Spin transport for spin diffusion lengths comparable to mean free paths

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In perpendicular-transport structures with magnetic and nonmagnetic layers, there is a resistance associated with changes in the value of current polarization throughout the structure. Starting from a Boltzmann equation, Valet and Fert [Phys. Rev. B **48**, 7099 (1993)] derived macroscopic transport equations to describe this effect. Their derivation is formally justified in the limit that the spin-diffusion length of each material is long compared to the mean free path of that same material, but appears to agree with experiment even for spin diffusion lengths comparable to the appropriate mean free paths. Here, numerical studies of the Boltzmann equation verify that their approach is accurate in this limit. In addition, we have examined the case of anisotropic spin-flip scattering. For reasonable parameters, there is only a small difference in the resistance from the isotropic case.

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I. INTRODUCTION

The resistance of magnetic multilayers depends on the relative orientation of the magnetizations of the constituent layers, an effect called giant magnetoresistance.[1,2] The effect occurs in two geometries, either with the current flowing in the plane of the multilayer or with current flowing perpendicular to the plane. The first of these has been the most heavily studied (see Ref. 3 for a review) in large part because giant magnetoresistance in this geometry is used in read heads in hard-disk drives. The second geometry was first investigated experimentally by a group from Michigan State University[4–6] and a group from Phillips Research.[7] Early theoretical work was done by Johnson and Silsbee.[8,9]. Much of the history of this topic is reviewed in Refs. 10 and 11. There has been a recent increase in interest in the perpendicular case because it may find application in the next generation of read heads, where a greater signal is needed. The perpendicular geometry also exhibits spin transfer torques (see Ref. 12 for a review), which are also of current interest.

The "standard theory" for perpendicular transport was developed by Valet and Fert.[13] In their theory, there are two contributions to the spin polarization of the current. The first is the spin dependence of the bulk conductivity in the ferromagnetic layers. The second is the diffusion of spins due to a spatially varying spin accumulation. Wherever the spin polarization of the current deviates from the value expected from the local bulk conductivity, there must be a spatially varying spin accumulation to compensate. Thus, whenever the materials change, there is spin accumulation near the interfaces. Associated with this accumulation is an excess resistance, which we refer to as an accumulation resistance to distinguish it from the interface resistance that results from a mismatch of electronic structures.[14,15] There are closely related accumulation resistances due to spin-dependent interface resistances and discontinuities in the spin chemical potential, but for simplicity we ignore these in this paper.

Valet and Fert derived macroscopic transport equations from the Boltzmann equation in the relaxation-time approximation. Their derivation assumes that the spin-flip diffusion length in each material is large compared to the electron mean free path in that same material. For most of the commonly studied materials this condition is well satisfied. However, for the case of $Ni_{80}Fe_{20}$, the spin-diffusion length is on the order of the electron mean free path.[16] Despite this, their model is able to explain experimental results in multilayers containing $Ni_{80}Fe_{20}$.[17]

In this paper, we investigate the validity of the macroscopic transport equations by comparison to an exact numerical treatment of the Boltzmann equation for the resistance caused by spin accumulation. The numerical results are based on a method described in an earlier paper[18] to solve the Boltzmann equation for a model electronic structure. We investigate N/F/N and F/N/F junctions, where F and N denote ferromagnetic and nonmagnetic layers. The middle layer in both cases is referred to as the spacer layer. In both cases, we compare results for different values of the spin diffusion length in the ferromagnetic layers because $Ni_{80}Fe_{20}$ is the principle material of interest.

We find the condition that the spin-flip diffusion length be large compared to the electron mean free path, a condition that Valet and Fert[13] gave for the validity of their derivation, can be relaxed for reasonable parameters. Even when the spin-flip diffusion length of the magnetic portion of the multilayer is some what less than the electron mean free path there, the macroscopic transport equations are found to be valid to a few percent or better.

We have also tested the importance of anisotropic spin-flip scattering. Previously, we studied the consequences of anisotropic non-spin-flip scattering in multilayers.[18] That process is of current interest because of theoretical suggestions[19,20] that such scattering gives rise to a dependence of the resistance of the multilayers on the ratio of the layer thicknesses to the mean free paths of the materials. Experiments[21] can be interpreted to show no such dependence, or the presence of such an effect.[20] As part of our present study of spin-flip scattering, we have tested a related but distinct issue—the consequences of anisotropic spin-flip scattering. We find that this anisotropy adds an additional term to the relaxation-time-approximation form of the Boltzmann equation as well as to the macroscopic transport equations. However, for reasonable values of the parameters, the

exact solution of the Boltzmann equation agrees with the solution of the macroscopic transport equations, i.e., anisotropic spin-flip scattering yields essentially the same results as isotropic spin-flip scattering.

II. THEORY

The accumulation resistance is defined to be the change in the chemical potential relative to that due solely to the bulk resistivities of the layers divided by the current. We calculate the resistance from the Valet and Fert macroscopic transport equations as well as from our exact method. These two approaches are applied to our trilayer geometry of semi-infinite metal, spacer layer, semi-infinite metal.

We begin by briefly reviewing the theory of Valet and Fert.[13] In this approach, the Boltzmann equation is solved in each layer and the solutions for different layers are joined through boundary conditions. The model presently considered includes the simplest possible boundary conditions (no reflection and, hence, no interface resistance) so as to focus in the behavior within each layer. The Boltzmann equation for the case of current perpendicular to the interface, and including spin flip scattering, is written as

$$v_z\frac{\partial f_s}{\partial z}-eEv_z\frac{\partial f_0}{\partial \epsilon}=\int d^3v'\,\delta[\epsilon(\mathbf{v}')-\epsilon(\mathbf{v})]P_s[f_s(\mathbf{v}')-f_s(\mathbf{v})]+\int d^3v'\,\delta[\epsilon(\mathbf{v}')-\epsilon(\mathbf{v})]\times P_{\mathrm{sf}}[f_{-s}(\mathbf{v}')-f_s(\mathbf{v})], \tag{1}$$

where f_0 is the equilibrium distribution function, f_s is the distribution function for electrons of spin s at position z and velocity $\mathbf{v}$, $E=E(z)$ is the local electric field, and $\epsilon=(1/2)mv^2$ in the simplified electronic structure treated in this paper. $P_s(z,\mathbf{v},\mathbf{v}')$ and $P_{\mathrm{sf}}(z,\mathbf{v},\mathbf{v}')$ are the spin-conserving and spin-flip transition probabilities. In the relaxation-time approximation, these are assumed to be isotropic in velocity space. The spin-up and spin-down Fermi energies are assumed to be equal. That is, following Valet and Fert, we ignore the differences in the electronic structure of the majority and minority electrons characteristic of transition metal ferromagnets and simply include spin-dependent scattering rates. The linearized Boltzmann equation is obtained by writing the distribution function as

$$f_s=f^0+\frac{\partial f^0}{\partial \epsilon}[(\mu^0-\mu_s)+g_s], \tag{2}$$

where μ^0 is the equilibrium chemical potential and $\mu_s(z)$ is the local chemical potential for spin s electrons and $g_s(z,\mathbf{v})$ is the anisotropic part of f_s.

In the relaxation-time approximation to the Boltzmann equation, Valet and Fert find that

$$v_z\frac{\partial g_s}{\partial z}+\left[\frac{1}{\tau_s^{\mathrm{trans}}}+\frac{1}{\tau_{\mathrm{sf}}^0}\right]g_s=v_z\frac{\partial \bar{\mu}_s}{\partial z}+\frac{\bar{\mu}_s-\bar{\mu}_{-s}}{\tau_{\mathrm{sf}}^0}, \tag{3}$$

where the quantity $\bar{\mu}_s=\mu_s-eV(z)$ and $V(z)$ is the local potential. The relaxation times for spin-flip and non-spin-flip events are τ_s^{trans} and τ_{sf}^0, where τ_s^{trans} is the transport relaxation time for spin s, i.e., the relaxation time for the momentum, whereas τ_{sf}^0 is the ordinary relaxation time,

$$\frac{1}{\tau_s^{\mathrm{trans}}}=\int d\Omega P_s(\Omega)[1-\cos(\Omega)]$$

$$\frac{1}{\tau_{\mathrm{sf}}^0}=\int d\Omega P_{\mathrm{sf}}(\Omega)$$

$$\frac{1}{\tau_{\mathrm{sf}}^{\mathrm{trans}}}=\int d\Omega P_{\mathrm{sf}}(\Omega)[1-\cos(\Omega)]. \tag{4}$$

The last quantity, the transport spin-flip relaxation time, is used below. These equations lead the macroscopic transport equations within each layer

$$\frac{e}{\sigma_s}\frac{\partial j_s}{\partial z}=\frac{\bar{\mu}_s-\bar{\mu}_{-s}}{l_s^2} \tag{5}$$

$$j_s=\frac{\sigma_s}{e}\frac{\partial \bar{\mu}_s}{\partial z}, \tag{6}$$

where j_s is the current of spin s electrons, $l_s=[(1/3)(v_{\mathrm{F}}\lambda_s)\tau_{\mathrm{sf}}^0]^{1/2}$ is the spin-dependent spin-diffusion length, $\lambda_s=v_{\mathrm{F}}[(1/\tau_s^{\mathrm{trans}})+(1/\tau_{\mathrm{sf}}^0)]$ is the mean free path, and v_{F} is the Fermi velocity. The second macroscopic equation is Ohm's law. When these equations are combined, the characteristic length describing the changes in spin accumulation is the spin-diffusion length, $l_{\mathrm{sf}}^{-2}=l_{\uparrow}^{-2}+l_{\downarrow}^{-2}$. The approximations used to derive the macroscopic equations include the condition that the spin-diffusion length be long compared to the mean free paths.

In the case that the spin-flip scattering is not isotropic, we find that there is an extra term on the left-hand side of the Boltzmann equation (3) of the form $[(1/\tau_{\mathrm{sf}}^0)-(1/\tau_{\mathrm{sf}}^{\mathrm{trans}})]g_{-s}$, which leads to a modification of the macroscopic equation (6) corresponding to Ohm's law

$$j_s=\frac{\sigma_s}{e}\frac{\partial \bar{\mu}_s}{\partial z}+\left(\frac{\lambda_{-s}}{\lambda'_{\mathrm{sf}}}\right)\left(\frac{\sigma_s}{\sigma_{-s}}\right)j_{-s}, \tag{7}$$

where $\lambda_s=v_{\mathrm{F}}[(1/\tau_s^{\mathrm{trans}})-(1/\tau_{\mathrm{sf}}^0)]^{-1}$ and $\lambda_{\mathrm{sf}}=v_{\mathrm{F}}[(1/\tau_{\mathrm{sf}}^0)-(1/\tau_{\mathrm{sf}}^{\mathrm{trans}})]^{-1}$. The other macroscopic equation (5) is unchanged.

The theory for the numerically exact calculations is described in Ref. 18. The distribution function in each layer that satisfies the Boltzmann equation (without the relaxation-time approximation) takes the form

$$g_k=\sum_i{}'\,\xi_i\exp(-\lambda_i z)x_{ik}+\xi_0 z_k+\xi_1(z_k z-z'_k), \tag{8}$$

where k refers to a particular momentum. The ξ_i are coefficients determined by the boundary conditions. As discussed in Ref. 18, the λ_i and x_{ik} are the eigenvalues and eigenfunctions of a matrix that is directly related to the Boltzmann equation within each layer separately. The prime in the summation implies that the two zero eigenvalues are to be omitted from the summation; they are replaced by the terms that

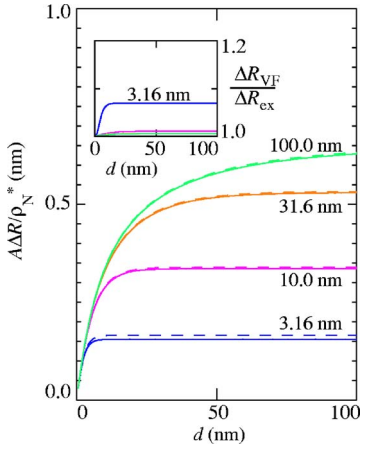

FIG. 1. (Color online) Accumulation resistance for N/F/N. The accumulation resistance vs spacer layer width is shown for various values of spin-diffusion length in the ferromagnetic layer (indicated next to each pair of curves). The solid curves are the numerical solutions of the Boltzmann equation, and the dashed curves are the solutions of the macroscopic equations due to Valet and Fert.[13] Shown in the inset are the ratios of macroscopic to the numerical calculations.

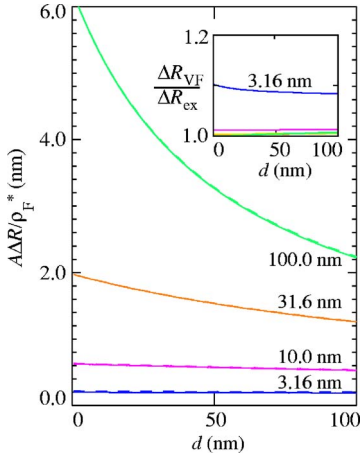

FIG. 2. (Color online) Accumulation resistance for F/N/F with antiparallel magnetizations. The accumulation resistance versus spacer layer width is shown for various values of spin-diffusion length in the ferromagnetic layers (indicated next to each pair of curves). The solid curves are the numerical solutions of the Boltzmann equation, and the dashed curves are the solutions of the macroscopic equations due to Valet and Fert.[13] Shown in the inset are the ratios of macroscopic to numerical calculations.

contain ξ_0 and ξ_1. The only term that carries current is the one proportional to ξ_1. In the case of current parallel to the interface there is no perpendicular current and $\xi_1=0$. The quantity z_k is a constant corresponding to a uniform expansion of the Fermi surface and z_k' is determined from the scattering probabilities as spelled out in the appendix of Ref. 18. For isotropic scattering, $z_k' \propto v_{z,k}$.

III. NUMERICAL RESULTS AND CONCLUSIONS

We examine the situation of semi-infinite metal, spacer layer, semi-infinite metal where the metals are Cu and $Ni_{80}Fe_{20}$. The cases considered are N/F/N and F/N/F, where in the latter case the F magnetizations are in opposite directions. We use parameters characteristic of N=Cu and F=$Ni_{80}Fe_{20}$,[17] $\lambda_N=110$ nm, $\lambda_{sf}^N=12\,000$ nm, $l_{sf}^N=470$ nm, $\lambda_F^{\uparrow}=5.5$ nm, and $\lambda_F^{\downarrow}=1.8$ nm. This junction composition is of interest as l_{sf}^F is estimated to be very small ($\approx$5.5 nm.), on the order of λ^F, which violates the Valet and Fert[13] assumption that $\lambda \ll l_{sf}$. We calculate the accumulation resistance ΔR and compare it to that found from the macroscopic transport equations of Valet and Fert for various values of the spin-diffusion length l_{sf}^F and interlayer thicknesses d.

The results are shown in Figs. 1 and 2 for the cases N/F/N and F/N/F, respectively. The accumulation resistance versus interlayer width is shown for various values of spin-diffusion lengths l_{sf}^F; also shown in the inset are the ratios of macroscopic to the exact calculations. The condition that the spin-dependent mean free paths λ^F be small compared to l_{sf}^F was used by Valet and Fert to derive their macroscopic transport equations is clearly more stringent than necessary as evidenced by the results for $l_{sf}^F=3.16$ nm, shown in both figures. We believe that the result that the Valet and Fert model works well even in regimes where the derivation is not justified is not very sensitive to the values of the other parameters chosen.

In Fig. 1, the accumulation resistance increases with the width of the F spacer layer because the polarization of the current in the F layer increases. It saturates when the width becomes larger than l_{sf}^F and the two interfaces do not interact. In Fig. 2, the oppositely directed magnetizations are closest when the N layer is thinnest. Spin relaxation in the N layer reduces the accumulation resistance from that of an $F_{\uparrow}/F_{\downarrow}$ interface to that due to two F/N interfaces. Again, there is saturation when the interface layers are sufficiently far apart.

The results shown in Figs. 1 and 2 are for the case that the impurity scattering (as well as the spin-flip scattering) in Cu and $Ni_{80}Fe_{20}$ is isotropic, in which case $\tau_s^{trans}=\tau_s^0$. Calculations for the case of anisotropic impurity scattering were also carried out, and the conclusions remain the same. Allowing the spin-flip scattering to be anisotropic modifies the macroscopic equations as indicated in Sec. II, but still yields results that are within a few percent of the isotropic case.

In summary, our numerical solutions of the Boltzmann equation show that the macroscopic equations derived from the Boltzmann equation by Valet and Fert[13] should be valid for most situations of interest. This result provides some justification for the good agreement found between experiments on materials with short spin-diffusion lengths and calculations based on Valet-Fert theory. In a previous paper,[18] we showed that anisotropic scattering can be accurately treated by replacing the mean free path with an appropriately defined transport mean free path. In this paper we have shown that the macroscopic transport equations work well even when the spin-diffusion length is comparable to the mean free path. We also tested anisotropic spin-flip scattering and found that it is inconsequential.

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