achieved by the measurement system. This section describes briefly the types of resistance standards that are presently used, the limitations on their accuracy, the types of measurement systems presently in common use, and the limitations on their uncertainties.

Parts of calibration system: artifact and measurement system

1.1. Present capabilities

1.1.1. What is calibrated?

The primary means used by industrial and primary standards laboratories to maintain the unit of resistance are resistors made from a length of wire wrapped round a special form. By far the most common values are 1 Ω and 10 k Ω , with 100 Ω resistors being used somewhat less frequently. While the most common resistors used in the United States are the 1 Ω Thomas-type resistors, the 10 k Ω resistors have found favor because they are closer to the middle of the range of resistances required by the end user, and therefore errors in scaling between the 10 k Ω and other resistance values are less than with the 1 Ω resistors.

1.1.2. Limitations on standard resistors

Wire resistors, however, do not provide invariant values of resistance: their values tend to drift with time as the wire from which they are made ages. Typically, the values of the 1 Ω (Thomas-type) resistors drift at a rate of ≈ 0.05 ppm per year, while the 10 k Ω resistors drift at rates of about 0.2 ppm per year¹. In addition, the resistance of the wire is a function of the temperature, and also of the stress on the wire, which can be affected by such factors as atmospheric pressure and humidity. For this reason, the most accurate resistors must be maintained in sealed enclosures, the temperatures of which are accurately controlled. While the temperature, pressure, humidity, and time-dependence of the resistances are generally predictable, the resistance of the wire occasionally jumps, or the slope of the drift changes in an unpredictable manner. For this reason, wire resistors must be periodically recalibrated at a national standards laboratory, such as NIST, where they are compared to a resistance standard whose value is known, usually in terms of the quantum Hall resistance.

Drift rates of wire resistors

While the particulars of the calibration procedures are often quite complex, they generally involve passing currents through two resistors to be compared (a standard and a resistor to be cali-

Effects of thermal noise

¹ R F Dziuba, P A Boynton, R E Elmquist, D G Jarrett, T P Moore, and J D Neal, "NIST Measurement Service for DC Standard Resistors," NIST Technical Note 1298 (Nov. 1992).

brated), and the voltages developed across the two resistors are compared with a detector. The ultimate uncertainty in the value of the calibrated resistor due to random effects (such as noise) that can be achieved with a given measurement time is then largely determined by the noise in the measurement system and the magnitude of the voltages across the resistors. To obtain the highest possible accuracies with the shortest possible measurement times, one wishes to either minimize the noise in the system or maximize the voltages across the resistors. The noise in the system is determined by the thermal, or Johnson noise in the resistors, the noise in the detector, the noise in the current sources, and the quality of the shielding in the cables and wires. Measurement systems can be built that utilize cryogenic detectors and have multiple layers of shielding, and consequently have extremely low noise (on the order of a few $\text{nV}/\sqrt{\text{Hz}}$), but these are extremely expensive to build and have not found extensive commercial application. The noise in most commercial measurement systems is quite large: the rms thermal noise in the resistors is as much as $13 \text{ nV}/\sqrt{\text{Hz}}$ (for a $10 \text{ k}\Omega$ resistor at room temperature), and the noise in most commercially available measurement systems (including the detector and current source) is usually considerably larger than the thermal noise in the resistors.

Self-heating limits
maximum power dissipation to 10 mW

As a result, the only way to achieve high accuracy calibrations in reasonable measurement times with presently commercially available measurement systems is to make the voltage across the resistors as high as possible, which requires passing the largest possible currents through them. The problem is that when current is passed through a resistor, heat is dissipated. If the power dissipated is sufficiently low, the form around which the wire is wound and the surrounding medium will remove the heat and prevent the temperature of the wire resistor from rising significantly, but if excessively large currents are used, the temperature of the resistor will rise, affecting its value. Most resistors in use today can dissipate as much as 10 mW of power without exhibiting significant self-heating. This limits the measurement currents to 1 mA for $10 \text{ k}\Omega$ resistors, and 100 mA for 1Ω resistors. With most commercially available measurement systems in use today, such currents permit calibrations with expanded relative uncertainties² with a coverage factor one of 0.05 ppm to 0.15 ppm in measurement times on the order of a half an hour.

1.1.3. Types of calibration systems currently used

DCC
Kelvin Bridge
Binary Voltage
Dividers
DVM Method
Accuracies = 0.1 ppm

There are four main types of resistance comparators that are used to calibrate resistors in industrial and governmental primary standards laboratories. The most common are commercially available direct current comparators and Kelvin Bridges, which are primarily manual systems at present (automated versions are under development in some laboratories). Recently, automated (i.e. computer-controlled) Binary Voltage Dividers and digital voltmeter (DVM)-based

² If a measured quantity R has an expanded uncertainty with coverage factor one equal to s_{e1} , then repeated measurements of the quantity R will lie within the interval $(R-s_{e1})$ to $(R+s_{e1})$ 68.3 % of the time. For more information about the determination of uncertainties, see the *Guide to the Expression of Uncertainty in Measurement* (International Organization for Standardization, Geneva, Switzerland, 1993). The quantity s_{e1} is also often called a one standard deviation or 1σ uncertainty. All uncertainties quoted in the rest of this manuscript will be one standard deviation uncertainties.

measurement systems have become available.³ These measurement systems are quite different in detail, but all share the common principle that currents are passed through the two resistors to be compared, and the voltages across the two resistors are compared. Since all of these measurement systems operate at room temperature and utilize room-temperature detectors to measure the voltage differences, the noise in these measurement systems is of such a magnitude as to require the use of the maximum possible measurement currents (see previous section) in order to permit calibration uncertainties of ≈ 0.1 ppm in measurement times of about half an hour when comparing resistors with nominally the same value. The uncertainty achieved when scaling (i.e. comparing resistors with nominally different values, such as 10 k Ω and 100 Ω) is usually much worse, being between a few tenths and 1 ppm for direct current comparators. The automated DVM-based measurement system developed at NIST⁴ has achieved relative combined uncertainties less than 0.1 ppm when comparing resistors with values that differ by as much as a factor of 4. Limitations on the accuracy, stability, and internal noise in currently available digital voltmeters, and the reproducibility of contact resistances and thermal voltages in the switches used to connect the DVM to the resistors will likely prevent this technique from achieving relative combined uncertainties much better than 0.03 ppm.

Generally, the noise in the (room-temperature) detector used to detect the bridge imbalance, which is proportional to the difference in the values of the resistors being compared, is much larger than the thermal noise in the resistors, and is the major factor limiting the ultimate accuracies and speeds with which measurements can be made. Because the one standard deviation uncertainties produced by most of these measurement systems are in the 0.1-1 ppm range, the drifts in the values of the standard resistors are usually within the general level of uncertainty in the calibration and are a fairly minor problem. As a result, the resistors are usually only returned to NIST every one or two years for re-calibration.

1.1.4. Conclusion

Present commercially available calibration systems are quite simple and inexpensive to use, as all the components are maintained at room temperature. These systems have two components: wire standard resistors, and measurement systems, both of which are limited in their ultimate accuracy. The values of the wire standard resistors drift with time, and are functions of the ambient temperature, humidity, and pressure. The uncertainties in the determinations of the correction factors associated with each of these effects limits the ultimate uncertainty in the determination of the values of these resistors. In addition, because the values of resistors, or the slopes of the dependences of the resistances on time and ambient conditions sometimes change erratically, periodic re-calibration of the wire standard resistors at a national standards lab is required. The

Limits to accuracy posed by standard resistors and measurement systems are about the same magnitude and are about 0.1-1 ppm

³ Descriptions of the particulars of the operation of these systems are beyond the scope of this manuscript. Excellent summaries can be found in the monograph of R F Dziuba, et al. (footnote 1) and in F Delahaye, "DC and AC Techniques for Resistance and Impedance Measurements," *Metrologia* 29 81-93 (1992).

⁴ M E Cage, D Y Yü, B M Jeckelmann, R L Steiner, and R V Duncan, "Investigating the Use of Multimeters to Measure Quantized Hall Resistance Standards," *IEEE Trans. Inst. Meas.* 40, 262-266 (1991) and K C Lee, M E Cage, and P S Rowe, "Sources of Uncertainty in a DVM-Based Measurement System for a Quantized Hall Resistance Standard," *J. Res. NIST*, 99(3) pp. 227-240 (May-June 1994).

values of these resistors, however, are quite sensitive to mechanical vibrations and variations in the ambient that normally occur in transport, so even with periodic recalibration at a national standards lab, the lowest uncertainty (1σ) that can be obtained with wire standard resistors is only slightly better than 0.1 ppm (for comparisons of resistors with nominally the same value).

The other factors limiting the ultimate uncertainty attainable with current calibration techniques are the limited resolution and accuracy of commercial measurement systems. These systems use room temperature detectors, and require that high currents (of 1 mA to 100 mA) be passed through the resistors to generate voltages high enough to permit calibrations to be effected with one standard deviation uncertainties of 0.1 ppm to 1 ppm in measurement times of 0.5 hour to 1 hour. Thus, in present practice, the resolution of the measurement system is of the same order of magnitude as the stability of the wire resistors, and the resistors must be periodically re-calibrated at NIST every few months to few years, depending on the accuracy required.

These limitations have led to the desire for an invariant standard of resistance that was simple and inexpensive enough to find application in industrial and governmental primary standards laboratories. The quantum Hall effect has been demonstrated to provide such an invariant standard of resistance, but despite over 10 years of work, this standard is still not commercially available. To understand the reason for this, it is useful to compare the situation of the quantum Hall resistance standard with that of the Josephson array voltage standard, which has been commercialized. Like resistance standards, voltage standards are comprised of two parts, an artifact which delivers a known voltage, and a measurement system used to compare other voltage sources with the standard. Prior to the advent of the Josephson array voltage standard, measurement systems (i.e. detectors) capable of comparing voltage differences with uncertainties approaching a few parts in 10^8 were commercially available. Two main impediments limited the uncertainty of calibrations to the part in 10^6 (ppm) range: the major one was the poor stability of the artifacts used as voltage standards, viz., standard cells and Zener diodes. In addition, since the artifacts produced only specific voltages, calibration of voltage sources with values different than the standard's value had to be accomplished using resistive voltage dividers. Drifts and inaccuracies in the determination of the values of the resistors in these dividers were the other factors that limited the accuracy of voltage calibrations.

The development of arrays of Josephson junctions as voltage standards greatly improved the accuracy and reduced the cost of maintaining and disseminating the volt for several reasons: the voltage produced by the arrays was stable and invariant; and the output voltage of the array could be adjusted, in small steps with known values, to be close to the nominal values of most of the common secondary standards (such as Zener diodes and standard cells), thus largely eliminating the need for the resistive voltage dividers. The elimination of the voltage dividers not only resulted in improved accuracy, it also eliminated the cost of periodically recalibrating the resistors in the dividers. Thus, Josephson arrays could be *directly substituted* for the voltage standards then in use (viz., standard cells and Zener diodes), and simpler, less costly, higher accuracy, *commercially available* measurement systems (viz. detectors for 1:1 voltage comparisons) could be used. In other words, the primary challenge in developing the Josephson array-based voltage standard was in the development of the *voltage standard*, viz. the Josephson array it-

Comparison of
Josephson Array
Voltage Standard
with QHE

self, and its associated cryogenic systems and microwave electronics, rather than in the development of the measurement system.

The situation with resistance standards is quite different. In present practice, the standard resistors used to maintain the unit of resistance have values that drift by amounts that are of the same order of magnitude as, or in some cases actually less than the resolution of the measurement systems. Thus, while the quantum Hall effect provides an invariant standard of resistance that can be used to replace the standard resistors currently used, no benefit would be obtained from doing so unless the accuracy of the measurement systems used were greatly improved. In contrast to the case of the Josephson array voltage standard, which only required the development of the standard (viz., the samples, the cryogenic systems, and the associated microwave electronics), a commercially viable quantum Hall resistance standard requires not only the development of a commercially viable quantum Hall resistor (including samples, cryogenic systems, and magnets), but *also* the development of a new, higher accuracy measurement system that can be commercially produced. In the last part of this section, the ultimate limitations on the quantum Hall effect as a resistance standard are described, and a set of specifications for a commercially viable quantum Hall effect-based resistance standards system, including the measurement system, are derived.

QHE standard requires new measurement system

1.2. Ultimate limits on the quantum Hall resistance

As the strength of the magnetic field is increased, the resistance of a quantum Hall device maintained at cryogenic temperatures will achieve, in certain ranges of magnetic field, constant values equal to h/ie^2 , where h is Planck's constant, e the electronic charge, and i an integer (see Fig. 1). For $i = 4$, this Hall resistance is 6 453.201 750 Ω ; for $i = 2$, it is 12 906.403 500 Ω ; and for $i = 1$, it is 25 812.807 Ω . The magnetic field at which the plateau is observed is inversely proportional to i : in other words, the $i = 2$ plateau occurs at twice the magnetic field strength at which the $i = 4$ plateau is observed. Because higher magnetic fields are more expensive and difficult to obtain, the $i = 4$ plateau is most commonly used. The $i = 2$ is the next most commonly used plateau, and is often used by national standards laboratories because the voltages across the device will be higher for a given current, and the larger the voltage, the larger the signal-to-noise ratio, and the less the averaging time required to obtain a given accuracy. Both of these plateaus have resistance values that are close to the value of 10 k Ω resistors, and are therefore quite convenient for calibrations of most commonly used resistors.

Unlike wire resistors, the resistance of a quantum Hall device does not drift with time, nor is it affected by pressure or humidity. It is, however, a function of the temperature of the device⁵ (see Figs 1, 4 and 5), but for most devices, this temperature dependence is not measurable at tem-

⁵ M E Cage, B F Field, R F Dziuba, S M Girvin, A C Gossard, and D C Tsui, "Temperature Dependence of the Quantum Hall Resistance," Phys. Rev. B30(4), 2286-8 (1984). This is discussed in more detail in Sec. 2.2.1.

peratures below about 1.1 K. The values of resistance obtained from different quantum Hall devices have been shown to be the same with an uncertainty of less than 0.003 ppm.⁶

Hall resistor has less noise than wire resistor

Because it is maintained at cryogenic temperatures, the noise in a quantum Hall device is much less than in a resistor of the same nominal value at room temperature: in fact, the rms thermal noise voltage in a quantum Hall device at 1.3 K on the 12 906.403 500 Ω resistance plateau is 30% less than that in a room temperature 100 Ω standard resistor and about a factor of 13 less than the noise in a room temperature 10 k Ω resistor.⁷ This makes it theoretically possible to achieve higher levels of accuracy with much shorter measurement times than are possible with room-temperature resistors.

To a certain extent, this advantage is offset by the limited current-carrying capacity of quantum Hall devices (presently 25 μ A to 100 μ A), which means that the voltages across the resistors being compared are smaller (between 161 mV and 645 mV for the $i = 4$ plateau) than with room temperature resistors which can easily carry 1 mA of current and provide voltages as high as 10 V. For currents above about 60 μ A for the $i = 2$ plateau and 80 μ A for the $i = 4$ plateau, however, not only is the rms thermal noise in a quantum Hall device less than in a 10 k Ω resistor, the signal-to-noise ratio of the Hall device is greater than that in a room temperature 10 k Ω resistor. Under these conditions the accuracy and stability of the quantum Hall resistor far exceed those of any wire resistance standard available. It is important to note, however, that the ability to take advantage of this low noise requires a measurement system that has low noise and high sensitivity: the primary limitation in most room-temperature measurement systems is the noise of the detector used to compare the voltages across the two resistors being compared, not the thermal noise in the resistor.

1.3. Proposed capabilities of a QHE-based standard system:

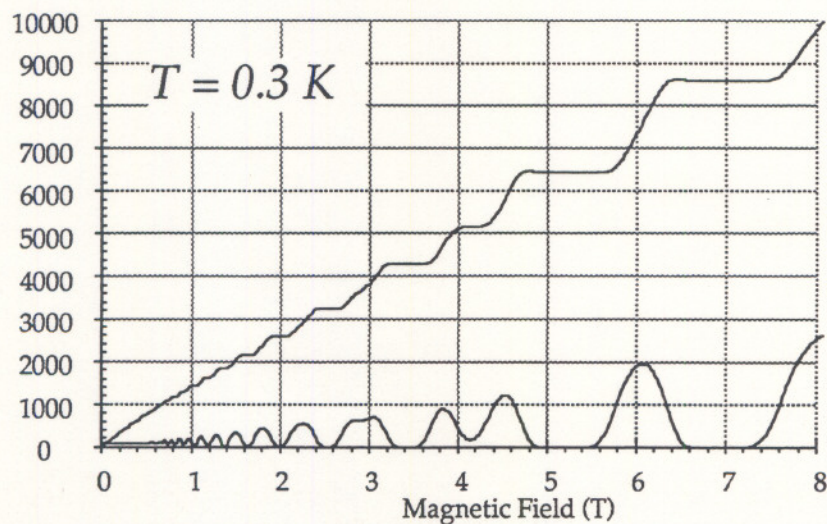
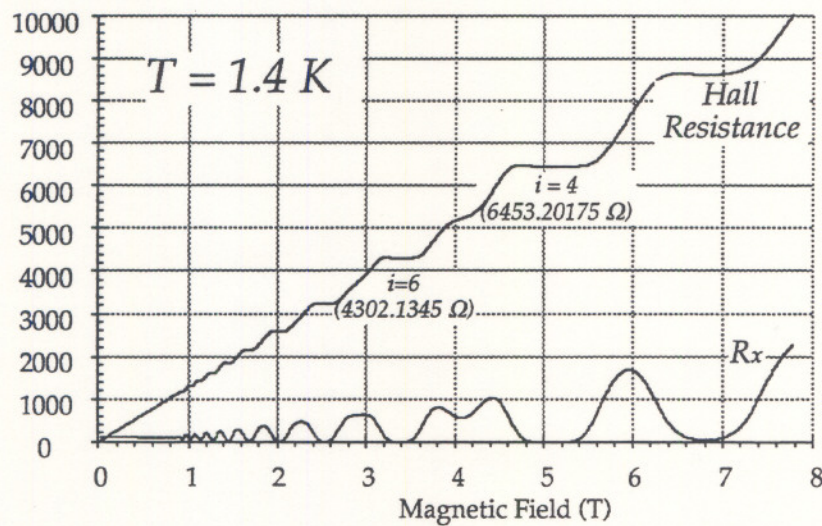
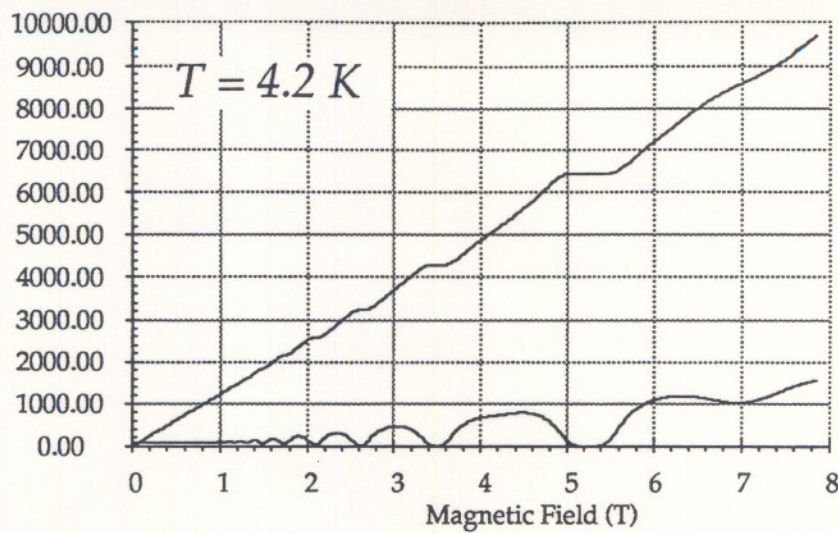
Commercially viable QHE resistance standard must be capable of ≤ 0.01 ppm

The quantum Hall effect provides an invariant standard of resistance that has many advantages over the wire standard resistors presently used for resistance calibrations, and as a result has been used for many years by national standards laboratories as a resistance standard. The measurement systems used by these laboratories, primarily potentiometric comparators and cryogenic current comparators, have such low noise and high accuracy that the quantum Hall resistors could be compared with uncertainties approaching a few parts in 10^9 . Such measurement systems, however, are very complex and must be assembled by hand in a very painstaking manner, and cannot be readily commercialized. With the commercial measurement systems commonly employed in industrial labs today, however, it scarcely makes sense to replace the wire standard resistor with a quantum Hall resistor, as these systems are capable of uncertainties of only 0.1-

⁶ See, for example, B Jeckelmann, W Schwitz, H J Bühlmann, R Houdré, M Ilegems, D Jucknischke, and M A Py, "Comparison of the Quantized Hall Resistance in Different GaAs/AlGaAs Heterostructures," IEEE Trans. Instr. Meas. 40 231-3 (1991).

⁷ F Delahaye and D Bournaud, "Low-Noise Measurements of the Quantized Hall Resistance Using an Improved Cryogenic Current Comparator Bridge," IEEE Trans. Instr. Meas. 40 237-40 (1991). The mean square thermal noise voltage is proportional to the resistance, so the rms noise voltage across the 100 Ω resistor is about a factor of 10 less than that across a 10 k Ω resistor at the same temperature.

Resistance (Ω)



Schematic Diagram of QHE Device

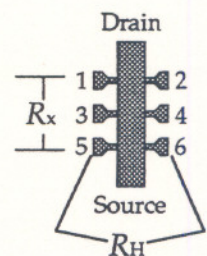


Fig. 1. Resistance of a quantum Hall device as a function of magnetic field at different temperatures. As the magnetic field is increased, the resistance R_H becomes constant over certain ranges of magnetic field, called Hall plateaus. On a plateau, R_x , equal to the voltage drop along the bar divided by the current through it, vanishes. As can be seen above, the resistance plateaus become much broader and flatter and the minima in R_x are closer to zero as the temperature of the sample is decreased.

1 ppm, and at these levels of accuracy, wire standard resistors are quite adequate as resistance standards. With measurement systems capable of uncertainties of the order of 0.01 ppm, however, the correction factors that would have to be applied for the drifts in the value of a wire standard resistor would significantly decrease the accuracy of the calibration, and the use of a quantum Hall resistor would be highly beneficial. If such high-accuracy measurement systems, in conjunction with quantum Hall resistors, were available commercially, great savings could be effected, for calibrations at the present levels of accuracy could be accomplished in orders of magnitude less time, the need and expense for periodic re-calibrations of the wire standard resistors at NIST would be largely eliminated, and industrial users would be able to utilize the much higher accuracy levels in new products.

The development of a commercial quantum Hall-effect based resistance standard system therefore has two components: the development of a commercially viable quantum Hall effect "resistance standard", and the development of a commercially viable measurement system. National standards laboratories have used the quantum Hall effect as a standard of resistance for many years, so such a system is possible: the challenge is to make the system commercially viable. The following sections describe in more detail the components of the QHE standard and the measurement system. Tables I and II summarize the characteristics that a commercially attractive QHE-based resistance standard and measurement system, respectively, might possess.

i) QHE Resistance Standard: Rather specialized conditions are required to observe the quantum Hall effect. A quantum Hall effect-based resistance standard is comprised of 3 main subsystems: a sample, a cryogenic system capable of maintaining temperatures less than 1.5 K, and a magnet capable of generating fields of about 4 to 8 T (40 000 to 80 000 Gauss). Present quantum Hall effect systems consist of large superconducting magnets bathed in liquid helium. The sample is cooled either by a ^4He refrigerator (i.e. the sample is bathed in liquid ^4He , which is cooled by causing it to evaporate by reducing the vapor pressure above it by means of a vacuum pump) or by a ^3He refrigerator (which operates on the same principle, but uses the helium isotope with atomic mass 3, rather than 4). These refrigerators are capable of producing temperatures of ≈ 1 K and ≈ 0.3 K, respectively, but both consume large quantities of liquid cryogenics, which are quite costly.

Components of cryogenic systems presently used for QHE standards:
- Sample
- Cryosystem
- Magnet

In order to be made commercially viable, a quantum Hall effect-based resistance standard would have to be less costly to acquire and maintain, which essentially translates to reducing the size of the magnet, and simplifying the cryogenic system by reducing or eliminating the requirement for liquid helium, which is quite costly. The lower the maximum field the magnet is capable of, the less expensive it is, but there are lower limits on the magnetic field imposed by the device requirements,

Table I: Parameters for a Commercial QHE-Based Resistance Standard.

PARAMETER	VALUE
Magnetic Field.....	4-8 T
Temperature	≤ 4.2 K
Resistance.....	≈ 10 k Ω
Current	≈ 80 μA
Refrigeration System	
Hold Time	≥ 9 h

which are discussed in more detail in Sec. 2.2.4 below. A practical range for the magnetic field is 4-8 T, as this permits the observation of high quality resistance plateaus and can be achieved with compact, inexpensive, commercially available superconducting magnets.

As regards the refrigeration system, both ^4He and ^3He refrigerators consume large quantities of liquid helium (typically $\approx 25\text{-}30$ liters/day), which is quite expensive ($\geq \$5/\text{liter}$ in the U.S.A., and as much as $\$20/\text{liter}$ elsewhere). A quantum Hall effect resistance standard that operated at 4.2 K (the temperature of liquid helium at atmospheric pressure) would not require the refrigerators, and therefore the need for liquid cryogenics would be greatly reduced, but not eliminated. Because such systems would still consume cryogenics it is important that the capacity of the dewar be sufficient to maintain the quantum Hall device at operational temperatures for at least one working day ("hold time" of about 9 hours), before having to be re-filled with liquid helium. If it should prove to be possible to cool the quantum Hall device with closed-cycle refrigerators that do not require cryogenics at all, the sample could be continuously cooled, and the "hold time" of the dewar would not be a problem (see Sec. 2.3.1 for more detailed discussion). A summary of the optimum requirements for a commercially viable quantum Hall effect-based resistance standard are shown in Table I. Cryogenic systems are presently commercially available that meet these criteria, and are not excessively expensive. The challenge in utilizing these systems in a quantum Hall effect-based resistance standard is in developing samples that can be operated at 4.2 K (see Sec. 2.2.1).

ii) Measurement System: Present commercial measurement systems cannot profitably be used with a quantum Hall effect-based standard resistor, for while they achieve relative uncertainties of about one part in 10^7 (0.1 ppm), they achieve these uncertainties with measurement currents (and hence voltages) more than an order of magnitude larger than the largest currents that can be used with quantum

Hall devices. At the lower currents required by quantum Hall devices, the uncertainties achieved by these systems would be much larger than 0.1 ppm. The measurement systems that have been used in national standards laboratories to maintain the standard of resistance, while capable of achieving extremely low uncertainties, cannot be readily commercialized because of their complexity. As a result, new measurement systems must be developed that are less costly to build, can be automated, and can achieve relative uncertainties on the order of 0.01 ppm with measurement currents of $\approx 80\text{ }\mu\text{A}$ in measurement times in the range of 10 to 30 min. Such a measurement system would permit resistor comparisons with uncertainties of the order of 0.1 ppm in significantly less time than current systems. There are several possible measurement systems that could meet these requirements and these are discussed in Sec. 2.4 below.

While the discussion has until now concentrated solely on dc measurements, the quantum Hall effect can also be used as an intrinsic ac standard of impedance.⁸ The development of the QHE as an ac impedance standard would require the development of new measurement systems, and is currently the subject of much research at national standards laboratories, including NIST. If suc-

Table II: Parameters for a possible commercial measurement system for a QHE-based resistance standard.

PARAMETER	VALUE
1σ Uncertainty	$\leq 0.05\text{ ppm}$
Measurement Time	10-30 min.
Current	$\approx 80\text{ }\mu\text{A}$
Operation.....	Automated

⁸ J Melcher, P Warnecke, and R Hanke, "Comparison of Precision ac & dc Measurements with the Quantized Hall Resistance," IEEE Trans. Instr. Meas., 42(2) 292-4 (1993).

successful, this work would make a QHE-based resistance standard even more attractive by making the maintenance of separate dc and ac impedance standards unnecessary.

2. QHE System Requirements

2.1. General aspects of a system

In order to design a QHE-based standard system with the specifications derived in Sec. 1.3, it is necessary to examine the factors that influence the performance of the different components of the system and determine whether they can be controlled sufficiently well to permit the realization of these goals. The three main components of a quantum Hall effect-based resistance standard system are:

- Sample (QHE Resistance Standard)
- Cryogenic System and Magnet (QHE Resistance Standard)
- Measurement System

Each of these components interacts with the others, so the task of designing the system is quite complex. For example, the particular strength of magnetic field, and hence the size of the magnet required, is a function of both the desired resistance as well as of the sample design. More importantly, the ultimate cost of the system, which largely determines the viability of a commercial system, is largely determined by the size of the refrigerator required, and the quantities of liquid helium consumed, which are in turn determined by the properties of the sample. In this section, the requirements that must be met by each of the components of the system in order to fulfil the specifications derived in Sec. 1.3 are described.

2.2. Sample requirements

In order to observe the quantum Hall effect, the conducting electrons must be confined to a thin, essentially two-dimensional sheet called a 2 dimensional electron gas (2 DEG). The highest quality samples that have been made to date create this 2 DEG in a potential well at the interface between thin layers of GaAs and AlGaAs (called a heterostructure) grown on a wafer of GaAs using the technique of molecular beam epitaxy (MBE). To prepare a quantum Hall resistor from such a heterostructure wafer requires (refer to Fig. 2) the shaping of the heterostructure into a pattern of a Hall bar and the fabrication of alloyed contacts with extremely low resistance. The design and quality of the heterostructure affect the values of magnetic field at which the various plateaus will be observed, and the temperature and current dependence of the plateau values. The geometry of the Hall device will also affect the magnitudes of the temperature and current dependence of the plateau values.

Structure of QHE device

In this section, the important effects that influence the performance of the Hall device will be discussed, including the present state of understanding of the causes of each of these effects, and, when possible, the relation between these causes and controllable sample parameters. A graphical summary of these relationships is shown in Fig. 3. The possibility of controlling each

of these effects sufficiently in order to achieve the design goals listed in Table I, namely, a sample that operates at temperatures near 4.2 K and currents near 80 μA , will be assessed.

2.2.1. Temperature dependence

When current is passed through a Hall device as shown in Fig. 2, one can measure the voltage drop along the axis of the Hall bar, denoted V_x , the Hall voltage across the bar, V_H , and the voltage between the current carrying contacts, which are also called the "source" and "drain" contacts. When this latter voltage is divided by the current, one obtains the 2-terminal resistance of the device. Under *ideal* conditions, the voltage drop V_x is zero, and V_H is equal to the source-drain voltage. Since the source-drain voltage in practice includes the voltage drop across contact resistances, and since the Hall voltage V_H is equal to the current times h/ie^2 , the Hall voltage is usually used to compute the resistance of the device. At higher temperatures (generally, above about 1.2 K), however, V_x does not vanish over a broad range of magnetic field, but rather, as shown in Fig. 4b, is "U" shaped. The plateaus in the Hall voltage are not flat and constant, but, as shown in Fig. 4a, have a slight slope, i.e., $dV_H/dB \neq 0$. All three quantities, viz. V_x , dV_H/dB , and the deviation of the value of V_H , measured at the magnetic field at which V_x is minimum, from its ideal value of h/ie^2 , $\Delta V_H/(Ih/ie^2) \equiv [(V_H - Ih/ie^2)/(Ih/ie^2)]$, all vary with temperature, reaching essentially constant values equal to their ideal values only at the very lowest temperatures (generally, below 1 K).⁹

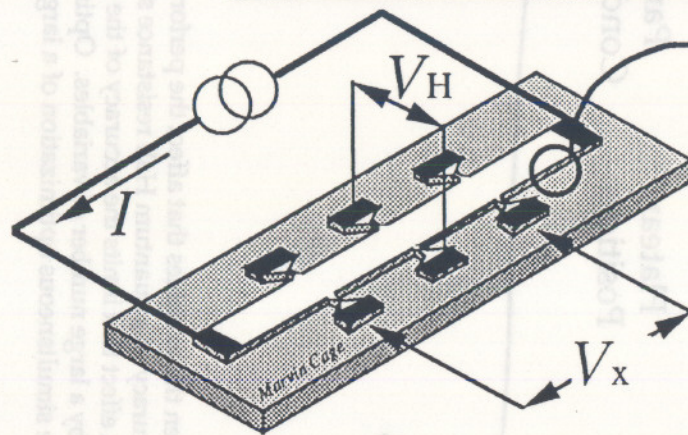
The explanations for these temperature dependence effects are not clear even today. Numerous complex explanations have been advanced, but none seem entirely satisfactory. It appears furthermore, that the temperature dependences of V_x , the deviation of V_H from its ideal value $[\Delta V_H/(Ih/ie^2)]$, and the Hall plateau slopes (dV_H/dB), are all due to different effects. These temperature dependences are not caused solely by defects in the heterostructure, but are affected by the geometry of the sample, the homogeneity of the 2 DEG, and numerous other properties.¹⁰ In addition, the exact form of the temperature dependence is different every time a sample is cooled down. Surprisingly, why this should be so is actually understood. At room temperature, there are many free carriers present in the donor and capping layers, and as the temperature is cooled, these carriers become bound to donor ions, and also to defects or trap impurities in the spacer layer and donor layer. It is generally difficult to cool the sample uniformly, so if one side gets "splashed" with liquid helium before the other side, the carriers on that side will "freeze out" before those on the warmer side, resulting in an inhomogeneity in the electron density in the 2 DEG. Since it will be impossible to reproduce this exact cooling procedure a second time the inhomogeneities in the 2 DEG will be different on subsequent cool-downs. Since the temperature dependence is a function of the inhomogeneities in the 2 DEG, the exact form of the dependence of V_x , $\Delta V_H/(Ih/ie^2)$, and (dV_H/dB) on temperature, and even the magnetic field at which the plateaus are observed, will be different for each cool-down.

Temperature dependence different each time sample cooled down

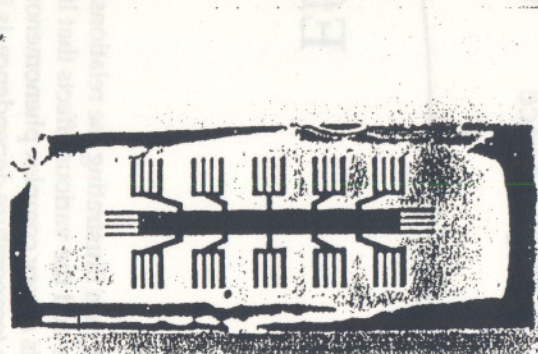
⁹ M E Cage et al., *ibid.* (footnote 5).

¹⁰ W van der Wel, C J P M Harmans and J E Mooij, "A Geometric Explanation of the Temperature Dependence of the Quantized Hall Resistance," *J. Phys. C*, **21** (7) L171-5 (1988).

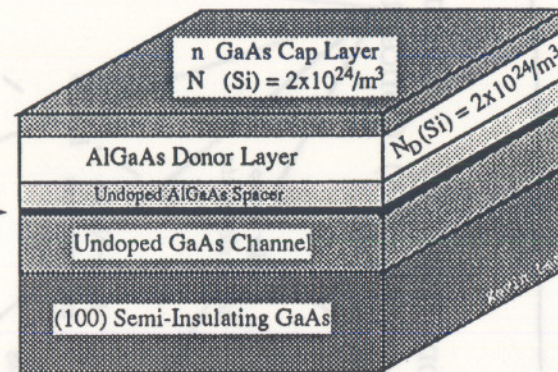
Quantized Hall Resistor in GaAs/AlGaAs Heterostructure



After Etching



2 DEG



Completed Device, after alloying



5 mm

Kevin Lee

Fig. 2. The quantum Hall effect is observed when the conducting electrons are confined to a 2-dimensional sheet by a potential well created in a GaAs-AlGaAs heterostructure (at right above). The Hall bar geometry shown at left is etched into this heterostructure, and alloyed contacts are made to the potential probes.

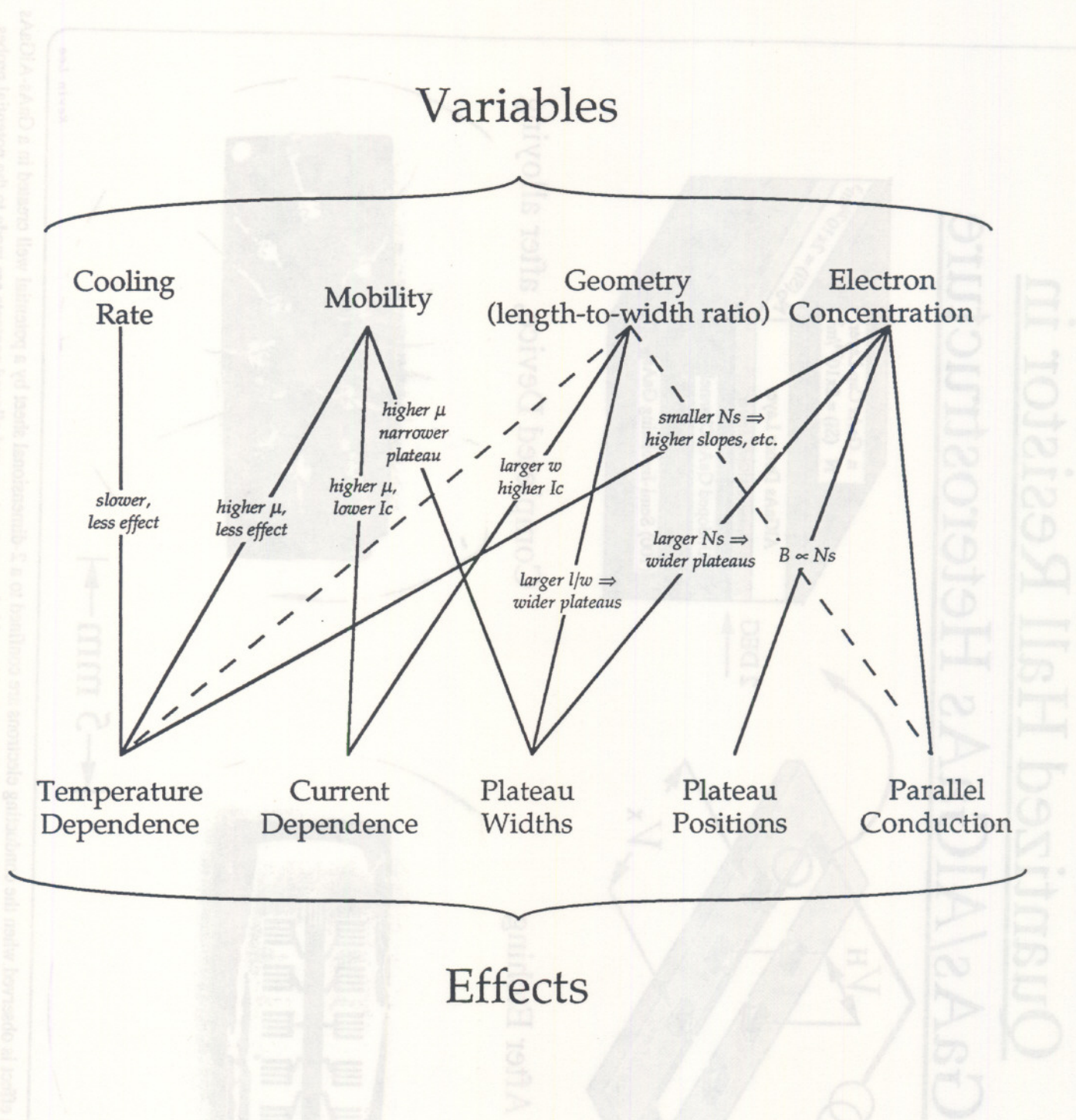


Fig. 3. Schematic diagram illustrating the relationships between the variables that affect the performance of a quantum Hall device and the various effects that limit the accuracy of a quantum Hall resistance standard. As can be seen, this is a very complex phenomenon, and each effect that limits the accuracy of the quantum Hall resistor, such as temperature dependence, is influenced by a large number of variables. Optimizing the performance of a quantum Hall resistor therefore requires the simultaneous optimization of a large number of parameters, which is a very difficult task.

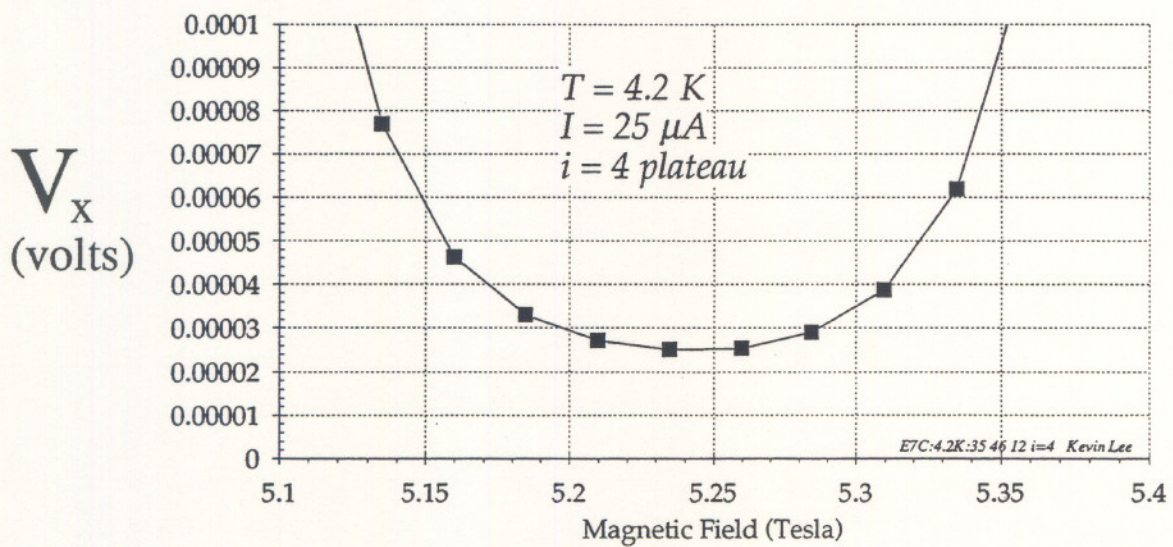
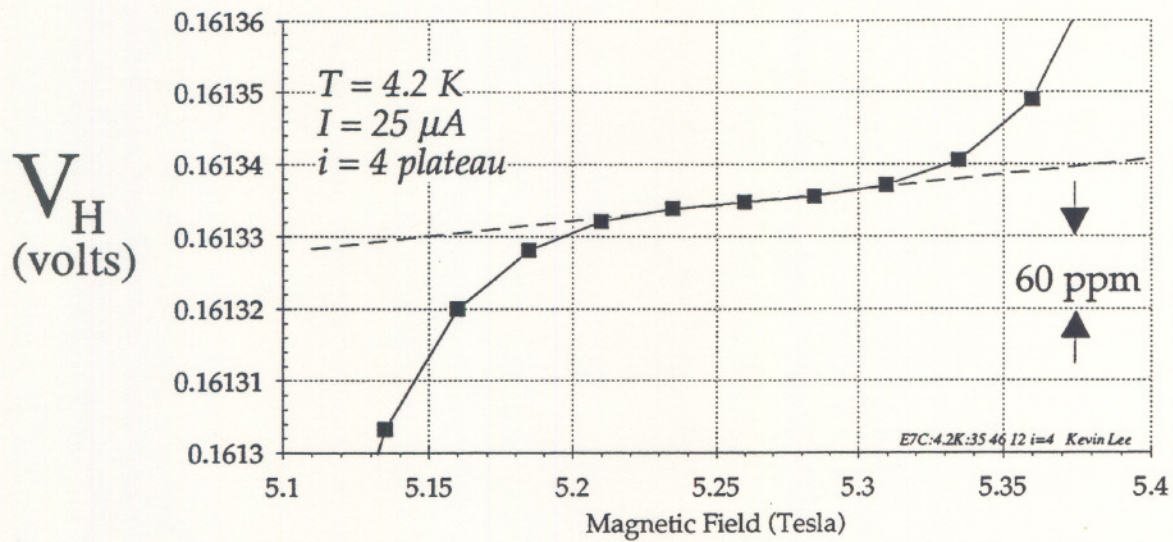


Fig. 4. Traces of Hall voltage (a, top) and V_x , the voltage drop along the Hall bar (b, bottom) as a function of magnetic field at 4.2 K. (4a): The Hall plateau has a non-zero slope, as indicated by the dashed line. (4b): V_x does not go to zero at 4.2 K, resulting in a temperature-dependent correction to the Hall resistance. It appears that for certain sample designs, the ratio between the correction to the Hall resistance measured at the magnetic field which minimizes V_x (5.25 T in this figure), to the minimum value of V_x , is a constant, independent of cool-down, and can be determined accurately for each sample. See discussion in Sec. 2.2.1.

In order to produce a quantum Hall device that can be used as a resistance standard near 4.2 K, as specified in Table I, it is either necessary to eliminate the temperature dependence, or to control it in such a way that it is reproducible each time the sample is cooled down, so that a temperature dependent correction factor can be accurately derived and applied to the resistance of the Hall device. While one sample was prepared in Germany that reportedly did not exhibit measurable temperature dependence at temperatures as high as 4.2 K, this work has not been reproduced.

Relation between $\Delta V_H/(Ih/e^2)$ and V_x is independent of cool-down

There does appear, however, to be two possible ways to control the temperature dependence of the Hall resistance. The first relies on an observation of M E Cage et al. [see footnote 5] on two samples. While the actual forms of the temperature dependences exhibited by these samples were different each time they were cooled from room temperature, the value of $\Delta V_H/(Ih/e^2)$ was always a constant multiple of V_x at any temperature (see Fig. 5c). The value of this multiple was independent of cool-down and dependent *only* upon the particular probe pairs used to measure V_H and V_x . Subsequent work by van der Wel (footnote 10), however, indicated that this behavior may be reproducible only for samples with wide potential probe contacts. If this should prove to be correct, it would appear to be possible to use such a quantum Hall device as an accurate resistance standard at temperatures as high as 4.2 K by performing the following procedure:

- Set the magnetic field at the minimum in the V_x vs B curve;
- Accurately measure V_x using the same current that will be used to measure V_H .¹¹
- Determine the correction factor ΔV_H by multiplying V_x by the pre-determined constant;
- Compare the voltage across the resistor being calibrated with the Hall voltage measured at the magnetic field at which V_x is minimum (set in the first step), determine the value of the resistor relative to the Hall resistance, and apply the correction factor derived above.

More research would have to be done, however, to determine the conditions under which the relationship between $\Delta V_H/(Ih/e^2)$ and V_x is independent of cool-down, and under which the above procedure could be reliably used.

The second possible technique for controlling the temperature dependence relies on an observation of d'Iorio and Wood¹² that a single quantum Hall device that was cooled very slowly and uniformly from room temperature to liquid helium temperature exhibited no observable $\Delta V_H/(Ih/e^2)$ at temperatures below 3.8 K. A cooling rate as slow as the one used by d'Iorio and Wood is rather difficult, but not impossible, to produce in cryogenic systems in which the sample is bathed in liquid helium. In contrast, if a closed-cycle refrigerator were used to cool the sample instead of liquid helium, such slow cooling rates would readily be achieved, for the refrigerator would have to cool both the sample and the magnet, which would take as long as 24 hours or more with present refrigerators. More research must, however, be done in order to determine the optimum cooling rates, and to determine how reproducible these effects are for a given sample, and whether they can be observed reproducibly in different samples.

$\Delta V_H/(Ih/e^2)$ may be eliminated by cooling very slowly

¹¹ The form of the temperature dependence is also a function of the current flowing through the Hall bar.

¹² M D'Iorio and B M Wood, "Temperature Dependence of the Quantum Hall Resistance," *Surf. Sci.* **170** 233-237 (1986).

Slope of V_H vs B

As noted above, the slope of the Hall plateau, dV_H/dB , also varies with temperature, and can be quite large at temperatures as high as 4.2 K. For most presently available samples, the value of this slope is immeasurably small at temperatures below 2.2 K. It has been shown that the higher the mobility of the heterostructure, the higher the temperature at which the slope becomes immeasurable,¹³ so it is theoretically possible to prepare a sample that has negligible dV_H/dB at 4.2 K. The problem is that generally, the higher the mobility of the sample, the lower the maximum current that can be passed through the sample before it breaks down (see section on current dependence below) and the narrower the plateaus in the Hall voltage.

It should be emphasized that to date, very little work has been done on temperature dependence, and the theoretical understanding of the factors that influence it. The relation of these factors to controllable sample parameters is as yet poorly understood. Much work remains to be done to understand the effects of contact width, cooling rate, mobility, and even measurement current on the dependences of $\Delta V_H/(Ih/e^2)$, V_x , dV_H/dB on temperature, and of $\Delta V_H/(Ih/e^2)$ on V_x . There are however, several promising techniques for controlling this temperature dependence that have been demonstrated to work on isolated samples, but more research must be done to understand the conditions under which these techniques can be used reliably and reproducibly.

2.2.2. Current dependence

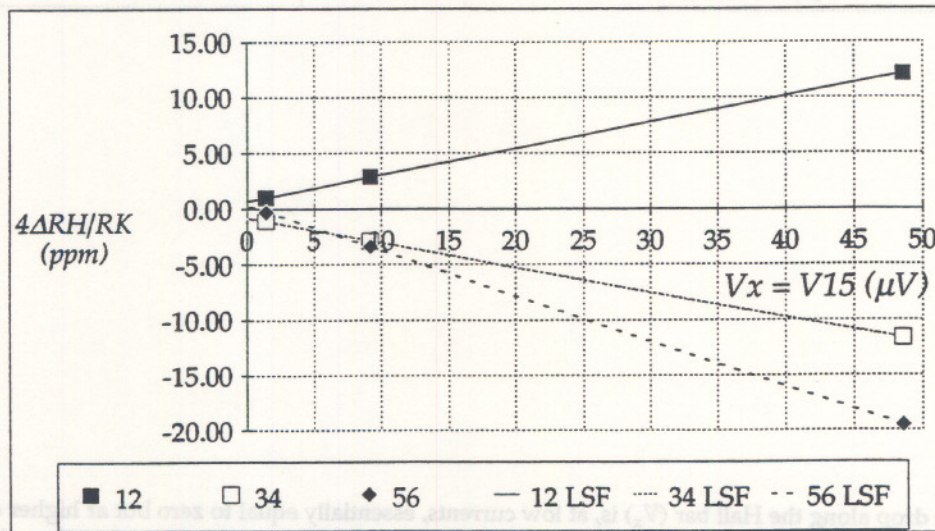
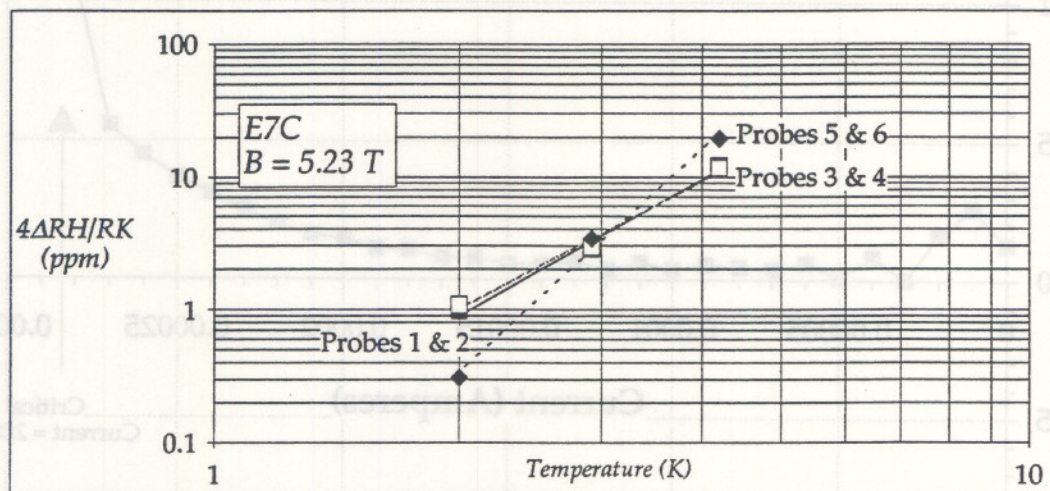
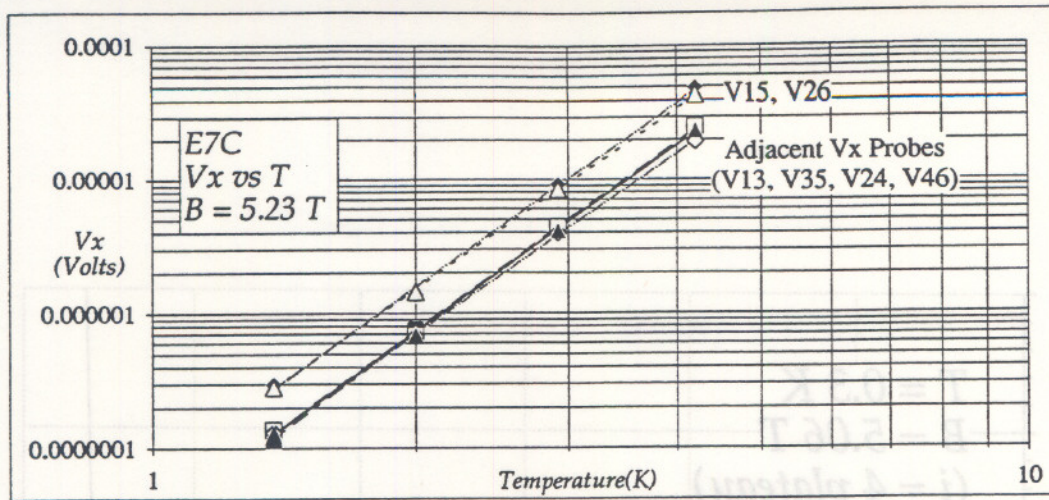
Deviations of V_H from h/e^2 possible even for $I < I_{crit}$

In the absence of temperature effects, the resistance of a Hall device is equal to the ideal value of h/e^2 only when sufficiently low currents are passed through it. If the current exceeds a certain critical value, V_x suddenly increases dramatically, and the mechanisms responsible for current transport at lower currents break down (See Fig. 6). The large values of V_x cause deviations of the Hall resistance from its ideal value of h/e^2 , and the device can no longer be used as a resistance standard.¹⁴ This phenomenon is known as breakdown. M E Cage, et al. showed that even at currents approaching the critical breakdown current at which V_x is quite large, the deviation of the Hall voltage from its ideal value, $\Delta V_H/(Ih/e^2)$, can be negligible.¹⁵ Even though these observations seem to indicate that quantum Hall devices can be used as resistance standards at currents of several hundred microamperes, this may not always be the case. It has been observed that breakdown can occur in isolated regions of the sample giving rise to rather large noise at currents less than the critical current [see van der Wel, footnote 13], an effect that is particularly pronounced in high mobility samples.

¹³ W van der Wel, E G Haanappel, J E Mooij, C J P M Harmans, J P André, G Weimann, K Ploog, C T Foxon, J J Harris, "Selection criteria for AlGaAs-GaAs heterostructures in view of their use as a quantum Hall resistance standard," J. Appl. Phys. 65(9) 3487-97 (1989).

¹⁴ It is important to note that while a relation between V_x and V_H exists in this case, it is *different* than the relationship that exists between these two quantities when *temperature* is the cause of non-zero V_x (as discussed in Sec. 2.2.1), as the physical origin of the two effects is quite different.

¹⁵ M E Cage, R F Dziuba, B F Field, E R Williams, S M Girvin, A C Gossard, D C Tsui, and R J Wagner, "Dissipation and Dynamic Non-linear Behavior in the Quantum Hall Regime," Phys. Rev. Lett. 51(15) 1374-7 (1983). In this work, the critical breakdown current was over 325 μA , but it was observed that at currents as high as 300 μA , $\Delta V_H/(Ih/e^2)$ was only -0.1 ppm. Indeed, M E Cage found that the sample used in this work exhibited no appreciable difference in the Hall voltage from its ideal value at currents less than 170 μA even though V_x was quite large (several microvolts) at these currents.



Schematic Diagram of QHE Device

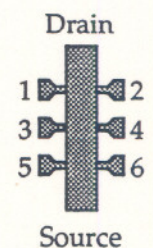


Fig. 5. Temperature dependence of the quantum Hall effect for the $i = 4$ plateau of EUROMET sample E7C: (5a) The voltage drop along the Hall bar (V_x) decreases as the temperature is lowered, and becomes unmeasurable (< 1 nV only below $= 0.5$ K for the sample shown here). (5b) The non-zero value of V_x gives rise to a temperature dependent deviation $[\Delta R_H / (R_K/4)]$ of the Hall resistance from its ideal value of $R_K/4 = h/4e^2$. (5c) While the exact forms of the dependences of V_x & $[4\Delta R_H / R_K]$ on temperature are different each time the sample is cooled down, the relation between $[4\Delta R_H / R_K]$ and V_x shown in this graph has been found to be the same, independent of cool-down on a few, but not all samples. For the samples for which the relation is cool-down independent, the relationship between $[4\Delta R_H / R_K]$ and V_x for a given probe pair can be determined once and used to extrapolate the ideal value of the Hall resistance from measurements taken at temperatures at which V_x is non-zero on subsequent cool-downs. Note that $[4\Delta R_H / R_K]$ tends to zero as V_x tends to zero, so that only when V_x is zero does the deviation of the Hall resistance from its ideal value vanish. The slight deviations of $[4\Delta R_H / R_K]$ from zero in this figure are due to the limited accuracy of the measurements (1σ uncertainty $\leq \pm 0.7$ ppm). In the above figures the symbols V_{ij} , where i and j are integers, refer to the voltages measured between probes i and j , indicated on the schematic diagram on the right. The notation LSF in Fig. 5c indicates Least Squares Fits to the data.

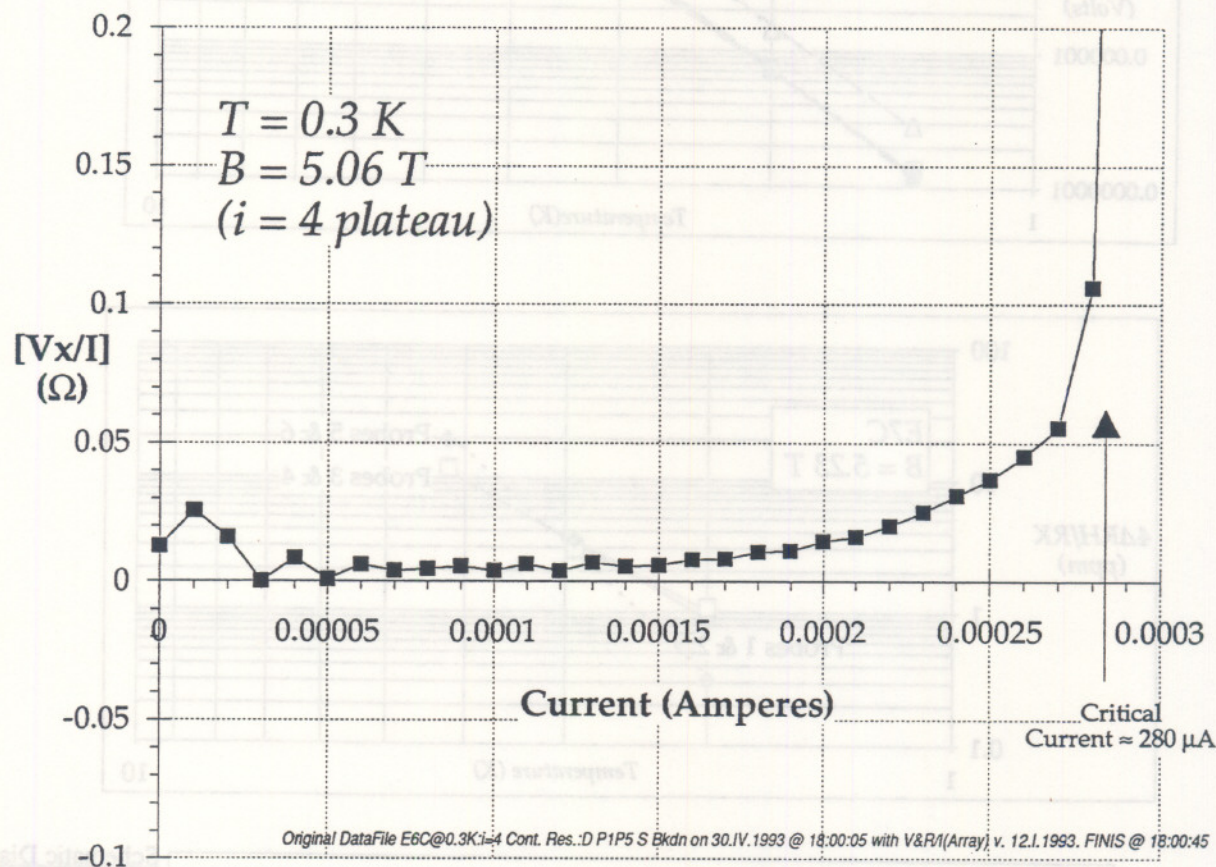


Fig. 6. The voltage drop along the Hall bar (V_x) is, at low currents, essentially equal to zero but at higher currents the device will suddenly break down, and V_x will become quite large. This figure shows V_x/I as a function of current through the bar. This resistance should be zero at low currents, but is not in this graph because of thermal voltages and noise (due to extremely short averaging time for each point). It is clear that V_x begins to increase at currents far below the critical current at which the device breaks down ($280 \mu\text{A}$). As V_x becomes non-zero, the value of the resistance of the quantum Hall device may deviate from the ideal value of h/ie^2 , so difficulty might be experienced in using this device as a resistance standard at currents as low as $100\text{--}150 \mu\text{A}$.

At present, samples are available that have critical currents as high as 300 μA when measured at temperatures less than 1.2 K. Unfortunately, the critical current tends to decrease with increasing temperature, and may be considerably less at 4.2 K than at 1.2 K. Presently, national standards laboratories perform resistance calibrations with currents much less than the critical current: NIST currently uses 25 μA , 40 μA , and 60 μA , and is presently trying 80 μA . Research would have to be conducted to determine the exact form of the temperature dependence of the critical current, and determine the maximum currents that can be used for high accuracy calibrations at different temperatures.

As mentioned in Sec. 1.2 above, one would ideally like to use currents of 80 μA or greater, in order to increase the voltages across the Hall device and thereby decrease the measurement time required to achieve a certain accuracy. This goal appears to be possible, but requires compromises in selection of the sample parameters. While high mobility samples tend to have lower critical current densities (200 μm wide samples with mobilities over 100 $\text{m}^2/\text{V-s}$ can have critical currents as low as 1-25 μA – see van der Wel, ref. 13), the use of lower mobility samples with higher critical current densities is not always desirable, for, as mentioned in the previous section, the lower mobility samples exhibit more pronounced temperature effects. At present, the optimum sample mobilities are in the range of 10 to 40 $\text{m}^2/\text{V-s}$ (see Sec. 2.2.4 below). For a given mobility, the critical current tends to increase linearly with the width of the sample (for sample widths $\geq 25 \mu\text{m}$), but in order to prevent geometrical effects from causing corrections to the Hall voltage, one must increase the length in the same proportion as the width. Since critical breakdown currents (and also temperature dependences, plateau widths, etc.) are very sensitive to inhomogeneities in the 2 DEG, and since increasing the area of the device increases the probability that large inhomogeneities will be present in the device, there is an upper limit to the size of the device. The actual limiting size is a function of the quality and uniformity of the heterostructure material from which the device is made.

Current dependence a function of geometry

The current dependence of $\Delta V_H/(Ih/ie^2)$ is also a function of the quality and design of the contacts through which current enters and leaves the sample, and of the homogeneity of the donor density and the uniformity of the spacer layer thickness in the heterostructure from which the quantum Hall device is made. Some research would have to be conducted to determine the optimum dimensions of a Hall device and the optimum design for the contacts and heterostructure, but it appears from the experience with present samples, that it is probable that a sample that will operate at currents in the range of 80-100 μA can be made. Whether such a device could be operated at 4.2 K at these currents is another question, which would require further research: generally, the current dependence effects are much less than temperature dependence effects, so the problem of producing such a device reduces to the problem of controlling temperature dependence discussed in the previous section.

2.2.3. Parallel conduction

In order that the resistance of the quantum Hall device equal h/ie^2 , all the current must flow through the 2 DEG. If any of the other layers in the heterostructure, such as the donor layer or cap layer, conduct electricity, then the resistance of these layers will contribute to the resistance of the device, and cause a deviation of the resistance from the ideal value of h/ie^2 . This deviation

tion is often temperature dependent, and is called "parallel conduction" (see Fig. 7). It can be effectively eliminated either by appropriately choosing the thickness and donor density in the donor and cap layers, or by etching away the capping layer, and, if necessary, part of the donor layer until the parallel conduction vanishes, although in practice this is a difficult and very time consuming job.

2.2.4. Plateau positions and widths

The range of magnetic field over which any given plateau is observed is determined by the electron density in the 2 DEG, which in turn is determined by the design of the heterostructure, specifically, the thickness of the spacer layer, and the donor density in the donor layer. The choice of the magnetic field range, however, involves a compromise between a number of different factors. Quantized Hall resistance plateaus with resistances less than 6.45 k Ω (i.e., $i > 4$) do not extend over a broad range in magnetic field, and V_x does not generally go to zero, which results in large deviations of the Hall plateau value from the ideal h/ie^2 , making these plateaus unsuitable for use in resistance calibrations. In addition, the widths and slopes of these plateaus are strongly temperature dependent and the lower resistance plateaus may not even be observed at temperatures around 4.2 K (see Fig. 1). The higher resistance plateaus ($i = 4, 2$, and 1), on the other hand, are very broad and flat, and V_x vanishes over a significant portion of the plateaus, particularly for the $i = 1$ and 2 plateaus. In addition, for a given current, the voltage across the Hall device is larger for the higher plateaus, making it possible to perform highly accurate resistance comparisons more quickly. As a consequence, these plateaus are used at national standards laboratories for resistance calibrations.

The higher resistance plateaus, however, occur at much higher values of magnetic field than the lower resistance plateaus. While the electron concentration in the 2 DEG can be varied to cause any plateau to occur at any value of magnetic field, the width of the plateau decreases and the temperature dependent slope dV_H/dB increases markedly as the magnetic field at which it is observed decreases. For this reason, a practical lower limit to the magnetic field at which the $i = 4$ plateau is observed is about 4 T. Because the $i = 2$ and $i = 1$ plateaus are inherently broader than the $i = 4$ plateau, it is possible that samples exhibiting these plateaus at fields below 4 T, and possibly as low as 2-3 T, could be used as resistance standards, but some research would have to be done to verify this. To maximize the width of the plateau and minimize V_x (thereby minimizing the deviation of the Hall resistance from its ideal value of h/ie^2), the electron concentration should be chosen to cause the plateau to occur at the maximum possible magnetic field. Higher magnetic fields, however, require larger, more expensive superconducting magnets. In addition, if one attempts to make the plateau occur at too high a field, such as above ≈ 8 T for the $i = 4$ plateau or ≈ 4.5 T for the $i = 6$ plateau, the electron density in the 2 DEG will have to be so high that higher sub-bands in the potential well will be occupied (see Fig. 7), the conditions required to observe quantized Hall resistances equal to h/ie^2 will not be met, and the sample will not be usable as a resistance standard.

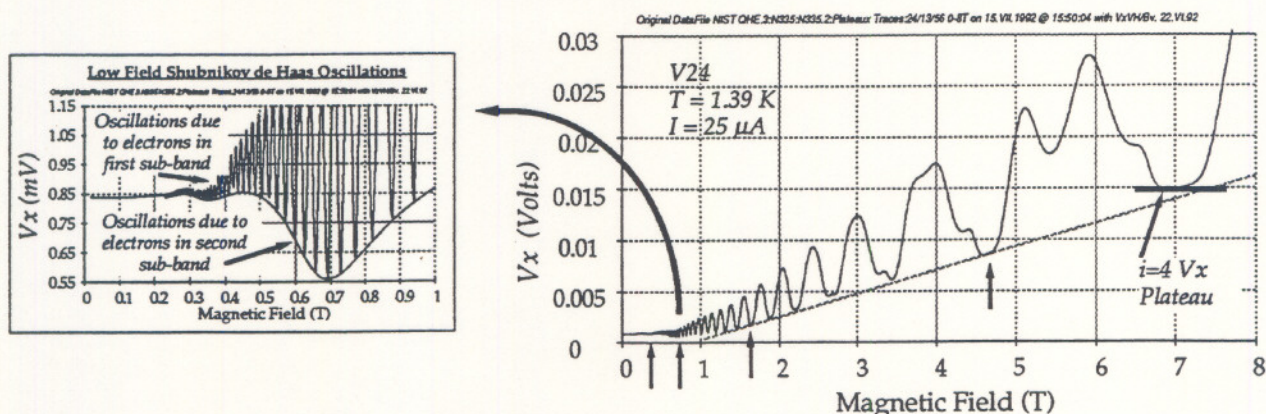
Experience has shown that the optimum magnetic field range for observing the $i = 4$ plateau is 4 to 8 T, a field range that is readily achieved with fairly inexpensive NbTi superconducting solenoids. Standards-quality samples can also be made with electron density such that the $i=2$ or $i = 1$ plateaus are observed in this magnetic field range. The field range over which a given

$i = 1, 2$, and 4 are
broadest and best
plateaus for resis-
tance calibrations

Width of a plateau is
proportional to the
field at which it is ob-
served

Optimum field range is
4-8 T

N335.2 $V_x(24)$ vs B



N335.2 R_H vs B

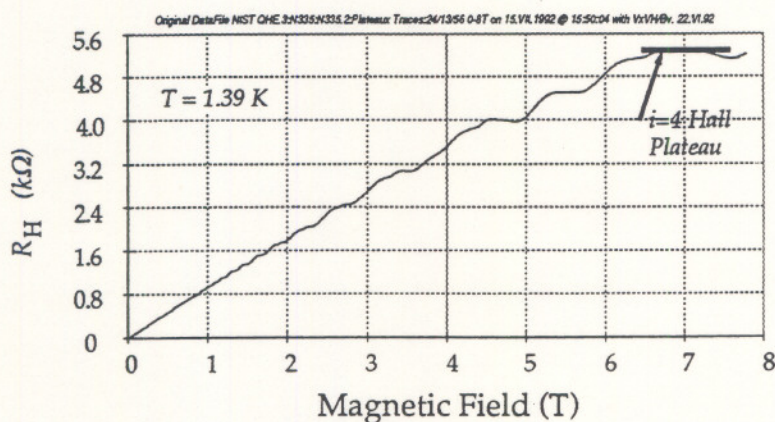


Fig. 7. As described in Sec. 2.2.4, the higher the electron concentration, the broader the plateau, and the higher the field at which it occurs. Too large an electron concentration, however, causes electrons to populate higher sub-bands. This in turn has a number of complex effects on the characteristics of the Hall device, as shown above. (7a) In samples in which only the lowest sub-band is populated, one only observes the high-frequency oscillations of V_x with increasing magnetic field, and the minimum values of V_x vanish at higher magnetic fields. In samples in which the higher sub-bands are filled, an additional lower-frequency oscillation in V_x is observed, as shown in the inset at left above. As the grey line in the figure at right indicates, these longer-period oscillations are still visible at higher fields (the minima of this longer period oscillation are indicated by arrows). This, coupled with parallel conduction in one of the upper layers of the heterostructure prevents the minimum in V_x from vanishing on the $i=4$ Hall plateau. (7b) Conduction by electrons in the second sub-band and by electrons in the upper layers of the heterostructure has the same effect as adding a resistor in parallel with the Hall device, and lowers its effective resistance. The $i=4$ plateau observed on this particular sample, N335.2, gives a resistance of only about 5.1 k Ω , which is a very significant deviation from the ideal value of $h/4e^2 = 6\,453.201\,750$ Ω .

plateau is observed is a characteristic of a particular heterostructure and a particular sample: in other words, one can not make a single standards-quality sample that will exhibit different plateaus in a given field range.¹⁶

The widths of the Hall plateaus are also functions of the mobility of the sample. Surprisingly, the higher the mobility, the narrower the widths of the plateaus [see van der Wel, footnote 13]. The optimum plateau widths occur with mobilities in the range of 10-40 m²/V-s. This appears to be the optimum mobility for quantum Hall devices, for samples made with mobilities larger than this, in addition to having narrow plateau widths, tend to have very low breakdown currents (cf. Sec. 2.2.2 above).

Plateau widths inversely proportional to mobility

The plateau widths are also influenced by the geometry of the sample, and the widths and geometry of the potential contacts on the side of the Hall bar. Samples with large length to width ratios tend to exhibit broader plateaus,¹⁷ while samples with wide potential contacts tend to exhibit a large decrease in Hall voltage at the high magnetic field end of the plateau, which decreases the width of the plateau.¹⁸ While some research would have to be done to determine the optimum geometry for a sample to operate at 4.2 K and 80 μ A, sufficient information is available that it should be possible to prepare a sample that exhibits the plateau in the desired magnetic field range, and does not require significant geometrical correction factors.

Plateau width proportional to length to width ratio

It should be noted that it is of the highest importance that the donor density in the donor layer and the spacer layer thickness be of the highest uniformity. Significant variations in either can cause large variations in the electron gas density, which in the most extreme cases can cause plateaus in V_x and V_H measured between adjacent probes to occur at different values of magnetic field (see Fig. 8). In such cases, large deviations of the Hall resistance from its ideal value of h/ie^2 will be observed, and the sample will not be usable as a resistance standard. Less extreme variations in the electron gas density give rise to enhanced temperature dependence effects and possibly to lower critical currents.

Importance of uniformity of electron density

2.3. Cryogenic system

The primary purpose of the cryogenic system is to provide the conditions necessary for the observation of the quantum Hall effect. The sample must be cooled to sufficiently low temperatures

¹⁶ This is not strictly correct: if one makes a gate on top of the heterostructure, one can vary the electron density in the 2-DEG by applying a potential to the gate. This has been done routinely in quantum Hall effect research, but has not been applied to standards-quality samples to date. The reason is that leakage resistances and currents between the gate and the potential probes can give rise to significant corrections to the Hall resistance, and no research has been done to determine their magnitude, their stability with time, and other properties. The Hall effect is a very complex phenomenon, and all aspects of it are not yet understood even for samples without gates, so it is unlikely that gated standards-quality samples will become available in the near future.

¹⁷ K von Klitzing, G Dorda, and M Pepper, "New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance," *Phys. Rev. Lett.* 45(6) 494-7 (11.Aug.1980).

¹⁸ R Woltjer, M J M de Blank, J J Harris, C T Foxon, and J P André, "The Influence of Contacts on the Quantized Hall Effect," in *High Magnetic Fields in Semiconductor Physics II*, ed. G. Landwehr (Springer, Berlin, 1989), pp. 66-75.

and must be placed in a sufficiently high magnetic field that plateaus in the resistance are observed, and that deviations from the ideal h/e^2 value of resistance are small. The actual conditions necessary to achieve these requirements are strong functions of the sample, as has been discussed in the previous section. A variety of cryogenic systems exist that can be used to produce a wide range of conditions under which the quantum Hall effect can be observed. In this section, possible cryogenic systems are discussed and evaluated with particular attention paid to the goals discussed in Sec. 1.3.

2.3.1. Temperature

In order to use the quantum Hall effect as a resistance standard, the temperature of the sample must be at least 4.2 K, and preferably, for most samples presently available, 1.2 K or less.

Cooling a sample to 4.2 K is very easy to do, as this is the temperature of liquid ^4He at its normal boiling point. To cool the sample to temperatures of 1.2 K or less, ^4He or ^3He refrigerators are used. These refrigerators cool a bath of liquid ^4He or ^3He by reducing the vapor pressure over the bath by means of a vacuum pump. The following section discusses the advantages and disadvantages of these refrigerators.

Recent advances in closed-cycle refrigeration technology have led to great interest in using them to cool a quantum Hall resistance standard. Present closed-cycle refrigerators have many attractive advantages: they have a low cost of acquisition and maintenance, they do not require liquid cryogens, they cool the sample continuously, and only consume electricity. They are, however, only capable of producing minimum temperatures around 4.2 K, which is somewhat high for observing the quantum Hall effect. The following paragraphs summarize the advantages and disadvantages of these different cryogenic systems.

- **LIQUID HELIUM-BASED REFRIGERATORS:** The systems presently used at NIST and in most other national standards laboratories are based on dewars filled with liquid ^4He . The sample is placed in a separate chamber inside the dewar, called the "insert", which is filled with either liquid ^4He or ^3He . Special vacuum systems reduce the pressure in the insert, causing the liquid helium to evaporate and its temperature to drop to 1.2 K in the case of ^4He , and 0.3 K in the case of ^3He .

These systems have a number of advantages: the sample can be maintained at temperatures that are so low that the temperature effects discussed in Sec. 2.2.1 above are quite small, or completely negligible and critical breakdown currents are quite high, so that the quantum Hall resistor performs close to its theoretical limits of accuracy. Because of the high heat conductivities of liquid ^3He or superfluid ^4He , the power dissipated in the sample, which can approach 100 μW at high currents for the $i = 1$ and 2 plateaus, is readily removed by the liquid helium that surrounds the sample, thus reducing the noise in the device. In addition, the superconducting magnets required to produce the high magnetic fields can be quite compact and inexpensive when they are cooled by liquid helium.

These refrigerators, however suffer from a number of rather severe disadvantages that have made them less attractive for use in commercial standards laboratories. Since the system must maintain the sample at the base temperature (1.2 K or 0.3 K depending on the type of system) for at least one working day (called the "hold time"), and since about half the liquid helium boils

^3He and ^4He refrigerators

Advantages: At low temperatures, QHE device is close to theoretical limits of accuracy

Disadvantages: expensive, complex

N256.2: Effect of heterostructure inhomogeneity on Hall resistance

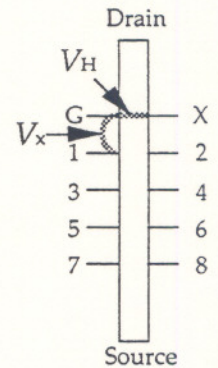
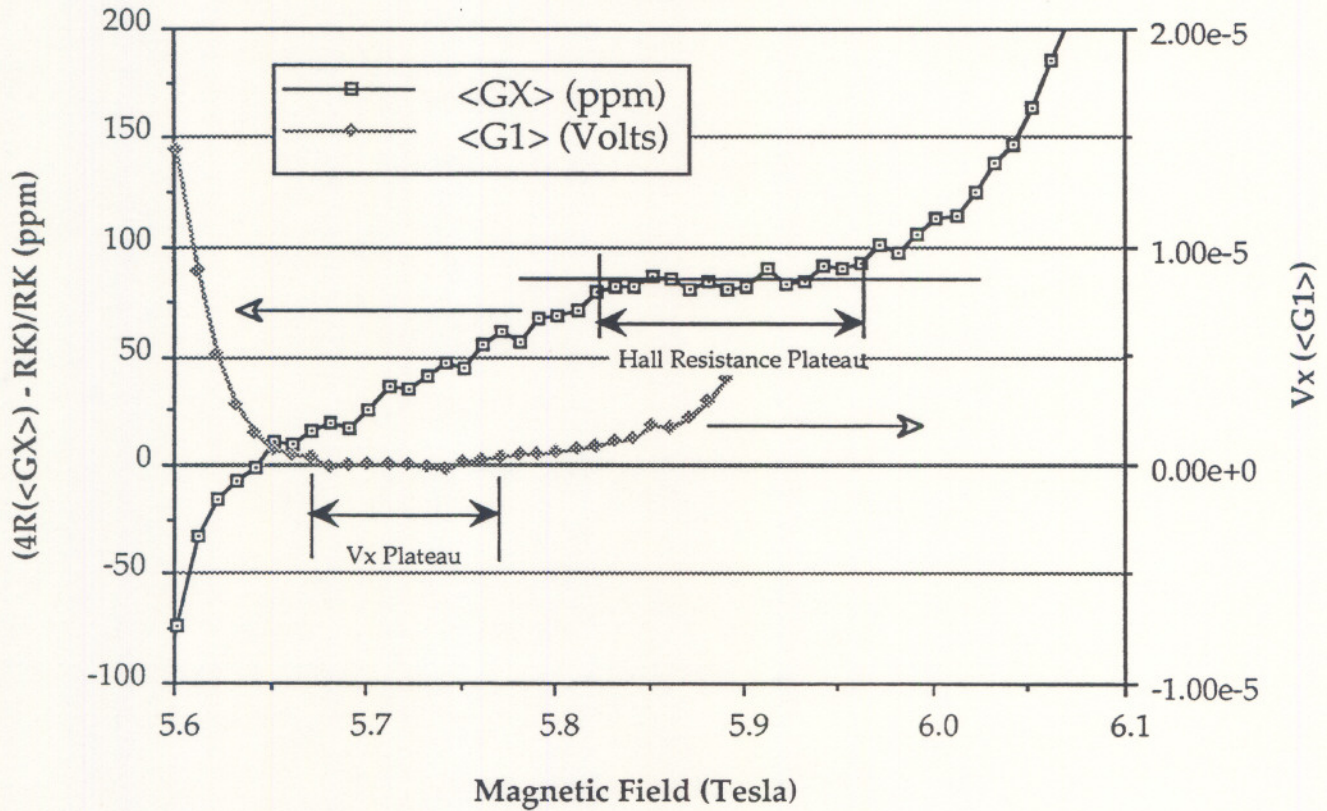


Fig. 8: Illustration of the effect of sample inhomogeneity on the Hall voltage for sample N256.2. The plateau in the Hall voltage (V_H in the inset) measured between probes G and X and that in the voltage drop along the bar (V_x) between G and the adjacent probe 1 occur at different magnetic fields. Since the magnetic field at which a plateau is observed is proportional to the electron concentration, this effect is due to strongly varying electron concentration in the Hall bar. One of the effects of this inhomogeneity is to cause the current flow in the bar to be misaligned relative to the axis of the bar, creating an effective V_x voltage drop between the Hall probes (on opposite sides of the bar), which adds to the Hall voltage and results in the observed deviation of the Hall resistance from its nominal value (see van der Wel, footnote 13). In this plot, the difference between the Hall resistance and its nominal value ($h/4e^2$) is plotted as a function of magnetic field, and one can see that the non-zero V_x at magnetic fields at which the Hall plateau is observed cause the Hall resistance between probes G & X [$V_H/I = R(\langle GX \rangle)$] to be about 80 ppm higher than its ideal value of $R_K/4$.

away when the bath is initially cooled from 4.2 K to 1.2 K, fairly substantial quantities of liquid helium are consumed:¹⁹ the liquid helium requirements for even conservative systems can be as high as 25 liters per day, which translates to an annual cost of over \$50 000 for liquid helium alone (assuming a cost of \$8/liter). Both ^3He and ^4He refrigerators are fairly expensive to assemble: while the dewar can be fairly inexpensive, the vacuum system and the means for controlling and measuring the pressure in the insert (in which the sample is kept) contribute to the high cost of the system. Liquid helium is continually consumed by these systems, so periodically, new liquid helium must be transferred into them, a complex and time consuming process. This causes large thermal shocks to the system, and calibrations must be halted for a period of at least several hours after a transfer to allow these thermal gradients to be eliminated.

• **"LOW-LOSS" DEWAR:** Since the largest costs in acquiring and operating the refrigerated systems described above are the costs of the refrigerator and the liquid helium consumed in the refrigeration process, the easiest way to reduce the cost of the cryogenic system is to eliminate the refrigerator and operate the system at 4.2 K, the temperature of liquid ^4He at its normal boiling point. Such a system would consist simply of a heavily insulated dewar filled with liquid ^4He . While consuming vastly less liquid ^4He than the ^4He or ^3He refrigerators described in the previous paragraph, a low-loss system would still consume liquid ^4He : heat conducted down the magnet leads and down the electrical leads to the sample would cause the liquid helium to boil off. The rate of consumption of liquid ^4He could be greatly reduced either by establishing the desired magnetic field, placing the superconducting magnet in persistent current mode, and removing the magnet leads, or by installing a cooling device connected to a closed-cycle refrigerator at the top of the dewar to re-condense the helium vapors, and prevent any loss of liquid helium. Such arrangements are commonly used on dewars for superconducting magnets used with magnetic resonance imaging systems, and hold times of more than 6 months can be achieved readily.

4.2 K liquid helium-based system

Low-loss dewars would retain many of the advantages of refrigerated systems: the magnet would still be bathed in liquid helium; the sample would still be bathed in liquid ^4He , although liquid ^4He at 4.2 K is not a superfluid and does not have quite as high a heat conductivity as superfluid ^4He , so the sample may be somewhat noisier than in one of the refrigerated systems described above. Because of the long hold times, liquid helium would have to be transferred into the dewar only once or twice a year, as opposed to once per day, so the daily "dead time" of several hours required with the liquid helium refrigerators described previously could be eliminated, permitting essentially continuous operation of the quantum Hall device as a resistance standard. The greatest disadvantage to the use of low-loss dewars is that they can only provide temperatures of 4.2 K, and would require the development of samples that could be operated at this temperature. While such samples are not widely available at present, it appears that with some research, they could be produced (see Sec. 2.2.1 above), making it possible to consider the use of low loss dewars for maintaining a quantum Hall resistance standard.

Advantages: inexpensive, long hold times, simpler to operate

¹⁹ This is a characteristic of both ^4He and ^3He systems: in the case of ^3He refrigerators, the ^3He must first be cooled below 1.2 K by a ^4He refrigerator to liquefy it, so ^3He refrigerators must not only have a sufficient quantity of liquid ^4He , they must also have sufficient ^3He to last one day.

Closed-cycle
Refrigeration
Systems

• **CLOSED-CYCLE REFRIGERATOR:** Closed-cycle refrigerators are heat pumps based on either the Carnot cycle (Gifford-McMahon engines) or the Sterling cycle (pulse-tube refrigerators) that continuously remove heat from the system. They can produce temperatures as low as 4.2 K, but do not require liquid cryogen. Thus, while their initial cost of acquisition may be somewhat high, they do not consume liquid helium, and the cost of maintaining such systems can be significantly less than for any system that consumes liquid helium. These systems have only recently become available, however, and have not been tested for use in quantum Hall resistance standards systems.

Disadvantages: need
new magnet design,
higher temperature
samples

There are several difficult problems that would have to be resolved before closed-cycle refrigerators could be used in a QHE-based resistance standard system. First, a sample that could be used as a resistance standard at 4.2 K would have to be developed. This problem is more serious for closed-cycle refrigerator-based systems than for low-loss dewars, for in the latter case, the sample is bathed in liquid helium, which cools it, while in a closed-cycle refrigerator-based system the sample is mounted on a heat sink connected to the refrigerator. This creates the problem of removing the power dissipated in the sample during calibrations. Any heat generated in the quantum Hall device must be conducted through the substrate to the refrigerator. The heat conductivity of GaAs, however, is not very high at 4.2 K, so power dissipated in the 2 DEG layer on top of the quantum Hall device might not be readily conducted through the substrate to the cryo-cooler, and the sample could heat up, and might be very noisy, if it could be used as a resistance standard at all. Some research would have to be conducted to determine the maximum currents that could be used in the devices under such conditions.

Another problem that would have to be addressed is the cooling of the magnet. As discussed in Sec. 2.2.4 above, superconducting solenoids are required to produce the magnetic fields in the range of 4-8 T that are required for operation of quantum Hall devices as resistance standards. Present solenoid designs generally must be cooled with liquid helium during operation. In order to use a closed-cycle refrigerator for a QHE-based standards system, a new magnet design, in which the magnet was cooled by conduction through a heat sink, would have to be developed. Recently, superconducting magnets that can be cooled by closed-cycle refrigerators have been reported, so this problem can apparently be solved.²⁰ Yet another problem that would have to be investigated is whether vibrations from the refrigerator, which is mechanically attached to the cold head on which the sample is mounted, would affect the operation of the QHE device as a resistance standard. Research would have to be conducted to determine how these factors affected the use of the quantum Hall device as a resistance standard, and at present no work on any of these problems has been done so it is difficult to assess the probability that these efforts will be successful.

²⁰ G F Greene et al. reported on "Conductively Cooling a Small Nb₃Sn Coil with a Cryo-cooler" at the International Cryo-cooler Conference in Sante Fe, NM in Nov. 1992.

2.3.2. Magnetic field

As discussed in Sec. 2.2.4, fairly high magnetic fields of at least several tesla are required to observe the quantum Hall effect. There are a number of different ways of providing these fields. Generally, conventional room-temperature electromagnets are not desirable, as they can only produce fields of at most 2-3 tesla over fairly small volumes, and it is difficult to maintain the sample at the required cryogenic temperatures in such a configuration. Recently developed permanent magnets, made of such high magnetic energy density rare earth permanent magnet materials as Nd-Fe-B, can produce fields as high as 2 to 2.5 T at room temperature without the need for external power supplies.²¹ In order for these magnets to be of utility, one would have to have quantized Hall devices that produced standards-quality resistance plateaus in this range of field. While this has not been done to date, there is a possibility that with some research, samples might be produced that exhibit an $i = 1$ or 2 plateau that is of sufficient quality to be used as a resistance standard in this field range. Were such samples available, these rare earth permanent magnets would be the magnets of choice for quantized Hall resistance standards, as they would not even have to be cooled to cryogenic temperatures to operate, would not require any bulky power supplies, and would not require any current leads, which conduct heat into the cryogenic system and increase the consumption of liquid helium.

Magnetic fields can be produced by:
Electromagnets;
Permanent Magnets;
Superconducting Solenoids

Unfortunately, such devices would require a fair amount of research to produce, and are not presently available. As discussed in Sec. 2.2.4, currently available devices require magnetic fields in the range of 4 to 8 tesla (40 000 to 80 000 gauss), which can only be produced using superconducting solenoids. The type of superconductor, and size and design of the solenoid required are determined primarily by constraints imposed by the cryogenic system and the sample.

Super-conducting solenoid required

Stability: If the solenoid is to be used with a cryogenic system capable of producing temperatures less than 1 K, the solenoid need not produce a particularly stable magnetic field, for the quantized Hall resistance plateaus are so broad and constant in value at those temperatures (cf. Fig. 1), that small drifts in magnetic field will not affect the value of the Hall resistance. If the cryogenic system will only be capable of temperatures ≈ 4.2 K, then the resistance plateaus will not be broad, and if they have a slope, resistance calibrations will only be possible at the magnetic field at which V_x is a minimum. As a result, the magnet must be extraordinarily stable, capable of maintaining a set magnetic field without drifting for periods of at least 8 hours, for any drifts in magnetic field will result in drifts in the value of the resistance of the Hall device and an error in calibration. The exact limits on the drift in the field over this time depend on the slope of the Hall plateau, and the accuracy of calibration, and some research would have to be conducted in order to determine these limits. It is quite likely, however, that high quality superconducting solenoids available today will meet these specifications.

Plateaus have slope @ $T=4$ K so B must not drift

Physical Size: The size of the magnet is particularly important if a closed-cycle refrigerator is to be used to cool the sample and magnet. The smaller the magnet, the less its thermal mass, and the more quickly it can be cooled. A superconducting solenoid approximately 75 mm (3 inches) in

Small magnet; faster to scan, but higher heat leak

²¹ H A Leupold and E Potenziani II, "An Overview of Modern Permanent Magnet Design," Technical Report SLCET-TR-90-6 (U.S. Army Laboratory Command, Ft Monmouth, NJ, August 1990).

diameter and 125 mm (5 inches) long can produce a field of ≈ 8 T using fairly few turns. It will have a low inductance (≤ 1 henry), and the magnetic field can be scanned at a fairly high rate (several tesla/minute). As a result, the tests required to set up the calibration conditions (namely the determination of the magnetic field that minimizes V_x) can be performed quite quickly. Such magnets, however, use high currents (80-150 A) and require large current leads (≈ 7 -8 mm dia.) which conduct more heat into the refrigerator and increase the rate of consumption of liquid helium. In conventional dewar systems, the heat leak can be eliminated by increasing the current in the magnet to produce the desired magnetic field, placing the magnet in persistent current mode, and removing the magnet leads. If a closed-cycle refrigerator is used to cool the system, the heat leak can be reduced by thermally anchoring the leads to the upper stages of the closed-cycle refrigerator, which have larger cooling power than the last (low temperature) stage, but the problem cannot be eliminated.

Large magnet:
slower to scan,
but lower heat
leak

The heat-leak problem can be greatly reduced by reducing the magnet current required to produce the desired fields, and hence the size of the magnet leads. Such a magnet will be larger in size, have many more turns, have a much higher inductance (possibly in the range of 10-80 henries), and will be capable of much slower maximum scan rates (possibly on the order of 0.01 T/min to 0.1 T/min). The current required, however, will be significantly lower (possibly 5-20 A), and the heat leak posed by the magnet leads can be quite minor. Such a magnet design has many advantages that recommend it for use with low-loss dewars: not much heat is conducted down the magnet leads, which will increase the hold time of the dewar. Since such systems have very long hold times, the process of setting the magnetic field to the value that minimizes V_x will not have to be done very frequently, and the increased time required for this operation will not be a drawback. In the case of closed-cycle refrigerators, however, it must be determined whether a solenoid with many turns can successfully be cooled without liquid helium.

Material: There are two different superconducting materials commonly used for producing solenoids, NbTi and Nb₃Sn. NbTi has a superconducting transition temperature of about 10 K, and one must cool it to 4-5 K to get fields in the range of 4-8 T, so such a magnet would necessarily have to be cooled by the last stage of a closed-cycle refrigerator, placing a rather large load on it. The maximum field that can be produced by NbTi solenoids at 4.2 K is about 9 T. Nb₃Sn wire has a much higher critical temperature ($T_c = 18$ K with no applied magnetic field), and solenoids made from Nb₃Sn can produce fields as high as 9 T at temperatures as high as 10-14 K. This makes Nb₃Sn attractive as a material for producing magnets for systems cooled by closed-cycle refrigerators [see footnote 20], as the magnet could be cooled by the second stage of the refrigerator, which for typical present 3-stage refrigerators is at ≤ 15 K. Since the second stage of the refrigerator has a higher refrigeration capacity, the heat load on the final, low-temperature stage of the refrigerator is thereby minimized. Nb₃Sn solenoids are also attractive for use in liquid helium cooled systems (including low-loss dewars), as they can achieve fields of ≥ 12 T at 4.2 K and can therefore be used to perform resistance calibrations with both the 6 453.201 750 Ω ($i = 4$) resistance plateau (magnetic field between 4 T & 6 T), and the 12 906.403 500 Ω ($i = 2$) plateau (magnetic field between 8 & 12 T). Nb₃Sn solenoids are, however, considerably more expensive to produce than NbTi ones, and the choice of which material to use must be made on

economic as well as technical grounds: a low-loss dewar system can use either NbTi or Nb₃Sn solenoids, but a closed-cycle refrigerator may require a Nb₃Sn solenoid.

2.4. Measurement system

In order for a quantized Hall effect-based resistance standard to be competitive with current wire resistor-based systems, the measurement system used to compare resistors to the quantized Hall resistance will have to be capable of uncertainties of the order of 10^{-8} , as discussed in Sec. 1.3 above. National standards laboratories, including NIST, have developed a number of measurement systems that meet these requirements. Unfortunately, most of these systems are very complex and time consuming to build, but it is possible that they might be commercialized if the need exists. This section summarizes the different types of measurement systems that have been developed that meet the accuracy requirements in Table II.

2.4.1. dc measurement systems

Potentiometric Systems: NIST for many years used a potentiometric bridge to maintain the national standard of resistance using the quantum Hall effect.²² In this bridge, the resistor to be calibrated and the Hall resistor were placed in series, and the same current flowed through both of them. A battery-powered potentiometer was then adjusted to produce the same voltage as the voltage drop across the Hall resistor. The potentiometer was then connected across the test resistor (which was of nominally the same value as the Hall resistor), and the difference between the potentiometer voltage and the voltage drop across the test resistor was measured using a low-noise (room-temperature) detector. The difference between the resistances of the test resistor and the Hall resistor was then determined from the detector output. This bridge was capable of uncertainties (due to random effects) of about 0.01 ppm (depending on the noise of the detector used) in measurement times of about 10 minutes. Its main drawback was that it could only compare resistors with nominally the same value. Resistors of other values could only be calibrated against it using special networks of 4-terminal resistors, called Hamon networks, that could be connected in series and parallel to produce large and small resistances (See Dziuba et al., footnote 1). This calibration must be performed using other measurement systems, such as Kelvin bridges. The need for using the Hamon networks decreases the calibration accuracy and increases the time and cost of calibrations. Indeed, NIST itself has recently stopped using this technique because of the time and care required to use it.²³

Capable of
0.01 ppm in
10 min;

Only for 1:1
comparisons

DVM Method: This system is a variation of the potentiometric system (See footnote 4). Instead of using a potentiometer to compare the voltage drops across the test resistor and the Hall resistor, a high accuracy digital voltmeter is used. This technique is presently capable of one standard deviation uncertainties (see footnote 2) of about 0.1 ppm, but measurement times of about 8 hours (with 25 μ A measurement current) are required. It is probable that the measurement

Capable of
0.1 ppm in 2 h
to 8 h;

Scaling to 4:1

²² G Marullo-Reedtz and M E Cage, "An Automated Potentiometric System for Precision Measurement of the Quantized Hall Resistance," J. Res. Nat. Bur. Stand. 92(5) 303-310 (Sept. 1987).

²³ R F Dziuba and R E Elmquist, "Improvements in Resistance Scaling at NIST Using Cryogenic Current Comparators," IEEE Trans. Instr. Meas., 42(2), 126-130 (April.1993).

Design Criteria for Commercial QHE Resistance Standard

times could be reduced to ≤ 2 hours by using currents as high as $50 \mu\text{A}$, and that the one standard deviation uncertainty could be improved to 0.03 ppm , but this has not yet been experimentally verified. The main advantages of this system are that it is inexpensive, can be assembled entirely from commercially available pieces, and can achieve uncertainties $\leq 0.1 \text{ ppm}$ when comparing resistors with values that differ by as much as a factor of 4 (and perhaps even more, though determination of the maximum scaling factor would require more research). This system is therefore the most readily commercialized measurement system available. The main disadvantage is that the DVM noise and stability and the quality of the switches in commercial scanners limit the ultimate accuracy of this technique to only $0.03 - 0.1 \text{ ppm}$, which is only slightly better than the accuracy presently being achieved with wire resistors and conventional commercial measurement systems (see Sec. 1.1).

Capable of
 0.01 ppm in $\leq$
1 min, accurate
scaling;

Extremely
complex

Cryogenic Current Comparators: NIST presently uses a cryogenic current comparator to maintain the national standard of resistance using a quantized Hall resistor. This measurement system is, however, very complex, and took many years to build. The test resistor and the Hall resistor are placed in separate circuits, and current is supplied to each by two separate current sources. The currents are chosen so that if the two resistors had their nominal values, the voltages across them would be exactly equal. The difference between the actual value of the resistor being calibrated and its nominal value causes the voltage across it to be slightly different than that across the Hall resistor, and this difference is monitored with a detector. The detector output is amplified and fed back to a circuit that increases the current in the secondary resistor until the voltages across the resistor and the Hall device are equal. The magnitude of this correction current is proportional to the difference between the actual value of the resistor being calibrated and its nominal value. Since the detectors can be all cryogenic [superconducting quantum interference devices (SQUIDs) maintained at 4.2 K], noise in the current comparator and detector, which is one of the primary limitations of the other techniques described above, is extremely low. This system is capable of uncertainties (due to random effects) of the order of 0.01 ppm with measurement times of less than 1 minute. Furthermore, it is capable of comparing resistors with different values without sacrificing accuracy, and is clearly superior to any of the other measurement systems available. Because of its complexity, however, it is difficult to manufacture, and is not readily commercialized.

2.4.2. ac measurement systems

Low-frequency room-temperature ac current comparator: One of the reasons for the complexity of dc measurement systems is that they are sensitive to thermal voltages and some types of noise, and special measurement procedures must be followed to minimize the influence of these effects on the calibration. Since low frequency ac measurement systems are insensitive to these effects and since it has been demonstrated that the resistance of quantum Hall devices is independent of frequency (over a limited range of low frequencies), it would appear to be possible to calibrate standard resistors using a quantum Hall resistor and an *ac* bridge, provided that the resistance of the standard resistor was the same for dc as well as ac. While there is a difference between the dc resistance of a wire resistor and its ac impedance, called the ac-dc difference, it is usually very

Design Criteria for Commercial QHE Resistance Standard

small (typically a few parts in 10^9 at frequencies ≤ 1 Hz), and the BIPM has recently developed a low-frequency ac bridge to calibrate resistors against the quantum Hall resistor.²⁴

This bridge is based on a current comparator circuit, but instead of cryogenic detectors, it uses tuned room temperature ac detectors to minimize the effect of noise. Such low-frequency ac current comparator bridges are capable of highly accurate (uncertainty due to random effects ≤ 0.01 ppm) comparisons of resistors in ≤ 20 minutes. The ultimate accuracy of these bridges is limited by the ratio error in the current comparator (which can be made as low as 0.001 ppm), noise in the detector, and the ac-dc difference of the resistor being calibrated. The low-frequency ac current comparator built by BIPM has been demonstrated to be capable of comparing $1\ \Omega$ resistors with a one standard deviation uncertainty of several parts in 10^9 , but this accuracy has not yet been demonstrated for comparisons between a quantum Hall resistor and a $10\ \text{k}\Omega$ or $100\ \Omega$ resistor. While the ac bridge achieves accuracies only slightly worse than the cryogenic current comparator without the need for expensive cryogenic fluids to cool it, it is, however, still quite complex, requiring special tuned circuits and carefully guarded detector coils that have complex electrostatic and magnetic shields. Research would have to be done to determine the ultimate accuracies achievable with this bridge when comparing quantum Hall resistors to $10\ \text{k}\Omega$ and $100\ \Omega$ resistors, and to determine the magnitudes of the ac-dc differences for $10\ \text{k}\Omega$ and $100\ \Omega$ resistors.

ac Bridge: An ac bridge could permit the quantum Hall resistor to be used as an ac impedance standard at frequencies of up to $1.5\ \text{kHz}$. AC bridges would not be used to calibrate dc standard resistors, since their ac-dc difference at these frequencies is quite high. Rather, an ac bridge would be used with a quantum Hall resistor as an intrinsic standard of ac impedance in the audio frequency range, and could possibly replace other systems, based on standard capacitors and inductors, that are currently in use. AC bridges are currently under development at NIST, and much research remains to be done to determine their ultimate limits of accuracy.

3. Possible QHE-based resistance standard systems

In order to be commercially viable, a quantum Hall effect-based resistance standard has to offer advantages in performance, cost of acquisition and operation, or both, over the resistance standard systems presently on the market. The first section of this manuscript described presently available resistor calibration systems, and set performance limits that a quantum Hall effect-based system would have to meet in order to be competitive with them. The problem of producing a quantum Hall effect-based system that meets these performance limits was shown to reduce to three separate problems that were each discussed in detail in the second section of this manuscript:

- i) the problem of producing a suitable QHE device capable of meeting the specifications shown in Table I;

²⁴ F Delahaye and D Bornauid, "Accurate AC Measurements of Standard Resistors Between 1 and 20 Hz," IEEE Trans. Instr. Meas. 42(2) 287-91 (April. 1993).

- ii) the problem of developing an inexpensive, preferably closed-cycle refrigerator-based, cryogenic system capable of cooling the sample and a superconducting solenoid to temperatures of around 4.2 K without significant adverse effects on the performance of the sample; and
- iii) the problem of developing a suitable high accuracy measurement system with specifications shown in Table II.

This section considers the problem of integrating existing and developing commercially available cryogenic and measurement technologies to produce specific QHE-based resistance standard systems. A QHE-based resistance standard system can be assembled today entirely from commercially available parts (see Sec. 3.1), but it would not meet all of the requirements discussed in Sec. 1.

In order to meet those requirements, new technologies presently under development must be used. Table III lists the different cryogenic, magnet, and measurement system technologies that are available or under development. These technologies are at different stages of development: the ones shown in normal font near the top of the table are currently available commercially; those shown in italic font are not yet mature, and require further development, with the technologies at the bottom of the table requiring the most time to develop. A QHE-based resistance standard can be assembled from any combination of these sub-systems.

The simplest system, which is the least expensive to assemble today with off-the-shelf technology, would use the second cryogenic system (^4He refrigerator) and the magnet and measurement system shown in the first line of the table. Such a system is described in Sec. 3.1, but it would only achieve uncertainties comparable to those available presently with much less expensive and much less complex non-QHE-based resistance standards, and would not therefore find a large market. A more sophisticated system could be assembled using the low-loss dewar (col. 1 of Table III), a superconducting solenoid (col. 2 of Table III) and the DVM-based measurement system (col. 3 of Table III), but some research and development would have to be done for this system to meet the requirements for commercial viability. The challenges involved in realizing this system are described in Sec. 3.2.

Sec. 3.3 describes a more ambitious system utilizing a closed-cycle refrigerator, superconducting solenoid, and cryogenic current comparator that would greatly exceed the performance of presently available non-QHE-based resistance standards systems, and might potentially be competitive in cost with those systems. This system, however, requires the full development of a number of technologies that are presently being developed, and this section describes some of the research that would have to be undertaken in order to successfully build this system. The most ambitious QHE-based resistance standard system would use all of the technologies at the bottom of Table III, including a closed-cycle refrigerator, a permanent magnet, and a room-temperature ac comparator, none of which are commercially available today. If all these systems could be made to work as planned, this system would be extremely cost-effective, yet would have a performance that far exceeded anything commercially available today, and would even rival the performance of the systems presently in use at national standards laboratories. The realization of such a system would, however, require the solution to many very challenging problems, as described in Sec. 3.4.

Table III: Possibilities for Commercial QHE-based Resistance Standard System

List of the possible technologies that can be used for each of the sub-systems in a QHE-based resistance standard. The technologies in normal type are available today; those in italic type are presently being developed, and may be available commercially within 5 to 10 years. The technologies are independent: for example, any of the measurement systems can be used with a QHE standard cooled by any of the different cryosystems listed. To a certain extent, however, the type of magnet used is a function of the type of cryosystem, as is described in the text (Sec. 2.3.2). All uncertainties are one standard deviation uncertainties (see footnote 2).

Cryosystem	Magnet	Measurement System
<u>³He Refrigerator:</u> $T \geq 0.3 \text{ K}$; Expensive to acquire; Consumes liquid ⁴ He: dewar must be refilled with $\approx 25 \text{ l}$ every few days.	<u>Superconducting:</u> <u>Nb-Ti Solenoid:</u> $T \leq 4.2 \text{ K}$; $0 < B < 9 \text{ T}$.	<u>DVM Method:</u> (ref. 4) Inexpensive to acquire; Uncertainties between 0.03 ppm and 0.1 ppm; Resistance Ratios up to 4:1; Measurement times between 30 min and 8 h
<u>⁴He Refrigerator:</u> $T \geq 1 \text{ K}$; Expensive to acquire; Consumes liquid ⁴ He: dewar must be refilled with $\approx 25 \text{ l}$ every few days.	<u>Superconducting:</u> <u>Nb₃Sn Solenoid:</u> $T \leq 10\text{-}15 \text{ K}$; $0 < B < 13 \text{ T}$ (@ 4.2 K).	<u>Potentiometric System:</u> Expensive to acquire; Uncertainties $\leq 0.01 \text{ ppm}$; [†] Measurement times $\approx 10 \text{ min}$ to few hours; Only calibrates resistors with nominally the same value.
<u>Low-loss Dewar:</u> $T = 4.2 \text{ K}$; Uses liquid ⁴ He, but much less than refrigerators above; Requires sample that can be operated at $\geq 4 \text{ K}$.	<u>Permanent Magnet:</u> <u>Nd-Fe-B cylinder</u> $T = 4.2 \text{ K to Room}$ Temperature; $0 < B < 2 \text{ to } 2.5 \text{ T}$.	<u>Cryogenic Current</u> <u>Comparator:</u> Expensive to acquire; Consumes liquid ⁴ He; Uncertainties $\leq 0.01 \text{ ppm}$; [§] Measurement times: 1 min to $\leq 30 \text{ minutes}$; Calibrates resistors with wide range of values (up to 100:1).
<u>Closed-Cycle</u> <u>Refrigerator:</u> $T \geq 3.8 \text{ K}$; No liquid ⁴ He; Requires sample that can be operated at $\geq 4 \text{ K}$.		<u>Room Temperature ac</u> <u>Comparator:</u> Expensive to acquire, rather difficult to build; Uncertainties $\leq 0.01 \text{ ppm}$; [‡] Measurement times $\approx 20 \text{ min. to}$ 30 min; Calibrates resistors with wide range of values (up to 100:1).

[†] G Marullo-Reedtz and M E Cage, *ibid.* (Ref. 22). 1:1 comparisons can be performed with relative combined uncertainty of $\leq 0.007 \text{ ppm}$, but uncertainties in comparing resistors with different values (using Hamon networks) are $\approx 0.01 \text{ ppm}$.

[§] R F Dziuba and R E Elmquist, *ibid.*, (Ref. 23). Comparisons of $R_H(i=4) = 6\,453.201\,750 \, \Omega$ to a $100 \, \Omega$ resistor can be effected with an uncertainty of 0.006 ppm .

[‡] F Delahaye and D Bournaud, *ibid.* (Ref. 24).

In each of the following sections, the capabilities of each of these systems, the hardware that would be required to build them, and the challenges required to realize them are described. For each challenge, the approximate time required to conduct the research to solve the problem, assuming 1 or 2 people are working on the problem, is given. These times are very approximate, because the problems are difficult and very little is understood about the quantum Hall effect, so that it could easily take much longer to solve these problems.

3.1. Presently feasible system using ^4He refrigerator

This system could be built today, entirely from commercially available components. It uses basically the same cryogenic system that is used in the QHE-based resistance standards systems presently in use in national standards laboratories, and therefore the problems concerning the cryogenic system, the magnet, and many of the sample fabrication problems would *not* have to be addressed. This proposed system utilizes a measurement system that has already been developed at NIST, and demonstrated to deliver one standard deviation uncertainties of at least 0.1 ppm. The main problem that would have to be solved to make this system commercially available is that of increasing the current through the sample to 50 μA to 100 μA so that measurement times could be reduced. Even if this problem is resolved, the system presently does not deliver uncertainties significantly better than those that can be achieved using conventional, non-QHE resistance standards systems that are much less complex and costly.

3.1.1. Capabilities

Accuracy: One standard deviation uncertainties ≤ 0.1 ppm in < 8 hours with 25 μA sample current for comparison of resistors with values differing by as much as a factor of 4; measurement time could be reduced if the current were higher: with 50 μA , the same accuracy could be obtained in 2 hours, and with 100 μA , the measurement time would be reduced to 30 min.

Sample Current: 25-50 μA , possibly as high as 100 μA , although this must be investigated.

Temperature: ≤ 1.2 K.

Magnetic Field: 5-6 T for QHE measurements, (8 T maximum field magnet).

3.1.2. System

Cryogenic System: ^4He refrigerator consisting of a dewar (either super-insulated or with liquid nitrogen jacket) cooled with liquid helium, with the sample in a separate insert evacuated to 67-133 Pa (0.5-1 Torr, or $T \leq 1.2$ K).

Magnet: 8 T NbTi superconducting solenoid.

Measurement System: DVM method.

3.1.3. Challenges

Sample: If one standard deviation uncertainties of about 0.1 ppm in 8 hour measurement times are suitable, this system can be delivered immediately with no modification, but it should be noted that much cheaper non-QHE-based resistor calibration systems are available that can meet this level of uncertainty with shorter measurement times. The main challenge in producing the QHE-

based system described above lies in reducing the measurement time by increasing the current through the sample. This is not a simple task, for, as mentioned in Sec. 2.2.2, V_x can be non-zero and the sample can exhibit erratic noise voltages at currents significantly less than the critical current. Presently available samples do not exhibit these effects at currents as high as $60\text{ }\mu\text{A}$, but may exhibit them at currents above $80\text{ }\mu\text{A}$. To produce a sample that can be used at currents of $80\text{ }\mu\text{A}$ and higher, the following research projects would have to be completed:

- The effects of width, size, and mobility on the critical current will have to be investigated.
- Precision measurements will have to be made to determine the relation between the critical current and the much lower current at which V_x becomes large enough to prevent precision measurements.
- The variation of V_x with current and the variation of the difference between the Hall resistance and its ideal value $[\Delta V_H/(Ih/ie^2)]$ with current and the variation of $\Delta V_H/(Ih/ie^2)$ with V_x will have to be determined in order to ascertain whether it is possible to derive a correction factor for the Hall resistance based on measurements of V_x . If this is possible, the dependence of this factor on thermal cycling, and sample design and geometry will have to be determined.

Time Scale: This system is possible today. The investigation of higher currents can be performed in a period of about a year. There is a very high probability that samples that can operate at $60\text{ }\mu\text{A}$ can be reliably produced; it is likely that samples can be made that can be operated at currents of up to $80\text{ }\mu\text{A}$, but the probability of success of producing samples that operate at higher currents is difficult to assess at present.

3.2. Possible "low-loss" dewar-based system

This system utilizes a commercially available low-loss dewar. Both the sample and magnet would be bathed in liquid helium, so conventional magnet designs could be used, and heat dissipation in the sample would not pose a problem. This system would, for simplicity, use the same DVM-based measurement system as the previously described system (see Sec. 3.1). The main components of this system, viz. the cryogenic system, magnet, and measurement system, are presently available commercially; the main challenge in delivering this system lies in producing a sample, and would require the solution to all of the sample fabrication problems outlined in Sec. 2.2 above, including the control of the temperature and current dependence of the sample.

3.2.1. Capabilities

Accuracy: One standard deviation uncertainties ≤ 0.03 to 0.1 ppm in ≤ 2 hours for resistance ratios of up to 4:1.

Sample Current: $50\text{--}80\text{ }\mu\text{A}$ (assuming current dependence problems have been solved).

Temperature: 4.2 K .

Magnetic Field: $5\text{--}6\text{ T}$ for QHE measurements (8 T maximum field magnet).

3.2.2. System

Cryogenic System: "Low-loss" dewar: super-insulated dewar, filled with liquid helium. The dewar can have a cold finger cooled by a closed-cycle refrigerator to re-condense helium vapors in the dewar, so that no helium is lost. Such dewars are currently available commercially.

Magnet: 8 T NbTi superconducting solenoid.

Measurement System: DVM Method.

3.2.3. Challenges

Sample: The main challenge in providing this system is to provide a sample that can be used at 4.2 K. This can be done either by making a sample that has negligible temperature correction at 4.2 K or by making a sample that has an invariant, reproducible temperature dependence that can be accurately determined so that a correction factor can be derived that can be accurately applied to the resistance. Since the phenomena that cause the temperature dependence are poorly understood, the possibility of actually eliminating it appears to be remote at present. The determination of a correction factor, however, appears to be far more feasible, but would entail solving the following research problems:

- i) It has been established (Cage et al., footnote 5) that the deviation of the Hall resistance from its ideal value is linearly related to the value of V_x by the relationship:
$$\Delta V_H / (Ih/ie^2) = s V_x$$

(see Fig. 5c). The following things have to be determined:
 - How much does s vary from cool-down to cool-down?
 - How accurately can s be determined?
 - What is the effect of the rate at which the sample is cooled on the magnitude of s ?
- ii) The effect of the carrier mobility in the heterostructure from which the samples are made, and the size and geometry of the Hall bar on the slope of the Hall plateaus must be ascertained.
- iii) The constancy of the Hall plateau slope from cool-down to cool-down, and the dependence of the slope on the rate at which the sample is cooled must be determined.
- iv) It is assumed that the critical current problems described in the previous section (3.1.3) were solved at lower temperatures. They must be re-investigated at 4.2 K to determine the maximum currents that can be used at this temperature. Presumably, relationships between the maximum usable current and device properties derived at lower temperatures will still be valid, so that the results of the earlier research can be used to develop new Hall bridge structures that can be used with higher currents, if the existing bridge designs should prove inadequate.

Measurement System: An analysis of the sources of uncertainty in the DVM-based measurement system indicate that it should be possible to achieve uncertainties of the order of 0.03 ppm (footnote 4), but in order to do so, improvements would have to be made to the current source and switch scanner, and tests would have to be done to verify that the system could in fact deliver these uncertainties.

Time Scale: Investigation of the reproducibility of the temperature dependence [tasks (i) & (iii) above] would take about a year. Research into the effect of heterostructure design, heterostructure materials, etc. on the temperature dependence, has not yet been done, and could take at least a year or more. If the problems of the current dependence of the Hall resistance described in Sec. 3.1.3 were not solved previously, then they would have to be solved, and would (as stated in Sec. 3.1.3) take about another year. If a closed-cycle refrigerator were to be used to re-condense helium vapors evaporated from the dewar, some research would have to be done to ensure that noise and vibrations from the motors in the refrigerator do not affect the measurements. The cryogenic system is already commercially available.

3.3. Possible closed-cycle refrigerator system

This system is more ambitious than the previous one, for whereas the components of the previous system were all commercially available, and the only challenge was in producing a suitable sample, the *components* of this system, while demonstrated in the laboratory, are not yet all commercially available. This system would use a closed-cycle refrigerator to cool the sample, a superconducting magnet, and a cryogenic current comparator as the measurement system. Because this system would not consume liquid helium, it would be extremely cost-effective to operate, and yet would provide uncertainties at least an order of magnitude better than present non-QHE-based measurement systems. The main advantage is that in addition to providing higher accuracies than present systems, the QHE-based resistance standard does not drift, and for the first time, an *intrinsic* standard of resistance would be available commercially, although the cost would be considerably higher than that of present non-QHE-based resistance standards. To build this system, all of the sample fabrication problems would have to be solved (these problems presumably were addressed in the projects 3.1 and 3.2 described above), a new magnet design would have to be developed, and a new measurement system designed and constructed. In addition, new problems peculiar to the use of a closed-cycle refrigerator (such as the effect of vibrations and temperature fluctuations in the cold head on the use of the sample as a resistance standard) would have to be solved.

3.3.1. Capabilities

Accuracy: One standard deviation uncertainties ≤ 0.01 ppm in 10-30 min, and possibly ≈ 0.003 ppm in ≈ 8 hours for resistance ratios up to 100:1.

Current: 25-100 μA .

Temperature: ≥ 3.8 K.

Magnetic Field: 4-6 T for QHE measurements.

3.3.2. System

Cryogenic System: 3-stage Gifford-McMahon refrigerator with Joule-Thompson expansion unit, possibly a new 2-stage refrigerator, if one is available that reaches 4 K. The use of pulse-tube refrigerators should be investigated, as they have no moving parts, and should in principle be extremely reliable.

Design Criteria for Commercial QHE Resistance Standard

Magnet: Two magnet designs are possible: either a Nb₃Sn solenoid cooled by the second stage of a 3-stage refrigerator, with the sample cooled by the third stage to 4.2 K; or a NbTi solenoid cooled with the sample by the last stage of the refrigerator to 4.2 K.

Measurement System: Cryogenic current comparator (CCC) or low frequency room temperature ac (≤ 2 Hz) current comparator bridge.

3.3.3. Challenges

Sample: All of the sample fabrication problems described in Sec. 3.1.3 and Sec. 3.2.3 would have to be solved. In addition, since the sample will be mounted on a heat sink rigidly attached to the end of the closed-cycle refrigerator, the effect of vibrations and temperature fluctuations that occur naturally during the refrigerator's cycle on the performance of the sample as a resistance standard would have to be investigated. In addition, the sample would not be bathed in liquid helium, so the problem of removing power dissipated in the sample would have to be addressed. As discussed in Sec. 2.3.1, the sample is bathed in liquid helium in present systems, and this removes the heat dissipated in the sample very effectively. This will not be the case in a closed-cycle refrigerator, and the maximum device currents that can be used without causing significant self-heating of the quantum Hall resistor will have to be determined.

Magnet: One of the two big challenges in realizing this system is to design a magnet that can supply a magnetic field of at least 6 T without being cooled by liquid helium. The problem is to devise a method for removing heat generated by the magnet when the magnetic field is increased to the Hall plateau value so as to maintain the magnet temperature below the critical temperature at which it ceases to be superconducting. Usually, magnets are bathed in liquid helium, which is an efficient conductor of heat. To provide such a bath of liquid helium with a closed-cycle refrigerator would require one with a large capacity, which would be expensive, noisy, require frequent maintenance, consume much electricity, and require the storage of large quantities of helium gas. Alternatively, a heat sink capable of rapidly absorbing heat generated by the magnet would have to be developed. In addition, highly reliable closed-cycle refrigerators would be required, for a refrigerator failure could result in damage to the magnet. Magnet manufacturers are presently investigating such magnet designs, and some preliminary successes have been reported (see footnote 20).

Measurement System: The second big challenge in realizing this system is to develop the cryogenic current comparator (CCC) or ac current comparator technology. Such comparators have been built at NIST (cryogenic comparator) and the BIPM (ac current comparator), but these systems are extremely complex, and scarcely amenable to commercialization at the moment. In addition, present CCCs must be cooled to liquid helium temperatures. In order to minimize the costs of maintaining this system, all liquid helium-consuming components must be eliminated. Since CCCs are extremely sensitive devices, however, these comparators could not be cooled with closed-cycle refrigerators, as noise and vibrations from the compressors in the refrigerator would prevent the CCC from operating. Cryogenic current comparators using high- T_c superconductors are presently under development at NIST, and these might be operated at liquid nitrogen temperatures, which can be attained far more easily and inexpensively. The ac current comparator, on the other hand, has been demonstrated to operate with *only* room temperature

components, but much research remains to be done to determine the ultimate accuracies attainable with it when comparing quantum Hall resistors to 10 k Ω and 100 Ω resistors. The magnitudes of the ac-dc differences for different 10 k Ω and 100 Ω resistor designs will have to be determined. Finally, it will have to be determined whether the complex shielding and guarding arrangements required by the ac current comparators can be simplified sufficiently to make them commercially viable.

Time Scale: Most of the sample fabrication problems were described Sec. 3.2.3 above, and as mentioned there, they will take about 4 years or more to solve. The additional sample problems specific to the use of the closed-cycle refrigerator described above will have to be addressed, and may take a year or more to investigate. At present it is not possible to predict whether this effort will be successful. Since it is not clear whether a sample can be made that can operate at temperatures *above* 4.2 K, such as at 5.5 K (the temperatures produced by state-of-the-art two-stage refrigerators), the closed-cycle refrigerator would definitely have to be able to produce temperatures of 4.2 K or below. The development of the current comparators could take an additional several years.

3.4. Possible universal impedance standard system

This is essentially the same system as discussed in Sec. 3.3 above, except that a new *ac* measurement system would have to be developed that would permit the quantum Hall resistor to be used as a standard of impedance.

3.4.1. Capabilities

Accuracy: *dc*: One standard deviation uncertainties ≤ 0.01 ppm in 10-30 min for dc calibrations of resistors with values up to 100:1;
ac: (frequency ≈ 1.5 kHz): ≤ 1 ppm in 8 hours.

Current: ≥ 100 μ A.

Temperature: ≥ 3.8 K.

Magnetic Field: 4-6 T for QHE measurements with 8 T maximum field superconducting solenoid, though *possibly* might be able to use 2 to 2.5 T field.

3.4.2. System

Cryogenic System: Closed-cycle refrigerator as described in Sec. 3.3.2 above.

Magnet: (as above, Sec 3.3.2) If samples can be developed that exhibit usable resistance plateaus at fields as low as 2 T to 2.5 T, the use of a Nd-Fe-B permanent magnet can be considered.

Measurement System: *dc*: cryogenic (dc) or low frequency ac (≤ 2 Hz) current comparator;
ac: ac bridge, quadrature bridge, or higher frequency current comparator operating at ≈ 1.5 kHz.

3.4.3. Challenges

Sample, cryogenic system, and magnet: All of the sample fabrication problems described in Sec. 3.3.3. would have to be solved. In addition, if samples that exhibited usable plateaus between 2 and 2.5 T could be made, then some research would have to be done to determine if variable field 2 T rare-earth permanent magnets could supply magnetic fields that were strong enough, and homogeneous enough for resistance standards work (see footnote 21).

Measurement System: The challenges involved in realizing the dc measurement system were described in Sec. 3.3.3; to realize this system the primary challenge would be designing and building the ac measurement system. The development of the ac quantum Hall effect as an ac impedance standard is currently an active area of research both here at NIST and in Europe (see footnote 8), and much work and time will be required before it will be possible to use the quantum Hall effect as an ac impedance standard.

Time Scale: The development of the ac quantum Hall effect as an ac standard of impedance could take 5 years or more, as work is just beginning on this subject. Once it was established that the effect could be used as an intrinsic impedance standard, a commercial prototype of the system would have to be designed and built, which would take another several years, so this is quite a long-range project. On the positive side, a closed-cycle refrigerator-based "dc" standard of resistance using the QHE could at a later date be upgraded to perform ac calibrations simply by adding a new measurement system, without altering the cryogenic system or sample.

4. Conclusion

Resistance standard systems consist of two main components: a resistance standard with a known, reproducible value of resistance, and a measurement system for comparing other resistors to the resistance standard. Presently, wire resistors are used as resistance standards, but their values drift by as much as several tenths of a part per million with time and environmental conditions. These drifts are, however, of the same order of magnitude as the resolution of current measurement systems (0.1 ppm to 1 ppm). Since the quantum Hall resistance does not drift with time, and is insensitive to environmental factors, it can be used as a resistance standard with uncertainties less than 0.003 ppm. While it would appear to be desirable to simply replace wire standard resistors with quantum Hall resistance systems, the high cost of the cryogenic system and superconducting magnet required to observe the quantum Hall effect have prevented quantum Hall resistance standards from being widely used. Another factor that has inhibited the widespread dissemination of the quantum Hall effect as a resistance standard is that only very low currents can be used with it, and the measurement systems presently in wide use would have difficulty achieving accuracies of even 1 ppm with such low currents.

The problem of making a quantum Hall effect-based resistance standard that would be competitive with systems currently in use therefore requires the solution of three problems: the development of suitable quantum Hall devices, of an inexpensive cryogenic system for cooling the quantum Hall resistor, and the development of a new, simple, automated measurement system capable of uncertainties of the order of 0.01 ppm (about an order of magnitude better than current systems). Section 2 of this manuscript discussed the various technical problems that would have

to be solved to realize these goals, and in Sec. 3, several possible quantum Hall effect systems that could be commercialized were presented, together with an assessment of the difficulty in making them.

It is clear that a quantum Hall device that can be used as a resistance standard at 4.2 K with currents approaching 80 μA to 100 μA would permit the use of inexpensive cryogenic systems and would go a long way toward making a quantum Hall effect-based resistance standard commercially viable. Unfortunately, such a system is not presently deliverable, and would require a significant research effort to realize. A resistance standard utilizing a low-loss dewar and operating at 4.2 K and currents approaching 50 μA , while still requiring a fair amount of research to realize, appears to be somewhat more feasible at present. Such a system would be far less costly than present liquid helium refrigerator-based systems, but may not be as cost-effective as closed-cycle refrigerator-based systems might prove to be.

As regards measurement systems for calibrating other resistors against this quantum Hall effect-based standard, several are available that can deliver uncertainties between ≤ 0.01 ppm and 0.1 ppm. The DVM-based measurement system, capable of calibrating resistors with values that differ by as much as a factor of 4 with uncertainties between 0.03 and 0.1 ppm, is the simplest and most easily commercialized of the lot, but offers only moderate improvement over present wire-resistor-based calibration systems that are capable of calibration uncertainties of 0.1 ppm, and are far less costly. The other systems that are available can achieve uncertainties of ≤ 0.01 ppm, which far out-performs current wire resistor-based calibration systems, but these systems are vastly more complex and some research would be required to simplify these systems to make commercialization cost effective.

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