SQUID Operational Amplifier

K. D. Irwin
Mail Stop 814.03
National Institute of Standards and Technology
325 Broadway
Boulder, CO 80303

M. E. Huber
Dept. of Physics
University of Colorado at Denver
Denver, CO 80217-3364

IEEE Trans. Appl. Supercond. 11(1): 1265-1270, Mar 2001

Presented at
Applied Superconductivity 2000
Sept. 17-22, 2000
Virginia Beach, Virginia

SQUID Operational Amplifier

K. D. Irwin and M. E. Huber

***Abstract*—The nonlinear response of Superconducting Quantum Interference Devices (SQUIDs) has limited their usefulness. We propose the SQUID Operational Amplifier, which consists of several stages of SQUIDs with high open-loop current gain. When connected in a negative-feedback configuration by passing some of the output current through a feedback coil connected to the first stage, the response is linearized. An analog of the semiconductor op-amp, the SQUID op-amp can be used in superconducting equivalents of op-amp circuits such as current amplifiers, current-to-voltage converters, and differentiators. We present experimental results with a ×10 current amplifier as well as a 750 Ω current-to-voltage amplifier which can couple directly to a room–temperature amplifier without a transformer or a feedback line.**

***Index Terms*—Feedback Circuits, SQUIDs, Superconducting Devices.**

I. INTRODUCTION

THE SQUID operational amplifier (op-amp) combines the high sensitivity of SQUID amplifiers with linear, predictable analog circuit performance, low cost per channel, and high bandwidth. In most SQUID applications, a feedback signal is applied by a room temperature amplifier to linearize the response, which adds complexity and cost and limits the bandwidth. To simplify the room-temperature electronics and increase the bandwidth, several groups have combined SQUIDs with superconducting digital logic coupled to the feedback loop, which linearizes the device on-chip [1]. The complexity and high power dissipation of this approach has limited its adoption. Another approach has been to couple the analog output of the SQUID through its own feedback coil in order to linearize the output [2]. This technique leads to some nonlinearity due to the limited loop gains achievable with feedback coils of practical size.

The SQUID op-amp uses several stages of DC SQUIDs to achieve a high open-loop gain with simple design and fabrication and low power. Like the semiconductor op-amp, when connected in a negative-feedback configuration, the SQUID op-amp can be used in a variety of circuits with predictable performance. SQUID op-amp I-to-V converters which use a 100-SQUID series-array [3] as the final stage can couple directly to a room-temperature amplifier without a feedback line or transformer, allowing linear operation with low cost per channel and high bandwidth. Further, SQUID op-amp current-to-voltage (I-to-V) converters can in principle be combined with cryogenic CMOS multiplexers to read out many channels with one output channel.

Manuscript received September 19, 2000.

K. D. Irwin is with the National Institute of Standards and Technology, Boulder, CO 80305-3328 USA (telephone: 303-497-5911, e-mail: irwin @boulder.nist.gov).

M. E. Huber is with the Department of Physics, University of Colorado at Denver, Denver, CO 80217-3364 USA (telephone: 303-556-3561, e-mail: martin.huber@cudenver.edu).

II. SQUID OP-AMP CONCEPTS

A. Basic SQUID Op-Amp Circuit

A circuit diagram of a three-stage SQUID op-amp is shown in Fig. 1a. Each SQUID is voltage biased, and its output current is passed through the input coil of the next stage. In this example, the voltage bias is applied by passing a bias current through a shunt resistor, R_s, (with a small resistance compared to the biased impedance of the SQUID), in parallel with a series combination of the SQUID bias line and the input coil of the next stage. Since the output voltage of a SQUID floats with respect to its input coil, the shunt resistors of each stage in Fig. 1a can be wired in series, allowing them all to be biased with a single current supply from room temperature. The input coil of the third stage is shunted by a resistor, R_d, to provide dominant-pole compensation, as will be described later.

The differential current gain of each stage is the mutual inductance between its input coil and the SQUID loop, M, multiplied by the flux-to-current response of the voltage-biased SQUID, $dI/d\Phi$. The total open-loop differential current gain of the three-stage SQUID of Fig. 1a is thus

$$A = \left(dI_3/d\Phi_3\right)M_3\left(dI_2/d\Phi_2\right)M_2\left(dI_1/d\Phi_1\right)M_{in}. \quad (1)$$

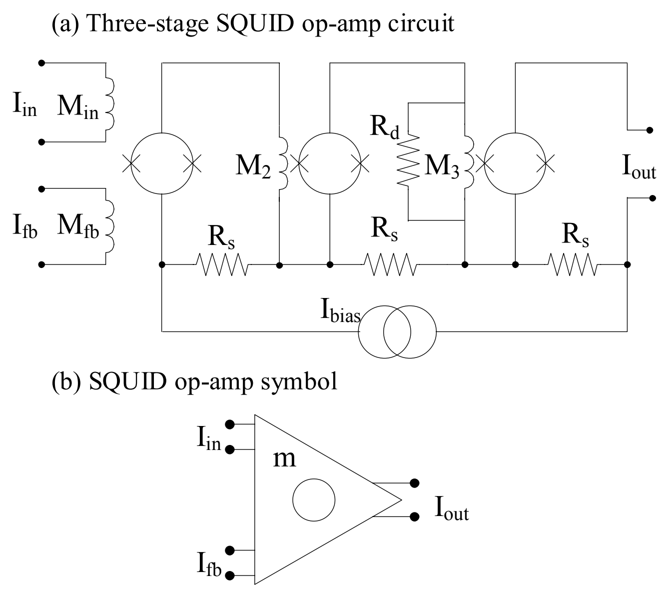

Fig. 1. (a) A circuit diagram for a three-stage SQUID op-amp. (b) A symbol for the SQUID op-amp. The ratio, *m,* of the mutual inductance between the input coil and the first stage SQUID, M_{in}, to the mutual inductance between the feedback coil and the SQUID, M_{fb}, is recorded beside the input lines.

The open-loop gain of a SQUID op-amp can be increased either by increasing the number of turns on the input coils of any stage, or by adding additional stages. The mutual inductance (and current gain) of a single SQUID increases linearly with the number of turns of the input coil, whereas after a moderate number of turns, the chip area used scales as the square of the number of turns. In contrast, the total current gain increases exponentially with the number of stages of a SQUID op-amp, whereas the chip area used increases linearly with the number of stages. If a higher open-loop gain is desired, after a moderate number of turns it is more efficient to increase the gain by adding additional stages.

The symbol for the SQUID op-amp in Fig. 1b will be used throughout this paper. The symbol for the SQUID op-amp has twice as many leads as a semiconductor op-amp since currents require two leads.

B. Finite-Current-Gain SQUID Amplifiers

The SQUID op-amp can be configured as a current amplifier of finite closed-loop differential gain by feeding back some of the output current into the feedback coil of the first stage SQUID (Fig. 2a). The loop gain is *AB/m*, where *A* is the open-loop gain, $B = I_{fb}/I_{out}$ is the fraction of the output current from the SQUID op-amp that is passed through the feedback coil, and $m=M_{in}/M_{fb}$ is the mutual inductance ratio (Fig. 1). The closed-loop differential gain is then

$$G = A/(1 + AB/m). \tag{2}$$

Consider a SQUID op-amp current follower (Fig. 2b). The output current of a voltage-biased SQUID is a periodic function of flux, $I_s(\Phi)$, which suffers from random flux offsets, $\Delta\Phi$, due to stray fields. For the 3-stage SQUID op-amp of Fig. 1a, the open-loop output current is

$$I_{out}(\Phi_{in}) = I_s(\Delta\Phi_3 + M_3 I_s(\Delta\Phi_2 + M_2 I_s(\Delta\Phi_1 + \Phi_{in}))) \tag{3}$$

where the applied flux $\Phi_{in} = M_{in} I_{in}$, and I_{in} is the input current. When configured as a follower, the SQUID op-amp has a stable bias point where there is a solution to the recursive function

$$\Phi_{fb} = M_{fb} I_{out}(\Phi_{fb} + \Phi_{in}), \tag{4}$$

and where the feedback is negative. With proper design, the periodic nature of the response of each stage will lead to stable solutions for any applied flux.

The results of a computer model of a SQUID op-amp current follower are presented in Fig. 3. The three-stage SQUID op-amp from Fig. 1a is used in the current-follower configuration of Fig. 2b. A SQUID with I_c = 100 μA and loop inductance L_s=20 nH is assumed. Both the feedback and input coil of the first stage are 10-turn coils with 200 pH mutual inductance to the SQUID loop and 2 nH self inductance. The second and third-stage input coil have 100 turns with 2 nH mutual inductance and 200 nH self inductance. The open-loop current gain is ~100,000, and the closed-loop current gain is unity. A finite-difference algorithm was used to model the time response of the current follower as the input flux is varied. In Fig. 3, the input current (I_{in}) is plotted vs. the output current (I_{out}) for a low-frequency variation of I_{in}.

In a semiconductor op-amp voltage follower, the high loop gain ensures that the circuit will lock where the feedback voltage is almost exactly equal to the input voltage. A SQUID op-amp *differential* current follower with high enough loop gain will lock where changes in the feedback current are almost exactly equal to changes in the input current, but since there are flux offsets and multiple stable bias points, the absolute value of the feedback current is unpredictable. This behavior is acceptable for many SQUID applications, which measure changes in field or current or reference to a known zero field or current condition.

A SQUID op-amp current amplifier may be useful in combination with a SQUID multiplexer [4] (MUX). If a current amplifier is placed in front of the switching element of a SQUID MUX, it will buffer the detector from the switching element, make a multi-pole anti-alias filter possible, and provide current gain. Configured as a flux-to-current converter (e.g. with no input coil), it may be useful as an initial gain stage before a SQUID MUX for a magnetic calorimeter [5]. Since the amplifier noise dominates in a magnetic calorimeter, the energy resolution would be degraded by multiplexing without such a gain stage.

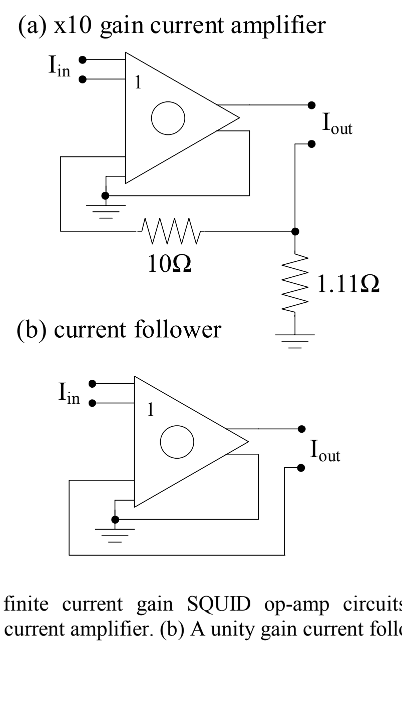

Fig. 2. Two finite current gain SQUID op-amp circuits. (a) A ×10 differential gain current amplifier. (b) A unity gain current follower.

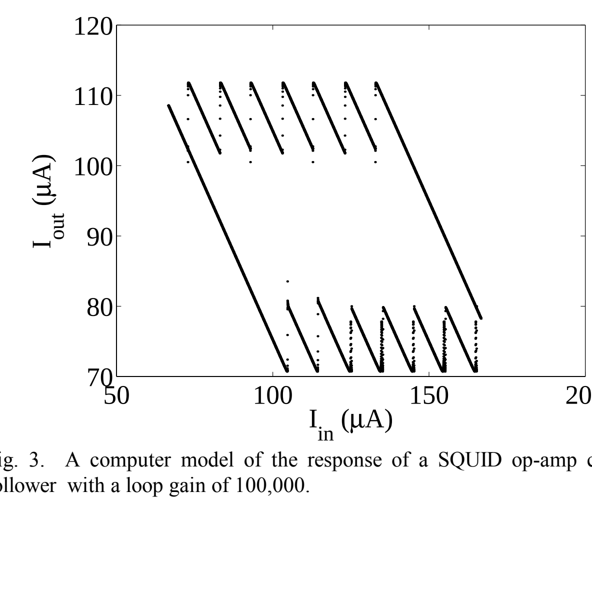

Fig. 3. A computer model of the response of a SQUID op-amp current follower with a loop gain of 100,000.

C. Dynamic Range and Auto-Biasing

In Fig. 3, there are several stable current branches for any applied current, each of which has negative feedback and satisfies (4). When the follower is first turned on, it settles unpredictably to one of the current branches. It will remain on that current branch as long as the applied flux remains within the dynamic range of the branch. The dynamic range is exceeded when the $dI/d\Phi$ of one of the SQUID stages (most often the final stage) switches sign, at which time the follower will lock on the next branch. The earlier SQUID stages can switch sign as well as the last one, which leads to an additional branch structure with a much larger period.

The input current dynamic range of a finite-gain SQUID op-amp circuit is

$$\Delta I_{in} = \Delta I_{\max} / G \,, \tag{5}$$

where $\Delta I_{\max}$ is the maximum output current swing of the output SQUID stage and G is the closed-loop current gain. Even if the dynamic range is sufficient for an application, when the op-amp is first turned on, it may settle near the edge of the current branch, and be driven off of that branch as the input flux changes. It is usually necessary to lock the amplifier on the correct branch by varying the applied flux through the intended range of input fluxes, a technique previously demonstrated using single SQUIDs with some self-feedback through their own input coil [2]. This auto-biasing technique causes the op-amp to settle on a current branch that can accommodate the entire input flux range. Rather than varying the input signal, which may not be convenient, the auto-biasing flux can be applied by an additional one-turn coil on the first-stage SQUID. It also may be possible to auto-bias by varying the SQUID voltage bias.

D. Variations in Open-Loop Gain

The current gain of each SQUID stage, $M\, dI/d\Phi$, varies depending on where it is biased on its I-Φ curve, causing variation in the open-loop gain of the op-amp. If the loop gain is high enough, this effect does not result in significant variation in closed-loop gain. However, these variations can result in changes in bandwidth.

Variations in bandwidth can be minimized by separately linearizing the response of each stage. This linearization can be done by passing a small amount of the output current of the SQUID stage through its own feedback coil [2], or by using SQUIDs with asymmetric shape [6] or different junction critical currents. Self feedback on the second stage SQUID is used in the experimental results presented in Section III.

E. Dominant-Pole Compensation

Above some frequency, an L/R low-pass filter due to stray and coil inductances and the SQUID resistance will cause the open-loop gain to roll off at 6 dB/octave and the open-loop phase shift to approach 90°. Inductive loads on additional stages can cause the phase shift to reach 180° while the gain is above unity, causing the amplifier to oscillate when negative feedback is applied.

In a semiconductor op-amp, a similar problem is resolved by adding an extra capacitor, which reduces the frequency of one pole so that the gain is less than unity before a 180° phase shift is reached. While this dominant-pole compensation reduces the open-loop bandwidth of the amplifier, the bandwidth is recovered when the op-amp is configured for negative feedback. The analogous solution for the SQUID op-amp would be to add a large inductor to make a dominant pole, but such an inductor would typically be too large to fit on a SQUID chip. Instead, a dominant pole is created by adding a small damping resistor, R_d, across one of the SQUID input coils (Fig. 1a). Since the damping resistor adds Johnson noise current to the op-amp, it is advisable to add it to the last amplifier stage, so that the total output noise is still dominated by the first SQUID stage. Alternatively, the dominant pole could be established by a shunt resistor on the first-stage feedback coil. In this configuration, the shunt resistor could be located off chip, allowing the compensation to be varied externally depending on the loop gain used in the particular circuit.

F. Current-to-Voltage Converter

A SQUID op-amp can be configured as an I-to-V converter by passing some of its output current through a resistor in series with the FB coil. The output signal is the voltage drop across the resistor. The ratio of closed-loop output voltage change to input current change is then

$$Z_{iv} = RG \,, \tag{6}$$

where R is the value of the resistor, and G is the closed-loop current gain from (2). The value of R should typically not exceed the dV/dI of the output SQUID so as to avoid significant loading of the output, which reduces the dynamic range of the circuit.

An I-to-V converter can be designed to couple directly to a room-temperature amplifier without a transformer or room-temperature feedback electronics. In Fig. 4a, a 1 kΩ I-to-V converter is shown with R = 1 Ω, G = 1000, and from (6), Z_{iv} = 1 kΩ. An input current noise of several pA/√Hz would thus have an output noise of several nV/√Hz, which could couple directly to a low-noise room-temperature amplifier without significant degradation. The drawback of this circuit is its limited dynamic range. From (5), assuming reasonable parameters ΔI_{out} =25 μA and M_{in} =2 nH, the flux dynamic range is only 0.025 Φ_0. This dynamic range is insufficient for many applications.

The dynamic range can be improved significantly by using a SQUID op-amp with a series-array SQUID [3] final stage. A SQUID op-amp with a 100-SQUID series array final stage is shown in Fig. 4b. The voltage bias on the series array must be 100 times higher, so the shunt resistor is 100 times larger than the shunts for the single-SQUID stages. Note that if the op-amp will always be driving a ~ 100 Ω load, the final stage series-array can optionally be current biased in parallel with the load resistor, rather than voltage biased in series with it.

A 1 kΩ I-to-V converter with a 100-SQUID series array is shown in Fig. 4c. Because its output impedance is 100 times larger than a single SQUID [3], this circuit can drive a 100 Ω

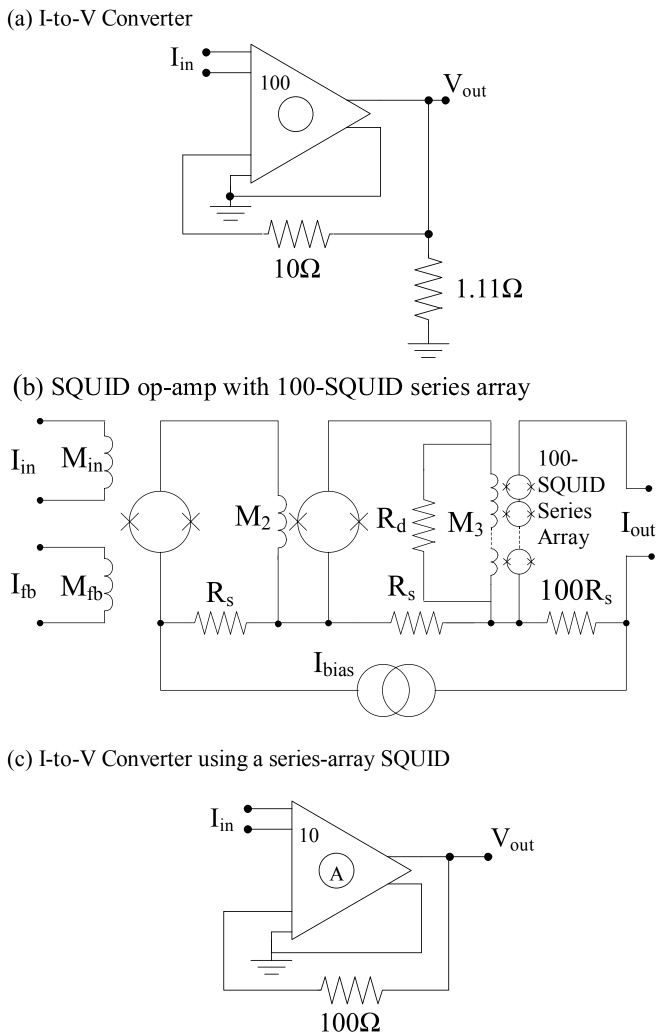

Fig. 4. (a) 1 kΩ I-to-V converter using a single-SQUID final stage driving a 1 Ω load with ×1000 current gain. (b) A SQUID op-amp with a 100-SQUID series-array final stage. (c) 1 kΩ I-to-V converter using a 100-SQUID series array final stage driving a 100Ω load with ×10 current gain. The 'A' in the symbol signifies that the final stage is a series array.

resistive load. The I-to-V converter in Fig. 4c has G=10, R=100 Ω, and from (6) Z_{iv} = 1 kΩ. Assuming ΔI_{out} =25 μA and M_{in} =2 nH, this circuit has a flux dynamic range from (5) of 2.5 Φ_0, where Φ_0 is the magnetic flux quantum.

The I-to-V converter of Fig. 4c would be ideally suited for many applications using low impedance superconducting transition-edge sensors (TES) for microcalorimeters and bolometers [7]. If the mutual inductance of the input SQUID is chosen optimally for this application, the dynamic range of the I-to-V converter will be sufficient if the ratio of flux dynamic range to flux noise is larger than the ratio of signal power to noise equivalent power (NEP) of the TES:

$$\Delta\Phi_{in}/\Phi_n > P/NEP\,. \qquad (7)$$

Assuming that the I-to-V converter of Fig. 4c has $\Delta\Phi_{in}$=2.5 Φ_0 and a reasonable flux noise of 1 $\mu\Phi_0$/√Hz, the I-to-V converter dynamic range ratio is 2.5 x 10^6 √Hz. Taking values of P = 1 pW and NEP = 5 x 10^{-18} W/√Hz for a sensitive 100 mK TES bolometer, we arrive at a TES dynamic range ratio of 2 x 10^5 √Hz. Thus, the I-to-V converter of Fig. 4c has a dynamic range an order of magnitude larger than is necessary for typical low-temperature TES applications. The dynamic range could be increased by using more than 100 SQUIDs in the series array.

The use of the I-to-V converter of Fig. 4c has significant advantages over a series-array SQUID with a feedback signal applied from room temperature. The cost of implementation is less, since the cryogenic feedback makes the room-temperature feedback electronics unnecessary. The theoretically achievable bandwidth is also higher, since the cryogenic feedback eliminates the cable delays introduced by sending a feedback signal from room temperature.

G. SQUID / CMOS Multiplexer

SQUID op-amp I-to-V converters can in principle be integrated with a cryogenic CMOS multiplexer [8] (MUX) in order to read out large numbers of SQUID channels with one output channel. In this approach, the output voltage of an array of first-stage I-to-V converters is applied across a CMOS MUX and the input coil of a series-array SQUID op-amp I-to-V converter (or a series-array SQUID with feedback from room temperature), which couples directly to a room-temperature amplifier. The voltage of the on channel causes a current to flow through the CMOS on resistance and the input coil of the second stage amplifier.

The value of Z_{iv} of the first-stage I-to-V converters can be chosen so that the noise is dominated by the first SQUID stage while multiplexing. It should be possible to multiplex a large number of channels since the second-stage amplifier is very quiet. In this circuit, a low-pass anti-aliasing filter can be provided by the dominant-pole rolloff of the first-stage I-to-V converter. Depending on the on resistance of the CMOS MUX and the required dynamic range of the input current, it may be necessary to use a series array of more than one SQUID as the output stage of the first-stage I-to-V converter. Additionally, if space and power constraints allow, first-stage I-to-V converters with a many-SQUID series-array final stage can couple through the CMOS MUX directly to a room-temperature amplifier.

The SQUID / CMOS MUX scheme has both advantages and disadvantages compared to conventional analog SQUID time-division MUX techniques[4]. It does not require complex, switching digital feedback circuitry. Feedback is done cryogenically, so cable delays are removed from the feedback loop. Since CMOS MUX are fast, it may be possible to make a SQUID / CMOS MUX operate faster than a standard SQUID MUX. Unfortunately, the dynamic range of the SQUID / CMOS MUX is less than that of a conventional analog SQUID MUX. Further, the power dissipation of the SQUID / CMOS MUX is larger, since several SQUIDs are always on for every input channel, whereas in the standard SQUID MUX, only the SQUIDs that are being read out are on. As such, the SQUID / CMOS MUX may not be suitable for very large format arrays. The SQUID / CMOS MUX may be useful in applications requiring modest sized arrays with high bandwidth at each pixel, such as high-count-rate TES x-ray microcalorimeters for materials analysis or x-ray astronomy, or TES optical detectors.

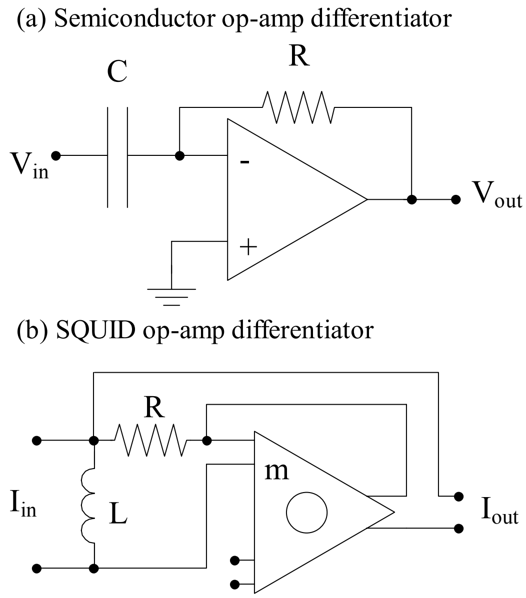

Fig. 5. SQUID op-amp circuits are the dual of semiconductor circuits. (a) A semiconductor op-amp differentiator. (b) A SQUID op-amp differentiator.

H. *Other Circuits*

A wide range of SQUID op-amp circuits can be designed by analogy with semiconductor op-amp circuits. SQUID op-amp circuits can be considered the dual of semiconductor op-amp circuits, with inductance interchanged with capacitance, current interchanged with voltage, and series circuits interchanged with parallel circuits. For example, the dual of the semiconductor op-amp differentiator (Fig. 5a) is the SQUID op-amp differentiator (Fig. 5b).

III. Experimental Results

We have fabricated two different types of SQUID op-amp chips. They are designed to be configurable by on-chip wirebonds. The first chip design consists of two single-SQUIDs, each with two input coils. One coil is configurable as 10, 20, 50, or 100 turns, and the other is configurable as 2, 4, 10, or 20 turns. The maximum (100-turn) option of the first coil was measured to have a mutual inductance of 0.85 nH, and the maximum (20-turn) option of the second coil was measured to have a mutual inductance of 0.16 nH. Each coil has an optional dominant-pole compensation resistor of values 0.001Ω, 0.01Ω, or 0.1Ω which can be wirebonded in parallel with the coil. The SQUID loop inductance is 20 pH, the SQUID shunt resistance is 0.5Ω, the normalized capacitance is $\beta_c = 0.3$, the normalized inductance is $\beta_L = 1$, and the critical current is 100 μA. The second chip style has one single-SQUID stage with parameters as described above, and a 100-SQUID series array. The series array has two coils (8 turns per SQUID and one turn per SQUID). The series-array SQUID has a ~100 Ω resistor which can optionally be wirebonded in series with one of the coils for use in an I-to-V converter to couple to a room-temperature amplifier. Any number of the two chip styles can be wirebonded together to allow multiple-stage SQUID op-amps to be tested.

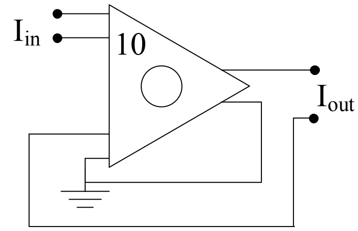

Fig. 6. The ×10 gain SQUID op-amp current amplifier.

The SQUID op-amp chips have been tested in two configurations: a ×10 gain current amplifier and a 750 Ω I-to-V converter with a 100-SQUID series array as the final stage.

A. *×10 Gain Current Amplifier*

A SQUID op-amp chip with two single SQUIDs was wirebonded as a ×10-gain current amplifier (Fig. 6). A 100-turn first-stage input coil was selected, along with a 10-turn first-stage feedback coil, giving a mutual inductance ratio $m \sim 10$. A 100-turn input coil was selected for the second-stage SQUID. The second-stage output current was passed through a ten-turn feedback coil on the second stage, partially linearizing the output of the second stage and reducing the second-stage open-loop gain. A 0.01 Ω dominant-pole-compensation resistor was wirebonded parallel to the input coil of the second-stage. Although this compensation resistor was not strictly needed with two stages, it was configured for use in the three-stage I-to-V converter described later.

The output current of the amplifier was measured by passing it through the input coil of a 100-SQUID series array which was linearized by room-temperature feedback. The circuit was cooled to 4 K in a liquid helium dip probe. I_{out} vs. I_{in} data was recorded using a digital oscilloscope (Fig. 7).

The ×10 gain current amplifier performed as expected. In Fig. 7, three current branches are evident. The output current swing was measured to be 50 μA. The input-flux dynamic range is 2.1 Φ_0, demonstrating a dynamic range larger than a flux quantum. The loop gain of this first test circuit is relatively low (AB/m<100) and the ×10 gain current amplifier has nonlinearity of about ~0.1% over its full dynamic range (Fig. 8). It should be possible to greatly improve this nonlinearity by increasing the loop gain, but it is already good enough for many applications.

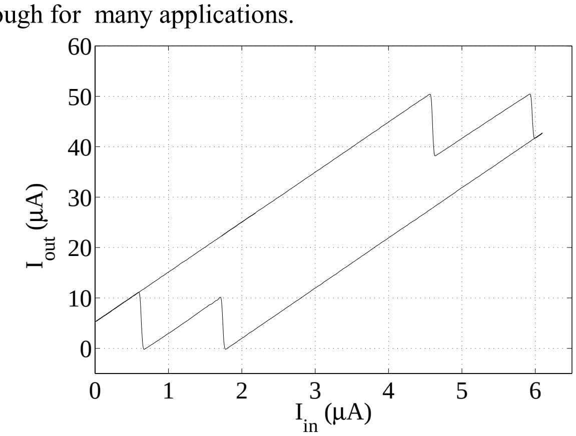

Fig. 7. Experimental current response of a SQUID op-amp ×10 current amplifier. Three current branches are evident.

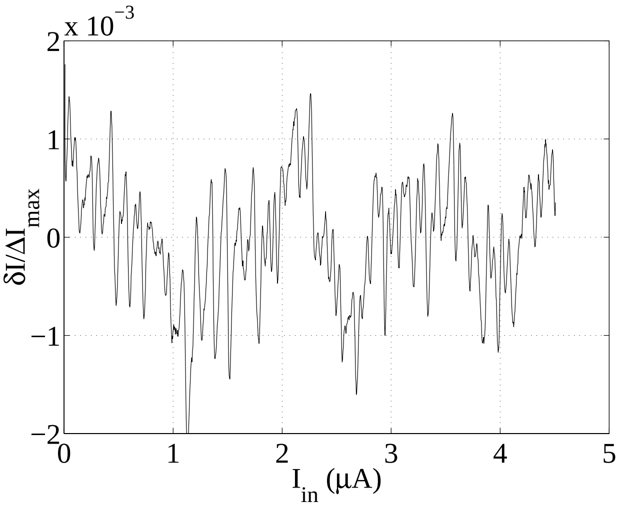

Fig. 8. Fractional deviation from linearity within one current branch of Fig. 7. The difference between the measured current response and a linear fit, δI, is divided by the maximum measured output current swing of 50 μA. Nonlinearity of about 0.1% is measured over the full dynamic range.

The auto-biasing technique was tested by putting a 3 μA sine wave with a DC offset current into the input of the current amplifier. For all the offset currents tested, the current amplifier successfully auto-biased. After one period, the full 3 μA signal remained on one current branch.

B. 750 Ω Series-Array I-to-V Converter

A third-stage 100-SQUID series array was added to the two-stage SQUID op-amp circuit described above. The output of the second-stage was passed through the one-turn-per-SQUID input coil of the series array. A 75 Ω resistor was wirebonded in series with the first-stage feedback coil (Fig. 9), giving a current-to-voltage response of Z_{iv}=750 Ω. The I-to-V converter was coupled directly to a room-temperature amplifier and digital oscilloscope. A sinusoidal current was passed through the input coil and the output voltage was measured (Fig. 10).

The I-to-V converter operated stably. Ten current branches are evident within the current swing. The total output voltage dynamic range was ~0.55 mV. When measured separately, the output voltage range of the 100-SQUID series array is 4 mV. Even loaded with a 75 Ω shunt resistor, we expect to be able to achieve an output swing of several millivolts. The

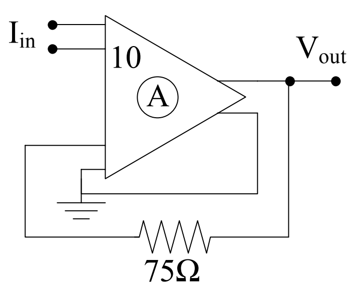

Fig. 9. The 750 Ω SQUID op-amp I-to-V converter with a 100-SQUID series array final stage.

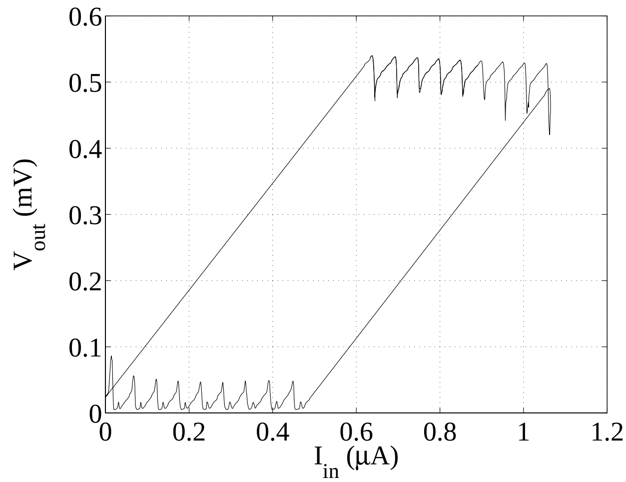

Fig. 10. Experimental current response of the I-to-V converter of Fig. 9.

output voltage swing is small in this I-to-V converter because a one-turn-per-SQUID input coil was used on the third stage. The 50 μA output current swing of the second stage can only provide a small fraction of a flux quantum in the series-array SQUID, severely limiting the output voltage dynamic range of the I-to-V converter. Wirebonding the series-array with more turns on its input coil is expected to significantly increase the output voltage swing of the I-to-V converter.

IV. Conclusions

The SQUID op-amp has been proposed and successfully demonstrated in a current amplifier and an I-to-V converter. Further experiments in a new probe will allow measurements of noise performance and frequency. Tests of optimized SQUID op-amps with higher loop gains are planned.

References

[1] N. Fujimaki, H. Tamura, T. Imamura, and S. Hasuo, "A single-chip SQUID magnetometer," *IEEE Trans. Elect. Dev.*, vol. 35, pp. 2412-2418, December 1988.

[2] R. P. Welty, *Integrated SQUID Array Amplifiers*, thesis, University of Colorado, Boulder, 1995, pp. 51-53.

[3] R. P. Welty and J. M. Martinis, "A series array of dc SQUIDs," *IEEE Trans. Magn.*, vol. 27, pp. 2924-2926, March 1991.

[4] J. A. Chervenak, K. D. Irwin, E. N. Grossman, John M. Martinis, and C. D. Reintsema, "Superconducting multiplexer for arrays of transition edge sensors," Appl. Phys. Lett., vol. 74, pp. 4043-4045, June 1999.

[5] J. S. Adams, S. R. Bandler, C. Enss, A. Fleischmann, S. Hunklinger, Y. H. Kim, J. Schönefeld, and G. M. Seidel,"Particle Detection Using Cryogenic Magnetic Calorimeters," *Physica B*, vol. 263, p. 604-606, March 1999.

[6] T. A. Fulton, L. N. Dunkleberger, and R. C. Dynes, "Quantum interference properties of double Josephson junctions," *Phys. Rev.* **B**., vol. 6, pp. 855-875, August 1972.

[7] K. D. Irwin, "An application of electrothermal feedback for high resolution cryogenic particle detection," *Appl. Phys. Lett.*, vol. 66, pp. 1998-2000, April 1995.

[8] Some commercially available, low-on-resistance CMOS multiplexers have been found to work at cryogenic temperatures. Personal communication: J. M. Martinis and M. W. Rabin, National Institute of Standards and Technology, Boulder, CO, 2000.